A TEXT BOOK

OF THE

PRINCIPLES OF PHYSICS

BY

ALFRED DANIELL, M.A., LL.B., D.Sc., F.R.S.E.

OF THE INNER TEMPLE, BARRISTER-AT-LAW
MEMBER OF THE FACULTY OF ADVOCATES IN SCOTLAND
FORMERLY LECTURER ON PHYSICS IN THE SCHOOL OF MEDICINE, EDINBURGH

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PREFACE TO THE THIRD EDITION.

I HAVE tried, in revising this volume, to maintain the characteristics of the book, to improve it where it seemed to need improvement, to correct any errors into which I had fallen, to fill up any gaps which seemed important, and to keep it up to date. I have considered every equation from the point of view of its Dimensions, and have had to modify some of the equations accordingly; I have tried to remove some difficulties from the student's path by the adoption of a uniform notation; and I have endeavoured to make it clear to him what the precise physical quantities are to which reference is being made in each particular case.

Whatever may be my own shortcomings in respect of the execution of this revision, I hope that what I have been able to do may at any rate tend in the direction of accuracy and precision; and I also hope that the additional labour which I have bestowed upon this book may be accepted as an earnest acknowledgment of the uniform kindness with which the work has been received.

A. D.

8 NEW COURT,
LINCOLN'S INN,
LONDON, W.C.

PREFACE TO THE FIRST EDITION.

In the following pages I have endeavoured to give, in terms as simple as the nature of the subject will permit, a connected account of the leading principles of modern physical science.

My aim has not been to build up a mere compendium of physical facts, but rather to put the reader in possession of such principles as will enable him with small difficulty to apprehend and to appreciate those facts.

I am regretfully aware of many material omissions. The subject of Natural Philosophy is so vast that many things which in themselves are by no means devoid of importance — but to which different writers would perhaps be inclined to attribute different degrees of importance — must necessarily be laid aside in the course of the preparation of a text-book of limited size. One of these omissions, which my own love of the developmental history of science made me decide upon with extreme unwillingness, is that of the history and the personal aspect of scientific discovery. As a general rule, the names of discoverers, even where they are mentioned, play a very subordinate part in the text.

At the same time, I trust that the reader of this work will find that, after assimilating its contents, he

is in some measure prepared for the reception of further information in the course of that wider reading and practical study to which I hope the following pages will be found fitted to serve as an elementary introduction.

It is wholly beyond question that to him who desires to become a physicist, Practical Laboratory Work is absolutely essential. Thorough knowledge must be drunk in by the eyes and the ears, and absorbed by the finger-tips; and the true use of a book of this kind is, I take it, not to replace practical work but to economise the labours of the student. This it may do by so furnishing his mind with a store of general principles, that when he comes to enter a physical laboratory he may there find around him, in the concrete form, a collection of pieces of apparatus, the construction and the action of which he is able, by the application of the principles already familiar to him, promptly and intelligently to comprehend. Bearing the necessary limitations of the usefulness of any mere book steadily before me, I have endeavoured, as far as possible, to simplify and generalise all descriptions of apparatus, and in the same way to simplify and generalise the accompanying diagrams; and thus I have tried not only to adapt the work to the requirements of those who may use it as a stepping-stone to further attainments, but also to render it a suitable text-book for that larger circle of readers who, having no distinct desire to follow out the special study of physics, may yet wish to possess an elementary acquaintance with the modern aspect of natural philosophy.

This book was primarily designed as a contribution to Medical Education, and as such I hope it may be

found useful. That arrangement, which still prevails in some of our Universities, under which a student of medicine may even proceed to the degree of M.D. without any adequate knowledge of physics, is self-evidently opposed to common sense, and to the exigencies of physiological study and of medical practice. Such an anomaly cannot, it may be anticipated,* endure much longer. Before many years are over it will be universally acknowledged in practice, as it already is in theory, that knowledge of natural philosophy is an essential part of the mental equipment of the medical student and of the properly-trained medical man. The needs of the intelligent student of physiology have been kept constantly before my mind, as I hope those of my readers who are already physiologists will recognise; but I have been careful not to make the book one suited for admission only into a medical class-room; my aim has been to produce a work useful at once to the Student of Medicine, the Student of Science, and the General Reader.

The plan of the work is that of a gradual progression from the simplest to the more complex. No preliminary knowledge of physical principles is assumed, and every effort has been made to attain to absolute lucidity of expression, even though this be found occasionally to necessitate the frequent repetition of a single word in the course of a single sentence. While the reader is expected as he reads each page to remember the contents of the preceding pages, I trust that I have sufficiently carried

* It is gratifying that this anomaly has ceased to exist, in the United Kingdom, since the year 1892, in which year the new regulations of the General Medical Council came into force.

out my intention of nowhere setting before him anything of the nature of an unsolved riddle, so far as that could be guarded against by my own efforts on his behalf.

I have endeavoured to secure intelligible continuity throughout the paragraphs printed in larger type, and thereby to enable the reader on his first perusal to confine his attention to the more prominent portions of the text.

However imperfectly its design may have been executed, I shall be glad if this work be found to contribute in any degree to the extension of that mode of teaching Natural Philosophy for the establishment of which we have come, directly and indirectly, to owe so much to the advocacy and example of Professors Thomson [now Lord Kelvin] and Tait—a mode of teaching under which the whole of Natural Philosophy is regarded as substantially a single science, in which scattered facts are connected and co-ordinated by reference to the principles of Dynamics and the great experimental Law of the Conservation of Energy.

20th February 1884.

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A Area.
B Varying coefficient of Kinetic Friction.
C Electrostatic Capacity or Permittance.
D Conductance, in electrostatic units.
E Potential-Difference between two points, in electrostatic units.
F A Force or Stress, generally.
G The Weight of a Body, Gravitational Attraction.
H Heat (in ergs): Driving Head of a Liquid or Gas; H_s and H_p, p. 310.
I Current-Intensity or Strength, in electrostatic measure.
K Specific Inductive Capacity, or Permittivity.
L Coefficient of Self-Induction, or Inductance; Latent Heat of Expansion, Coefficient of.
M Coefficient of Mutual Magnetic Induction; Moment of a Couple.
N Moment of Inertia.
P A Total Pressure, generally.
Q Quantity of Electricity, in electrostatic measure.
R Resistance of a Conductor, in electrostatic measure.
T Period of S.H.M. or of a complete oscillation.
U Resistance to Fluid Flow, in cm. of liquid column.
V Electric Potential, in e.-s. units.
W Work: Energy.

A Number of Ampères.
E Linear Extension on Stretching.
F Statical Frictional Resistance.
G Temperature-Gradient.
H Height of Barometer-Column.
L Coefficient of Linear Expansion by Heat.
R Kinetical Frictional Resistance.
S Energy of Surface-Film per sq. cm.
T Superficial Tension of Surface-Film, per linear centimetre.
V Number of Volts.

B Total Electromagnetic Induction.
F Total Force, in some particular direction.
H Total Magnetic Force, across an Area.
I Total Electrostatic Induction across an Area.
P Total Pressure, directed, across an Area.
T Total Tension, directed, across a cross-section.
V The Electromagnetic-Electrostatic Ratio; the Velocity of Light etc.

$\mathfrak{I}$ Intensity of Magnetisation.
$\mathfrak{M}$ Magnetic Moment.
$\mathfrak{K}$ Thermodynamic Constant, $=k-c$.

a Amplitude in Periodic Motion.
b Coefficient of Kinetic Friction.
c Thermal Capacity at constant Volume, in ergs per gramme.
ca Calories (gramme-calories).
d Distance: Diameter of Tube.
e Potential-Difference in magnetic measure; Epoch in S.H.M.
f Force or Stress per sq. cm., in general; Focal Distance.
g Acceleration due to Gravitation: Intensity of Gravity.
h Vertical Height; Height of Liquid Column.
i Current-Intensity or Strength in magnetic measure.
k Thermal Capacity at constant Pressure, in ergs per gramme; Coefficient of Transpiration; Poiseuille's Coefficient; generally, a Coefficient.
l A Length.
m Mass: Coefficient of Inertia.
$\overline{m}$ Average Mass of a Molecule.
n Frequency: Number of Turns in a Coil.
o Area of Cross-Section.
p Pressure per sq. cm., general; or Hydrostatic Pressure per sq. cm.
p_0 "Electric Tension" per sq. cm.
$\dot{p}$ Small increment of Pressure, dynes per sq. cm.
q Quantity of Electricity, in magnetic units.
r Radius: Resistance of a Conductor in magnetic units.

s Distance traversed.
t Time: Temperature in °C.
v Velocity or Speed in general.
w Activity, ergs per second.

c Curvature.
k Viscosity *log. dec.*
n Coefficient in S.H.M.

b Magnetic Induction per sq. cm.
f Force in a stated direction, per sq. cm. of Area.
h Magnetic Field-Intensity, per sq. cm.
i Electrostatic Induction per sq. cm.
p Pressure per sq. cm., in a stated direction.
s Displacement in a given straight line.
t Tension per sq. cm. of cross-section.
x Displacement along axis of x.
v Velocity in a given straight line.
y Displacement along axis of y.

$\mathfrak{f}$ Electric Force in magnetic units.
$\mathfrak{h}$ Horizontal component of Earth's Magnetic Field (per sq. cm.).
$\mathfrak{k}$ Coefficient of Resistance to Compression.
$\mathfrak{l}$ Length of the Simple Pendulum.
$\mathfrak{m}$ Quantity of Magnetism.
$\mathfrak{n}$ Coefficient of Rigidity to Shear.
$\mathfrak{r}$ Impēdance.
$\mathfrak{t}$ Coefficient of Rigidity to Twist.
$\mathfrak{v}$ Volume.
$\dot{\mathfrak{v}}$ Small increment of Volume.
$\mathfrak{y}$ Coefficient of Rigidity to Stretching (Young's Modulus).

D Conductivity.
N No. of grooves per cm. in Diffraction-grating.
R Resistivity.

П Atmospheric Pressure, per sq. cm.
þ Thermometric Coefficient of Thermal Conductivity.

a Acceleration in general.
β Index of Refraction; β_∞, do. do. when wave-length $\lambda = \infty$.
γ Gravitation-Constant.
δ An Angle of Deviation.
δx A small increment of x.
ϵ 2·718281...
θ Angular Displacement: Angle of Shear; Critical Temperature.
ϑ Calorimetric Coefficient of Thermal Conductivity.

η Coefficient of Viscosity.
ι Radius of Inertia: Angle of Incidence.
ι_0 Rad. of Inertia round Cent. Gravity.
ξ An Angle.
κ Magnetic Susceptibility.
λ Wave-Length: Latitude: Coefficient of Restitution: Latent Heat of Evaporation.
μ Magnetic Permeability: Coefficient of Statical Friction.
ν Velocity of Wave-Propagation.
π 3·1416...
$\tilde{\varpi}$ Critical Pressure.
ρ Density of a Mass: Angle of Reflexion.
ϱ Angle of Refraction.
σ Electric Surface-Density: Specific Heat in *ca* per gramme.
ς Magnetic Surface-Density.
τ Temperature on the Absolute scale.
$\dot{\tau}$ Small increment of Temperature.
φ Strength of Magnetic Shell: Entropy.
ϕ Critical Volume.
χ Critical Angle in Statical Friction.
ψ Critical Angle in Kinetical Friction.
ω Angular Velocity: a Solid Angle: = "Ohms."

$\boldsymbol{\alpha}$ Acceleration in a given Direction.
$\boldsymbol{\sigma}$ Mass per unit surface of a Film or Shell.
$\boldsymbol{\phi}$ Electric Force (per sq. cm.) = Potential-Slope or Gradient.

Γ Galvanometer-Constant.
Δ Current-Density.
Θ Dynamical Coefficient of Thermal Conductivity.
Λ Coefficient of Extensibility.
Π Total Atmospheric Pressure.
Σ "Sum of all the x's" $= \Sigma(x)$.
Ω Magnetic (scalar) Potential.

/ "divided by."
∞ "Infinity."
$\propto$ "varies as;" *i.e.*, "= some *constant* ×."
$\sim$ "Numerical difference between:" always positive.
$10^9 = 1000,000000$; nine ciphers.
$10^{-9} = 1 \div 10^9 = 0{\cdot}000,000,001$: *eight* ciphers after the point.
$a^{\frac{1}{2}} = \sqrt{a}$; $a^{\frac{3}{2}} = \sqrt{a^3}$; $a^{-\frac{1}{2}} = 1/a^{\frac{1}{2}}$.
a_t "Value of a at end of time t."

THE PRINCIPLES OF PHYSICS.

INTRODUCTORY.

NATURAL PHILOSOPHY or PHYSICS may be briefly defined as the **Science of Matter and Energy.** This definition is one which is obviously comprehensive enough to include within its range the whole of Chemistry and of Biology as well as of Chemical and Physiological Physics. Chemistry is in truth but a colony of facts closely related to one another, and classified by us on principles which depend almost entirely upon our ignorance of the fundamental nature of the relation between those apparently different Forms of Matter which we know as the various Chemical Elements; and the consummation of Chemistry, a full and accurate knowledge of the inner mechanism of all chemical reactions, would probably result in the absorption of all Chemistry in the wider science of Molecular Physics. In the meantime the fundamental unity of the two nominally distinct sciences, Chemistry and Physics, is shown by the extent to which they overlap one another in the field of Chemical Physics.

Physiology, again, or in a wider sense Biology, is concerned with the matter and the energy of living beings; and if it ever come to attain its highest ideal, even Biology must thereupon necessarily merge in Natural Philosophy. Already we see that while physiological research is steadily conquering the unknown, that which it succeeds in thoroughly explaining falls out of its grasp and comes to form a part of ordinary physical or, it may in the meantime be, of ordinary chemical knowledge.

We may more amply define Natural Philosophy or Physics as the systematic exposition of the Phenomena and Properties of Matter and Energy, in so far as these phenomena and properties can be stated in terms of definite Measurement and summarised by the formulation of mechanical principles or Laws.

Here, again, we must admit that our definition is, in the present state of our knowledge, too ideal. A perfect and accu-

rate knowledge even of the simplest actual phenomenon would imply absolute omniscience. Often we find that we can measure but cannot systematise the phenomena of Nature; and we find at the very outset of our exposition that we are compelled to confess entire ignorance as to the very nature of our subject-matter; for we do not know what Matter is.

To us the question, What is Matter?—What is, assuming it to have a real existence outside ourselves, the essential basis of the phenomena with which we may as physicists make ourselves acquainted?— appears absolutely insoluble. Even if we became perfectly and certainly acquainted with the intimate structure of what we call Matter, we would but have made a further step in the study of its properties; and as physicists we are forced to say that while somewhat has been learned as to the properties of Matter, its essential nature is quite unknown to us.

As little can we give any full and satisfactory answer to the question, What is Energy? As a provisional statement we may say that Energy is the Power of doing Work; a rifle-bullet in motion, a coiled watch-spring, possesses the power of doing Work upon other bodies suitably arranged; but plainly this power depends upon the relation into which the matter which is said to possess it is brought with reference to other matter, and it ultimately depends upon the position of one set of particles of matter with reference to other sets. Since Energy depends, then, upon the relative position of particles of Matter, we are not able to explain its own essential nature, though we may acquire a considerable amount of information as to its very remarkable properties.

These properties of Energy, those of Matter, their mutual relations, and the laws of these properties and relations, constitute the subject-matter of Natural Philosophy; and these have been ascertained by observation, by measurement, and by judicious reasoning upon the data supplied by investigation.

In the investigations upon which Natural Philosophy is founded, the guiding principle is a belief, based on the recorded experience of the human race, in the **Constancy of the order of Nature.** This does not mean that things are to continue for ever as they are at present. If a closed boiler containing water be heated to a certain temperature, the Constancy of Nature would not be interfered with by the consequent explosion of the boiler; it would be seriously infringed if the boiler did not burst. So, again, if volcanic eruptions thrust up mountain

ranges through a flat plain, as in the case of the Rocky Mountains, or if a crack in the earth's crust allow a flood of lava to flow over a wide region, as in the geological history of Idaho, such a cataclysm would appear to be an awful break in the uniformity of Nature; yet if the earth's crust be so pressed upwards that it can resist no further pressure, the Constancy of Nature is confirmed by its giving way. On this belief in the Constancy of Nature are based all rational calculations of eventualities, and all our arrangements from day to day, which are subject to the transpiry of facts unknown or unforeseen at the time when these arrangements were made.

This belief finds formulated expression in the **Law of Causality,** which affirms that every effect has a sufficient cause. If we observe any given phenomenon, we conceive ourselves entitled as the result of all experience to enquire into its cause, and conversely, to affirm that if there be no cause tending to produce change in any particular respect in the present condition of things, there will in that respect be no change. It is scarcely necessary here to investigate the meaning of the word Cause itself; it will be quite sufficient to point out that for us the relation of Cause and Effect is one of Sequence, found to be invariable if not interfered with by the intervention of circumstances which render cases dissimilar. In similar cases, the same causes are observed to be followed by the same effects. It is plain, however, that the same effects are not always and necessarily the results of the same causes; and when different causes are found to produce the same effects they are equivalent in effectiveness to, and may be substituted for, one another.

Again, the principle may be stated that **the cause** is **equivalent** and in proper terms of measurement **numerically equal** to the **effect** produced by it. Apparent exceptions to this statement arise only when the problem is not of the extremely simple form in which one cause, and one cause alone, is brought into play. It is not, except in a loose popular sense, the heat of the spark which causes the explosion of a magazine and consequent destruction of property; it is not drawing the trigger which is *the* cause of the bullet's leaving the gun. The heat of the spark, the drawing of the trigger, is necessary as one cause out of several; but the problem is not here so simple that these can be adduced as cases in which the effect is greater than the cause. They only point out an extended statement, that the **total effect** produced is **equivalent to the effective sum of the causes**

acting; and when one of the causes acting is an arrangement of matter which is explosive or in unstable equilibrium, ready to topple over so as to assume a stable position, the effect produced, though apparently greater than the small disturbance which disarranged the unstably-balanced matter, must be traced not to it only, but to all the conditions and circumstances involved, including the unstable equilibrium, the antecedent cause of which may itself be sought for.

If several causes act simultaneously, each produces only a part of the aggregate effect, and the total effect is equal to the sum of the acting causes. Under the name of **Galileo's principle** this is one of the fundamental truths of physics, and is thus enunciated:—If a body be acted on by two or more Forces (*force* being meanwhile defined as *any cause of motion*), each of these forces acts independently, and produces its own effect without reference to the others, the total effect produced being ascertained by finding, in any appropriate way, the sum of the effects due to the several forces. A cannon-ball, for instance, fired from a height, is, as it passes through the air, executing movement due to at least two forces or causes of motion: the force exerted upon the ball during the explosion sends the ball forwards, that of gravity continuously draws it downwards. If, for the sake of convenience, we neglect the resistance of the air, and enquire what would be the path pursued by a shot travelling *in vacuo*, we would find by making use of this principle that the position of the shot at any moment would be found by enquiring (1) How far outwards the shot would have been projected had there been no tendency to fall; and (2) How far the ball would have fallen if gravity had alone acted on it. For any specified instant a point may in this way be found, which, being both so far outwards and so far downwards, must be the position of the ball at the instant in question; and by thus finding the position of the ball at several separate succeeding instants of time, we may find the curved path which a ball fired *in vacuo* would traverse. This principle of the independence of simultaneously-acting causes was an experimental discovery of Galileo's: before his time it was held as self-evident truth that one cause must cease to act before another can commence to do so; and it was accordingly believed that when a projectile was shot into the air, the force of projection must be expended and dissipated before any tendency to fall to the earth could assert itself.

Experimentation. — When we learn that a certain phenomenon is due to a congeries of causes, we may arrange matters so as to prevent one of the ordinarily-acting causes from producing its effect, and then we may observe in what respect the resultant phenomenon now produced differs from that usually seen. Thus we may find the way in which a given cause acts. Again, we may directly arrange matters so that a given cause, and, as far as possible, that cause alone, shall act, and we may then observe what happens. The principle of the Constancy of Nature shows us that like causes will always produce like results; and if we find that by ingeniously varied interrogations of Nature we have obtained as reply an assurance that certain causes are allied to certain effects, we feel assured that the same causes and the same effects will continue to be so allied. This assurance is the only basis of the art of Experimentation. By this art we become acquainted with the constant modes in which events follow one another in the material world, these modes being the Laws of Nature, arbitrarily appointed, and only to be learned by us through the instrumentality of our own experimental enquiry, or else through attentive consideration of the varying phenomena of the Universe, "experiments made at Nature's own hand."

Newton's Laws of Motion or Axioms. — If a body be at rest it will remain at rest: if in motion it will continue to move until stopped by friction or some external force. Here we find the word Force meaning not only that which causes motion, but also that which arrests motion. Experiment shows us that this is true in reference to bodies which are at rest, for they remain at rest if undisturbed; but it also shows that among bodies which are in motion, it is only those which are moving in a Straight Line that retain their course unaffected when allowed to move unexposed to the action of any disturbing cause. Bodies which are moving in curved paths, such as sling-stones, do not retain their curved paths when liberated, but continue their course in a straight line in that direction in which they happened to be moving at the instant of release from constraint. Hence Newton, in his First Law of Motion, says, "Every body tends to persevere in its state of Rest or of Uniform Motion in a Straight Line unless in so far as it is acted on by impressed Force," and this is tersely expressed by saying that "**Matter has Inertia.**"

If a single force act upon a body which is at rest, the body will begin to move in a straight line; and, further, the greater

the force, the more rapid will be the motion of the body acted upon. If the body be already in motion, the force acting upon it will cause it to move more rapidly or more slowly in the same straight line, or else in a deflected course. Experiment shows that every force has a definite direction in which it tends to cause a body to move, whether that body be already under the action of other forces or not. Thus the words of Newton, in his Second Law of Motion, are: "Change of Motion is proportional to the impressed Force, and takes place in the direction of the Straight Line in which the force acts."

The word Motion in this law is now rendered Momentum (p. 19).

The third of the Laws of Motion which Newton formulated as axiomatic is the following:—"To every Action there is always an equal and contrary Reaction; or the Mutual Actions of any two bodies are always equal and oppositely directed." The truth of this statement is based upon experimental evidence, but its universal applicability is, after consideration, seen to be reasonable enough; and in this sense Newton uses the word Axiom.

When a shot is fired from a gun, if the gun be free to move there is considerable recoil, the shot moving forward and the gun backwards. If the gun be fixed to the ground, the shot is apparently the only thing which moves. If the shot were held fast and the gun were free to move, the gun would move backwards. In this case we see, then, that to the action which impels the shot forward there is a contrary reaction which impels the gun backwards; and in the sequel we shall learn what evidence there is for the statement that that reaction is equal to the action.

When a man walks on firm ground, the action of his legs in locomotion tends to separate his body from the ground at each step. The action which tends to raise his body is contrary to the reaction tending to depress the earth, and at every step the earth is pushed down as a whole, or else if the soil be soft it yields locally and the foot sinks. Hence the difficulty experienced in getting out of boggy soil; the soft mud yields under the foot at each effort made by the individual, so that every step causes him to sink more deeply.

When a horse is loosely harnessed to a car, it may sometimes be observed that an inexperienced animal starts forward quickly; but suddenly the traces tighten, the car is jolted forward, and the horse is jolted backwards.

If a locomotive with a heavy train be suddenly started, it will be seen that its wheels may uselessly turn round; it has given a sudden pull to the carriages, and their reaction upon it is equivalent to a backward pull given to a moving engine.

The earth attracts the moon, and the moon equally attracts the earth. The former attraction mainly keeps the moon in her orbit, and the latter is one of the causes of tidal phenomena.

When a stone is thrown upwards from the earth, the earth is thrown back by recoil, and moves downwards to a very small extent so long as the

stone continues to ascend: when the stone is at its highest point the earth is at its lowest, and as the stone falls the earth ascends to meet it. This is, of course, not the result of direct observation, but is deduced by way of inference from Newton's third law of motion, which is confirmed by all phenomena, terrestrial and astronomical, by which it can be put to the test.

The next statement generally applicable is that of the **Indestructibility of Matter.** This is, that Matter, as we at present know it, cannot be destroyed by any process with which we are acquainted. The limitations of this statement should be borne in mind, for there is no scientific warranty for saying that Matter is absolutely indestructible, and more than one consideration indicates that the structure of Matter may be such as to denote that in its present form it has had a beginning and may have an end. Within our experimental knowledge, however, Matter cannot be destroyed: and when it apparently disappears, as when a candle is burned in the air, Chemistry charges itself with the explanation of that disappearance, and shows what new forms the matter has assumed.

Another principle of the greatest possible use, and entirely the result of experiment, is that of the **Indestructibility** or the **Conservation of Energy.** Energy has been provisionally defined as the Power of doing Work; and this doctrine states that this power of doing work may alter its form but is never destroyed. A coiled watch-spring possesses power of doing work in virtue of its distortion; when it uncoils, it seems to lose this power of doing work, but the Energy thus lost is transferred to other bodies, while Heat, Light, or Sound produced, Work done, Electrical Condition set up, Friction overcome, etc., present the missing Energy in several apparently dissimilar forms, which may all be reduced, however, to two types: Energy due to Motion; Energy due to Displacement. The Energy of a body depends on the advantage which that body possesses either of motion or of position: the loss of that advantage can only occur through some other body or bodies simultaneously acquiring either motion or an advantage of position. If Energy disappear in one form, it will reappear in one or several others, and none of it is ever lost, though it may assume such a form that it is no longer a power of doing work available to man, namely, the form of uniformly diffused Heat. The principle of the Conservation of Energy, which is so important that the whole of Natural Philosophy may be said to be a commentary on it, will be better understood when the laws of Energy have been discussed, as they will be at greater length in Chapter IV.

A corollary to this principle takes the form of a statement of the belief that **The Perpetual Motion is impossible:** if the sum of the Energy in the Universe be constant, no machine in which this energy is employed in doing work, in which friction is overcome, in which sound is produced, and so on, can possibly go on for ever, for the reserve of energy at its disposal will ultimately be exhausted and become useless to that machine. Even the tides will ultimately cease, as the earth loses speed — we know it is at present losing an aggregate of 22 seconds in the course of a century — in its rotation round its own axis.

It cannot be too strongly insisted on that these general principles, the Constancy of Nature, the Law of Causality, Galileo's principle, the three Laws of Motion, the Indestructibility of Matter and of Energy, are of no value for us except in so far as they are supported by experimental evidence. They are grouped together here, for the statement of them is necessary for comprehension of the results which have been obtained through their aid. We are not here called upon to go through the steps by which they have been arrived at, but we must bear in mind that no *a priori* deduction of them by any metaphysical reasoning is for a moment admissible. The doctrine of the Conservation of Energy is very simple when stated as the result of experiment, and its simplicity has led to statements that the contrary is unthinkable, and that a belief in this doctrine is deeply grounded in the constitution of the mind of man; but all conclusions derived from such reasoning must be regarded with suspicion, for we must take warning by the example of the ancients, who believed circular motion to be perfect and heavy bodies to fall faster than light ones, until experimental evidence was adduced to the contrary. The truth of these principles must be proved by their perfect accord with the phenomena which we may actually observe, and by their enabling us to predict results of hitherto untried experiments which agree with those actually obtained. Exact science depends directly for its facts and indirectly for its principles upon experimental evidence, and the true place of speculative imagination in scientific work is the conception of new combinations of circumstances, and hence of new fields of experimental Research, as also the construction of Hypotheses, which explain and co-ordinate observed facts, and which, when they are found to do this consistently and with but a few reasonable and simple assumptions, are raised to the rank of accepted Theories.

CHAPTER I.

TIME, SPACE, AND MASS.

So far as man's knowledge of phenomena occurring around him has become accurate, it has been obtained by means of precise Measurement; and the Fundamental Units in terms of which every measurement must be executed are those of **Time, Space,** and **Mass.**

The **unit of Time** is usually taken as one Second, and the time during which phenomena appear or are observed is reckoned in seconds, unless motives of obvious convenience cause it to be reckoned in minutes, hours, days, years, or centuries. The second is usually a second of mean solar time — that is to say, the $\frac{1}{86400}$th part of the average length of a solar day.

The solar day is the period which elapses between the sun's crossing the meridian, or being situated directly south (or in the southern hemisphere, directly north) of a place, and the next occasion on which it crosses that line. The sidereal day, in the same way, is the interval between two successive transits of any fixed star. The sidereal days are shorter than the solar; they are nearly constant in length, for a sidereal day is the time of one complete rotation of the earth round its axis; but the solar day is not constant in length. A clock can keep time with the stars, and keep good "sidereal time;" but a clock of ordinary construction does not always indicate noon when the sun is highest in the heavens; it is sometimes apparently 14½ minutes fast, and sometimes appears to be 16¼ minutes slow. A good clock, however, is one which measures off and indicates as twenty-four hours a period of time equal to the average length of the solar day for a year or a century or an age, and such a clock is said to keep "mean solar time;" while the Second used in physical measurements is the second as indicated by a clock such as this. Astronomers reckon by the sidereal day, which is equal to 86164·092 mean solar seconds: and they use shorter pendulums in their clocks, so that these keep not mean solar, but sidereal time. The astronomer's second is then equal to $(86164{\cdot}092 \div 86400) = 0{\cdot}99727$ mean solar second.

Space. — When a single point moves it describes a Line: if it travel by the shortest distance between two points, its path is a straight line; and a straight line is an example of space of

one dimension. Movement and measurement may be effected in a forward or a backward direction along it, but as a line has neither breadth nor thickness there can be no other.

Distance along a straight line may be measured in one direction arbitrarily chosen; let this be, for instance, the direction from left to right; if, then, a point travel towards the right its motion is positive, if to the left, negative. If it move a inches to the right and then b inches to the left, its distance from the starting-point becomes $a - b$; while if it first go b inches to the left and then a to the right, its position will become $-b + a$ from that point; and these two positions are the same, for $a - b = -b + a$. Hence we learn that if a point move backwards and forwards by varying amounts along a line, it does not matter in what order it performs these operations: the spot ultimately arrived at will be the same in all cases.

In order to effect measurements along lines, we require a **standard of length.** This is taken as the Foot or the Mètre. The British standard yard, which is equal to three feet, is defined by law as "the distance between the centres of the transverse lines in the two gold plugs in the bronze bar deposited in the office of the Exchequer" at the temperature of 62° F. A number of authorised copies of this have been made and are deposited at the Royal Mint, the Royal Observatory at Greenwich, the New Palace at Westminster, and under the care of the Royal Society of London.

The Mètre is the distance, at the temperature of melting ice, between the ends of a platinum rod preserved in the Archives, and of which copies, to regulate French commerce, are preserved at the Ministère de l'Intérieure in Paris. It was originally intended to represent the ten-millionth part of the distance from the Equator to the Pole: the measurements of Delambre and Méchain, from which Borda made the standard metre according to a law of the French Republic passed in 1795, have been found not to be quite correct, for the earth's quadrant is now known to measure 10,000,880 mètres.

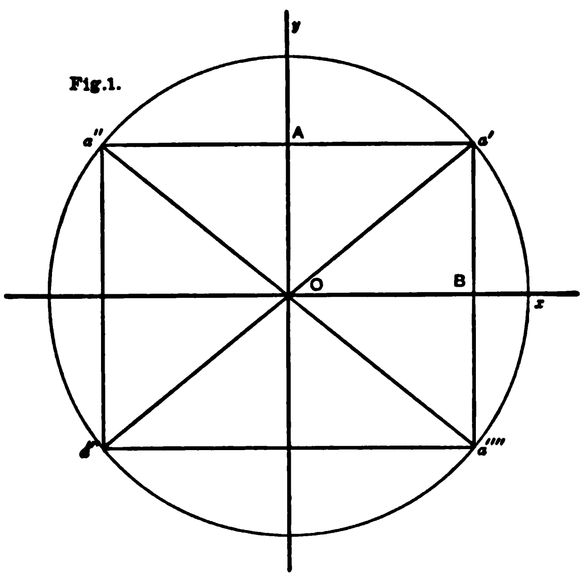
The metric system of measurement of length is decimal; each metre contains 10 decimetres, 100 centimetres, or 1000 millimetres: 1000 metres make a kilometre, which is, roughly speaking, about $\frac{5}{8}$ ($\frac{3281}{5280}$) of a mile; one metre is equal to 39·37043196 inches, or 3·28087 feet; a decimetre very nearly corresponds to 4 inches (really 3·937043196); a millimetre is very nearly equal to the twenty-fifth of an inch. For the pur-

pose of physical measurement it is customary and convenient to make use of the **Centimetre** * (·3937043196 inch) as a unit of length. One English foot is equal to 30·47972654 centimetres, and an inch to 2·53993, or very nearly 2·54 centimetres.

A plane Surface has length and breadth but no thickness, and is therefore said to be space of t w o d i m e n s i o n s. Two terms are always necessary for the precise statement of the position of any point on a surface. The position of a ship at sea is determined when its latitude and its longitude are known.

The position of a point a on a plane surface is determined by choosing a fixed point O as the origin; then two axes, Ox and Oy, are chosen, generally at right angles to one another; aA is drawn parallel to Ox, and aB

Fig.1.



parallel to Oy, and the point a is said to be situated at a distance OA along the axis of y, and OB along the axis of x. If a point lie at the same time three miles to the north and four miles to the west of a given place, its true position (at the distance of five miles) can be easily indicated on a chart. The symbols + and − are also used here to denote that the measurement is to one side or the other of the point assumed as the origin. Points to the right of O have a positive, points to the left a negative, value of Ox; points above O have a positive, points below a negative, value of Oy. Thus (Fig. 1) the point a' has a b s c i s s a (or line cut off along the axis of x) OB, and ordinate (cut off along the axis of y) OA; the point a'' has abscissa −OB and ordinate +OA; the point a''' has abscissa −OB and ordinate −OA; that at a'''' has abscissa +OB and ordinate −OA.

* It is worth remarking that a French ten-centime piece measures 3 centimetres across, while a five-centime piece has a diameter of 2¼ centimetres. Similarly, an English halfpenny measures an inch, while a penny measures an inch and a fifth.

The Area of a Surface may be measured if we fix upon a **standard unit of area.** The unit of length may be made use of in order to obtain this. If a square be constructed, one of whose sides is one foot or one centimetre, we shall have a unit-surface whose area is known as one square foot or one square centimetre; and the areas of other surfaces may be measured by comparison with these standards.

A Solid has length, breadth, and thickness, and is said to occupy space of three dimensions. The position of any point in tridimensional space requires three numerical terms for its exact statement. The position of a balloon, for instance, will be definitely known if the latitude and longitude of the spot over which it stands and its height above that spot be ascertained.

Three terms are also required to define the position of a star: the telescope has to move so much "in azimuth" round a vertical axis; then so much in "altitude" round a horizontal axis; and thirdly, the distance of the star in a straight line must be known.

A cube whose side is one foot or one centimetre — that is, a cubic foot or a cubic centimetre — serves as the **unit of volume.** For convenience' sake other units of volume are often chosen, such as the cubic inch, the cubic decimetre (otherwise known as the liquid measure, one Litre), the cubic metre, and so forth.

The remaining fundamental idea involving measurement is that of **Mass, or quantity of Matter.** The notion implied in this term is quite distinct from that of Weight. The weight of a certain quantity of matter depends upon the presence and nearness of other matter, which attracts it according to the well-known law of Gravitation. This may and, even within our terrestrial observation, does vary; the effect of gravity on a given mass — that is to say, its Weight — is greater as we near the Poles than it is at the Equator; and the weight of a substance varies, therefore, according to local causes, while the mass or quantity of matter in it remains the same. *Cœteris paribus*, however, equal masses will everywhere counterpoise one another in a balance, and we may define the **unit of mass** as that quantity of matter which will counterpoise in a balance a certain standard mass known as a standard Pound or Gramme.

The British standard Pound is a piece of platinum preserved in the same place as the standard yard, while authorised copies of it are preserved at the same institutions. The French stand-

ard is the Kilogramme (= 1000 grammes), made of platinum, and preserved at the Archives in Paris. This is intended to have the same weight as a cubic decimetre of water at its temperature of maximum density — that is, 3·9° C. Since a kilogramme contains a thousand grammes, and a cubic decimetre a thousand cubic centimetres, it follows that the **gramme** is intended to be equal to the mass of one Cubic Centimetre of water at 3·9° C. Comparison of the actual standards shows, however, that a litre of water weighs, at 3·9° C., 1·000013 kilogrammes, and a cubic centimetre of water at 3·9° C. weighs therefore not one gramme, but 1·000013 grm. For most practical purposes the intended value may, however, be taken as correct. The British pound avoirdupois weighs 7000 grains, while the standard kilogramme weighs, according to Prof. W. H. Miller, 15432·34874 grains, and the gramme 15·43234874 grains.

It may be noticed that the British fluid ounce of water at 62° F. weighs one ounce avoirdupois; that the British pint of water (20 fluid ounces) weighs therefore a pound and a quarter, and the British gallon of water ten pounds. A French franc-piece weighs, when new, five grammes.

In British measurements the Foot, the Pound and the Second may be used as the fundamental units. In British Magnetic Observatories the units employed till lately were the Foot, the Grain and the Second.

The C.G.S. System. — For the international convenience of scientific men the C.G.S. or Centimetre-Gramme-Second system of units and measurements is in current use.

The gramme is chosen as a unit rather than the kilogramme, the centimetre rather than the metre; firstly, because the use of smaller units diminishes the need for working with decimal fractions; and, secondly, because on the C. G. S. system the density of water (p. 220) is equal to unity, which is a distinct advantage. If the kilogramme and the metre had been employed as units, the density of water — the number of kilogrammes in a cubic metre — would have been 1000.

The introduction of coherent systems of units for the measurement of all physical quantities has been an enormous stride in advance. When we have a problem to solve numerically, if we take care to put in all the terms in C.G.S. measurement, the answer comes out in C.G.S. units, ready for use without further reduction.

CHAPTER II.

NOTIONS DERIVED FROM THE PRECEDING.

WHEN a physical particle changes its position, it effects **Motion.** This Motion or Change of Position must be performed by passing along a definite continuous path—continuous because it is not possible for any physical particle to occupy two consecutive positions without traversing the intermediate space.

In this respect the path of a physical particle differs from many mathematical curves which abruptly end at one point and recommence their course at another. Obviously the path described by the moving particle may have any form, straight or curved; and the shortest possible path between the initial and final positions is a straight line.

We may remind the reader of Newton's use of the word Motion in the sense of Momentum (pp. 6, 19).

A moving body may travel rapidly or slowly: the rate at which it travels along its path is called its Rate of Motion, its rate of change of position, its **Velocity.** The Velocity of a moving body may be stated in units of length per unit of time, *e.g.* feet per second; and a body is moving with Unit velocity when it moves one foot per second, or one centimetre per second (the latter, the C.G.S. unit, being one 'Kine'). It will be observed that it is necessary for us to make consistent use of the British or of the C.G.S. units of measurement, and not to use them confusedly within the limits of the same problem.

A body which moves sixty feet in five seconds has a mean velocity, evidently, of twelve feet per second. The velocity is equal to sixty divided by five—that is, to the whole space traversed divided by the time occupied in the movement. In algebraical language this may be expressed thus: $v=\frac{s}{t}$, where the velocity, space, and time are denoted by their initial letters. Multiplying both sides of this equation by t, we get $vt=s$; the

space traversed in a given time is equal to the velocity per second multiplied by the number of seconds.

If we consider Motion and Velocity in any one particular **direction,** we may emphasise this by using black-faced type for our symbols; our equation then becomes $\mathbf{v}=\mathbf{s}/t$; the Velocity in any given direction is the Space traversed in that direction divided by the Time.

A Velocity in general, without reference to its direction, is sometimes called a **speed,** v; while the term **velocity** is then restricted to velocity, $\mathbf{v}$, in some particular Direction. In this volume we shall, for the most part, distinguish Speeds in general from Velocities in particular directions by the use of the symbols v or $\mathbf{v}$, as required.

Digression as to mathematical formulæ and the theory of Dimensions. — Each such formula is a kind of generalised shorthand blank form, waiting to be applied to particular cases by being consistently filled in with appropriate numbers. In words at full length we may affirm that the Number expressing a speed or **velocity** is equal to the Number expressing the **space** traversed divided by the Number expressing the corresponding **time** taken; all these being, of course, systematically measured in consistent units. The numbers themselves in any particular case we may not know at present, and in the meantime we may not even care to know; for such a verbal formula is of a higher order of generality, of wider value than a mere statement of the particular numbers in any particular case. By way of rough jotting we may shorten the phrase "Number expressing a Velocity" down to the simple word "Velocity," and so on. Then we have the condensed note "Velocity = Space ÷ Time." This may be still further shortened by using initial letters only, in which case the symbols "$v = s \div t$," or "$v = \frac{s}{t}$," or "$v = s/t$," suffice to express the law; or we may agree that these unknown numbers shall for the time being be represented by letters arbitrarily chosen. Thus if we agree that the letter a shall stand for "number expressing velocity," or, as it is more usually phrased, that a shall represent velocity; and similarly that b shall represent space traversed, and c the corresponding time, the condensed expression of our law becomes $a = b \div c$. To apply this to any particular case we must know what the numerical values of two of the terms actually are; this much being determined, it is only an arithmetical matter to find the numerical value of the third term. For example, let v (the number expressing a velocity) be 12 (ft. or cm. per sec.), and let $s = 60$ (ft. or cm.), then replacing v in the equation by 12 and s by 60 we get $12 = 60 \div t$, and t cannot have any other value than 5 (sec.); all in units of the same system.

One great advantage attending the use of mathematical formulæ is their susceptibility to algebraic transformation. The above equation may be written $s = vt$ or $t = s/v$, either of which modes of expression, when translated into words at full length, is found to present the subject from so fresh a point of view as practically to amount in each case to the enunciation of an independent truth.

When a is stated by a formula to depend upon or to be "a function of"

b, *c*, *d*, and of these only, it seems, when put into words, a truism to affirm that *a* is independent of variations in the values of *e*, *f*, *g*, etc.; yet this often leads to the enunciation of valuable principles, *e.g.* p. 21, line 16.

The Theory of Dimensions. — The number expressing a Velocity is the number expressing a Space divided by the number expressing a Time; $v = s/t$, as we have seen before. But there underlies this mode of expression a tacit understanding that we adhere consistently to some known system of units. The numbers must vary with the units conventionally employed, even when the same facts have to be expressed. Consequently we may, if we have in our minds a possible change of units, write such an equation as $v\,[V] = s\,[S] \div t\,[T]$, where the italic initials represent numbers and the corresponding bracketed letters the respective conventional units. If v, s, and t in the above equation become all $= 1$, that equation becomes $[V] = [S/T]$, an equation which refers to the conventional units only. Such an equation is technically known as an equation of Dimensions. Then if we change our conventional units from [V], [S], and [T] to others, say, $[lV]$, $[mS]$, $[nT]$, the last written equation must still hold good, and the new unit $[lV]$ is equal to $[mS/nT]$, or to $m/n\ [S/T]$; that is, the new unit of velocity is equal to m/n times the old unit. The numerical value of any given velocity is, inversely, n/m times as great when expressed in terms of the new units as it was when expressed in terms of the old units; that is, it varies inversely as the unit employed; just as a sum of £40,000 seems greater (one million) when expressed in the smaller French unit, the franc. Let us now set ourselves a problem: What is the ratio between the British and the C.G.S. unit of velocity? The former is 1 ft. per sec., the latter is 1 cm. per sec. Here $[V] = [S/T] = [\text{Foot}/\text{Second}] = [30{\cdot}478\ \text{cm.}/\text{second}] = 30{\cdot}478[\text{cm.}/\text{second}]$; the British unit is 30·478 times the C.G.S. unit. Consequently a velocity of 3047·8 cm./secs. would be a velocity of only 100 if measured in ft./seconds.

But the Equation of Dimensions is not limited to this interpretation and use. It far more frequently means, in actual use, to adhere for example's sake to the equation $[V] = [S/T]$, that the Numerical Measure of any Velocity is some Number of Units of Space (or Length) divided by a corresponding Number of Units of Time: and where we have, for example, the Dimensions of a Quantity of Electricity in magnetic measure given as $[q] = [M^{\frac{1}{2}}L^{\frac{1}{2}}]$, it means that the numerical measure of any given quantity of electricity on the magnetic system of measurement is the square root of some Number of Units of Mass multiplied by the square root of a corresponding Number of Units of Length, the Numbers themselves necessarily varying inversely as the units employed.

The equations of Dimensions thus explained were an invention of Fourier's, and were brought into prominence by Clerk Maxwell. Their use is twofold: (1) as a means of converting physical quantities expressed in one set of units into the same quantities expressed in other units; and (2) as a means of checking our equations, for the dimensions must agree on both sides, as will be seen in very simple examples on p. 60.

Velocity (resumed). — If a body move through equal spaces in equal times, its velocity is said to be **uniform.**

We are familiar with instances in which a body such as a railway train is said to be running at a certain time with a

velocity of (say) thirty miles an hour. This indicates that if the train ran for a whole hour at the rate at which, and in the same direction as, it was travelling at the instant of observation, it would at the end of an hour be thirty miles away from the point which it occupied at the beginning of it. But the train may possibly not have run more than a mile on the whole. The statement means, then, that during (say) a minute it ran half a mile, and that therefore during sixty minutes it might, at the same speed, have run thirty miles. But even during a minute it may have gained or lost speed, so as to render its motion not uniform but **variable**: the statement would be still more exact if we knew that in six seconds it ran the twentieth of a mile, or in one second the hundred-and-twentieth; for when the interval of time is very short, there is less possibility of variation during that interval, and the speed approximates more nearly to uniformity. Hence the variable velocity of any moving body at a particular instant is found by observing the amount of motion effected in a certain very short interval of time, and finding what movement would be effected in one unit of time if the velocity were to remain uniform during that period.

If a body move over a certain space, s (say thirty feet), in time t (say ten seconds), the equation $v=\frac{s}{t}=\frac{30}{10}=3$ feet per second shows what the mean or average velocity is during the motion. The **mean velocity** of a train which travels fifteen miles in one hour is a quarter of a mile a minute, or $\frac{1}{240}$ part of a mile in a second, although during some seconds or minutes it may be travelling at the rate of sixty miles an hour, at others may be standing still, and at others may be actually going backwards.

All velocities, mean and constant, uniform and variable, may be expressed in feet or in centimetres per second, and can, when so expressed, be compared with one another.

All measurable velocities are Relative; we know nothing about Absolute velocities in space, for we have no standard of comparison.

Problems.

1. If a body move 144 feet in 3 seconds, what will be its mean velocity? —*Ans*. 48 feet per second.

2. In the previous question: What will be its mean velocity during the second second if it travel 16 feet in the first second and 80 feet in the third? —*Ans*. 48 feet per second.

3. A body moves with a uniform velocity of 40 miles 1600 yards per hour: what is its velocity in feet per second; and how many feet will it traverse in 10 seconds?—*Ans.* 60 feet per second; 600 feet.

4. A railway train explodes two detonating signals placed on the rails at a distance from one another of 176 feet; an interval of exactly 2 seconds elapses between the explosions. Compare the velocity during that interval with the mean velocity, which is indicated by the statement that the train takes an hour and a half to perform the journey between two stations 45 miles distant from one another.—*Ans.* It is twice the mean velocity.

5. Which is the greater velocity, 40 miles an hour or 12 metres per second?

6. A train travels 10 miles at a velocity of 20 miles per hour; then 4 miles at an average rate of 30 miles per hour; then 6 at a uniform rate of 40 miles per hour; it takes 1 mile to come to rest, running at an average speed of 20 miles an hour; it stands for 7 minutes; it starts and runs for 20 minutes at the average speed of 21 miles an hour. What has been its mean velocity?—*Ans.* 32 feet per second.

Acceleration.—When the rectilinear velocity **v** of a moving body varies, the Rate of Change of Velocity is called its Acceleration. In popular language this word indicates increase of speed, but it is in this connection used to signify the rate of change of the velocity, whether that change be an increase or a diminution. If a body be moving at the rate of ten feet a second at the beginning of a certain second of time, and at the end of that second be found to be moving in the same line at the rate of eleven or of nine feet, it is said that its motion has been accelerated during that second, positively in the former case, negatively in the latter, by one foot per second. Acceleration is usually indicated by the symbol α, and the Unit of Acceleration is the acceleration observed when a body alters its velocity in a given direction by one unit of velocity *every second.* A body, then, which has its velocity in a given direction increased or diminished by one foot in one second, two feet in two seconds, and so forth, is undergoing, in that direction, a unit acceleration, in British units; if by one cm. per sec., n cm. in n seconds, it is undergoing a unit acceleration (one 'Spoud') in C.G.S. units.

The initial velocity may be zero, the body being originally at rest; in such a case the body will undergo unit acceleration in a given direction, if in that direction it acquire unit velocity in one second, or a velocity of n units in n seconds.

There are some cases in which the apparent effect of acceleration is to change the direction of motion; but there is no essential difference between such cases and those upon which the definition here given is based; and such a result will be readily understood after we have discussed the Composition of Velocities and of Accelerations.

If a body move with velocity $\mathbf{v}_0$, and at the end of t seconds with velocity $\mathbf{v}_t$, the total change of velocity during t seconds is $\mathbf{v}_t - \mathbf{v}_0$, the time during which this change is effected is t, and the acceleration per second is $\frac{\mathbf{v}_t - \mathbf{v}_0}{t}$. This is the mean acceleration, in the direction of motion, during the time t: and the acceleration may during that period t be uniform or variable, but an approximation to its value at any instant may be found by making the interval t as short as possible.

Accelerations, being measured by the velocities imparted per unit of time, are stated in terms of units of length per second, *per second*.

Problems.—1. A body starts from rest under the influence of a force which produces acceleration $a = 2$ ft.-per-sec. per second: when will it have a velocity of 1000 feet per second?—*Ans.* At the end of the 500th second.

2. A body travels at 12 feet per second; in 10 seconds it is moving 7 feet per second: what is the mean acceleration?—*Ans.* $-\frac{1}{2}$ ft.-per-sec. per second.

3. If in the last question the acceleration had been $+\frac{1}{2}$ ft.-per-sec. per second, what would have been the rate of movement at the end of 10 seconds?—*Ans.* 17 feet per second.

4. A body moves, in the first second during which it is under observation, through a space of 16 feet; in the fourth second through 112 feet: what is the acceleration per second?—*Ans.* 32 ft.-per-sec., so that during consecutive seconds it moves 16, 48, 80, 112 feet. At the end of each successive second it moves with a velocity of 32, 64, 96, 128 feet per second respectively.

5. A body as it moves is made to record its own velocity: it is found that at a certain instant it is moving at the rate of 112 feet a second; after an interval of $\frac{1}{20}$ second its velocity is 114 feet per second: what is its acceleration?—*Ans.* $\frac{\mathbf{v}_t - \mathbf{v}_0}{t} = \frac{114 - 112}{\frac{1}{20}} = 40$ ft.-per-sec. per second.

6. A particle moves, during the first second, with diminishing velocity, at the *mean* rate of 10 centimetres per second; the next second it moves at the mean rate of 8 centimetres per second; the acceleration is constant: how far will it travel, and what will it do when it has come to rest?—*Ans.* It will go on for $5\frac{1}{2}$ seconds, will traverse 30·25 centimetres, and will return, arriving at every point on its previous path with the same speed as that with which it left it, and will retrace the 30·25 centimetres in another $5\frac{1}{2}$ seconds, passing the starting point with a reversed velocity of 11 centimetres per second.

Momentum.—When a body whose mass is m moves with a speed or velocity v, it is said that the total *Momentum* or Quantity of Motion is the product of these two terms, namely, mv units of Momentum. The greater the velocity or the greater the mass moved, the greater the quantity of motion. The C.G.S. unit of momentum is called a Bole.

Force.—When a body which is at rest is set in motion, or one which is in motion is accelerated (positively or negatively)

or deflected from its straight course, we commonly attribute these effects to impressed force, or simply to Force. This is sometimes defined as any Cause which tends to alter a body's state of rest, or of uniform motion in a straight line. It is better defined as it is by Newton, not as a cause, an existing reality of any kind, but simply as an observed Phenomenon, a measurable Action upon a body, under which the state of rest of that body, or its state of uniform motion in a straight line, suffers change.

The presence and the mutual influence of at least two bodies are always essential to the production, in any one of them, of those effects of displacement which we commonly attribute to Force; and such displacement is always associated with a transformation or a redistribution of Energy.

Forces considered as measurable Actions are measured by the Masses set in motion, and by the Velocities imparted to them in unit of time — that is to say, by their Accelerations; and the equation $F = m\alpha$ enables us to measure any Force F, as the product of a Mass m into the Acceleration α imparted to it, as found by observation.

Any observed acceleration must necessarily be acceleration in some particular direction, at any rate at the instant of observation. We shall have occasion from time to time to emphasise this by writing α as $\boldsymbol{\alpha}$, where the black-faced type draws attention to the fact that the quantity symbolised is a directed quantity; and when we do so, our equation becomes $\mathbf{F} = m\boldsymbol{\alpha}$; *i.e.* the Force in a particular Direction is equal to the Mass $m \times$ the Acceleration acquired in that direction.

Force may also be measured as an Observed Rate of Change of Momentum. Acceleration $\boldsymbol{\alpha}$ = rate of change of velocity $\mathbf{v}$; $\mathbf{F} = m \times \boldsymbol{\alpha} = m \times$ rate of change of $\mathbf{v}$ = rate of change of $m\mathbf{v}$ = rate of change of momentum. A uniform Force is therefore numerically equal to the Amount of Momentum gained or lost by a body during each Second. (See further p. 41 and p. 47.)

The product of a force acting into the time during which it acts measures the momentum imparted during that time; and this product is known as Impulse — a term of frequent use in the study of the working of machinery.

By a convenient form of speech a given Force is said to act upon a given body and to impart to it a given acceleration. It must be constantly borne in mind, however, that a Force is not a physical entity, and the word Force is not in itself an explanation of anything. Force can never be measured until we already know, absolutely, or by comparison, the mass acted upon and the acceleration actually imparted to it; and force may be increased or diminished by varying the arrangement of the bodies to whose

mutual actions it corresponds; as in the case of the Hydraulic Press, where the ordinary action presents an apparent increase of Force, while if the action be reversed, Force seems to be destroyed.

If a body weighing three pounds be set in motion so as at the end of one second to have had a velocity of four feet per second imparted to it, then $F = ma = 3 \times 4 = 12$ Poundals or British units of force. The same force would have imparted to a two-pound mass an acceleration of six feet, to a one-pound mass an acceleration of twelve feet, and to a twelve-pound mass it would have imparted in one second a speed of one foot per second.

If the mass moved be a unit, and the acceleration acquired by the mass be unity, the product $ma = \mathbf{F}$ is also unity; and hence the **Unit of Force** is to be defined as that observed when a unit of mass is found to acquire unit velocity in the course of one second.

It will be observed that this definition of the Unit of Force is absolute, is not affected by local variations in the intensity of gravity, and is hence everywhere the same.

If the unit of length chosen be the centimetre, and the unit of mass the gramme, the Unit of Force will in one second cause a gramme-mass to acquire a velocity of one centimetre per second; and the unit of Force so defined is called a **Dyne.** Any force may be stated to be equal to so many dynes.

One million dynes make one Megadyne.

Problems.

1. A certain force acts upon m units of mass of matter, and at the end of a second that mass is found to be moving with a velocity of 32 feet per second: what velocity will be produced if the same force act upon $32\,m$ units of mass for the same period?—*Ans.* One foot per second.

2. How many dynes of force are required to set a mass weighing 50 kilogrammes in motion with a velocity of 12 metres per second, the force being supposed to act for precisely one second?—*Ans.* 60,000,000.

3. How many dynes are required to make a gramme-mass move with a velocity of 9·81 metres per second, the force measured in dynes being supposed to act for precisely one second? what if it act for two seconds?—*Ans.* 981 dynes; 490·5 dynes.

4. Compare the velocities produced by the action on masses of 2 kilogrammes, 750 grammes, and one gramme respectively, of forces measuring respectively 300,000, 112,500, and 150 dynes.—*Ans.* All equal; 150 centimetres per second if the action endure for one second.

5. Equal forces act upon the masses specified in the last question: what will be the relative accelerations produced?—*Ans.* 3 : 8 : 6000.

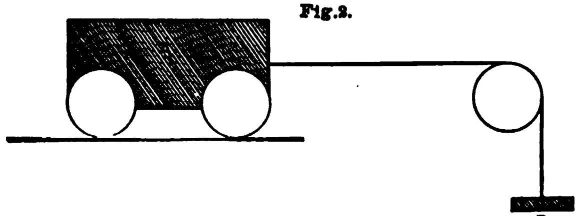
Weight.—Experiments made near the earth's surface show that every mass of matter acquires, if gravity act freely upon

it for one second, a downward velocity of nearly 981 cm. (32·2 feet) per second. This downward acceleration is experimentally found to be independent of the nature and of the size of the falling body; but it is not the same at all parts of the earth's surface, for between the Poles and the Equator it presents a difference of about $\frac{1}{2}$ per cent. Whatever be the local value of this gravitational acceleration g, we find that the equation $F = ma$ takes the form $G = mg$, where G is the local Force or measurable Action or downward Pull of Gravity upon a body of mass m; and this is known as the Weight of that body. Necessarily, if the downward acceleration g differs from place to place, the local weight of a given mass will vary in like proportion.

But if we take the acceleration due to terrestrial gravitation as $g = 981$, which is somewhat nearer its Paris than its Greenwich value (see p. 205), the Gravitational Force acting on a gramme-mass, being its Mass × its Acceleration, is (1 gramme × 981 units of acceleration), or 981 Dynes. The Weight of a gramme-mass, that is to say, is equal to 981 dynes; and conversely, a C.G.S. unit of Force, one Dyne, is equal to the Weight of 1/981 gramme.

Similarly, if we use British units, we find, writing pound for gramme, and 32·2 feet for 981 cm., that the Weight of a pound-mass is equal to 32·2 British units of Force, or Poundals; the Poundal is therefore nearly equal to the Weight of half-an-ounce.

Fig. 2.



If a unit-mass be divided into two parts, one of which, A, weighs $(g-1)/g$ units, and the other, B, $1/g$ unit, and if the weight of the smaller be employed to set both in motion, then the whole mass set in motion is $m=1$, and the Force acting is the Weight of $1/g$ unit-mass, that is, $1/g \times g = 1$. Hence, $m = 1$, $F = 1$; and since $F = ma$, the Acceleration must also be unity, and at the end of a second the whole mass would, apart from friction, be found to be moving with unit velocity.

The local Weight of a gramme-mass is called the local Intensity of Gravity; and this is equal to g dynes.

We can state, then, that any given *Force* is *equal to the Weight* of so many units of mass at a certain definite place.

The engineer's unit of Force is the Weight of 1 lb. or that of 1 kilogramme. He accordingly speaks of, say, a "Force of 8 lbs.," where the physicist would say a "Force equal to the local Weight, at some particular place, of a Mass of 8 lbs." His unit of force is therefore a variable unit: whereas the physicist's unit of force does not in any way depend on local variations in the force of gravity. Again, if we seek the analogue of the equation $F = ma$, we find it to take the form $P = ma/g$; and this is awkward. Still, if properly understood, such expressions as "a Force of 8 lbs." are compendious and not wanting in convenience, even though they do lead to encumbering some of the engineer's formulæ with an unnecessary divisor g; they facilitate the immediate expression of certain results in foot-pounds or in kilogramme-metres (p. 41); and the error which they may introduce, through the variability of the engineer's unit of force from place to place, is practically well within one-half per cent.

Stress.—The word Force is limited to the case in which some movement of masses or of particles is produced, varied, or checked: what is popularly known as the force tending to bring a spring back to its original form, but not actually doing so, is a Stress; and the condition of the spring under such circumstances is a Condition of Stress. A spring, when its form is altered, tends to resume its original form, and it exerts a pressure or a pull upon any object so placed as to prevent its doing so; but this object also exerts continuously an equal but opposed pressure or pull upon the spring. This mutual pressure or pull will cause motion if the bodies pressed upon or pulled become free to move; if not, the pressure or pull is continuously applied without producing movement, and such an inactive Mutual Pressure or Pull is called a **stress.**

In popular language a Stress is called a Strain, as where it is said that a bridge or wire being exposed to too great a strain gives way and breaks or snaps. Properly the word Strain means Deformation of a body.

Every such stress implies at least two fixed points; these are either pressed together, or else the material stretched between them is in a condition of tension. In the former case, when the condition of stress ceases, the body previously compressed expands; in the latter case, when set free it contracts. If both ends of a stretched body be simultaneously liberated, the resultant movement is towards the centre; if one end only be set free, the movement is towards the end which remains fixed; and conversely for a body exposed to compression.

Stress therefore always implies mutual Action and Reaction; and we might, with Tait, paraphrase Newton's third Law thus: "Every action between two bodies is a Stress." A stress is always numerically equal to either the Action or the Reaction, as also to the Force which is necessary to produce it, or to that which is developed when the condition of stress comes to an end. Stress cannot be said to be either positive or negative in the line

of its application, for it depends on extraneous circumstances which point or part of the stressed body shall be set free, and therefore what shall be the direction of the resultant movement; but it has a numerical magnitude, for it can be measured in dynes; and it may be numerically specified either (1) as Total Stress F or (2) as Stress f per Unit of Area of the common bounding surface between two bodies under mutual Action and Reaction.

In the first sense, Stresses or total stresses may be measured as equal to the Forces which produce them. A spring is pressed upon by a certain known weight; it yields to a certain extent; it is then caught by a ratchet, and the weight is removed. The Force necessary to cause the given yielding of the spring is known, for it is the numerical value, in dynes, of the Weight of the mass employed; the Total Stress established in the spring is numerically equal in dynes to the Force used. The whole upward pressure of the spring on the ratchet must be numerically equal to the weight of the mass removed; so must the downward pressure of the ratchet on the spring. The opposite extremity of the spring imposes an equal and downward pressure on its support, opposed to which is an equal upward pressure of the support upon the spring.

In the second sense, the numerical value of the stress (*i.e.* Stress per Unit of Area) is obtained by dividing the total stress (in dynes) by the area (in sq. cm.) over which it is distributed; and this is otherwise known as the Intensity of the Stress.

In the sequel we shall, except where the context makes it plain, avoid the use of the unqualified word Stress, and shall endeavour to make it clear whether in any particular instance we refer (1) to a Condition of Stress, (2) to a Total Stress of so many dynes, or (3) to a Stress of so many dynes per square centimetre.

Pressure. — Suppose a heavy slab of iron weighing 100 kilogrammes to be laid upon a flat slab of indiarubber of sufficient size; and let its under surface be flat and have an area of 1000 sq. cm. Its total weight is $G = 100{,}000$ grms. $\times 981 = 98{,}100{,}000$ dynes; and this is distributed over the underlying surface of the indiarubber as a Total downward Pressure **P** of 98,100,000 dynes. When the arrangement is that specified, the indiarubber suffers (over 1000 sq. cm.) a downward pressure $\mathbf{p} = 98{,}100$ dynes on each sq. cm. acted upon; but it exerts on the iron an upward pressure of equal amount, for the pressure is mutual.

Now let the metal slab be mounted on four legs, whose joint cross-area is, say, 20 sq. cm.; and let the whole again stand upon the indiarubber. The total pressure **P** is the same as at first; but it is now distributed over an area of only 20 sq. cm., for which reason the indiarubber and the metal are now subject to a mutual pressure $\mathbf{p}' = 4{,}905{,}000$ dynes per sq. cm. across the area of contact.

Here, therefore, we have again a number of different meanings. The word Pressure may mean:—

(1.) Between two objects having a common bounding surface, a Total Mutual Pressure **P** of so many dynes over that whole surface, and at right angles to that surface.

(2.) From the point of view of one of the objects, the Total Pressure **P** suffered by it or exerted by it across the whole area of mutual contact and at right angles to that surface; the same number of dynes over the whole area as in the preceding case.

(3.) The Mutual Pressure per Unit of Area of the common surface and at right angles to that surface. (The "Intensity of Pressure," **p** dynes per sq. cm.)

(4.) The Pressure of the one body on the other across the common bounding surface, measured in dynes per sq. cm., or "Barads," and at right angles to that surface (Pressure of A on B, or of B on A per Unit of Area).

(5.) In the interior of a mass subjected to a uniform stress of so many dynes per unit area of its bounding surface, acting inwardly, there is a mutual pressure of all the parts of the mass upon one another, in all directions: this is an undirected or Hydrostatic Pressure; and if the applied stress or pressure be **p** dynes per sq. cm., at right angles to the bounding surface, this hydrostatic pressure is equal to $p = \mathbf{p}$ dynes per sq. cm. across any plane arbitrarily chosen in the mass considered.

The distinction between **P**, the Total Pressure, and **p**, the Intensity of Pressure, is a matter of importance. Compare a railway tie or sleeper, on which the rail rests by a wide chair-bearing, with one on which the rail rests by a narrow bearing; when the train crosses the latter, the whole pressure is borne by a small area, and the sleeper may give way locally. A knife or a chisel is an instrument for producing great intensity of pressure; when we "sharpen" it, we diminish the area of its edge.

We shall endeavour to make it consistently clear in which sense the word Pressure is used in each particular case.

Tension.—If a mass of, say, 100 kilogrammes be hung by a metallic rod of 1 sq. cm. cross-section, the metallic rod is under

longitudinal tension amounting to 98,100000 dynes across that one sq. cm. of cross-section. If the same mass had been suspended by a metallic rod of, say, 10 sq. cm. cross-section, the tension would have amounted to 9,810000 dynes per sq. cm. of cross-sectional area. In both these cases the Total Tension is equal to the Weight of the mass suspended, viz. 98,100000 dynes; and it is irrespective of the transverse-sectional area of the rod which is being acted upon. Here again we have thus to distinguish between a Total Tension (of **T** dynes) and a Tension per Unit of cross-sectional Area (**T**/Area = **t** dynes per sq. cm.); and here again we shall have, in the sequel, to make it plain to which of these reference is being made in any particular case.

As a rule the phrase The Tension of a Cord is supposed to mean the Total Tension **T** acting across any transverse-section of a cord; this is the same at all parts of a cord stretched between two points, whatever may be the local variations of the thickness of that cord. Pretty obviously the thinnest part of a cord thus stretched between two supports is exposed to the greatest tension *per unit of cross-sectional area;* for, the Total Tension being uniform, it necessarily follows that where the cross-area is least, there the Tension per Unit of Area (*i.e.* the quotient (Total Tension ÷ Cross-Area), otherwise known as the Intensity of Tension or the **Traction t**) is the greatest; and, accordingly, a stretched string is most liable to snap where it is thinnest.

It is scarcely necessary here to point out for the sake of clearness that there are three other distinct meanings of the same word Tension, which will duly come up in their respective places. These are: (1) the Surface-tension of a Liquid, p. 272; (2) Electric Tension, a name given to the self-repulsion of electrified surfaces, p. 582; (3) "in Tension," an old-fashioned and obsolete phrase denoting a certain arrangement of cells in a galvanic battery, p. 640.

CHAPTER III.

MEASUREMENTS.

In the foregoing chapters we have become acquainted with the units of Space, Time, and Mass, and with those, derived from the preceding, of Velocity, Acceleration, Momentum, and Force; and it is for us now to ascertain what principles are made use of in the various measurements effected in terms of these units.

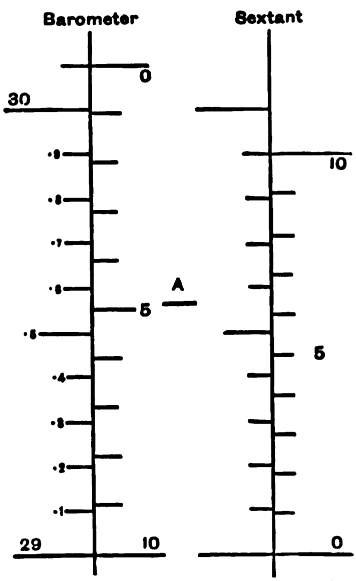
In the **Measurement of Lengths** two main methods are resorted to — Line measurement (*mesure à traits*) and End measurement (*mesure à bouts*). The former is the method in habitual use among carpenters, who lay off so many feet and inches by the aid of their pocket-rule; the latter is the method which they use when they measure the width of a cavity by means of a pair of callipers, which they open until it exactly fits the space.

Line Measurement. — The length of any line may be measured by a graduated scale, which may be, like the carpenter's pocket-rule, somewhat roughly graduated; or instruments on the same principle may be made use of, which are finely and very accurately divided. The measurement is effected by observing the nearest coincidence of the marks on the scale with the length of the object, and by then reading off the value on the scale. This is a familiar operation, but it will be observed that it depends on the accuracy of the sight. The eye requires to be held directly first over one, then over the other end of the object to be measured, together with the corresponding part of the scale; for the true coincidences would be disturbed if the scale and object were looked at obliquely. It is found that in estimating measurements which are very small the eye gets confused; and besides this, the difficulty of making accurately and very finely graduated scales increases with the accuracy required. Thus it happens that, while the ordinary 24-inch rule divided into various fractions of an inch, or Whitworth's very convenient 20-inch rule, decimally divided, or a measure divided to half or quarter millimetres, may be used for measurements involving differences of the hundredth of an inch, it is only with difficulty that they can so be applied, and it is much more convenient in cases involving minute measurements, such as observations of the height of the barometer or thermometer, to use a contrivance called a Vernier. This is a subsidiary scale which slides up and down past the main scale, and is differently divided from it.

There are two kinds of Vernier in use, those known as the Barometer-Vernier and the Sextant-Vernier, which require separate descriptions.

The Barometer-Vernier is thus graduated: a line is set off on the vernier equal to eleven divisions on the main scale. This line is divided into ten equal parts. Each of these parts is therefore equal to $1\frac{1}{10}$ division on the main scale. If the main scale be divided to tenths of an inch, the difference between a division on the main scale and one on the vernier is $\frac{1}{100}$ inch. Suppose the object measured to be more than 29·5 inches and less than 29·6 inches on the scale. The zero of the vernier is laid, as exactly as possible, opposite to the point whose position is to be found, the extremity of the object to be measured, or the height of the mercurial column in the barometer; then on looking *down* the vernier it will be found that at some point there is a coincidence between a graduation mark on the vernier and one on the main scale. The number of that mark on the vernier is noted, and that is the figure required in the second place of decimals. For example, let the point A be above 29·5, below 29·6 inches; the zero point 0 of the vernier is brought opposite to it: the point 6 of the vernier coincides with a division of the main scale; the length is 29·56.

Fig. 3.



In the Sextant-Vernier, which is the form more usually found in instruments of Continental make, the divisions on the vernier run in the same direction as those on the main scale. A line is set off on the vernier equal to nine divisions of the main scale; this is divided into ten parts, each of which is equal to nine-tenths of a division of the main scale. In the same way the vernier is moved until its zero point is brought opposite the end of the object to be measured, and the mark *up* the vernier which first coincides with a division-mark on the main scale gives the figure as before.

Frequently, as in the sextant, a little magnifying glass is so placed that the zero point of the vernier may be by its aid brought more accurately opposite the object to be measured. When still greater accuracy is required, a microscope is so placed as to ensure the greatest possible completeness of coincidence between the zero point of the vernier and the end of the object to be measured, both of which are simultaneously brought into the centre of the field.

The Cathetometer is an instrument whereby vertical heights are measured. It consists of a vertical rod on which a finely-graduated scale is engraved. This carries a sliding piece to which is attached a telescope. In this telescope is fixed a pair of spider threads or fine platinum wires arranged at right angles to one another, and so placed (in the focus of the eyepiece) as to be visible simultaneously with the object looked at through the lenses. The telescope-carrier is placed in such a position on the vertical rod that the lower end of the object to be measured is seen, when looked at

through the telescope, to coincide exactly with the point of crossing of the spider-threads in the field of view; then it is slid up until the upper end of the object to be measured appears to coincide with the same point; the distance along which the carrier has been slid along the vertical rod indicates the height of the object to be measured. Provision must be made in the construction of the apparatus for ensuring that the vertical rod is quite perpendicular to the horizon; this is effected by making it stand upon three screws whose heights can be adjusted until a spirit-level shows the base of the apparatus to be quite horizontal.

It may be necessary to compare a standard measure with the length of a body which is very nearly of the same length as the standard. In this case, a microscope may be placed at each end. Coincidence as perfect as possible is established between the images of the object and the standard measure in the field of the first microscope. If, then, the coincidence be perfect in the field of the second microscope, the object is of precisely the same length as the standard. This but rarely occurs, and the object in view frequently is to ascertain what the error amounts to. The second microscope is provided with spider threads in the focus of the eyepiece, and the end of the object is brought exactly under the apparent crossing-point of these threads; then the microscope is moved along until the end of the standard appears to be in the same position; the extent to which the microscope has been moved indicates the difference between the two lengths compared. Since, however, the amount to which the microscope has been moved may be exceedingly small and difficult to measure, the methods hitherto described may be insufficient in accuracy, and we have to resort to those more delicate devices which depend on the properties of the Screw.

The Screw, as will be seen on examination of any specimen, presents a spiral coiled round a cylinder. If a screw, having twenty threads to the inch, be inserted in a fixed body, and turned round exactly once, its point will have advanced the twentieth part of an inch. If the head of the screw be connected with a pointer fixed on it at right angles, which can indicate on a graduated circle the amount of rotation of the screw, there will be no difficulty, even with roughly made apparatus, in causing the screw to execute a rotation of half a circle, a quadrant, 45°, 5°, or even 1°. If a screw, then, which has twenty turns to the inch be turned through one degree ($\frac{1}{360}$ of a complete turn), its point will have advanced or been retracted by $\frac{1}{360} \times \frac{1}{20} = \frac{1}{7200}$ inch. But this is rough measurement. By making the head of the screw part of a large wheel with graduated circumference and using a fixed vernier, rotation of the screw to the extent of half-a-minute of arc can be easily observed, and this would correspond to onward motion on the part of the point of the screw of $\frac{1}{864000}$ inch. The principle of the screw thus enables us to detect and to measure very small quantities of motion. If the second microscope in the last paragraph be connected with a graduated screw of this kind, the amount of its motion, indicating the difference of length of the two objects measured, can be very exactly determined.

In Sir Joseph Whitworth's measuring machines advantage is taken of another principle for producing and measuring very slight motion. The screw (twenty threads to the inch) is driven by a "worm-wheel," a wheel bearing 200 teeth on its circumference: this is propelled by a tangent-screw, a screw whose threads fit between the teeth of the worm-wheel: each turn of the tangent-screw sends each tooth of the worm-wheel forward into the

position previously occupied by the tooth immediately before it — that is to say, causes the worm-wheel itself to revolve through the two-hundredth part of 360°, and to press the point of the screw forward by the four-thousandth of an inch. But the tangent-screw is itself driven by a wheel divided into 250 parts, so that if this wheel be turned round only one division, the tangent screw is rotated $\frac{1}{250}$ of a turn, and the point of the "worm-wheel" screw is thus pressed forward the 250th part of $\frac{1}{4000}$, *i.e.* the millionth part of an inch.

If a screw be fixed at each end so that it can rotate but not progress, the "thread" of the screw will appear to travel when the screw itself is turned. If any object (the slide-rest of a lathe, or the like) have a female screw* cut in it, and be by means of that screw fitted upon a rotary but otherwise fixed male screw; and if it be then placed between guides so as to be free to move backwards and forwards along the fixed screw but in no other direction: if then the fixed screw be rotated, the object borne by it will travel along it in one direction or the other, according to the sense of the rotation. This mechanism will be thoroughly understood on looking at the traversing-screw and slide-rest of a lathe. If the travelling carrier bear a pencil or a diamond, and mark paper or glass at equal intervals, as indicated by equal rotations of the driving wheel, we shall have a contrivance illustrating the main principle of the Dividing Engine which is used for graduating thermometer-tubes, etc.

End Measurement. — If a couple of rods, exactly ten feet in length, be placed on the ground end to end; if then the first rod be taken up and carefully laid down endways at the other end of the second; and if the second be taken up and placed in the same way beyond the first and just in contact with it, and so on: then a very accurate setting off of any multiple of ten feet can be easily effected, provided that the rods themselves be exactly ten feet long. Measurement of a given length can also be thus effected: if there be an odd number of feet and inches, they can be measured by a set of smaller rods, or by an ordinary tape measure.

In measuring or setting-off in this way, it is plain that we depend upon the sense of touch for the perception of the contacts set up between the ends of the rods. The sense of touch is found to give more satisfactory results in many ways than the sense of sight; for if one object be intended to fit into another, and have a diameter $\frac{1}{5000}$ inch less than what is exactly necessary, its fit will be perfectly loose. The eye could not perceive this directly without the intervention of lenses.

The Callipers used by carpenters can be opened out so as exactly to fit into a cavity, or exactly to grasp an object. They are usually made so that the one end serves for inside, the other for outside measurement. They are useful in comparing the dimensions of objects which should be of the same size; but it is difficult to take very accurate measurements off a scale with them.

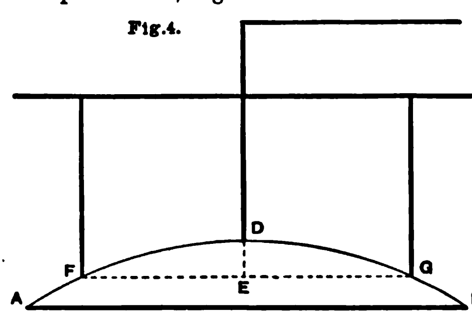
Gauges are made of known sizes, and the size of the object to be measured is compared with that of the gauge by trying the fit. If the gauge be made conical, then from the extent to which it penetrates a given aperture can the width of that aperture be determined.

* A screw cut out of a solid mass, through which another screw, the "male," passes. In the ordinary nut and bolt, the bolt bears the male screw, the nut the female.

The Spherometer consists of a disc of metal with graduated circumference. This is supported on three equal legs, which are furnished with hard steel points, equidistant, and rounded off so as not to pierce any object on which the instrument is set. In the axis of it is a screw, the steel point of which is also rounded, and forms a fourth foot. Any instrument which stands on three feet is certain to be steady, because three points are always in some one plane: one which stands on four feet will only be steady if the point of the fourth foot be exactly in the same plane as the other three. If it be above this plane the instrument does not rest on it at all; if it be below this plane the instrument can never stand on more than three feet at a time, and may be rocked from one set of three to another. If the spherometer be set upon a piece of glass, it will stand steadily; if the central screw be turned so as to bring down the fourth foot, the instrument will be easily rocked if it be brought down too far. The hand in perceiving and the ear in hearing this rocking, just at its commencement, concur in detecting very small motions of the screw just at that part of its movement. The instrument also becomes easy to spin on its centre-screw. By means of a pointer attached to the head of the screw the exact position of the screw which corresponds to the commencement of rocking can be observed on the graduated scale. Suppose the thickness of a piece of microscopic cover glass is to be determined. It is placed under the fourth foot. This central foot of the spherometer is brought down upon it until the whole rocks; the central screw is then raised until the rocking ceases; it is turned back again till it just commences, and, as before, the position of the screw corresponding to the commencement of rocking can be observed by means of the pointer and the graduated scale. If the pointer had stood at 75° when the instrument stood on the plain glass, and at 3° when the central point was on the piece of thin glass, the difference of position of the pointer corresponds to 72°, or $\frac{72}{360}$ of the circumference; and if the screw itself have twenty turns to the inch, the thickness of the glass is $\frac{72}{360} \times \frac{1}{20} = \frac{1}{100}$ inch.

The curvature of a lens may be determined by this instrument, for if the lens ABD be placed under a spherometer, Fig. 4 shows that the amount of curvature determines the length of the line DE; and the radius r of the sphere of which the lens may be considered a part is related to the line DE (represented by l) and the distance a between the equidistant tripod feet, by the formula $2r = \overline{a^2/3l} + l$.

Fig. 4.



In Whitworth's Measuring Engine a bar representing the unit of length is placed between two jaws, which are made to move towards one another so as, without pressure, just to grasp it: they are then separated from one another, and the standard unit removed: the bar to be measured is placed instead of it, and the jaws are again brought together so as to grasp it in the same way. The jaws are brought together by fine screw adjustments, such as those previously described, so that the difference of the millionth part of an inch in two bars of metal can be detected. The

precise position at which the jaws grasp objects without pressure is determined by a plane piece of metal, which is included along with them between the jaws, with its edges in a vertical plane. If the grasp be too loose, this piece of metal can be moved freely, and will fall back when lifted and let go; if the grasp be too tight, this metal plane cannot be moved; if it be exact, the metal plane can be raised, and will remain in any position in which it may be placed.

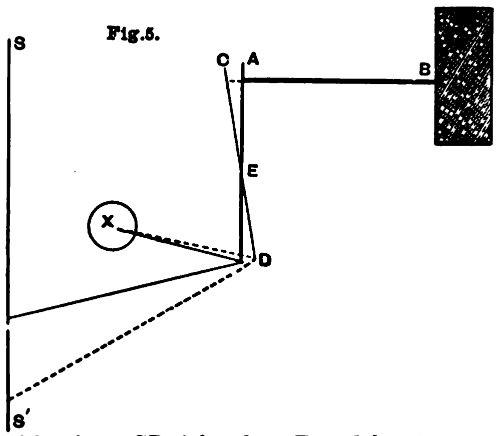


Fig. 5.

Another plan by which an alteration in the length of a bar may be determined is the Optical. The end A of a bar AB rests against a strong framework at B, so that any alteration in its length may only affect the position of the point A. At A the bar is in contact with a lever CD, jointed at E, and bearing a mirror at D. A lamp at X casts a ray of light on the mirror; this is reflected to a screen SS′. If BA alter in length, or if another bar of slightly different length be substituted for it, the bar CD assumes another position, and the spot of light on the screen SS′ is deflected. From the amount of deflection may be calculated the alteration in length of the bar BA.

Good linear measurement, in whatever way effected, ought to present an error less than $\frac{1}{10000}$ %, or one-millionth of the whole.

Measurement of Surface.—If a surface be bounded by straight lines at right angles to one another, the parallelogram may be measured by the product of two adjacent sides: if it be of any other form bounded by straight lines, it can be broken up into triangles, and its area be found by the rules of trigonometry: if its boundary be a regular curve, its area can generally be found: but if the surface be bounded by an irregular curve, the determination of the area involves the following principle.

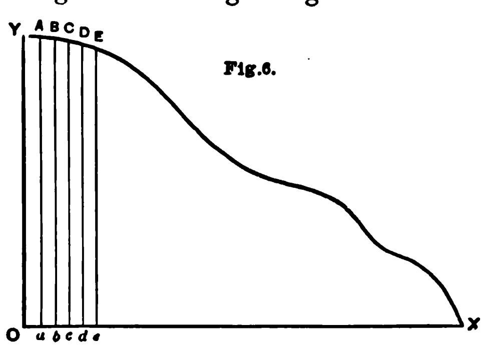
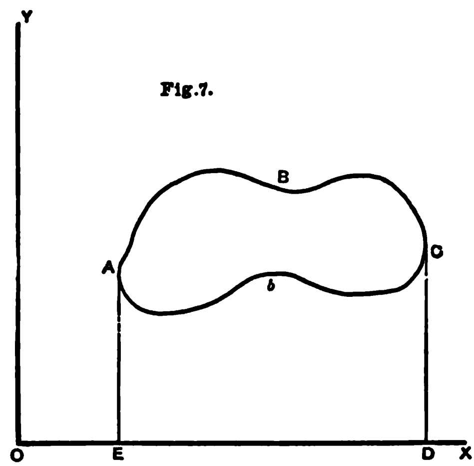


Fig. 6.

Let the figure YXO be bounded by the two rectangular

straight lines OY and OX, and the curve YABCDEX. Find its area. Draw a series of lines parallel to OY; these will cut the curve in the points A, B, C, D, E, and so forth. Then the area YXO is divided into a number of narrow parallelograms, OYA*a*, A*ab*B, B*bc*C, etc. Each of these is equal to the product OY × O*a*, *a*A × *ab*, etc.: these being all found and added together give the area of the surface.

If now the surface be completely bounded by an irregular curve, as in Fig. 7, the area ABCDEA is first found by the above method, then the area A*b*CDEA. The difference between these represents the area of the curved surface ABC*b*. This method is very difficult in actual practice, but all the mathematical methods of integration are based upon this principle. For actual work a convenient means of measurement of surface, which gives very fair results, and which is specially useful in those cases in which mechanical contrivances have registered their own performances on paper, is the following: —



The paper on which the curve is drawn is laid on a flat board, and the outline of the surface very carefully traced by a sharp-pointed penknife, so as to cut out the part of the paper bounded by that outline: this is then weighed and its weight compared with that of a standard area, say a square inch of the same paper. This method is not unexceptionable, but it often gives a very useful approximation to the value required.

An instrument called a planimeter is also used for this purpose.

Measurement of Volume. — The volume of a substance may often be found by calculation from its form if that be a known geometrical figure; but the volume of a mass of irregular figure is best ascertained by the rough method of immersing it in water or any liquid which will not affect it, and by observing how much more bulk the whole now occupies than the water alone had done.

If, for instance, a piece of metal be placed with three fluid ounces of water in a measure, and if the whole measure exactly four fluid ounces, the piece of metal must occupy exactly the same bulk as one fluid ounce or $\frac{1}{80}$ British gallon of water; that is, since a gallon of water occupies 277·274 cubic inches, (277·274 ÷ 80) or 3·466 cubic inches; and so for fractional parts of the units of liquid measure. The volume of a flask may be ascertained, in cub. cm., by weighing the water it can contain; 1 gramme, at 3·9° C., occupies 1 cub. cm.

Measurement of Time. — It is not possible or necessary to do more in treating of this than to suggest one or two leading principles. A simple water-dropper, consisting of a vessel of water in the bottom of which there is a minute hole, through which the water falls, drop after drop, into a dish, was used anciently under the name of the Clepsydra. The water which fell through was kept in the lower vessel: the amount there accumulated, or equally the loss of level in the upper vessel, indicated approximately the lapse of time. It was found, however, that the flow of water from a vessel of this description was far from uniform. The use of Wheelwork set in motion by some constantly acting force was a fruitful suggestion: setting the wheels to indicate the amount of their own rotation by means of pointers connected with their axles was a plan early adopted; the train of wheelwork was set in motion by a falling weight; but there wanted yet some regulating contrivance by which the motion might be rendered uniform. A heavy flywheel was adapted to the mechanism, but without the desired result being fully attained; and it was only after Galileo's observation of the fact that the Pendulum oscillates from side to side in almost exactly equal periods of time, whether its arc of oscillation be great or small, that it was suggested that this property of the pendulum might be rendered available for regulating clockwork. This was effected by Huyghens; and the action of all pendulum clocks, however various the trains of wheelwork, depends on their regulation by an isochronously — *i.e.* in equal times — oscillating pendulum. The simplest mode in which this regulation may be effected is the following: — One of the wheels of the train of mechanism bears on its circumference an appropriate number of teeth. The descent of the weight would, if there were no pendulum attached, cause the mechanism to run on continuously until the weight had run down to its lowest possible point; but at every stroke of the pendulum one of the teeth of the wheel is caught and the progress of the wheelwork arrested.

The isochronism of the oscillations of the pendulum is not

sustained; variations in the external temperature cause changes in the length of the pendulum, and hence in its rate of motion. The contrivances by which compensation is made for this cause of error, so that the rate of oscillation is maintained practically uniform, will be explained under Heat.

The measurement of small intervals of time is of great importance. A tuning-fork, if a writing-point be attached to it, will, when vibrating, describe wavy lines on a piece of smoked glass or paper drawn under the writing-point. If the tuning-fork vibrate 400 times per second, the time taken to draw each wave on the paper must be the four-hundredth part of a second; and if any other phenomenon be so produced and arranged as to record its own performance by a line on the paper or the glass, parallel to the wavy line of the tuning-fork, its duration may be estimated by counting the number of recorded vibrations of the tuning-fork to which that duration corresponds.

Measurement of Mass. — Masses are compared with one another by means of the Balance. The accurate and expeditious use of a delicate balance involves attention to certain practical rules, which will be found set forth in Walker's *Balance*.

Measurement of Force. — There are four main methods of measuring any force. These may be stated as —

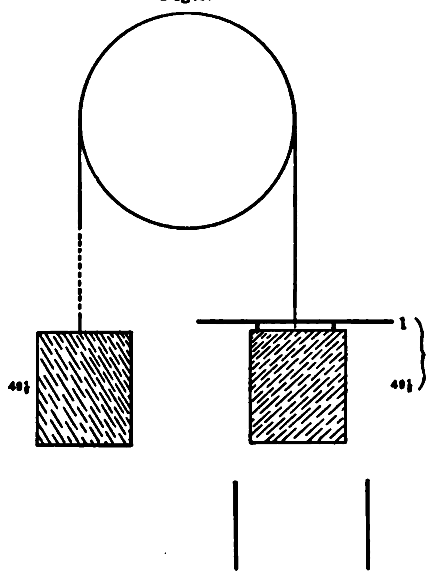
1. Direct Observation of Mass and Acceleration.
2. Direct Counterpoising.
3. Indirect Counterpoising.
4. The Method of Oscillations.

The first, the method of **direct observation** of the mass moved and the acceleration imparted to it by the force to be measured, is based on the equation $F = m\alpha$; and if m the mass and α the acceleration be known, F, the Force acting, can easily be found. This method presents, however, serious practical difficulties in the observation of the acceleration produced.

One important problem to be solved by this method is the determination of the force with which Gravity acts upon a unit mass of matter at any place. The equation $F = m\alpha$ shows that if we use a unit mass, $F = \alpha$; thus we need only find the acceleration produced. This is effected roughly by Attwood's Machine. In this the weight of one gramme is used as the force which sets a larger mass in motion. If it set only its own mass in motion, a velocity is acquired so great as not to be easily observed: if this limited force set a larger mass in motion, the speed acquired is less, varying inversely as the aggregate mass, for $\alpha = F/m$. If a gramme in falling set a mass of 100 grammes (including its own substance) in motion, it can only acquire a velocity one-hundredth that which it would have acquired if it had fallen

alone. The essential part of Attwood's machine consists of a wheel over which two masses are suspended. Let these masses be 49½ and 50½ grammes. The total mass set in motion is 100 grammes, and the force acting is the excess in weight of the heavier mass over the lighter — that is, 50½ – 49½ = the weight of one gramme. Let this gramme not be a fixed part of the heavier mass, but merely a piece of wire which can be removed by making the weighted mass fall through a metal ring. A pendulum which beats seconds regulates a timepiece; attached to the wheelwork of the timepiece is an "excentric," which works a lever; this lever, at a pre-arranged instant, pushes or pulls away a little plate which supports the heavier mass; this mass suddenly finds itself freely exposed to the action of gravity; the excess-weight of the little gramme-load imparts to the whole mass a certain velocity; the ring is placed at such a position as to catch the wire exactly at the end of one second, this being indicated by the sound of the timepiece and pendulum coinciding with the click of the wire upon the ring which catches it as it falls. Thereafter there is no unbalanced force acting, and the mass of 99 grammes continues to move uniformly according to the first law of motion. Its speed can be observed by comparing the distance it travels with the ticking of the timepiece. This is done by placing a little plate to receive the falling body. A slight sound will be made by the falling body touching this plate.

Fig. 8.



If this sound and that of the pendulum coincide, the plate is in the right position; if not, that position must be found by a process of trial and error. It is found that if a pair of masses of 49½ grammes each be suspended over the pulley, and one of them be loaded with one gramme so that the whole mass to be moved weighs 100 grammes; if the overweight be taken off at the end of one second by a ring; if the balanced masses be allowed to move onward with their then acquired velocity for one second; if a plate be so adjusted under the ring as to check this motion precisely at the end of a second — it is found that that plate must be 9·81 centimetres below the ring. This shows that the force acting (the weight of one gramme), acting for one second, is able to impart to a mass of 100 grammes a velocity of 9·81 cm.-per-sec. Hence by the equation $F = ma$, F, which is equal to the Weight of one gramme, is equal to 100 grammes × 9·81 cm.-per-sec. per second = 981 dynes. This method can give no more than an approximation to the value required.

Much greater accuracy is attained by the use of the Pendulum. The time of oscillation of a pendulum, as we shall afterwards learn, varies inversely as the square root of the force of gravity at the place where the observation is made. The time of the oscillation of any pendulum can be

very accurately learned by observing the time taken to perform a certain sufficiently large number of oscillations, and dividing that time by the whole number of oscillations. From this observation can be deduced the local acceleration of gravity.

Measurement of Force by Direct Counterpoising.—In an ordinary balance whose arms are perfectly equal, the force with which gravity acts on the mass in one pan is equal to that with which it acts on the mass in the other. For one of these we may substitute another force of any kind but of equal amount. If, for instance, we use a balance with glass pans, we may lay one of the glass pans on the surface of mercury and determine what mass must be put in the other pan, to pull the first from the mercury. Let this be 47 grammes, and the area of the glass pan 25 square centimetres. Then a force equal to the weight of 47 grammes is necessary to pull 25 square centimetres of the surface of glass away from mercury—that is, 1·88 gramme per square centimetre; and the force of adhesion between mercury and glass is, for every square centimetre, equal to the weight of 1·88 grammes—that is, a force of $1{\cdot}88 \times 981 = 1844{\cdot}28$ dynes.

A soap film tends to contract. If we find how much mass must be suspended on a soap film of a certain size in order to prevent it from contracting, the force of contraction will be equal to the weight of the mass which the film supports, and that force can hence be measured in absolute units.

This method, as well as the next, lends itself so readily that no special explanation is necessary, to the measurement of stresses, pressures, tensions.

Measurement of Force by Indirect Counterpoising.—Let us suppose that we have access to a standard unit of mass. This is hung upon a spiral or spring of steel wire. It is observed to lengthen the spring by a certain measured amount. If another mass be hung upon the same spiral, and if the lengthening produced be the same, the inference is that the action of gravity upon the second mass is equal to that on the first, and hence, if the two observations be made at the same place, that the second mass is itself equal in quantity to the first. This is the principle of the Spring Balance. Different known masses may be suspended on such a spiral, and the elongations produced may be recorded on a scale attached to the instrument. If a mass of unknown amount be attached to the spiral, its weight may be found by reading on the scale the number of standard pounds and ounces, etc., requisite to produce the same distortion as the unknown mass causes when hung upon the spring.

The instrument known in one form as a spring balance is known in another as a D y n a m o m e t e r. The form of the steel spring used is quite independent of the principle involved, which is that if two forces produce equal distortions in a body, these forces must be equal to one another. If a man can pull a spring out two inches, and if 200 lbs. must be hung on the spring to produce the same distortion, the man's pull is equal to the weight of 200 lbs.; similarly the force required to pull the spring out two inches is equal to that which must be exerted to raise a weight of 200 lbs.; and these can be translated, when we know the local acceleration of gravity, into forces measured in absolute units. If he can give it a blow which will compress it for a moment to the same extent as the Weight of 140 kilogrammes placed on it would do, the force of his blow is equal to the weight of 140 kilogrammes — that is, $140{,}000 \times 981 = 137{,}340000$ dynes. If he can, by closing his hand firmly, distort a spring to a certain extent, it can easily be ascertained what amount of weight acting on the spring is capable of producing the same distortion. This is usually done beforehand, and the instrument is provided with a graduated scale which indicates what amounts of weight — at the place where the instrument is made, be it remembered — correspond to the various readings of the pointer. When his flexor muscles contract so as to force the pointer of the dynamometer to indicate 84 kilogrammes, the distortion produced by them is equal to that which would be produced by the Weight of 84 kilos., at Paris if the instrument have been made there; that is, since $G = mg$, 84,000 grms. $\times$ 981 = 82,404000 dynes.

Illustrations of this principle abound. The attraction of magnetism may be measured in a similar way. Let a magnet attract a piece of iron, which is attached to a spiral, to such an extent that the spiral is lengthened, say one inch, when the magnet is at a distance of a tenth of an inch from the iron. It is found that, say, 2 lbs. 3 oz. must be hung on that spiral to produce the same distortion; the magnetic attraction is equal to the local Weight of a mass of 2 lbs. 3 oz. This is an undesirable method from the practical point of view, but it shows how magnetic and other attracting forces can be compared with forces whose absolute amounts we know.

If an electromagnet can hold ten pounds of iron, but cannot support ten pounds and a grain, the force of attraction is equal to the weight of ten pounds; for instead of the magnetic attraction, we might have used, in order to prevent the ten-pound

mass of iron from falling, another ten-pound mass connected with it by a cord passed over a pulley.

If we take a bar of metal, suspend it on centres at each end, fix it firmly at one end so as to prevent that end from rotating, and hang a known mass over the side of that end which is free to rotate, we find that the bar is twisted; this effect is measurable. Whatever other force will produce the same effect must be equal to the known Weight which caused it. If the body to be twisted be a glass or silk fibre, the amount of force required to twist it is small. To twist such a fibre through a certain number of degrees, a certain fractional number of grammes' weight must be applied at unit-distance from the centre. If an electric attraction be applied to a body suspended by such a silk fibre, the suspended body is attracted, the suspending fibre may be twisted; to produce the observed torsion or twist, a certain number of grammes' weight must be applied; the electric Attraction can be stated to be equal to the Weight of so many grammes, and therefore to so many absolute Units of Force.

Ruhelage: Equilibrium-position.— It is often advantageous to measure the force acting on a displaceable object, by balancing that displacing force against another force, so adjusted as to bring the displacement back to zero value. A magnetic needle deflected by a current is twisted back into its original position by a twisted suspending fibre; the torsion imparted to the suspending thread is measurable and represents a known number of dynes. The force acting on the needle is thus measured. The advantage of this method is that we obtain precisely what we wish, the full force exerted by the current on the needle when in its original position, not the force acting on it in any other position; and we thus eliminate any disturbance produced by such variations in that force as may be due to variations in the position of the suspended needle itself.

The fourth method is that of **oscillations.** If a magnet be brought near another magnet it oscillates from side to side. If it be brought near a stronger magnet it oscillates more frequently. It can be proved that the velocities produced vary as the square root of the forces causing the oscillations. Hence we count the number of oscillations in a given period in two cases, and the ratio of their squares is the ratio of the two forces. If, for instance, a magnetic needle oscillate fifteen times a minute in the presence of a magnet A, and sixty times in presence of a magnet B; the forces acting in the two cases are as the square of 15 is to the square of 60, or as 1 to 16. In this way we are able to compare the forces acting under the given conditions, but we do not learn the absolute amount of either. That must be ascertained by one of the methods previously discussed.

CHAPTER IV.

WORK AND ENERGY.

Work. — When a force "acts upon" a body, and that body moves in the direction of the force, that force is said to Do Work, and the work said to be done *by* it is measured by the product of **F**, the force acting in a certain direction, into **s**, the space through which the body has moved in that direction;

$$\text{Work} = \text{W} = \mathbf{Fs} = m a \mathbf{s}.$$

For example: Steam exerts on the piston of a cylinder a mean force or pressure of, say, 30 lbs. per square inch; the area of the piston is, say, 30 square inches; the whole pressure exerted is thus equal to the weight of 900 lbs. The piston is thrust through, say, 16 inches. The work done is 900 lbs.-wt. × $1\frac{1}{3}$ ft. = 1200 foot-pounds at each stroke.

Conversely, when a force acts upon a body and that body moves or is moved in a direction *opposed* to that of the force, that force is said to be Resisted, and work is said to be done *against* it; and $\mathbf{Fs} = \text{W}$; the product of the force resisted, into the space traversed against that force, represents the Work said to be done *against* the force so resisted.

When a ten-pound mass is raised ten feet against gravity, the work done against gravity is equal to the product of the space traversed into the force resisted — *i.e.*, 10 ft. × wt. of 10 lbs. = 100 foot-pounds. In this case Work = $\mathbf{Fs}$ as usual; but $\mathbf{F}$, the force resisted, is the Weight of a mass m, and therefore $\mathbf{F} = G = mg$; consequently the work done, $W = \mathbf{Fs} = mg\mathbf{s} = mgh$.

Suppose a man to walk against a heavy gale of wind, the mean pressure of which is 40 lbs. per square foot. If the surface presented to the wind-pressure be virtually 5 sq. ft., the total pressure of the wind will be 200 lbs., and the effort of walking against it will be the same as if the man pulled a weight of 200 lbs. out of a pit by means of a cord thrown over a pulley. If the man make his way for a mile, he will have resisted a mean pressure of 200 lbs. through a space of 5280 ft. He will, therefore, have done 1,056,000 foot-pounds of work; an amount of work which, otherwise directed, would have sufficed to lift him up (his total weight being supposed to be 150 lbs.), to twice the height of Snowdon.

There is no Work done against or by the force acting unless there be actual Motion. We might imagine machinery to be

driven by an avalanche during its fall; but not before, and not after. Gravity does no Work upon a resting stone: it does work upon a falling stone.

If **Fs** = 1, we have the Unit of Work. This is the case when **F** = 1 and **s** = 1; that is, a unit of work is done when a body acted on by unit force moves through a unit distance in the direction of the force. In C.G.S. measures the unit of work is done by raising $\frac{1}{981}$ gramme (mass whose weight at Paris = 1 Dyne) to the vertical height of one centimetre. This is the **Erg.** The erg is, however, a very small unit of work, and for many purposes it is convenient to use the Megalerg, which is equal to 1,000,000 Ergs and would therefore be the amount of work done in raising $\frac{10000}{981} = 10{\cdot}19$ grammes through one metre; or the Ergten, 10^{10} or 10,000,000,000 Ergs; or the Joule, 10^7 or 10,000,000 Ergs.

In British measures 32·2 units of work are done in raising a pound-mass through one foot. Such units of work are called foot-poundals. British engineers are in the habit of using the foot-*pound* (the work spent in raising one pound one foot) as a unit of work. This would be satisfactory if foot-pounds were equal over the whole earth, but g, the acceleration of gravity, varies from place to place. Hence the foot-pound is from place to place a variable measure, varying between the Equator and the Poles by about one-half (0·512) per cent.; and it has to be reduced for each place to absolute units of work by the equation — Work = Force overcome × Space = Weight × **s** = mg**s**, and the foot-pound is equal to g foot-poundals, where g is measured in ft./sec.^{2} ($g = 32{\cdot}2$ nearly). The foot-pound is equal to 13,562,691 Ergs, when $g = 981$ cm./sec.^{2} The kilogramme-metre, or French engineers' unit of work, is 1000 grms. × g × 100 cm. = 98,100,000 Ergs.

Any amount of work may be specified as the product of two numbers, which represent respectively a Force and a Displacement. These may vary, but if they have the same product the amount of work done is the same. A pound raised 100 feet, 100 pounds raised one foot, fifty pounds raised two feet, four pounds raised twenty-five feet, all represent the same amount of work, namely, 100 foot-pounds, it being here assumed that the force of gravity is uniform within heights of 100 feet.

Since Work = **Fs**, it follows that **F** = Work ÷ **s**; whence Force is the number of Units of Work done upon or by a body moving in a straight line, divided by the number of Units of Length traversed by that body in that line. Force in a given direction is therefore a **rate** at which **work** is observed to be done, **per unit** not of time but of **space traversed** in that direction.

This looks like a definition obtained by reasoning in a circle; but if it be presented in the equivalent form — Force is the rate at which a moving body gains or loses either potential or kinetic Energy per Unit of Space

traversed — we shall presently understand that it is not a truism, for Energy is a physical entity. In this view, the Force in a given direction is the **Energy-Slope** in that direction.

The Mean Rate of Doing Work is the whole Work done in a given time divided by the Time. If an engine can raise 1,980,000 pounds vertically one foot in an hour, its mean rate of doing work, its Power, or, as Lord Kelvin phrases it, its **Activity** (French *puissance*), is 33,000 foot-pounds per minute, or 550 foot-pounds per second. This particular mean rate is known by British and American Engineers as a **Horse-power**; and an engine of one horse-power can do this amount of work. A horse can, according to General Morin, do 26,150 foot-pounds per minute, and a labourer from 470 (lifting earth with a spade) to 4230 (raising his own weight, treadmill exercise) per minute. The French horse-power (cheval-vapeur) is 75 kilogram-metres, or 7,500,000 g=7,357,500,000 Ergs per second; whilst the British horse-power is equal to 7,459,480,050 Ergs per second, when $g=981$ cm./sec.^{2}

If a man weighing 14 stone run upstairs at such a rate as to gain 3 feet in vertical height every second, his muscular system is doing every second the work of carrying 196 lbs. up 3 feet, *i.e.*, 588 foot-pounds. If this could be kept up for a minute, $60 \times 588 = 35{,}280$ foot-pounds would be done, and the man would be, in the case supposed, undergoing an exertion which for the moment would be much greater than a horse can keep up, and seventy-five times that which a continuously-toiling labourer, lifting earth with a spade, can sustain; and in the most favourable circumstances, a labourer, raising his own weight merely, can only keep up one-eighth of this effort.

The Activity, or Power, or Effective Horse-Power of an engine must be distinguished from its Nominal Horse-Power, which is a term based upon certain dimensions of the cylinder, and has no well-defined experimental meaning.

The Unit of Activity is frequently taken as one **Watt**, which represents 10 Megalergs per second. The British horse-power is thus equal to 746 Watts nearly, the French to 735$\frac{3}{4}$.

A thousand Watts are one kilowatt. Inconveniently enough, the Congrès International de Mécanique Appliquée, 1889, recommended as a unit of activity a 'Poncelet'=100 kilogramme-metres per second= 98,100,000 ergs per second=9·81 Watts.

Activity is also measured as $\mathbf{Fv}$, Force × Velocity in the direction of the force; for $\mathbf{v}=\mathbf{s}/t$; and, therefore, Activity$=W/t=\mathbf{Fs}/t=\mathbf{Fv}$.

Energy. — When a body weighing ten pounds is raised ten feet, and prevented by a catch from falling, the work done upon it — 100 foot-pounds — can be recovered by permitting it to fall upon a train of mechanism. If the mechanism were perfect, the work would be so restored that another ten-pound mass

might be projected by it to a height of ten feet, a fifty-pound mass to a height of two feet, and so on. The fact that we cannot obtain perfect mechanism does not affect the principle. The body at a height has therefore a power of doing work equal to the work done upon it in lifting it. In this case the power of doing work has been conferred upon a body by the separation of it from the earth against the action of gravity: as it remains in its elevated position, there is a stress, or pull, or attraction, tending to draw it down, and it is only in virtue of this stress that it has any power of doing work. If the earth and the elevated body ceased to attract one another, the body would, if liberated, not fall down, and would not restore the 100 ft.-lbs. of work spent upon it. We know that the Work done in raising a mass m through a height h against gravity is mgh: the energy stored up in the body is therefore equal to mgh, and is seen to depend on the mass of the body, the height at which it is placed, and the local accelerative effect of gravity. Energy or power of doing work, stored up in this way, is called **Potential Energy**, or Static Energy, or Energy of Position, or Energy of Stress. As an example of Potential Energy we may take that stored up in a mill-pond. The number of units of Energy in such a pond may be found by taking the product of the quantity of water in it and the average height at which it is placed, and multiplying that product by the local value of g. A small quantity of water at a great height may obviously have the same amount of energy stored up in it as a larger quantity at a lesser height. If the question be put, How much work could be got by appropriate mechanism from the rise and fall of the tide?—we consider (1) the total amount of water carried into the area which can be brought within the range of the mechanism, (2) the average height to which it rises, and (3) the local value of g.

We have also energy stored up in such bodies as watch-springs. Work is done upon them in distorting them, and producing a movement, not of their mass as a whole, but a relative displacement of their parts. This work is restored and utilized in producing movement of the mechanism attached. When a watch-spring is distorted and held fast so that the distortion or strain persists, the whole mass remains in a condition of Stress, and tends at the first opportunity to restore the work done upon it.

If we look at our previous example of the earth and a stone lifted from its surface, we see that the phenomenon is on the

large scale one of the same order. The earth and the stone together constitute a system: when this is deformed by pulling the stone away from the earth, the system tends to return to its original form, and there is a stress between the earth and the stone, which continues until the stone is allowed to fall back to the earth. If the stone had been connected with the earth by a band of indiarubber, we would have seen the indiarubber to be stretched or under stress, and would easily see that if the stone were liberated it would be pulled back towards the earth; but the question is, What is under stress in the actual case? for there is no visible connecting cord between the stone and the earth. If we could state what this was, we would be able to arrive at the cause of Gravitation. As it is, our knowledge ceases. That there is some medium, and that it may be under stress, is a theory necessary for the exposition of Electricity, of Light, of Magnetism, and of Heat; but we are by no means, as yet, entitled to say that stress in this medium is the cause of Gravitation.

Work may be done, then, in altering the relative configuration of a system, whether this consists of large masses or of smaller particles. If this system be what is known as a "Conservative System," in which a stress may be established depending upon the configuration, and only upon the configuration (not in any degree upon the history of any antecedent deformations through which the configuration in question may have been arrived at), the system will tend, when work has been done upon it, to return to its original form, and to restore the work done upon it. If its relation to surrounding objects be such that it cannot so return, it will be under stress, and will continue under stress until its relations to surrounding objects have become such as to permit it liberty of restitution; then, at the first opportunity, it will restore the work done upon it.

The change in its relations to surrounding objects necessary to render this restitution possible may be very small; for example, a heavy mass may be prevented from falling by a very small catch, but when the catch is removed the body falls. The cause of the body falling is not simply the release of the catch, but also the previously existing conformation of the distorted system.

Similarly, the ingredients of Gunpowder have a tendency to combine: its particles are chemically separate, but chemically attract one another, and therefore possess potential energy; the application of a very small amount of heat, as by a spark, liberates these particles, which can rush together and form new and stable compounds, which have no longer any tendency to alter their chemical constitution, being no longer under the same stress, having

no longer the same potential energy. As it happens that in this special case the new and stable compounds formed are mainly gaseous at the ordinary temperature and pressure, the products of combination occupy a much larger bulk than the original gunpowder, and the result is an explosion. The spark only produces its own small effect; the previous arrangement of the particles of the powder is responsible for the rest.

Cases abound in which energy is stored up in mechanical arrangements. The Air-gun consists of a volume of air which has been, by work done upon it, compressed into a small bulk, and which tends to return to its original dimensions. When permitted to do so, it suddenly expands, and may be made, in propelling bullets, to restore work done upon it. When a Clock is wound up by pulling up the 'weights,' work is done upon the system; this is restored by the whole system returning to its original form, the weights descending to their lowest position. It takes a definite number of days or hours to do this, according to the mechanical arrangements devised. The work done in bending a Bow is swiftly restored as the bow returns to its original form, and may be spent in imparting motion to the arrow.

A Non-conservative System is one in which, when the system is deformed, there is no stress established tending to restore the original arrangement. Such a system is exemplified by a gun and bullet. When the bullet has left the gun, Newton's first law applies, according to which the bullet tends to go straight on at a uniform rate, unless acted on by impressed forces. The bullet forms a part of two systems, one conservative and the other non-conservative; its motion will necessarily be that due to its relations to both. Let it be fired obliquely upwards: in virtue of its separation from the earth, with which it forms a conservative system, a stress is established which brings it back to some part of the earth's surface: it does not, in virtue of its separation from the gun, tend to return to the barrel of the gun, but goes on until it is stopped. The question, What causes one system to be conservative, another not to be so? is scarcely to be answered at present. The presumption is that a body if set in motion will, according to the first law of motion, travel onwards in a straight line and with uniform velocity, unless acted on by impressed forces; in other words, that all systems are non-conservative. A shot fired vertically upwards should, according to this law, pass on in the same direction without ceasing; but experience shows that it does return, that some impressed force does act upon it, and this, which is another expression for the attraction of gravitation, is at present not explained. Similarly, the particles of a distorted spring undoubtedly form a conservative system; stress is established between them: but the explanation of this fact would imply a knowledge of the constitution of those particles and of their

actions upon one another, a knowledge which we do not yet possess.

Kinetic Energy. — Power of doing work is possessed also by bodies which are in Motion. If, for instance, a rifle bullet be received on an appropriate mechanism, the jolt suffered by the instrument might be utilised in producing a certain amount of work. Or otherwise, the bullet, in whatever direction flying, might, by a cord passed over a pulley, be attached to a weight which it pulled up. The simplest case of this problem is, How far can a shot fired from a rifle carry itself vertically upwards, in virtue of the power of doing work possessed by it because it is in motion? It is known that a body travelling upwards against gravity, and passing a certain point with a speed v, will rise to a height $h = v^2/2g$ above that point. The power of doing work possessed by the bullet in virtue of its motion (its Kinetic Energy, or Energy of Motion, or Actual Energy) is competent, then, to raise its own mass m through a height $h = v^2/2g$ against gravity whose local acceleration is g. The work done is $mg \cdot h = mg \cdot v^2/2g = \frac{1}{2}mv^2$. The Kinetic Energy, then, of a body moving in any direction with speed v depends only on its Mass m and on its Speed v — *not at all on the local intensity of gravity;* and it is **independent of the direction of the motion.**

When the bullet arrives at the top of its course it has no velocity, and therefore no kinetic energy; but it will easily be seen that if it be caught when "at the turn," it can be retained on a ledge, and will there possess potential energy. This we know how to express as mgh. The kinetic energy which the bullet has lost it still retains under the form of potential energy. If it be allowed to fall, it will lose its potential energy, and will (*in vacuo*) have acquired, in a reversed direction, the original speed v as it passes the point of observation.

Let us suppose a body weighing 10 lbs. to leave the ground, starting upwards with a velocity of 64·4 feet per second; let $g = 32{\cdot}2$ ft./sec.^{2} Then the body will ascend $v^2/2g$, or $(64{\cdot}4)^2/\overline{2 \times 32{\cdot}2} = 64{\cdot}4$ ft. The body whose mass $m = 10$ lbs. will rise 64·4 ft., and if caught at the instant when it comes to rest will have a potential energy of 644 foot-pounds. The absolute value of this amount of energy depends on the local force of gravity, but as g is taken $= 32{\cdot}2$, the potential energy may be expressed absolutely as 20,736·8 foot-poundals. The kinetic energy which the body possessed at the moment of starting was $\frac{1}{2}mv^2 = \frac{1}{2}(10 \times (64{\cdot}4)^2) = 20{,}736{\cdot}8$ foot-poundals,

measured directly and irrespectively of the local force of gravity. Hence the kinetic energy lost by the bullet in ascending is exactly equal to the potential energy gained by it. At any intermediate point, where it has less velocity but some potential energy, it will always be found, in the case supposed, that the sum of the kinetic and potential energies is 20,736·8 foot-poundals. The one kind of energy, the potential, is transformed into another, the kinetic, and there is in the system (earth and stone) neither gain nor loss of energy during the transformation. This is the simplest case of a widely applicable principle, that of the **Conservation** or **Indestructibility of Energy.**

This principle is, that if a system of bodies have a certain amount of energy in one form, it must retain that energy in one form or another unless it come into such relations with other bodies as, together with them, to form a larger system in which the energy becomes differently distributed; and if the system be so large that there is no other body with which it can enter into such relations — that is, if the system which possesses the energy be the whole Universe — that system cannot gain or lose energy by sharing with other bodies, and hence the total amount of Energy in the Universe is invariable and numerically constant.

If we take the instance just discussed, that of the earth, the bullet, and the gun pointed upwards, these three bodies possessed before the explosion a certain amount of energy, potential in the gunpowder: just as the bullet left the gun, kinetic in the bullet: when the bullet was detained at the summit of its course, potential between the bullet and the earth, but always equal in amount — the same number of foot-poundals. While the kinetic energy was being transformed into potential, work was being done in the conservative system. During this period the bullet and the earth were relatively moving, and the acceleration associated with the transformation of one kind of energy into another is attributed to a Force acting during that period. Force is associated with a variation in the rate of change of the configuration of a system under which the energy in that system is altered in its distribution and form, and it is said to act only as long as that variation continues; and every part of a system tends to move so as to get rid of potential energy in the shortest time by the shortest path.

Transformations of Energy. — Energy, however, assumes other forms than the two discussed. If the bullet in the case adduced be allowed to fall to the ground, it falls more and more

rapidly until it regains its original velocity, and therefore its whole kinetic energy. But this bullet may suddenly strike the ground and lose all its kinetic energy: it has already lost all its potential energy; what has become of the energy of the system? We find that the bullet and the part of the earth on which it has fallen are warmed, and we learn from a wide induction of similar cases that Heat is one of the forms of Energy. It is proved to be so by the observation that the same amount of work, if entirely spent in producing heat, will always produce the same amount: 772·55 foot-pounds of work were found by Joule to correspond to an amount of heat capable of raising the temperature of a pound of water from 60° to 61° F. The Heat possessed by a body is explained as being the Energy possessed by it in virtue of the motion of its particles. Just as a swarm of insects may remain nearly at the same spot while each individual insect is energetically bustling about, so a warm body is conceived as an aggregation of particles which are in active motion while the mass as a whole has no motion. Heat is therefore a form of Kinetic Energy: and the more heat is imparted to a body the greater is the kinetic energy of each particle. If $\bar{m}$ represent the average mass of the particles, and $\bar{v}$ their average velocity, $\frac{1}{2}\bar{m}\bar{v}^2$ represents the average kinetic energy of each particle; and the sum of all the masses multiplied by half the square of the average velocity represents the intrinsic kinetic energy of the whole mass. The words "sum of" are expressed by the symbol Σ. Hence, Intrinsic Kinetic Energy $=\Sigma(\frac{1}{2}\bar{m}\bar{v}^2)=\frac{1}{2}\bar{v}^2\Sigma(\bar{m})=\frac{1}{2}m\bar{v}^2$, where m is the whole mass.

When a bullet possessing actual energy of motion impinges on a target there is a certain amount of Heat obtained, and the bullet may be partly fused: there is also a flash of Light and a certain amount of Sound. Light seems to be a phenomenon of wave-motion in that Ether whose existence throughout space is apparently a necessary hypothesis; so also is Radiant Heat, such heat as streams to us from the sun, or from a fire across a room; and in that Ether, partly swinging, partly distorted by the passing waves, the energy is partly kinetic, partly potential: thus we say that the Energy of Light — or, briefly, Light itself — is a distinct form of Energy.

When a tuning-fork is made to vibrate, work is done upon it in giving it in the first place a distorted form. Its arms swing like pendulums, but their vibration gradually dies away and the energy of vibration of the fork becomes converted into the partly

kinetic, partly potential energy of vibration of the air — that is, into the Energy of Sound; and ultimately it is converted into uniformly-diffused Heat.

Energy may appear, then, as Energy of Mechanical Position or Motion, as Heat, as the Energy of Light, of Sound, and again as that of Electrical or Magnetic condition; and a great part of our work is to study the modes in which the various forms of Energy are transformed and redistributed, and the Forces and the phenomena attributed to forces which are associated with these transformations and redistributions.

A few other examples of Transformation of Energy may here be added. A man, ascending a stair, gains some potential energy: it is found (Hirn) that he is perceptibly cooler for a moment. The heat of his body has been partly transformed into potential energy. Of course the exertion of his muscles and the excitement of his circulation cause him to become warm immediately afterwards. When he comes downstairs he sacrifices the potential energy which he had possessed when upstairs in virtue of his elevated position, and which he might conceivably have utilised by dropping himself out of the window on an appropriate machine placed on the pavement. This energy is not lost, for he is (Hirn) perceptibly warmer at the bottom of the stairs than he had been at the top. At every step downstairs he had arrested his own fall, and had consequently converted a part of his potential energy first into kinetic energy and then into heat.

When a quantity of water is decomposed by an electric current, the electric current is diminished and work is done in separating a certain number of particles or atoms of oxygen and hydrogen. These separated atoms tend to fall together again and form the stable compound, water. The mixture of oxygen and hydrogen thus formed by "electrolysis" possesses potential energy of chemical separation. When a flame is applied to the mixture, a process of recombination commences, and the whole of this potential energy is sacrificed as such, but appears in the form of heat, light, and sound, and may in an appropriate gas-engine be partly spent in doing mechanical work.

The heat and light produced by combustion and by chemical combinations in general are forms of energy obtained by transformation of the potential energy which the particles had previously possessed in virtue of their chemical separation and chemical affinity. Under certain circumstances this potential energy may not be transformed into heat or light, but, as in the galvanic battery, into the energy of a current of electricity, which may in its turn be made to do work, be transformed into heat, into light, into sound, or be spent in setting up magnetic condition, and so on.

When an engine goes round without doing work the steam remains hot. When the engine does work the steam is cooled, and the researches of Hirn have shown that the amount of work done is exactly equivalent to the heat which has disappeared.

The energy of an engine is derived from the heat evolved by the combustion of the coal. The coal of the furnace and the oxygen of the air rush together and sacrifice their energy of chemically-separate position, which was originally obtained by the action of the chlorophyll in the coal-producing plants.

When a plant is exposed to sunlight it has the power, by means of the chlorophyll or colouring matter of the leaves, of breaking up carbonic dioxide, CO_2, of evolving part of its oxygen in the free form, and of depositing the carbon in a less oxidised form in its own tissues. The work thus done by the plant in tearing asunder the constituents of CO_2 it is enabled to do by the energy supplied to it in the form of Light and Heat radiated from the Sun.

The Sun's radiant energy has next to be accounted for. This is not derived from combustion, for the sun would last but a comparatively short time if its energy were derived from any such source: its radiation of energy seems to correspond to 16,500 horse-power from every square foot, and such an enormous outflow would soon exhaust the store of energy if the sun were merely a huge fire: if of coal, it could not last much more than about 400 years. It has been suggested that the meteorites which fall into the sun in great numbers are capable of accounting for the sun's energy; of the thickening of the sun due to this cause a very small amount corresponds to a very large amount of energy. Those meteorites which strike our own earth's atmosphere are retarded and greatly heated in their course through the upper regions of the air. If they be small enough they are entirely broken up, and their dust, characteristically ferruginous, settles down on the surface of the earth, and may be recognised in the dust collected from some specially favourable spots, such as glaciers, roofs, and snowy wastes, and the bottom of the sea. The kinetic energy lost by a meteorite falling upon the earth becomes distributed between it and the earth in the system of which the meteorite becomes a part, and this contributes to the total energy possessed by the earth; while its material goes to increase the earth's mass. In this way, Nordenskjöld computes, the earth gains every year at least half-a-million tons. In the same way, the meteorites which fall on the sun must produce a flash of light, some heat, and a slight thickening of the sun. It has also been suggested that a very slight shrinking of the sun's mass would evolve a large amount of energy, its particles not being so far from one another after this contraction in bulk; and this view is confirmed by the fact that the sun's total heat-radiation is greatly more than can be accounted for by any permissible demand on the meteorite theory. The Sun must thus be considered as possessing a store of Energy, but as having been itself originally made up by the coalescence of widely scattered material.

The question next arises, How did the meteorites get their energy of motion, or the widely scattered material its potential energy? This would relegate us to the consideration of the Universe as a system of masses and particles containing as a whole a fixed quantity of energy: and this would bring us to the problem of the origin of this system.

Availability of Energy.—When a certain amount of energy has been spent in rubbing a button, the button is perceptibly warmed. The heat produced is exactly equal to the work done in rubbing. It is $\frac{1}{2}m\bar{v}^2$, where m is the hot mass and $\bar{v}$ the average velocity of its moving particles. All this we know. If a little time elapse, the button is no longer perceptibly warm: it has shared its heat with surrounding objects: their particles have been induced to oscillate more rapidly. Heat has thus a tendency to become uniformly diffused. It is then no longer

available *to man* for doing work. It ceases to be power of doing work as far as he is concerned; but none the less do the particles of a hot body set in motion the particles of a cooler body, and the energy which has thus been imparted to these they can in their turn share with the particles of other cooler bodies. The Heat of a hot body tends uniformly to diffuse itself throughout the whole material Universe.

In every Transformation of Energy we find that some energy is **wasted** through conversion into Heat, the result, direct or indirect, of friction, noise, flashes of light, and so on. This heat is presently distributed pretty uniformly among the surrounding objects, and can no more be made use of by us for the sake of producing work. A large quantity of the Energy of the Universe must have already assumed this relatively-useless condition, and in the course of time the whole of the Energy in the Universe will have assumed it. The Energy of the Universe is a constant amount: some of it is available, some is non-available: the former is in every phenomenon somewhat diminished but never increased: the non-available energy is constantly increasing: hence the Available Energy of the Universe tends to zero.

Lord Kelvin expresses this by saying that the **Motivity** (the proportion between the theoretically-available energy and the whole energy) of the Universe tends to zero.

If with this clue we trace back the history of the Energy of the Universe, we find, as we go back, less and less of the total Energy of the Universe to have become non-available. On going back far enough we arrive at a definite period when none of the total energy had become non-available. But in every actual phenomenon there is always Dissipation in this way of some part of the total energy of a system. Hence we find that we are forced to realise a precise instant before which there were no phenomena such as those with which we are now acquainted, and since which such phenomena as are due to those relations of matter and energy which are within our knowledge have been occurring: while in the future we have to contemplate a moment at which the whole physical universe will have run itself down like the weights of a clock, and after which an inert, uniformly-warm mass will represent the whole material order of things.

The only way of escape from this conclusion is to lay emphasis on the fact that one part of the total Energy of the Universe is unavailable *to man*,

and to suggest that at some time a state of things may supervene, as a result of which the molecular motion which is implied in a state of uniformly-diffused heat may be so arranged and directed as once more to produce a state of things such that particles may become aggregated into masses, in which all the particles may move on the whole in the same direction. This is what Clerk Maxwell's "Demon" is pleasantly imagined to do; he separates those particles which he prevents from going in one direction from those which he allows to go in another, so that ere long, without expending any work, he has the particles divided into two groups, moving in opposite directions. This is interesting, but it is not pretended that it is any other than a speculation.

"Conservation of Force" an erroneous phrase. — There is now no warranty for this expression. It was originally a translation of the German *Erhaltung der Kraft*, where *Kraft*, meaning strength or force, was used in 1847 by Professor Helmholtz, for want of a better term, to indicate what is now rigorously named *Energie* or Energy. Forces are of the same order as pressures exerted, pounds' or grammes' weight, resistances overcome; forces may be represented by *lines* which indicate their magnitude and direction. Energy is of the same order as work accomplished, as pounds' or grammes' weight or resistances overcome through a certain number of feet or centimetres, and it may be represented by *areas* which are independent of direction.

The Hydraulic Press apparently creates Force, and if its action be reversed, Force disappears; but the work done upon it must be the same as the work done by it, and though there is no Conservation of Force, yet there is strict Conservation of Energy in this as in all those other mechanical contrivances in which Force is altered in amount.

We have seen that Energy may be represented by **Fs**, the product of force acting or resisted through space **s**; by mgh where mass m is raised through height h against gravity whose local acceleration is g; by $\frac{1}{2}mv^2$ when a mass m has a velocity v imparted to it. We shall further see that Energy may be represented by the product $\frac{1}{2}QV$, where Q is a charge of Electricity and V a numerical quantity called Electric Potential; by the product $\mathfrak{v}p$ of a volume $\mathfrak{v}$ of fluid forced into a space against an average pressure p units of force per unit of area of the bounding surface of the fluid; by the product of a chemical affinity (which is equal to the work done in separating the atoms of an equivalent of a chemical compound) into the number of electro-chemical equivalents which enter into combination; and in other similar ways. These things will, however, find their explanation in due place.

Problems.

1. Energy is power of doing work: this depends on $\frac{1}{2}mv^2$; a body moving with a certain velocity v can pierce a plank of thickness t; if it move with velocity $v_{\prime}$, what thickness can it pierce? — *Ans.* $t_{\prime} = t(v_{\prime}/v)^2$.

2. A shot travelling at the rate of 700 feet a second is just able to pierce a 2-inch board. What velocity is required to pierce a 3-inch board? —*Ans.* $700 \times (\sqrt{3} \div \sqrt{2}) = 857{\cdot}42$ feet per second.

3. A shot travelling at a certain rate can bury itself 10 feet in sand: how far could a shot travelling with double that speed bury itself?—*Ans.* 40 feet.

4. If a mass of 154·51 pounds be allowed to fall 10 feet, but in its fall be made to set a train of mechanism in action, and if that mechanism do no other work than to stir up a pound of water with a paddle, how much will the water thus stirred up be warmed?—*Ans.* 2° F.

5. If a locomotive weighing 5000 kilogrammes run at the uniform rate of 10 metres per second round a circular railway whose radius is 2 kilometres, what will be its kinetic energy?—*Ans.* $m = 5{,}000{,}000$ grms.; $v = 1000$ cm. per second; $\frac{1}{2}mv^2 = 2{,}500000{,}000000$ Ergs, or 250 Ergtens. The energy does not depend on the radius of the circle, for it does not depend on the form of the path traversed, but only on the velocity at each instant along that path; kinetic energy is independent of direction.

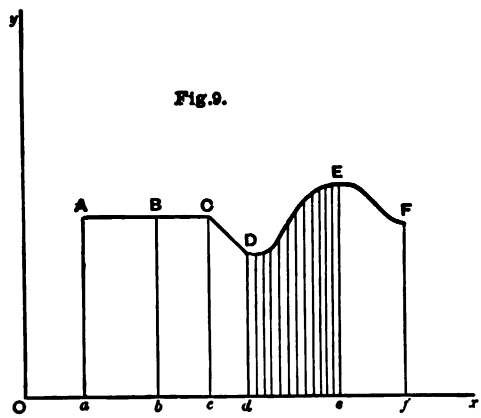
Graphic Representation of Energy.—The representation of work by the product **Fs** (force acting into the space through which it acts or is resisted) finds its graphical equivalent in the representation of work done as a rectangular Area, the product of two lines, of which one represents the Force acting and the other the Space through which a body has been moved. If any instrument can be devised which will mechanically describe such an area, the amount of work done by a moving body can be recorded; such an instrument is a Dynamometer. This name is, as we have already seen, applied to the apparatus in which an elastic spring is deformed, the extent of its deformation showing, by comparison with that produced by a given weight, the amount of Force acting on the instrument. The same name has, however, been given to instruments designed to record not only the force acting on the spring at any given instant, but also the whole Energy spent in producing the deformation, and measured by a simultaneous record of the force acting and of the space through which it has acted.

If a distorted spring have a writing-point attached to it, as the distortion of the spring varies the pencil will move backwards and forwards in one line; if a piece of paper be held against the writing-point as it travels back and fore, the tracing produced is not instructive, for it is simply a line traced over and over. If the paper be drawn past the writing-point at a uniform rate, the line drawn is a curve, from which may easily be deduced the mean value of the deforming force during the whole time of observation. If, however, the paper be moved not uniformly but at a varying rate, proportioned at every instant to

the space passed through by the moving body during given successive equal periods of time (that is, to the rate of change of deformation of the spring), then there are two factors recorded in the same tracing — first, the amount of Space passed through (this being indicated by the amount of paper unrolled under the writing-point) in a given period of time; and second, the Force which has acted in producing deformation (this being recorded by the oscillations of the writing-point attached to the deformed spring).

If the writing-point thus attached to the spring be supposed to draw the curve ABCDEF of Fig. 9, the various parts of the line give rise to the following discussion. The line O*abcd* shows the various spaces traversed by the body set in motion; the lines *a*A, *b*B, *c*C, etc., show the various pressures or forces in action at successive instants of time. The condition of affairs is more easily realised if we consider a cylinder, the steam in which pushes a piston. Then the expansion of the steam is correlated to the movement of the piston, which may be represented by distances along the line Ox. Then ab may be supposed to denote the expansion of the steam as its volume increases from Oa to Ob; aA or bB its pressure; and in that case the work done during the increase of volume ab would be represented by the rectangle aABb. When the working substance expands still further, so that its increase in volume is bc, the pressure or force acting is again constant, and the work done is represented by the rectangle bC. Next, when the volume Oc becomes Od, the pressure sinks from cC to dD; the average value of the pressure is $\frac{1}{2}(c\text{C} + d\text{D})$, and the work done (or average pressure $\times$ space cd) $= \frac{1}{2}(c\text{C} + d\text{D})\, cd$. This is the area of the figure cCDd, which accordingly represents the work done during the increase cd of volume. The area De is made up of numerous rectangles, and if these be made sufficiently numerous the line DE is a curved line. The area comprised between the

Fig. 9.



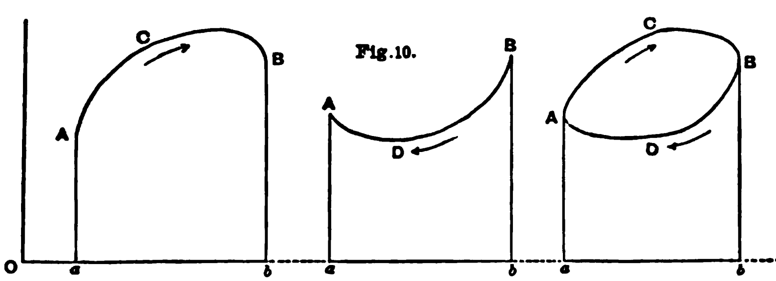
curved line DEF, the ordinates dD and fF, and the abscissa df, represents the work done during expansion from the volume Od to the volume Of. The area aAFfa represents the work done during expansion from volume Oa to volume Of.

The Indicator-Diagram. — Since the time of James Watt engineers have been accustomed to make their engines record their own working by the method just discussed. In the Indicator-Diagram, as the curve traced out is called, the two factors which it is desired to record are the Space traversed, which is measured by the amount of movement of the piston, and the Force acting, which is the pressure of the steam in the cylinder. The former factor, the space moved through by the piston, is determined by making the piston or any part of the machinery upon which it directly acts set in operation the mechanism that unrolls the paper upon which the record is to be preserved. This paper is drawn over the writing-point at a rate depending on the velocity of the piston: hence the spaces traversed by the piston during successive intervals of time are proportional to the amount of paper which is drawn under the writing-point.

The pencil is borne by the piston of a small side cylinder attached to the main cylinder. The steam is let into this, and presses the little piston outwards; it presses it against a spring until the resistance offered by that spring prevents further propulsion. If the pressure were constant, the little piston would remain at the same level; but as the pressure of the steam varies, the position of the piston also varies, as it lies between the opposing spring and steam. Its displacement, then, is at each instant proportional to the pressure of steam in the cylinder, which is the second factor. If the paper be rolled over the pencil-point when no steam has access to the side cylinder, a smooth line is drawn, the Line of No Pressure; if the steam be allowed to enter the side cylinder, the divergences of the line then produced, from the line of no pressure, measure the variations of the pressure of the steam.

While the machine is working, steam is not allowed to enter the side cylinder until the apparatus is ready to record. The small piston is consequently at rest. While the piston of the main cylinder is moving in one

Fig. 10.



direction, paper is rolled over the pencil-point; as the piston slackens speed the paper also slackens in speed; as the piston stops, the paper does the same; as the piston travels in the opposite direction, the paper travels in the opposite way at a proportionate rate. So long as the steam is not admitted into the side cylinder, the paper travels backwards and forwards over the pencil, and the same straight line is traced and retraced. The steam is ad-

mitted to the side cylinder for the space of one complete oscillation of the main piston, and the pencil-point itself travels in accordance with the varying pressure of steam during that period. The curve traced thereby is composed of two parts. The one, ACB, is produced during expansion. The work done by the steam in the cylinder during its expansion is the area ACB*ba* if O*ab* be the line of no pressure. When the piston has finished its stroke, it — and therefore the paper — stands for an instant at rest. Then the piston is pressed against the steam either by other steam or by the atmosphere, and the paper is drawn backwards. Work is thus done against the steam during the backward stroke, and it is represented by the area BDA*ab*, where BDA is the line recorded.

The difference between the areas ACB*ba* and BDA*ab* represents the excess of work done by the steam over that done against it: hence the total work done by the engine is represented by the area of the surface ACBD traced out by the pencil in the formation of the so-called Indicator-Diagram.

The pencil of the Indicator in tracing out such a curve mechanically performs an operation equivalent to that which the mathematician effects when he sums up areas by means of the algebraic processes of the Integral Calculus.

In some cases it is sufficient to know the mean force acting. In such cases the space traversed being a known quantity, the energy can be determined if the mean force alone be recorded. For example, if the mean force required to pull a vehicle be found, it is a very simple matter to multiply the recorded mean force by the space traversed in order to find the total amount of energy expended. In Marey's investigations (*Trav. du Laborat.*, 1875) into the comparative total work expended on a vehicle, according as an elastic spring is or is not placed between it and the draught animal, a capsule was so arranged that the air in it suffered irregular compressions and rarefactions, corresponding to the irregular jolts between the animal and the car. The writing-point, set in movement by the correspondingly-irregular oscillations of one of the walls of the capsule, which was flexible, described irregular lines when there was no elastic intermediary, and more regular ones, nearer the line of no disturbance, when there was such an intermediary introduced; from these it was found that the mean forces, and therefore the amounts of energy expended in the two cases were in the ratio of about 4 to 5, showing that the use of an elastic spring between a draught animal and the vehicle which it draws results in an economy of labour amounting to about 25 per cent.

CHAPTER V.

KINEMATICS.

To the part of Science which deals with Motion, considered *per se* and without reference either to the force producing it or to the body moved, is given the name of **Kinematics.** The nature of the questions discussed under this title is essentially mathematical; and though no great acquaintance with mathematical methods is presumed in the reader of this volume, it will be necessary to assume in him a certain amount of knowledge of the most elementary geometry and algebra.

General Propositions.

Direction. — There cannot be Motion without Direction; we cannot think of a body or a point as moving, and yet not moving in any direction. If it move at all, it must either move so as to travel constantly in the same direction, in which case it is moving in a straight line; or else the direction of its motion must change as it proceeds from point to point of the path traversed, so that the body travels in some kind of curved line.

In the great majority of those curves which possess physical interest as being those in which bodies actually do move, it is possible to draw at any point of the curve a straight line known as the Tangent to the curve at that point. The Tangent to the Circle at any point is familiar enough, and is easily understood to be a straight line at right angles to a radius connecting the centre of the circle with that point of the circumference at which the tangent is to be drawn; and the characteristic property of the line as a tangent is that it touches the circle without cutting it. Tangents may in a similar way be drawn to most curves, so as at any determined point to touch but not to cut the curve unless the curve changes its curvature beyond the point at which the tangent touches it.

If a circle be drawn on a very large scale, and a tangent be drawn to it at any point chosen, it will be found that the larger the scale the more nearly will the circle appear to coincide with the tangent at the point of contact. This can easily be seen by actually drawing such a figure. In fact, if a circle be drawn on a very great scale, any very little part of its circumference will appear to be practically straight. Of course it is not straight, but by drawing the circle sufficiently large, and by diminishing the size of the little part of the circumference considered, the approximation to perfect straightness in the little part or "element" considered may be rendered as close as may be desired. Such a circle may, then, be considered as a polygon, having an infinite—greater, that is, than any definite assignable—number of sides, the length of each of which is indefinitely small, and each of which coincides for an infinitesimal distance with the tangent which is drawn past it.

What is true of a large circle is true of a small one, and hence motion in a circle may be considered as motion round a polygon of an indefinite number of sides; whence the following proposition.

As a body or point moves round a circle, the Direction of its motion is that of the **Tangent** at each successive instant. Similarly, the direction of motion of a body which travels in any other curve is, at each successive instant of time, the same as the direction of the Tangent to the Curve at the point of the curve momentarily occupied by the moving body.

Velocity. — We have already anticipated some kinematical statements in discussing the velocity of a moving body. This was defined as the distance passed over in a unit of time by a body in motion; and if we consider, not the moving body, but the motion itself, we may say that one of the necessary properties of pure Motion is Velocity. It is not possible to think of Motion without thinking of a corresponding definite Rate of motion, which, if there be motion at all, cannot be zero, and on the other hand cannot be infinite, so long as Space and Time are related to Motion in the way in which experience shows them to be; and the idea of Rate of Movement is as necessary a constituent of the idea of Motion as is that of Direction.

Velocity may be **uniform** or **variable.** The measurement of uniform velocity is simple enough; and it has already been explained on what principle the measurement of variable velocity is based. Whether the direction of motion be constant or variable — whether the moving particle travel in a straight line or in a curve — the principle involved is always the same, namely, that the velocity v of a moving particle is the length of path traversed by it in a unit of time, or the length of path which would have been traversed by it during a unit of time if the speed had remained uniform during that period. In the case of

motion in curved paths, there arise subsidiary mathematical difficulties in the estimation of the precise length of the path traversed, but these only arise in the determination of the value of one of the terms of the formula $v = s/t$, and do not affect the validity of the formula itself.

Velocity may be otherwise defined as the relation of change of position to change of time. If at a certain instant of time the moving point be at a distance $\mathbf{s}$ from a fixed point chosen as a standard of reference, then, as time goes on, the position of the body, and therefore the value of $\mathbf{s}$, changes. Let the time during which motion is going on be δt, a very small element of time, and the corresponding change of position be $\delta\mathbf{s}$, then this relation of the change of position to the change of time may be expressed by the fraction $\delta\mathbf{s}/\delta t$, which, when δt is chosen sufficiently small, becomes the function familiar to students of the Differential Calculus as $d\mathbf{s}/dt$. This change of $\mathbf{s}$ in accord with the passage of time may be very advantageously represented as $\dot{\mathbf{s}}$, where the dot above the letter indicates "the value of the change in unit of time of" the quantity expressed by the letter over which the dot is placed, when the unit of time chosen is very small. This is the notation employed by Newton in his *Fluxions*, $\dot{\mathbf{s}}$ being the change or Fluxion of $\mathbf{s}$. If the velocity ($\dot{\mathbf{s}}$) itself change, the change of velocity in unit of time — which we otherwise know under the name of Acceleration — would be represented by the symbol $\ddot{\mathbf{s}}$. So if the acceleration ($\ddot{\mathbf{s}}$) itself varied, the change of acceleration per unit of time would be represented by the symbol $\dddot{\mathbf{s}}$. Such a number of dots as this rarely occurs in physical problems.

We may here make use of previous discussions to bring together some of the symbols used to express frequently-recurring terms.

Distance of particle from point of reference $= \mathbf{s}$.
Velocity, $\mathbf{v} = \dot{\mathbf{s}} = \mathbf{s}/t$.
Acceleration, $\mathbf{a} =$ change of $\mathbf{v}$ in unit of time $= \mathbf{v}/t = \dot{\mathbf{v}} = \ddot{\mathbf{s}}$.
Force $\mathbf{F} = m\mathbf{a} = m\dot{\mathbf{v}} = m\ddot{\mathbf{s}}$.
Work $\mathrm{W} = \mathbf{F}\mathbf{s} = m\ddot{\mathbf{s}}\mathbf{s}$.
Rate of doing work (Lord Kelvin's *Activity*, Newton's *Actio Agentis*) $= \mathrm{W}/t = \mathbf{F}\mathbf{s}/t = \mathbf{F}\mathbf{v} = m\ddot{\mathbf{s}}\dot{\mathbf{s}}$.
[Action (Maupertuis) $= \mathbf{v}\mathbf{s}$ or $\Sigma(\mathbf{v}\mathbf{s})$; held by him to be always a minimum in unguided motion of a conservative system: shown by Hamilton to present, in unguided motion between fixed positions, either a minimum or a maximum value or else a value little affected by slight variations in the path traversed.]
Energy $= \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{\mathbf{s}})^2 =$ Kinetic energy.
Do. $= \mathrm{W} = \mathbf{F}\mathbf{s} = m\mathbf{a}\mathbf{s} = m\ddot{\mathbf{s}}$ = Potential energy.

Dimensions. — Distance of a particle from a point of reference may be represented by a straight line, which is measurable in units of Length. Space traversed in a straight line is therefore said to be of one dimension in Length, and may be represented by the symbol [L]. Velocity is a Length (space traversed) divided by a Time, and may be represented by the symbol [L/T]. Momentum is Mass × Velocity; its dimensions are [ML/T]. Acceleration, the velocity acquired per unit of time, is a Velocity divided by a Time, and the Dimensions of Acceleration are [L/T] ÷ [T], or [$\mathrm{L/T^2}$].

Force is a Mass × Acceleration, and its dimensions are accordingly $[ML/T^2]$. Weight and Total Pressure have the same dimensions as Force. Force, f, and pressure, p, per unit area, have dimensions $[M/LT^2]$. Energy, if we take the expression $\frac{1}{2}mv^2$, has the dimensions $[M]\ [L/T]^2$ or $[ML^2/T^2]$; while if we take the expression mas, it is found to have the dimensions $[M]\ [L/T^2]\ [L]$ or $[ML^2/T^2]$, the same result. The dimensions of Work are the same as those of Energy. Energy per unit volume has dimensions $[M/LT^2]$, like f and p above. Those of Activity are Work done ÷ Time = $[ML^2/T^3]$.

Examples. — 1. How many British units of force is a Dyne equal to? The Dyne = $[ML/T^2]$ = [Gramme × Centimetre / Second^2] = $\left[\frac{15{\cdot}432}{7000}\text{ lb.} \times \frac{1}{30{\cdot}48}\text{ ft.} / \text{Second}^2\right] = \left(\frac{15{\cdot}432}{7000} \times \frac{1}{30{\cdot}48}\right)[\text{lb.-ft./second}^2] = \frac{15{\cdot}432}{7000 \times 30{\cdot}48}$ British units.

2. Suppose it were affirmed that Force is Rate of gain or loss of Energy as Time goes on; test the statement. Using a dimensional equation, we would have $[F] = [W/T]$ or $[ML/T^2] = [ML^2/T^2] \div [T]$, which is obviously wrong.

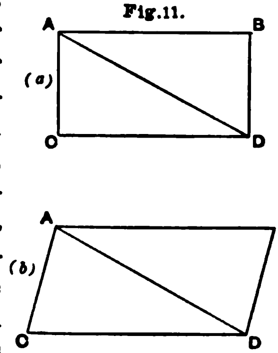
3. Test the statement that Force is measured by Time-rate of Change of Momentum. Similarly $[F]$ = [Mom.] ÷ $[T]$, or $[ML/T^2] = [ML/T] \div [T]$, which is consistent.

4. Test the assertion that Force is the Rate at which a body gains or loses Energy as it traverses Space. $[F]$ = [Energy/distance traversed] = $[W/L]$, or $[ML/T^2] = [ML^2/T^2] \div [L] = [ML/T^2]$, which is consistent.

Simultaneous Motions. — If a particle have by any means two separate independent motions communicated to it simultaneously, each will produce its own effect, and the total movement of the particle can be found by any process of summation which may be found mathematically appropriate. It will always be found as the result of experiment on bodies which may be taken to represent particles, that if the motions imparted to the particles be themselves **constant in velocity and direction,** the result of their concurrence is a single motion in a straight line with a single velocity and direction. The single motion which is produced as the result of the concurrence of two motions is called their **Resultant,** and they with regard to the resultant are called its **Components.** If a ship travel from west to east and a man on board also walk from west to east, the speed of the ship and the speed of his walking will have to be added together to find the rate at which he is moving eastwards: if the ship travel from west to east and he walk along the ship from east to west, the difference between his own speed and that of the ship is the rate at which he is travelling eastwards. In the latter case the result may be positive — *i.e.* he is really going eastward; negative — *i.e.* he is really going westward; or zero,

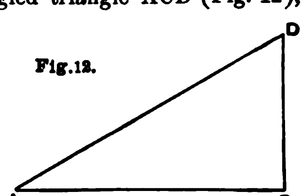
in which case he has no movement at all, the ship carrying him east just as much as he walks to the west, so that he is really beating time in the same place. If a steamer travel to the east and be at the same time carried to the north by a current, the path traversed by the steamer will be a line which is the diagonal of a **parallelogram** whose sides represent the eastward and northward velocities respectively. The steamer will describe this diagonal line in the same time as it would have taken to have steamed or to have drifted along one or the other side of the parallelogram if the steaming or the drifting respectively had been the only cause of its movement. Hence, to find the Resultant of two simultaneous Velocities, the rule is: Construct a parallelogram whose adjacent sides represent in magnitude and direction the velocities produced, and the Diagonal which lies between these adjacent sides represents the Resultant Velocity. If the lines AB, AC (Fig. 11), represent in direction and, on any conventional scale, the magnitude of the velocities simultaneously imparted to the particle A, the particle A will move along the line AD to the point D in the same time that, under the influence of the velocity AB alone, it would have taken to reach B, or, under the influence of the velocity AC, to reach the point C. The actual construction of the diagram representing the resultant of any two velocities is an easy matter: the calculation of the value of the resultant — that is, of the length of the diagonal — involves a little geometrical working.

Fig. 11.



When the two components are at right angles to one another, we resort to Eucl. I. 47, which shows that in a right-angled triangle ACD (Fig. 12), of which the right angle is at C, $AD^2 = AC^2 + CD^2$. In the parallelogram Fig. 11 (a) it is plain that $AD^2 = AC^2 + CD^2$; but $CD = AB$; hence $AD^2 = AC^2 + AB^2$; or in words, the square of the resultant is equal to the sum of the squares of the two components, if these be at right angles to one another.

Fig. 12.



Again, if they be not at right angles to one another, they must make either an acute or an obtuse angle. In the former case, we resort to Eucl. II. 12, which shows that if the parallelogram be drawn (Fig. 13) and the side AC be produced so far that a line DE can be drawn at right angles to it from the point D, the equation $AD^2 = AC^2$

$+ CD^2 + 2AC \cdot CE$ is true; this enables us to find the value of AD if we know that of AC and AB (which is equal to CD), and if we can find that of CE. In the latter case, where the angle BAC between the components

Fig. 13.

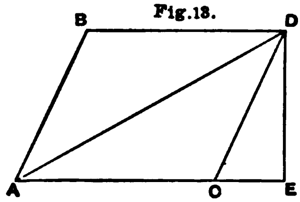
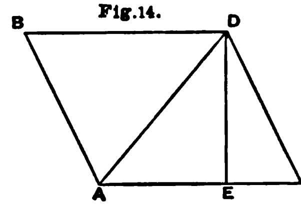


Fig. 14.



AB and AC is obtuse (Fig. 14), Eucl. II. 13 shows us that if we drop a perpendicular DE from D upon the base AC, the equation $AD^2 = AB^2 + AC^2 - 2AC \cdot EC$ holds good, and enables us to find the value of AD.

Problems.

1. A person on board a ship which is going eastwards walks back and fore at the rate of 4 miles an hour relative to the ship: the ship is travelling at the rate of 12 miles an hour. What is his eastward velocity when he is walking forward? what when he is going aft? what is his average eastward velocity?—*Ans.* 16 miles an hour; 8 miles an hour; 12 miles an hour.

2. A point moves with velocity a eastwards and velocity b westwards simultaneously. What is its eastward velocity?—*Ans.* $a - b$.

3. Interpret the result if b is greater than a.—*Ans.* The eastward velocity $= a - b$; this is negative: the velocity must therefore be westward and $= b - a$.

4. Interpret the result if $b = a$.—*Ans.* The eastward velocity $= a - b = 0$; or the body is at rest.

5. If in a railway carriage compartment a man walk across at the rate of 5 miles an hour while the train goes forward at the rate of 12 miles an hour, what will have been his real path and velocity relative to the railway line underneath?—*Ans.* In Fig. 11 (a), if $AB = 12$ and $AC = 5$: the real path is AD, which has a value of 13.

6. To the same particle are imparted a velocity of 12 and one of 6 feet per second in directions which stand to one another at an angle of 60°: what is the direction and the amount of the resultant velocity?—*Ans.* In Fig. 13, if AB represent the velocity of 6 feet per second, and AC on the same scale that of 12 feet, the angle BAC being one of 60°, then the line AD will indicate the direction of the resultant movement, and the equation $AD^2 = 12^2 + 6^2 + \overline{2 \times 12 \times CE}$ (CE being seen, since the triangle CDE is half an equilateral triangle, to be equal to half CD—that is, to 3), or $144 + 36 + 72 = 252$, shows that $AD = \sqrt{252} = 15{\cdot}822$.

7. If in the last question the angle between the directions of the velocities had been 120°, what would the resultant velocity have been?—*Ans.* In Fig. 14, if AB be 6 and AC 12, the angle BAC being 120°, $AD^2 = 12^2 + 6^2 - \overline{2 \times 12 \times CE} = 144 + 36 - 72 = 108$; whence $AD = \sqrt{108} = 10{\cdot}392$ feet per second.

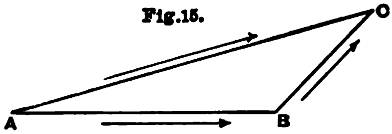
8. What would have been its direction?

In the same figure, 14, the triangle ADC has its sides $AC = 12$, $CD = 6$, and the angle $DCA = 60°$. By trigonometry we find that the angle DAC is

35° 16′, and hence the direction of the resultant motion is inclined to those of its components AC and AB at the angles of 35° 16′ and 84° 44′ respectively.

Triangle of Velocities. — If the figures just made use of be reduced to their simplest necessary elements, it will be seen that there is no need to describe a complete parallelogram in order to find the line which would be its diagonal. The three sides of a triangle are quite sufficient to express the relation between two component velocities and their resultant, and for the determination of the resultant of two velocities the rule may be thus stated: Take a starting-point; from it draw a line representing in magnitude and direction one of the component velocities; from the point thus arrived at — that is to say, the end of the line thus drawn — draw another line similarly representing the second component velocity. The third side may now be laid down, and the problem is reduced to the form which in trigonometry is simple enough, namely — Given two sides of a triangle and the angle between them, to find the third side, and the angles which it makes with the two sides given. In the triangle ABC (Fig. 15), if the sides AB and BC represent the component velocities in amount and direction, AC in the same way represents their resultant; and it will be observed that if the sides of the triangle be taken consecutively in the "cyclical" order, AB, BC, CA, the direction of the resultant is in this diagram opposed to that of the components.

Fig. 15.



Resolution of a Velocity into Components. — The converse proposition is one of very general utility. In the former case, by Composition, that single resultant was found which was the effect of two simultaneously-imparted movements. A single movement may, conversely, be considered as the resultant of two component movements which we may wish to find. The process of finding them is known as the Resolution of a motion into its components. A given pair of velocities can only have one resultant, for if two sides of a triangle be fixed, there is no scope for variation in the position or length of the third side; but if the resultant be given, it can be resolved into components in an indefinite number of ways, for there can be an infinite number of triangles made by supplying two sides when only one side is determinately fixed. Hence the question how to resolve a velocity into its components, set in this vague way,

never arises; but the question how to resolve a velocity into its components in certain fixed directions is of constant occurrence. Such a question is generally solved by construction in the following way:—Let AB (Fig. 16) be a line indicating the direction and rate of movement of a particle. It is required to know what are the corresponding components in two directions, LM, NO, arbitrarily chosen or determined by the conditions of the problem. Draw lines from both extremities of the line AB, parallel to the directions assigned. In this way a parallelogram will be formed in which AD, CB will be parallel to LM, and DB, AC to NO. In this parallelogram AB represents the single motion whose components are to be found: the length of AD or CB represents the proportionate value of the component parallel to LM, and DB or AC the proportionate value of that parallel to NO. Hence the problem is solved. Very little practice enables one to dispense with drawing a complete parallelogram, and to find the components by constructing either the triangle ABC or ABD. If numerical values are required, we can find them by means of the known angles which the directions of LM and NO make with that of AB: the values of these two angles, together with the numerical value of AB, give by trigonometry the numerical values of AD and DB, which represent the components. If we wish to resolve a single velocity into components at right angles to one another, the process is precisely the same, LM and NO being drawn at right angles to one another. A modified form of the problem which we very often encounter is—Given a velocity in a certain direction, what is the value of its component in another assigned direction? This is solved by the following construction:—Let AB (Fig. 17) represent the given velocity and LM the direction of the required component of AB. Draw through A a straight line, AC, indefinite in length, but parallel to LM. From the

Fig. 16.

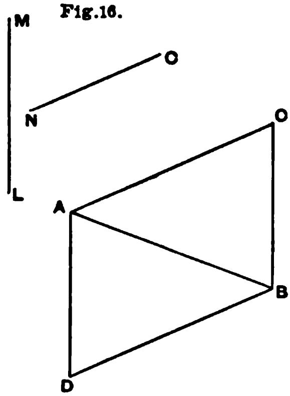
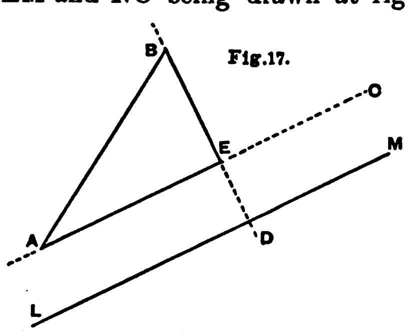


Fig. 17.



other extremity, B, of the line AB, draw a line BD *at right angles* to AC, cutting it in the point E. AE is the component required in the direction parallel to LM.

In Fig. 17, AE = AB·cos ξ; and BE = AB·sin ξ; where ξ is the angle BAE.

Problems.

1. A velocity of 30 feet per second: what is the value of its component in a direction which makes with its own an angle of 60°?—*Ans.* In Fig. 17, if the angle ABD be 60°, the line BA may represent the velocity given; BE represents its component at an angle of 60° with it. The triangle BEA is half an equilateral triangle, and BE is half of BA; it represents therefore a component velocity of 15 feet per second.

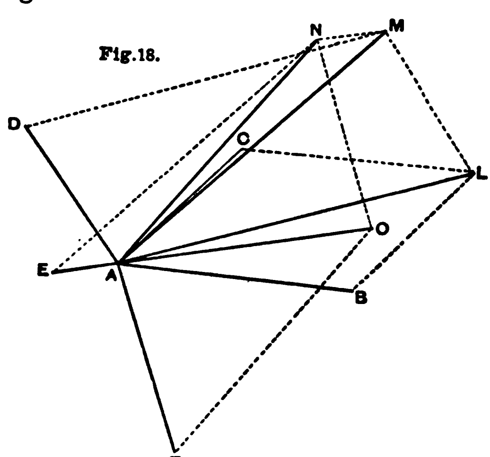
2. A velocity of 20 feet per second: what is the value of its component whose direction makes with its own an angle of 30°?—*Ans.* In the same figure, if the angle EAB be 30°, and AB represent the velocity of 20 feet per second: in such a triangle AE : AB : : $\sqrt{3/4}$: 1, and the value of the component is $\sqrt{3/4} \times 20 = \sqrt{300} = 17{\cdot}32$ feet per second.

3. A velocity of 60 feet a second: what is the value of its component at an angle of 45°?—*Ans.* $\sqrt{1/2} \times 60 = 42{\cdot}42$ feet per second.

4. A velocity v in a certain direction: what is its component at right angles to that direction?—*Ans.* It has none.

Composition of more than Two Velocities.—If more than two velocities be imparted to a body the resultant is always, if they themselves be uniform in amount and direction, a single uniform motion in a straight line. If the several velocities imparted be all in the same plane, their resultant may easily be found by finding the resultant of any two of them, compounding the resultant thus obtained with any other of the velocities imparted, and so on, till all the velocities have been taken into consideration, and the final resultant obtained. Let the several velocities which are imparted to a particle be represented by the lines AB, AC, AD, AE, AF (Fig. 18), all in one plane. It is required to find their resultant. The resultant of AB and AC is AL; the resultant of AL and AD is AM; the resultant of AM and AE is AN;

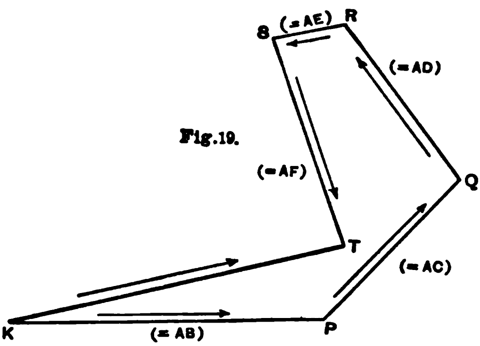
Fig. 18.



that of AN and AF is AO, the final resultant of the five velocities AB, AC, AD, AE, AF. It does not matter in what order they are compounded; it may be left as an exercise for the reader to show that the same result is always obtained whatever be the order followed.

The Polygon of Velocities. — If, in the last diagram, the figure ABLMNOA be traced out, it will be seen that it is a polygon whose sides represent the various velocities and the resultant: for these sides are AB, BL (= AC), LM (=AD), MN (= AE), NO (= AF), and AO, which represents the Resultant. Hence the method of finding the resultant of any number of forces in the same plane may be exemplified as follows: — Take a starting-point K (Fig. 19); from K draw the line KP, representing AB in magnitude and direction; from P draw PQ, representing AC; from Q QR, representing AD; from R RS, representing AE; from S ST, representing AF; then join KT. KT represents the Resultant sought. It will be seen that the direction of the resultant is opposed to that of the other sides of the polygon taken in cyclical order. The rule, then, for the composition of a number of velocities in the same plane is — Construct a polygon with lines representing them (it being a matter of indifference in what order they are taken, or whether they cross one another or not), and if there be a side missing, complete it; it will represent the magnitude of the resultant, and its direction will be opposed to that of the other constituent sides, taken in cyclical order. If the two points K and T coincide, then the line KT has no value, there is no resultant motion, and the result of the simultaneous velocities is, in such a case, a state of rest.

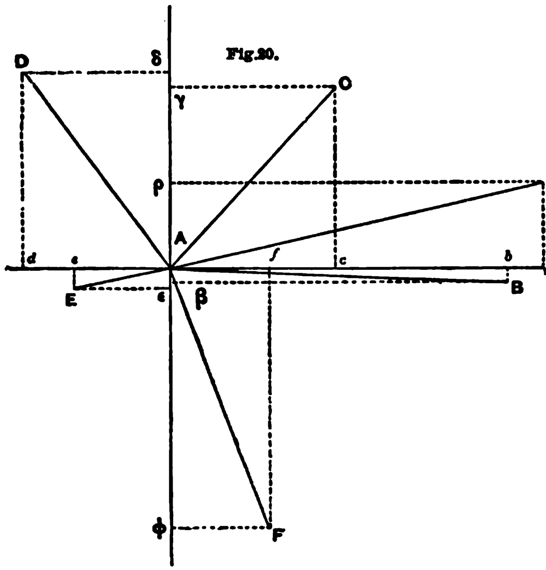
Fig. 19.



Reference to Axes. — It is often as convenient, or more so, first to resolve each velocity into two components, which are made parallel to arbitrarily chosen axes. Let the same velocities, AB, AC, AD, AE, AF, be supposed as in the previous paragraphs. Through the point A (Fig. 20) draw axes of x and y at right angles to each other. Resolve each velocity into its components parallel to these axes. AB is resolved into Ab and Aβ; AC into Ac and Aγ, and so on. The value of the resultant is found after

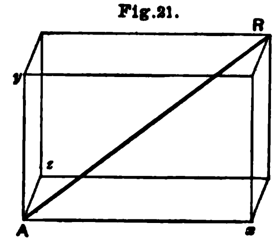
summing up with reference to each axis separately. The total result with reference to the axis of x is $(Ab + Ac - Ad - Ae + Af)$, which has a value, say, $+ Ar$. In the axis of y the total result is $(- A\beta + A\gamma + A\delta - A\epsilon - A\phi)$, which has the aggregate value, say, $+ A\rho$. The resultant therefore is to be drawn from A to a point R, which has co-ordinates, $x = + Ar$, $y = + A\rho$.

Fig. 20.



Velocities not in one Plane.—The same essential principles apply here as in the preceding paragraphs. In the case of a railway train travelling at the same time northwards, westwards, and upwards, the motion, while it may be represented by a straight line, is the resultant of three components at right angles to each other. The proposition in three dimensions, which corresponds to that known as the parallelogram of velocities in bidimensional space (in a plane), is called the **parallelepipedon of velocities.** If the three velocities, Ax, Ay, Az (Fig. 21), at right angles to each other, be compounded, the resultant is expressed by a line drawn from A to the opposite angle of that parallelepipedon of which Ax, Ay, Az measure the length, breadth, and thickness. If Ax, Ay, Az be at right angles to one another, $AR^2 = (Ax^2 + Ay^2 + Az^2)$, while if they be not at right angles to one another, AR is the diagonal of an oblique prism.

Fig. 21.



Any rectilineal velocity may be resolved into three components in an indefinite number of ways, for there may be an infinite number of prisms constructed on a given diagonal line; but there can only be one way of resolving such a movement into components if these must be at right angles to one another while the direction of any one of them is given, or if the directions of any two of them be assigned.

The **Polygon of Velocities** also applies when the component movements are not restricted to one plane, for a so-called "*gauche polygone,*" or

"skew-polygon," may be realised, no three of whose contiguous sides are in the same plane; the only essential criterion of such a polygon is that it shall be continuous and closed. If such a polygon whose sides represent velocities be realised, but be incomplete or "unclosed," the missing side represents the Resultant, and the direction of the resultant — opposed to that of the rest of the sides taken in cyclical order — and its magnitude are found in the same way as if the polygon had been restricted to a plane surface.

The method of **reference to axes**, illustrated by Fig. 20, is of special use when extended to tridimensional space. Of a number of velocities in different directions in space, each may be resolved into three components, parallel to the axes of x, of y, and of z, and the resultant is found after summing up the effects produced with reference to each of these axes respectively.

Change of Velocity. — This phrase is sometimes employed, as when the statement is made that a certain velocity has been changed into another, and the question is asked, What has been the "Change of Velocity?" Another way of stating the same is — A known component and an unknown one have produced a given resultant: *what was the value of the unknown component?* This is easily solved if the direction of motion have not changed; while if the direction have also changed, the question is answered by the aid of the triangle of velocities; the two sides being known, the third side is easily found.

Parallelograms, etc., of Accelerations. — What is true of simultaneous velocities imparted in general is true of velocities simultaneously imparted in unit of time — that is, of Accelerations, and hence, if a body receive two accelerations, these must be compounded in exactly the same way as two velocities. So every one of the geometric propositions just laid down with reference to velocities finds its exact counterpart in a proposition relating to accelerations, and we thus have such propositions as the Parallelogram, the Triangle, the Polygon, the Parallelepipedon of Accelerations.

Acceleration may therefore result in mere change of direction of motion: for the original velocity compounded with that produced in a given time by the acceleration may yield a resultant velocity which is the same in amount, but not in direction, as the original velocity: the triangle of velocities is then an isosceles triangle, the two equal sides in which represent the original and the resultant velocities respectively.

Problems.

1. If the same particle be simultaneously affected by a northward velocity of 10 feet per second, an eastward of 8, one towards the S.W. of 7, to

the W. of 8, to the S.E. of 5, and to the N.E. of 7, find the resultant movement, and show that it does not matter in what order the components are taken.

2. If the axes of x and y be drawn at right angles to one another through the common point A; if then the point A be supposed to be simultaneously affected by velocities represented by the following lines, viz., (a) one drawn making an angle of 15° with Ax, and of such a length as to represent a velocity of 10 metres per second; (b) one making an angle of 45° with Ax, and representing a speed of 15 metres per second; (c) one making an obtuse angle of 120° with Ax, and representing 8 metres per second; and (d) one at an angle of 195° with Ax, and representing a rate of 12 metres per second. Find the resultant velocity (1) by the polygon, and (2) by reference to axes.

3. If a body moving 10 miles an hour northward come to move at the same rate southward, what is the change of velocity?—*Ans.* 20 miles an hour.

4. If a body be moving with a velocity 4 miles an hour northward, and be after some time found to be moving at the same rate eastward, what is the change of velocity?—*Ans.* $4 \times \sqrt{2}$, acting towards the S.E.; the hypotenuse of a right-angled triangle.

5. If a body moving at the rate of 10 feet a second be found after some time to be travelling at the same rate, but in a direction inclined at an angle of 60° to its former one, what is the change of velocity?—*Ans.* 10 feet per second, making, with the original component and the resultant, an equilateral triangle.

Accelerated Motion.—If a body be moving, *in a straight line*, at a rate which increases or decreases with the time, its velocity is said to be *accelerated*. The acceleration is said to be *positive* when the velocity of the motion is increased, *negative* when it is diminished. It is measured by the amount of increase or decrease of the velocity per unit of time. If a particle be, at a certain initial instant, moving at a rate $\mathbf{v}_0$, and if its acceleration be $\pm \mathbf{a}$, in the same straight line, then its various rates of motion are—

At the initial instant . .	$\mathbf{v}_0$.
At the end of one second . .	$\mathbf{v}_0 \pm \mathbf{a}$.
At the end of two seconds .	$\mathbf{v}_0 \pm 2\mathbf{a}$.
At the end of t seconds . .	$\mathbf{v}_0 \pm \mathbf{a}t$.

Hence we arrive at a general equation expressing the relation between $\mathbf{v}_t$ the **velocity attained** at the end of t seconds, $\mathbf{v}_0$ the original velocity, and $\pm \mathbf{a}$ the acceleration, namely,—

$$\mathbf{v}_t = \mathbf{v}_0 \pm \mathbf{a}t, \quad (1.)$$

in which the + or the − sign is used according to the positive or the negative character of the uniform acceleration $\mathbf{a}$.

It is supposed that the acceleration is uniform, and hence the **average velocity** during any interval of time is the arith-

metical mean between the velocity $\mathbf{v}_0$ at the commencement and the velocity $\mathbf{v}_t$ at the end of the interval; that is to say, it is equal to half their sum or $\frac{1}{2}\{(\mathbf{v}_0)+(\mathbf{v}_0 \pm \alpha t)\} = (\mathbf{v}_0 + \frac{1}{2}\alpha t)$. This being the average velocity during the interval, the **space traversed** will be found by multiplying the average velocity by the time, and hence we have ($\mathbf{s}$ being the space traversed) —

$$\mathbf{s} = \tfrac{1}{2}(\mathbf{v}_0 + \mathbf{v}_t)t = t\,(\mathbf{v}_0 \pm \tfrac{1}{2}\alpha t) = \mathbf{v}_0 t \pm \tfrac{1}{2}\alpha t^2. \quad (2.)$$

From equations (1) and (2) we may eliminate t, and thus obtain a third equation —

$$\mathbf{v}_t^2 = \mathbf{v}_0^2 \pm 2\alpha\mathbf{s}, \quad (3.)$$

which expresses the relations between the original and acquired velocities, the space passed over, and the acceleration. Also,

$$\mathbf{s} = \mathbf{v}_t t \mp \alpha t^2/2 = (\mathbf{v}_t^2 - \mathbf{v}_0^2) \div 2\alpha. \quad (4.)$$

All elementary problems concerning accelerated movement in one direction, which give a sufficient number of terms to enable a conclusion to be arrived at, can be solved by the aid of these equations.

Problems.

1. If the velocity at the initial instant be 10 feet per second, and the acceleration be + 2 ft.-per-sec. per second, what will be the speed at the end of 13 seconds? — *Ans.* Here, by equation (1) ($\mathbf{v}_t$ being the unknown term),

$$\mathbf{v}_t = \mathbf{v}_0 + \alpha t = 10 + (2 \times 13) = 36.$$

2. If the acceleration be − 2 ft.-per-sec. per second, what will be the velocity? — *Ans.* $\mathbf{v}_t = 10 - (2 \times 13) = -16$; that is, 16 feet per second, in a direction opposed to the original velocity.

3. If the terminal velocity be 20 feet per second, the acceleration be 4 ft.-per-sec. per second, and the initial velocity 4 feet per second, what was the time spent in attaining the ultimate speed? — *Ans.* Here, by equation (1) (t being the unknown term), $20 = 4 + 4t$, whence $t = 4$.

4. A body travels with accelerated velocity; its attained velocity is 100 feet per second, its acceleration is 10 ft.-per-sec. per second, and it has been gaining speed for 8 seconds. What was the initial velocity? — *Ans.* By equation (1), $\mathbf{v}_0$ being the unknown term, $100 = \mathbf{v}_0 + (10 \times 8)$, whence $\mathbf{v}_0 = 20$.

5. A body falls from rest: its velocity increases by 32·2 ft.-per-sec. per second. What will be its speed at the end of 5 seconds? — *Ans.* By equation (1), $\mathbf{v}_t$ being the unknown term, and $\mathbf{v}_0 = 0$, $\mathbf{v}_t = 0 + (32{\cdot}2 \times 5) = 161$ feet per second.

6. What space will have been traversed, the terms remaining as in the last question? — *Ans.* By equation (2), $\mathbf{s}$ being the unknown quantity, and $\mathbf{v}_0 = 0$, $\mathbf{s} = 0 + \frac{1}{2}(32{\cdot}2 \times 25) = 402{\cdot}5$ feet.

7. What time will a body take to fall 502·5 feet if it be thrown down from a cliff at the initial rate of 20 feet per second, and if the acceleration of a falling body be 32·2 ft.-per-sec. per second? — *Ans.* Here, by equation (2), t being the unknown term, $502{\cdot}5 = 20t + 16{\cdot}1t^2$, a quadratic: whence $t = 5$ seconds.

8. If the initial velocity had been 20 feet per second *upwards*, how long would it take to fall? — *Ans.* Here the acceleration is opposed to the original velocity; equation (2) becomes $-502\cdot5=20t-16\cdot1t^2$; whence $t=6\cdot24$ seconds.

9. What speed is attained by a falling body if it start from rest and fall 1610 feet? — *Ans.* Here $\mathbf{v}_t$ is unknown, $\mathbf{v}_0 = 0$, and $\mathbf{a} = 32\cdot2$. By equation (3), $\mathbf{v}_t^2 = 0 + (2 \times 32\cdot2 \times 1610)$; $\therefore \mathbf{v}_t = 322$ feet per second.

10. If a body start with initial velocity $\mathbf{v}_0$, and the acceleration be $\mathbf{a}$, what will be the space traversed in the first, in the second, in the third, in the fourth seconds respectively; and what will be the space traversed in 4 seconds? — *Ans.* $\mathbf{v}_0 + \mathbf{a}/2$; $\mathbf{v}_0 + 3\mathbf{a}/2$; $\mathbf{v}_0 + 5\mathbf{a}/2$; $\mathbf{v}_0 + 7\mathbf{a}/2$; $4\,\mathbf{v}_0 + 16\mathbf{a}/2$.

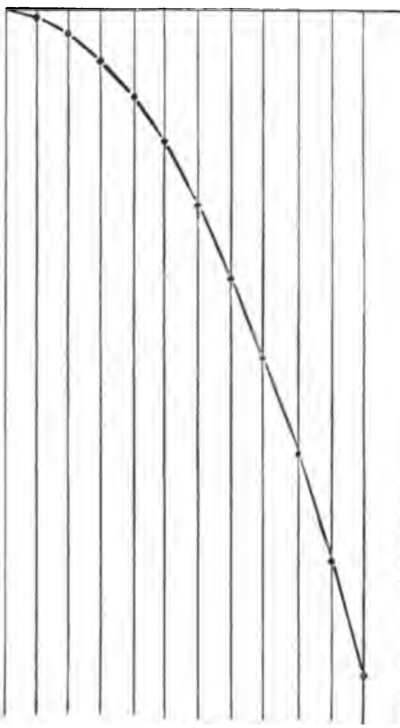
Composition of uniform with accelerated motion. — If a particle be affected with both a uniform and an accelerated motion, and if these be in the same straight line, we have simply the problem dealt with by the aid of the last four equations. When, however, the uniform and the accelerated velocities are not along the same line, but are in directions inclined to one another, the resultant must be found by a geometrical or an algebraical process of composition of velocities. If, for the sake of fixing our ideas, we consider such a motion as that of a projectile fired horizontally from a gun placed on a height, we see that the ball is affected with two simultaneous but independent motions, the one horizontal and uniform, the other vertically downward and accelerated. If we consider the positions reached by such a body in successive equal intervals of time, we find that while it passes forward, by reason of its horizontal component, over spaces varying directly as the time, the amount of its vertical drop due to the downward accelerated component is proportional (Equation 2, p. 70, where $\mathbf{v}_0 = 0$) to the square of the time during which it has been in motion; so that if we separately find its various positions at the end of successive small intervals of time, we can draw a line joining these positions, which line we find to be a curve known as a Parabola (Fig. 22).

If, again, we consider that the one movement is in the axis of x, and is uniform, so that at the end of time t the horizontal component motion has carried the body along the axis of x a distance $\mathbf{x} = \mathbf{v}t$; while the vertical fall represents the distance $\mathbf{y}$, along the axis of y, at which the body is situated at the end of the same time t, so that according to equation (2) above, $\mathbf{y} = \frac{1}{2}\mathbf{a}t^2$; then we find* that in time t (whatever this time be)

* This is an instructive example of a method frequently in use. One consideration leads us to the equation $x = vt$: another to the equation $y = \frac{1}{2}\mathbf{a}t^2$: the question is, what law governs the relations of x and y? The two equations are combined in any way so as to represent x as some multiple (or other "function") of y, and also, if possible, so as to eliminate a letter common to both equations. Here the value of $t\ (= x/v)$ derived from the first equation may be substituted for t in the second, thus making it $y = \frac{1}{2}\mathbf{a}\ (x/v)^2$, whence $x^2 \div y = const.$

the body moves to a position such that its vertical distance $\mathbf{y}$ from the starting point bears to $\mathbf{x}^2$, the square of its horizontal distance from the starting point, a constant ratio, or in symbols $k\mathbf{x}^2 = \mathbf{y}$, which is recognised as "the Equation to" a Parabola. This indicates that in order to preserve the given relation between the values of $\mathbf{x}$ and $\mathbf{y}$, the path of the body as it moves from point to point must be in a curve known as a parabola.

Fig. 22.



It is stated that the body will move in *a* parabolic not in *the* parabolic path; this is because there is an indefinite number of parabolic paths possible, there being an infinite number of parabolic curves (just as there may be an infinite variety in the forms of a jet of water expelled from a fire-engine), which resemble each other in having some constant proportion between the values of the one co-ordinate and of the square of the other, but which differ in the numerical value of that ratio.

Degrees of Freedom of a Particle.—If a particle be free to move in any direction in space, it is said to have three "degrees of freedom," because it may move in tridimensional space; it may move, *e.g.*, (1) up or down, (2) forwards or backwards, or (3) to the right or left; or more generally, it may move in the direction of any of the three axes arbitrarily chosen at right angles to one another, by reference to which we agree to specify any given direction in space, or it may move in any other direction, motion in which may be considered as the resultant of simultaneous motions in the three directions assumed as axes of reference. If the particle be restricted to a surface, it cannot move in a direction at right angles to that surface, and is accordingly said to have one degree of freedom less; it has now two degrees of freedom, for it may travel along the surface in two main axial directions (*e.g.*, (1) forward or backward, (2) to the right or left), or in any direction derived from the combination of these. If the particle be restricted to two surfaces, on both of which it must lie at the same time, it can lie nowhere but on the line in which these two surfaces cut one another, and it has now only the one degree of freedom implied in the possibiiity of moving (backwards or forwards) along this line. If the particle be restricted to three surfaces which cut one another in a point, the particle cannot leave that point without leaving one or other of the surfaces; its position is definitely fixed, and it has no degree of freedom to move in any direction.

Translation.—If there be a system of separate particles, all of which are affected with equal and parallel velocities, each particle will move in such a way as to retain its relative position with regard to its fellow-particles, and the system will move as a whole, undergoing no deformation, just as a company of soldiers, all the constituent units of which march in the same direction and at the same rate, retains its formation. If a straight line be drawn between any two of these particles when the system is in its initial position, it will be found that the line drawn between the same particles after such movement will always remain parallel to its former position, and will be unaltered in length. Motion in which every such line remains parallel to all its previous positions is called Simple Translation. If we study the motion of such a straight line or of the particles between which it lies, we shall have complete knowledge of the positions of the various particles of the system, if that system be restricted to a plane surface. If the system be not restricted to a plane surface, then it is possible that though one line and all lines parallel to it may continue to be parallel to their former positions, the whole system may have rotated round one of these lines as round an axis; and hence in this case it is necessary, before the motion of the system can be said to be a motion of simple translation, that not a line only, but any plane through the system—or, which amounts to the same thing, every line in any such plane—should retain parallelism to its initial position.

Rigid Body.—This is an ideal, not physically realisable. A rigid body may be regarded as a system of particles which may move as a whole with reference to surrounding objects, but in which there can be no displacement of its particles with reference to one another.

Centre of Figure.—There is in the case of every body of any shape whatsoever some one point occupying a definite position, which position may be described as the average of all the respective positions of the several particles of the body. A body suspended in the air somewhere towards the N.W. will have (generally within it) a point which is not only situated at an average distance to the north of the point of reference, but is also at an average distance to the west and at an average height; and this point is the Centre of Figure. Not only with respect to the planes chosen as those of reference is this point the centre of figure, and its distance from each of these planes the average of the several distances of all the particles, but it has this property with reference to any plane whatsoever.

The centre of figure of a straight line or linear body is its middle point; the centre of figure of a circle is its centre; the centre of figure of a sphere, of an ellipsoid, of a spheroid, is equally obvious; that of a hollow spherical shell is the centre of the corresponding solid sphere, and is therefore not within the substance of the shell; that of a parallelogram is the point at which the diagonals cross one another. That of any regular plane figure is obtained by dividing it into numerous thin strips and bisecting these; by joining the points of bisection a line is drawn in which the centre of figure must lie. By repeating the process another such line may be obtained. These two lines will cross one another in some point, and the point where they do so is the only point which lies in both the lines, and it is the centre of figure. This holds good only when the lines thus containing the centre of figure are straight; if they be not so the construction fails, and we may

modify one of the experimental methods described under the Centre of Gravity, farther on.

The importance of the Centre of Figure lies in this: that if a rigid body be subject to translation without rotation, the motion of the body may be quite effectively studied by considering the movement of the centre of figure, and, on the other hand, if a rigid body be subject to accelerations whose resultant passes through the centre of figure, the whole rigid body will participate in the movement of its centre, and there will be *translation:* while if the resultant of accelerations do not pass through the centre, there will be *rotation*. It is assumed in this that the body is uniform in density.

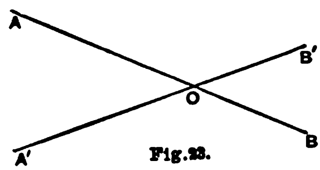
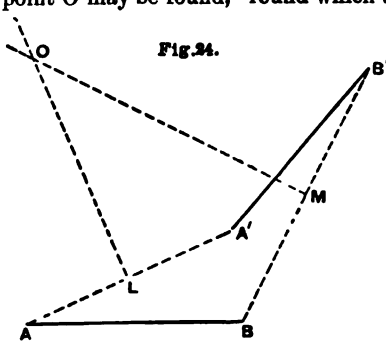


Fig. 23.

Rotation takes place when a straight line drawn through a moving body or system of particles does not continue to be parallel to its previous directions in space. Let us suppose the moving system to be restricted to a plane surface. Then a determinate line AB, arbitrarily chosen in the body, may move so that its ultimate position is A′B′ (Fig. 23). Obviously the line AB, and with it the system, has rotated round the point O. Again, the relative positions of the same line may be AB and A′B′ in Fig. 24. In this case a point O may be found,* round which the line AB has rotated so as to acquire its new position A′B′. If the lines AB and A′B′ be very nearly parallel to one another, construction will show that the point O is at a great distance. When the lines AB and A′B′ are perfectly parallel, the point O is at an indefinitely great, an infinite distance. Thus we see that though it is convenient to regard translation and rotation as distinct forms of motion, yet translation may be considered as a limiting case of rotation, effected round an infinitely distant centre.

Fig. 24.


Any translation of AB into a parallel position A′B′ may be resolved into a succession of rotations first round one extremity, as A, then to an equal extent but in an opposite direction round B: this is easily verified by construction. If the line AB cannot be brought into coincidence with the line A′B′ by a single pair of such rotations, a sufficient number of pairs of rotations will certainly effect this.

Composition of Rotations.—If by reason of a rotation round the point O, the line AB be brought into the position A′B′; if it be then rotated round a point O′, so as to assume the position A″B″: the two rotations can be compounded into one round a point O″, which is found directly by comparing the initial and final positions AB and A″B″, without reference to the intermediate position A′B′.

If a solid body, of which one point is fixed, move in any way whatsoever, the result is the same as if it had revolved round some definite axis

* Join AA′; bisect AA′ in L; draw LO at right angles to AA′. Join BB′; bisect BB′ in M; draw MO at right angles to BB′. LO and MO will intersect in O; O is the point required. Join OA, OA′, OB, OB′; the triangles OAB and OA′B′ are equal in every respect.

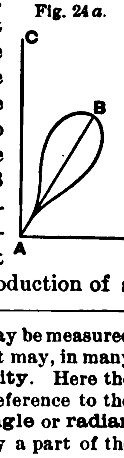
passing through that point. Any movement under this condition is equivalent to a single rotation.

If a body be subject to two or three simultaneous rotations round axes which meet in a fixed point, the resultant movement is rotation round a single axis, which is found by a construction precisely the same as that of the parallelogram or the parallelepipedon of velocities: the sides of the figure represent in direction the axes round which the rotations occur, and in length the amount of angular velocity;* the diagonal obtained represents in the same way the axis and the angular velocity of the resultant rotation. Similarly, a rotation round any axis can be resolved into component rotations round other axes passing through a point in the original axis.

The most indeterminate motion of a rigid body may always be resolved into the same motion as that of a screw in its nut, namely, a Rotation and a Translation. As the body continues to move, the axis of the imaginary screw may change its direction in space; but when considered at and for the space of any particular very small instant, it may be regarded as fixed, and is then called the Instantaneous Axis. As limiting cases, the translation may = 0, when there is Simple Rotation; or the angular velocity of rotation may = 0, in which case we would have Simple Translation. The ultimate position attained may always be reached by means of a single translation and a single rotation round some axis.

Precession. — When a new rotation is superposed on an existing one, the axis of the resultant rotation is more nearly parallel to the axis of the superposed rotation than that of the original rotation had been. Assume a top (Fig. 24 *a*) to be spinning at any particular instant round the axis AB. The top tends to fall: that is, to rotate round an axis through A, say at right angles to the plane of the paper. The axis AB therefore tends to work round, off the plane of the paper, so as to become more nearly parallel to the axis passing through A at right angles to the paper. The result is, that the axis AB pivots round the point A, and B describes a circle. This accounts for the gyration or Precession of a spinning-top. The angle BAC is the Angle of Precession. If the top be so acted upon as to make its movement of precession more or less rapid, this is equivalent to the introduction of a

Fig. 24 *a*.



* When a body moves in a circular path, its velocity in that path may be measured by the length of the path traversed divided by the time, as usual; or it may, in many respects, more conveniently be expressed in terms of **angular velocity**. Here the path is measured, not directly in terms of its own length, but with reference to the angle which it subtends and to the length of the radius. The **unit angle** or **radian** is that angle ($57^\circ{\cdot}29578 = 57^\circ{\cdot}17'{\cdot}44''{\cdot}8$ nearly) which is subtended by a part of the circumference equal in length to the radius. Hence the circumference $= 360^\circ = 2\pi$ radians, π being equal to 3·1416. Unit angular velocity is that under which a particle travelling in a circle whose radius = 1 would itself describe a path = 1 — that is, unit angular velocity is that of a rotating body which traverses the unit angle — in unit of time. If the radius be r and the angle traversed be θ, the part of the circumference passed over is $r\theta$, and if this be accomplished in time t, the linear velocity of a particle on the circumference is $v = r\theta/t$; that of a particle nearer or farther from the centre is proportionately less or greater; while the Angular Velocity of all the particles of a rotating wheel is the same, namely, $\theta/t = \omega = v/r$. The Dimensions of angular velocity are an Angle (= Arc ÷ Radius) ÷ a Time = $[(L \div L)] \div [T] = [T^{-1}]$; and this is manifestly correct, for angular velocity does not depend on the size of the circle, but only on the time taken to go round it. Similarly, **angular acceleration** $\dot{\omega} = a/r$; and it has Dimensions $[1/T^2]$.

new or third rotation, round AC; if the precession be made more rapid, the resultant axis now lies between AB and AC, and the top rises. If the point A be really some little way up the axis of the top, so that the point of the top tends to describe a little circle, say on a sheet of paper, the top gradually rises, if the spin be rapid enough, into a vertical position and "sleeps." The reason of this is that the spin tends to make the point of the top travel wheel-wise along the paper at a certain rate: the precession tends to make it describe a circle on the paper at a certain rate: if the former rate exceed the latter, the point is hurried on in its path on the paper: the energy of the spin is partly converted into energy of precessional motion: and this is equivalent to accelerating the precessory movement: so the top rises. In the same way pebbles, egg-shells, hard-boiled eggs, etc., rise up and spin round their longest axis if spun fast enough.

Precession may, in a freely-suspended rotating body, be caused by an unsymmetrical or unbalanced distribution of the rotating mass round the actual instantaneous axis of rotation. Thus the equatorial protuberance of the Earth enables the attractions of the Sun and Moon to exercise a tilting action which results in a precession whose period is about 26,000 years, and the angle of which is 23° 30′.

Nutation. — Variations in the tilting forces which give rise to Precession cause variations in the speed and angle of precession. Thus, in the case of the Earth, there are three simultaneous sets of fluctuations in the Angle of Precession: one of nineteen years' period, due to varying angles between the axis of rotation and the moon's orbit; one of a half-year's period, the sun's tilting action being zero at the solstices; one of a fortnight's period, the moon's tilting action being zero twice in the lunar month. These fluctuations in the angle of precession convert the precessional circle into a wavy line: and this phenomenon is called Nutation.

Degrees of Freedom of a Rigid Body. — When a rigid body is absolutely free to move in any direction in space, it is said to have six degrees of freedom. These are (1) three degrees of freedom of translation, like those of a simple particle; and (2) three degrees of freedom of rotation round three axes arbitrarily chosen at right angles to one another. Any such body may move, for example, (1) upwards or downwards, (2) to the N. or S., (3) to the E. or W., or it may rotate round (4) a vertical axis, (5) an axis lying N. and S., or (6) an axis lying E. and W. Any rotation not round these axes, or any translation not in the direction of these axes, may be resolved into its components, round or parallel to them; and as any change of position whatsoever may be produced by a single translation and a single rotation, any motion whatsoever may be effected by a body which has these six degrees of freedom.

If one point in a rigid body be fixed, there can be no translation, and three degrees of freedom are thus lost; the body has, however, unlimited freedom of rotation round any axis passing through the fixed point, and thus retains three degrees of freedom. If a line in the body be fixed in position, there can be no translation, and there can be no rotation except round this fixed line, and so there can be only one degree of freedom, which corresponds to that rotation. If a surface (or, which amounts to the same thing, if three points) in the body be fixed in position, there can be neither translation nor rotation, and the rigid body has no freedom.

If a point in the body be restricted to motion along a given line, there can only be one translation, but there may be any rotation, and so the rigid

body has four degrees of freedom. When a given line in the body must coincide with some part of a line assigned in space, there can be only one translation — that along the line assigned, and one rotation — that round the line; and here we find the rigid body to have two degrees of freedom. If a point in the body be restricted to a given surface, the only motion which is impossible is translation in a direction at right angles to the surface, and hence the body has in this case five degrees of freedom. If a line in the body be restricted to a given surface, one translation is impossible, as in the previous instance, and there are two rotations possible, the one round the line which is restricted to the surface, and the other round an axis at right angles to the surface: in this case there are accordingly four degrees of freedom. If three points in a body be restricted to a surface, there can be rotation round an axis at right angles to the surface, and there can be translation in any direction along the surface but not away from it, so that in this case we have three degrees of freedom.

Strain. — When a body is not rigid, its particles may so move with reference to one another that their displacement produces deformation, and such relative motion of the particles of which a body is made up is called a Strain of the body.

Suppose a circular plate to be expanded uniformly, as a disc of iron is when heated; the radius will enlarge in the ratio of (say) 1 to a; the area of the plate increases in the ratio $1 : a^2$. The linear expansion is the difference between the initial and the final length of the radius, *i.e.* $r(a-1)$ where r is the original radius, and is hence proportional to $(a-1)$. If the body have contracted, a is less than 1, and $a-1$ is negative; hence the linear expansion is negative. The superficial expansion is the difference between the areas before and after the strain, viz. $\{\pi(ar)^2 - \pi r^2\} = \pi r^2(a^2-1)$; * hence the superficial expansion is proportional to (a^2-1). A square similarly affected has its sides and area increased or diminished in the same ratios: so would a parallelogram or any other plane figure, if the linear expansion were the same in all directions. Again, suppose a globular body to be thus uniformly expanded; it increases in size and becomes a larger globe: if its radius increase in the ratio $1 : a$, its bulk will increase in the ratio $1 : a^3$, and its cubical expansion will be proportional to $a^3 - 1$. Cubes, parallelepipeda, and all other solid figures, would under the same circumstances become larger or smaller cubes, parallelepipeda, etc., whose sides and bulk would bear similar ratios to their original dimensions.

Suppose a square to be unequally dilated or contracted along axes parallel to its sides, the square will become a parallelogram. A circle will thus become an ellipse; an ellipse will become an ellipse of another form. As a circle is an ellipse of a particular form whose length (its major axis) is equal to its breadth (its minor axis), any ellipse may be converted by a strain into a circle, if its axes be in due proportion lengthened or shortened. If the expansions along the two rectangular axes be in the ratios $1 : a$ and $1 : b$, the area of the resultant parallelogram, ellipse, or circle, will be to that of the original figure in the ratio $ab : 1$.

If the body be a cube and be unequally expanded in directions parallel to its sides, it becomes an unequal-sided parallelepipedon. If the several

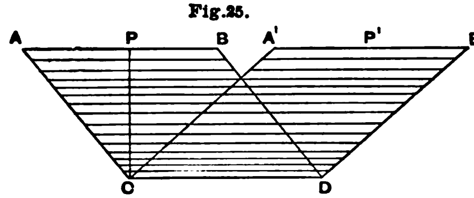
* $\pi = 3{\cdot}14159\ldots$, the ratio of the circumference of a circle to its diameter. The area of a circle $= {\cdot}7854 \times \text{diar.}^2 = 3{\cdot}1416 \times \text{rad.}^2 = \pi r^2$.

sides expand in the respective ratios $1:a$, $1:b$, and $1:c$, the bulk of the parallelepipedon will bear to that of the cube the ratio $abc:1$. A sphere strained in the same way will become an ellipsoid, a so-called "strain-ellipsoid": any ellipsoid will become an ellipsoid of a different form, and may become that particular kind of ellipsoid known as a sphere, an ellipsoid whose three axes are equal to one another.

It will be seen on drawing any of the figures just described that any two parallel lines drawn through the body in its original form will be parallel after the strain. In this kind of strain, called Homogeneous Strain, there are always three axes, which were at right angles to one another in the body in its original form, and which continue to be so after the strain. These are the axes of the strain-ellipsoid into which an imaginary sphere existing in the body would be transformed by the strain.

Shear. — If a body be so distorted that one plane passing through it is fixed while others move past this at rates proportional to their distances from it; — if, for example, the body ABCD (Fig. 25) be so distorted that CD remains in its original position, while the line AB travels into the position A′B′, the body is being sheared, or is undergoing the kind of deformation or strain known as a Shear.

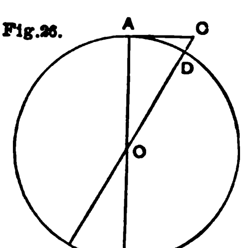
Fig. 25.



The greater the distance of any plane in the body from the plane passing through CD, the greater will be its displacement from its original position; and a body so sheared we may conceive as made up of an indefinite number of indefinitely thin layers which relatively move by slipping over one another. A shear is measured by the amount of relative motion between two non-distorted planes which are situated at a unit distance from one another, and which remain parallel. In Fig. 25 the amount of the shear of ABCD is the ratio of the displacement AA′ or PP′ to PC, the shortest possible line drawn between the two parallel planes AB and CD, and vertical to them both; that is, the Shear is equal to $PP'/PC = \tan PCP' = \tan \theta$.

Circular Motion. — If a body move in a circular path, as, for example, a stone whirled in a sling, its motion at every instant is compounded of a tangential motion and a motion towards the centre. If it pass the point A in Fig. 26 with such a velocity that it would in a unit of time, if not drawn towards the centre, have reached the point C, and if it be at the end of that interval found at the point D, it is evident that the acceleration towards the centre must have produced in unit of time the change of position represented by the line CD. In other words, the whirling sling-stone is constantly being drawn in from the tangential path, which, in virtue of its inertia, it would at every instant naturally take; and a planet in its orbit is constantly falling towards the sun, but does not proceed straight towards it, for the resultant of its tangential

Fig. 26.



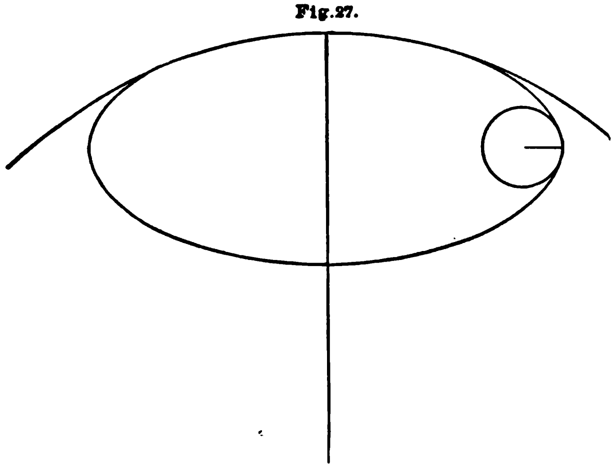
and its centripetal motions is an elliptical path which is approximately circular.

If the line CD be prolonged through the centre to the point E, Eucl. III. 36 shows that $CD \cdot CE = AC^2$. If v represent the velocity of the body in the direction AC, $\mathbf{v}$ the average velocity in the direction CD, and r the radius of the circle, we thus find $\mathbf{v}(2r + \mathbf{v}) = v^2$; or $2r\mathbf{v} + \mathbf{v}^2 = v^2$. (i).

If the unit of time taken be sufficiently small, the square of the small quantity $\mathbf{v}$ will be so small as to be negligible, and the above equation will become $2\mathbf{v}r = v^2$. (ii). But $\mathbf{v}$ is the average velocity of fall towards the centre O during the instant in question, and hence the velocity at the end of the interval is $2\mathbf{v}$; this is the velocity acquired in unit of time, and hence the acceleration towards the centre is $\mathbf{a} = 2\mathbf{v}$. Hence the equation (ii) may be written $\mathbf{a}r = v^2$ or $\mathbf{a} = v^2/r$; the **Acceleration towards the centre** of the circular path in which a body is moving is numerically equal to the Square of its Tangential Velocity v at any instant divided by the Radius of curvature. If a body be travelling in any other curve, there can at every instant be found a circle, a part of the circumference of which coincides, to an indefinite approximation, with the curve at the instant.

Curvature.—Any curve may be considered as made up of successive elements, each of which approximately coincides with a part of the circum-

Fig. 27.



ference of a particular "**osculating**" **circle**, which may always be found. For each element of the curve, the radius of the corresponding osculating circle—whose circumference coincides with that element, and which would have the same tangent—is called the instantaneous **radius of curvature**; and as the curve passes from point to point the osculating circle may be changed in respect of its radius or its centre. Thus in an ellipse, near the extremity of the major axis, the osculating circle of curvature is smaller than it is near the end of the minor axis, as is shown in Fig. 27. Accordingly, if a body move in a curved path, its acceleration at every instant towards the instantaneous centre of curvature is numerically equal to the square of the instantaneous tangential velocity divided by the instantaneous radius of curvature.

But in a curve, the Curvature is the angle through which the tangent sweeps round per unit of length of the curve, and this varies inversely as the

radius, as may be seen on comparing the circles in Fig. 28. The radius of I is twice that of II: the length $AA' = Cc'$ is supposed to be a unit of length. In I the tangent AB has swept round into the position A′B′ through an angle θ: the tangent CD has swept through twice as great an angle, the length of circular path traversed being the same: wherefore the curvature (as above defined) of the circle II is twice that of the circle I, and that of any circle is inversely as the radius: and since curvature and acceleration

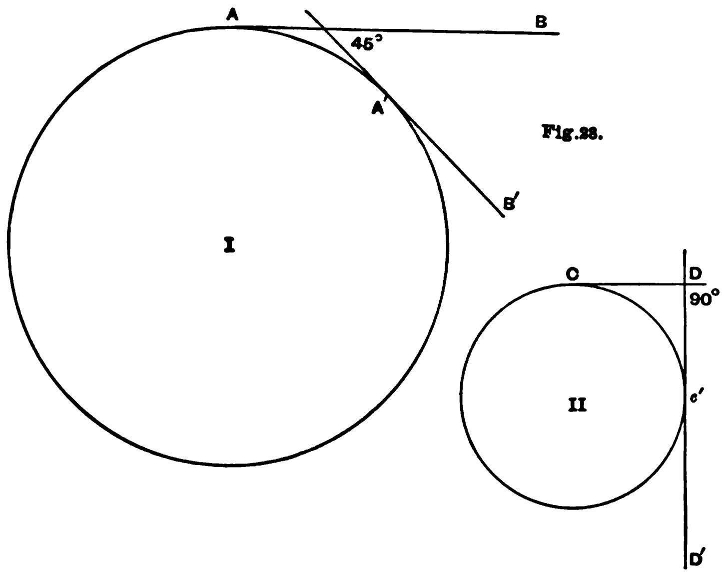


Fig. 28.

towards the instantaneous centre both vary inversely as the radius, they are proportional to one another, and therefore the acceleration of a body moving in a curved path is directed towards the instantaneous centre of curvature, and is equal to the product of the square of the instantaneous tangential velocity into the curvature. $a = v^2/r$; $1/r = c$; $\therefore a = v^2 c$, where c is the curvature. Hence a comet turning sharply round the sun, the curvature of its path being very great, has a very great acceleration inwards.

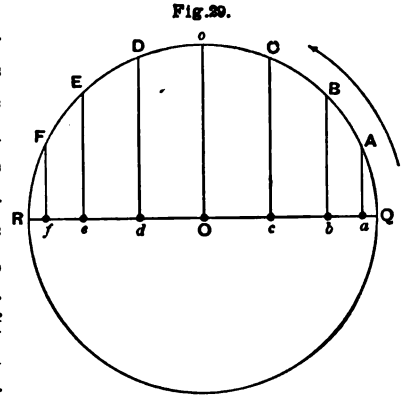
SIMPLE HARMONIC MOTION AND WAVE-MOTION.

Motion in a circle may be practically effected by a heavy ball suspended by a string, and set to swing in a circular path. A pendulum set to swing in this way goes by the name of the "Conical Pendulum." If the path of the bob of the so-called conical pendulum be looked at from above, it appears circular: if looked at from a point somewhat to one side, it appears elliptical: as the eye approaches the level of the plane in which the bob travels, its path appears to be an ellipse comparatively long and narrow; and as the eye is placed exactly on a level with that plane, the bob appears to travel backwards and forwards

in a straight line. In a similar way, the satellites of Jupiter, which travel round that planet pretty nearly in the plane of the Ecliptic,* and therefore astronomically on a level with ourselves, seem to travel backwards and forwards in lines nearly straight. The bob of the conical pendulum and the satellites of Jupiter appear to move very slowly at the end of their apparently linear courses. This is because the moving body is really travelling either towards the eye of the observer or away from it at the time when it appears to be at the end of its swing. When it is travelling right across the field of view, when it is in the middle of its apparently linear path, it seems to travel rapidly. Just in the same way a railway train seems to be moving very much faster when it runs right across the field of view than when it is coming or going round a curve, and is seen not broadside but end-on.

If we represent the circle in which the body is moving by the circle QAR, and its apparent linear path by the line QR, and if we represent a certain number of positions of the body in the circle by the points A, B, C, D, etc., we may define the apparent motion of the body in the straight line QR by finding the points *a*, *b*, *c*, etc., to which the respective positions of the body in the Circle of Reference correspond. This is done by drawing lines A*a*, B*b*, etc., at right angles to the line QR. It will be easily seen that if QA, AB, BC, C*o*, etc., be equal to one another, the corresponding lines drawn along QR are longer near the centre of that line than near its ends; and these represent the spaces apparently traversed in equal intervals of time.

Fig. 20.



A representation of the relative values of these lines Q*a*, *ab*, etc., is obtained as follows. If the line QR be taken as the axis of x, the line OA may be supposed to sweep round into the successive positions OB, OC, O*o*, etc. As it does so, it forms an increasing angle with the line OQ. Then the lengths of the lines O*a*, O*b*, O*c*, etc., bear to one another the ratios of the *cosines* of the angles QOA, QOB, etc.

* The bodies which make up the solar system may be said in a rough way to be situated in a plane fixed in space, and called the Ecliptic, from which they do not very widely depart. Objects moving in their respective orbits in this plane may appear to pass and obscure — *i.e.* eclipse — one another, like so many ships at sea.

These angles will, if the corresponding motion in the circle of reference be uniform, depend directly on the time. If the angle swept through in unit of time be ω, that swept through in time t is $t\omega$: hence, if the starting-point in time be that instant at which the body is at the point Q, the apparent distance $\mathbf{x}$ of the body from the point O will be proportional to $\cos t\omega$, and, if a be the radius, will be equal to $a \cos t\omega$. Hence $\mathbf{x} = a \cos t\omega$. When $t\omega = 0$, since $\cos 0^\circ = 1$, $\mathbf{x} = a$: when $t\omega = 90^\circ$, since $\cos 90^\circ = 0$, $\mathbf{x} = 0$: when $t\omega = 180^\circ$, since $\cos 180^\circ = -1$, $\mathbf{x} = -a$: when $t\omega = 270^\circ$, $\mathbf{x} = 0$: when $t\omega = 360^\circ = 2\pi$, $\mathbf{x} = a$. As the radius continues to sweep round, the values of $\mathbf{x}$ repeat themselves.

Such a motion as that apparently executed backwards and forwards along the line QR is called **Simple Harmonic Motion** or S.H.M. Such motion must be studied with great care, for actual instances of it occur throughout the phenomena of Optics and Acoustics, and of many other parts of physics.

The length OQ or OR of the swing from the median position O is called the **Amplitude,** a, of the S.H.M. Simple Harmonic motion, then, is motion which is a periodic* function of the time (*i.e.* repeats itself at regular intervals), which is effected backwards and forwards along a line, and which may be studied by comparison with uniform motion round a **circle of reference,** of which the line of S.H.M. is the diameter, and of which accordingly the Amplitude of S.H.M. measures the Radius.

The **Period,** T, of a S.H.M. is the *interval of time* which elapses between the passage of the moving particle over a certain point and the next passage of the same particle over the same point in the same direction. This corresponds to the time during which one complete revolution would be effected round the circle of reference; $T = 2\pi/\omega$.

It is understood that when the moving body appears to travel from left to right, its motion is positive; when it moves from right to left, its motion is in the negative direction. When the particle is at Q in Fig. 29, it is said to be in its position of greatest positive elongation: when at R, it is in its position of greatest negative elongation.

At any instant the position of the particle executing the S.H. Motion may be stated in terms of the **Phase** of the S.H.M. at that instant,—the Phase being the interval of time, the frac-

* If x vary when y varies, as, for instance, if $x = ay$, or if $x^2 = y^3 + ay^2 + by + c$, or $x = \log y$, or if in any other way whatsoever the value of x depend on that of y, x is said to be a *function* of y; and if x recur to the same value while y goes on uniformly increasing or diminishing, x is said to be a *periodic* function of y: if $x = \cos y$, or $= \tan y$, or $= \sin y$, etc., as y goes on increasing, x recurs to the same values, for $\cos y = \cos (2\pi + y) = \cos (4\pi + y) = \cos (6\pi + y) = \cos (8\pi + y)$, etc.

tion of a period, which has elapsed since the particle last passed through O, the middle point of its course, in the direction reckoned as positive.

The phase of a S.H.M., at any instant, may also be stated by specifying, for that instant, the corresponding angle swept round, in the circle of reference, past the point o; and the difference of phase between two S.H.M.'s may be stated by specifying the difference between two such angles, taken simultaneously.

If the starting-point in time be, not the instant at which the particle was at the point Q in the circle of reference (Fig. 29), but so many units of time after that instant that the angle traversed is not $t\omega$ but $(t\omega + e)$, then the displacement, or distance from O, along the axis of x is $\mathbf{x} = a \cos (t\omega + e)$. This term e is called the **Epoch.**

Acceleration in S.H.M. proportional to Displacement.— In Fig. 29 the moving particle, when it describes a circular path, does so under the influence of an acceleration v^2/r towards the centre. This may be resolved, when the particle is at any point A, into $(-\cos AOQ \cdot v^2/r)$ parallel to QR, and $(-\sin AOQ \cdot v^2/r)$ at right angles to it. The former component is alone effective in reference to a body moving in S.H.M. in the line QR, and, being always towards the centre, it is alternately in the same and in an opposite direction to the movement of the body. But this acceleration towards the centre $=(-\cos AOQ \cdot \mathbf{v}^2/r)=(Oa./r)\cdot(-\mathbf{v}^2/r)$ $= Oa \cdot (-\mathbf{v}^2/r^2) = Oa \times$ a constant negative number, for $\mathbf{v}$ (*i.e.* the maximum velocity in the line QR = the uniform speed v in the circle) and r are constant. The Acceleration when the particle is at any point a in its S.H.M. is thus proportional and also opposite to $Oa = \mathbf{x}$, the Displacement from O.

Again, the constant number $(\mathbf{v}^2/r^2)$ is equal to the square of $v/r = \omega$, the angular velocity in the circle of reference. Therefore

$$\omega = \sqrt{\frac{\text{acceleration at any point.}}{\text{displacement at that point.}}}$$

The acceleration, $a = \ddot{\mathbf{x}}$, at any instant being proportional and opposite to the displacement $\mathbf{x}$, we have $\ddot{\mathbf{x}} = -\,\mathrm{n}^2\mathbf{x}$, where n^2 is a factor always positive. This is a Differential Equation: the solution is, that at the end of any time t, the displacement $\mathbf{x}_t = a \cos \mathrm{n}t$, where a is the maximum value of $\mathbf{x}$. This agrees with the previous equation $\mathbf{x} = a \cos t\omega$, on the footing that $\mathrm{n}^2 = \omega^2$.

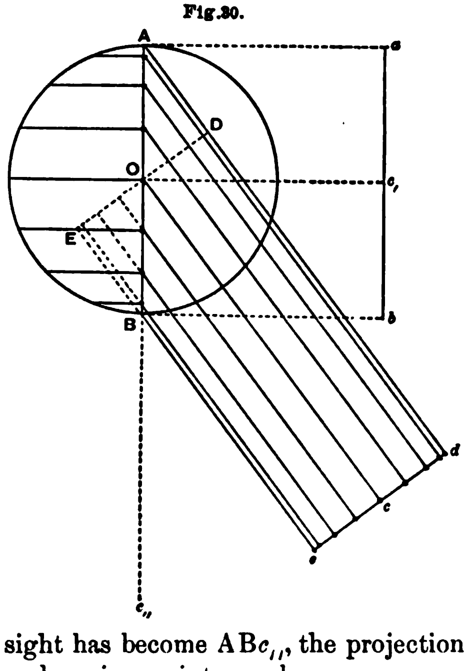
The **Frequency** of a S.H.M. is the number of periods per second, the number of revolutions round the circle of reference, or of complete to-and-fro oscillations per second. This is $n = 1/\mathrm{T} = \omega/2\pi = \mathrm{n}/2\pi$.

Isochronous S.H.M.'S.—Since ω is the angular velocity (page 75, *note*), and since the time taken to execute one complete revolution round the circle of reference is $\mathrm{T} = 2\pi/\omega$, then *if* ω, the angular velocity in the circle of reference, be constant,

the time T — that is, the period of the S.H.M. — is independent of the amplitude; for the amplitude does not enter into that formula which expresses the value of T, namely, $T = 2\pi/\omega$. This criterion, the constancy of ω, is satisfied if the quotient $\frac{\text{acceleration}}{\text{displacement}}$ be a constant number. In other words, if the acceleration with which a particle tends to return to its median position bear a fixed proportion to the displacement, the particle will execute a S.H.M. whose period is independent of the amplitude of oscillation. This proposition is one of high importance in the theory of the Pendulum, of Elastic bodies, of Sound, of Heat, and of Light.

Projection of a S.H.M. always an Apparent S.H.M. — It is understood that when a line AB is looked at from the position c in Fig. 30, that line appears to be shortened, and to assume the length DE, and the line DE, or de, at right angles to cO, is called a Projection of AB. There may be as many projections as there are possible directions of the line Oc. When the eye is placed somewhere in the line O$c_{,}$, the line AB does not appear to be shortened, and the projection ab of the line AB is equal to that line itself; when the direction of sight has become AB$c_{,,}$, the projection of the line AB, thus seen end-on, is a point merely.

Fig. 30.



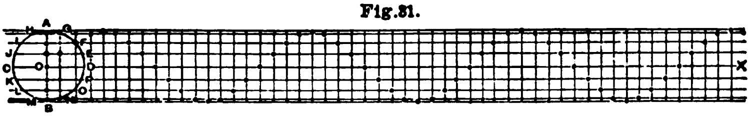
If the position of the point of view be intermediate between these extremes, as, for example, at c, the projection ed is to the original AB as the cosine of the angle AOD in the figure is to unity.

If now a body execute S.H.M. in the line AB, the diagram will show that, if regarded from c, it will appear to perform a S.H.M. corresponding in period and phase, though not in ampli-

tude, in the line DE; or, in other words, the projection of a S.H.M. is itself an apparent S.H.M.

Harmonic Curve. — If a S.H.M. in one line be compounded with a uniform motion in a direction at right angles to that line, the resultant path may be found by the following construction. Let A and B (Fig. 31) be the points of greatest elongation of the particle. Let the particle be also made at the same time to travel uniformly from left to right. Draw ACB, the circle of reference. On it lay off (say) sixteen equidistant points, D, E, F, G, etc.; lines drawn through these at right angles to the line AB determine the points on that line which define the positions of the particle, so far as these positions are determined by the S.H.M., at equal intervals of $\frac{1}{16}$ of the period. These lines being drawn as in the figure, other lines may be drawn parallel to AB and cutting the axis OX at equal intervals, each of which represents

Fig. 31.



the amount of motion from left to right during $\frac{1}{16}$ of the period of the S.H.M. The previous examples of composition of simultaneous motions will show us that the successive positions after successive intervals ($\frac{1}{16}$ of a period in this case) will be found by marking off points, such as those indicated in the diagram, the distances of which along the axis of x represent the displacements due to the uniform motion, and whose distances along the axis of y represent the displacements due to the S.H.M. If these points be joined they give rise to a very characteristic curve, the Curve of Sines, or the Harmonic Curve.

The geometrical property of this curve evidently is that, of any point in it the Abscissa (along OX) is proportional to the time, while the Ordinates (or distances from the axis of x) are proportional to the sines of angles, which are themselves proportional to the time. The ordinates, therefore, pass through positive and negative values alternately, while the abscissæ uniformly alter in value.

Take the closed figure bounded by any one half-period portion of the curve of sines lying entirely above or entirely below the base line OX, Fig. 31, together with the corresponding portion of the base-line cut off by it. The area of that figure is $2/\pi$ times the rectangle between the amplitude OA and the portion of the base-line so cut off. During such a half-period, therefore, the average value of the ordinates is $2/\pi \times$ the maximum value OA.

There may be several curves of sines, differing from one another in the amplitude of the S.H.M., or in the rapidity of

the uniform motion in the direction of the axis of x. Evidently, if the amplitude of the S.H.M. be greater or less, the undulations of the Harmonic Curve will be deeper or shallower; while, if the motion along the axis of x be slower or quicker, the undulations of the resultant curve will be closer together or farther apart.

When a pendulum is set to swing, its oscillatory motion is visibly quickest at the middle of its course and slackens towards each end of it; so that the motion of a pendulum is very much like S.H.M., and hence, if a pendulum be made to carry sand and to drop it as it travels, it will deposit a trace which is much thicker at each end of its course, where its bob is moving slowly, than it is at the middle where its course is rapid. If the pendulum be made to oscillate, while the frame which supports it is at the same time made to travel in a direction at right angles to the plane of oscillation of the bob, or — what amounts to the same thing — if the surface on which the sand is received be made to travel under the oscillating pendulum, which is suspended from a fixed support, the sand is deposited in a curve which can hardly be distinguished from the Harmonic Curve. But any motion which, when so compounded with a uniform rectilinear motion, produces the characteristic Curve of Sines must itself be a S.H.M., and hence the motion of the pendulum, projected on the plane which receives the sand, is approximately a S.H.M.

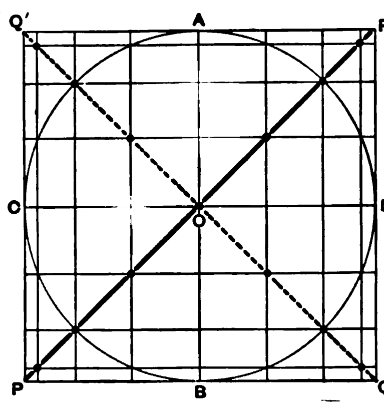
That the trace left by the falling sand does not with perfect exactitude coincide with any harmonic curve is due to the fact that though the motion of the bob in its curved path is nearly S.H.M., the necessary divergence between that curved path and the flat plane on which the sand is deposited expresses itself as a slight distortion of the resultant trace. If the arc of oscillation be small, this distortion is so very small that most of the properties of Simple Harmonic Motion can be practically demonstrated by the use of pendulums which record their own movement in some such way as that mentioned.

The trace left by such a moving body does not present, in the parts corresponding to the greatest positive or negative elongation, so steep an ascent or descent as it does when it crosses the axis OX (Fig. 31); this indicates that the body moving in S.H.M. is moving more rapidly at the centre of its course than at its ends.

Composition of Simple Harmonic Motions. — If the same body be subjected to two different S.H.M.'s, the problem of their composition may in general be solved with great ease by the use

of the respective circles of reference. (1.) Let the two motions be equal and in the same direction: the resultant will be a S.H.M. of double amplitude. (2.) Let the two motions be equal and in the same line, but differ from one another in phase by half a period: the resultant will be Rest. (3.) Let the two S.H.M.'s be equal, at right angles to one another (AB and CD, Fig. 32), and in the same phase, so that when the moving particle is at O it is moving in a positive direction with reference to both axes: its real course will be in a line PP′, making an angle of 45° with both AB and CD. (4.) If it be half a period behind in one of the S.H.M.'s, so as to be moving in the + direction (from O to D) with reference to one axis, and negatively (from O to B) with reference to the other, the resultant will be S.H.M. in the line Q′Q. (5.) If the one S.H.M. be a quarter period behind the other, so that while the moving particle is at the middle of (one, say) its vertical oscillation, it is only just leaving the point of greatest negative elongation, in respect to the other — its horizontal oscillation — its motion will be compounded of one forwards, from C towards O, and one upwards, past O towards A; the result will be that the motion of the body will be restricted to the circumference of the circle DBC, and the body will move round that circle in the direction CA. Similarly (6), if the horizontal movement be in advance of the vertical by a quarter period, so as to be already bringing the body back from its position of greatest positive elongation while it is still moving vertically upwards past O towards A, the body will travel in the circle DAC in the direction DA. Hence we have the very important proposition that motion in a circular path may be considered to be made up of two S.H.M.'s, the one a quarter of a period in advance of or behind the other, according to the direction in which the body travels in the circle.

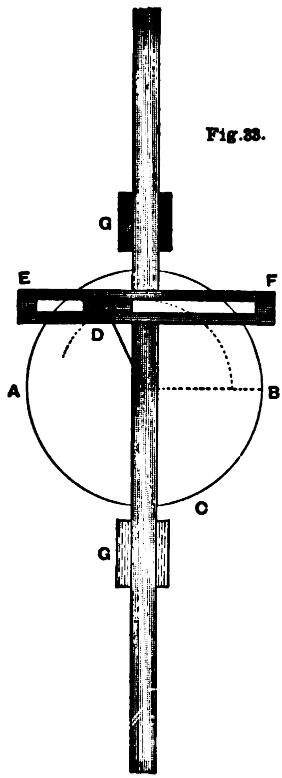
Fig. 32.



Though the conical pendulum shows this when its motion is watched, perhaps the simple piece of mechanism drawn in Fig. 33 may make it even plainer. The circular plate ACB has

a pin D set in it. This pin works in a sliding piece within a slot in the frame EF. The frame EF is connected with two sliding bars, which run between the guides G, so that lateral motion is impossible. Let the circular plate ACB be rotated uniformly: the frame EF will be moved upwards and downwards alternately, while the pin D will move in the slot from right to left and from left to right alternately. It will easily be seen on making a model, or on imagining the diagram to act, that the oscillations of D in its slot, and those by which it produces alternating ("reciprocating") motion of EF, do not agree in phase, but differ by a quarter of a period, the one being at the middle when the other is at the end of its course.

Fig. 33.

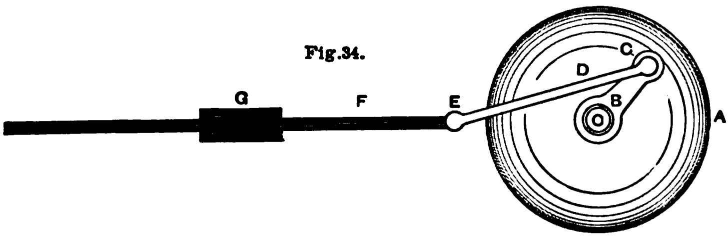


The circular motion of the pin D is, therefore, compounded of two S.H.M.'s, of which one is easily conveyed to the frame EF, while the other cannot, because of the arrangement of the guides G, be so conveyed. The apparent conversion of the circular motion of the disc ACB into the reciprocating motion of the sliding-bars is in reality, then, due to the suppression of one of its simple harmonic components; and the motion of the sliding-bars is exactly S.H.M. if D rotate uniformly.

Circular transformed into Reciprocating Motion.—In accordance with this principle, mechanism intended to transform rotatory into reciprocating motion is in reality mechanism which, with more or less completeness, suppresses one of the S.H.M.'s of each particle of a rotating body. The most usual device is that of a **crank**; this may be seen in one form or another in almost every piece of machinery worked by steam-power. In Fig. 34 the wheel A is rotated almost uniformly, and the crank B is turned round along with it; attached to the crank B by a joint at C is the rod D, which is, in its turn, attached by the joint E to the rod F; this runs between the guides G. Here the motion of the bar F between the guides is only approximately Simple Harmonic, but approximates more and more nearly to that condition the longer the bar D and the shorter the crank B, or, in other words, the less F and D together diverge from a straight line. The rod F may be made to work a pump-handle, a saw, or any such contrivance whose use requires reciprocating motion.

Conversion of Reciprocating into Circular Motion. — If in Fig. 34 the rod F be supposed to be pushed towards the crank B, then D will be pushed over towards C, and the wheel will turn until E, O, and C are in the same straight line. No further pushing will make the wheel A turn any farther; neither will pulling, when the crank is in this position, have any effect. If in the same figure the rod F be pulled instead of pushed, the points E, C, and O will come to be in the same straight line; and in this

Fig. 34.



position, again, neither pulling nor pushing will have any effect in making the wheel turn. There are therefore two positions or **dead points** in which a piston cannot by means of a crank set a wheel in motion. If, however, the wheel A be heavy, or, better still, if it be connected with a heavy fly-wheel, when it arrives at the dead points its Inertia, or that of the flywheel, manifests itself by the wheel A rotating past these unfavourable positions into others in which the reciprocating movement of F can act effectively; the wheel A is thus set in continuous motion. An example of this is furnished by the treadle and flywheel of a lathe or of a sewing-machine.

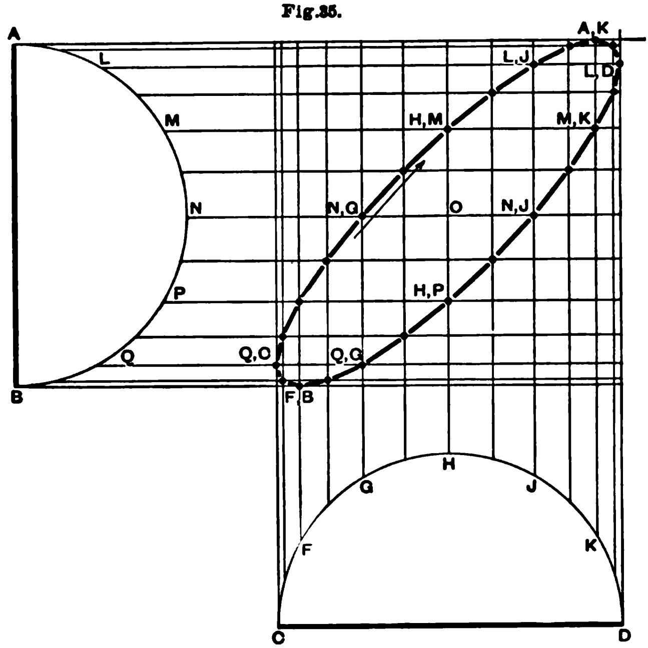
In the marine-engine, since there can be no flywheel on board ship, some other contrivance is necessary. Two cylinders, and therefore two reciprocating pistons, both acting on the same wheelwork, are so arranged that when the one crank is at its dead points the other is at the middle of its course, and therefore at its position of greatest advantage; or there may be three, at a mutual angle of 120°.

Composition of Simple Harmonic Motions (resumed). — Let the two S.H.M.'s differ by some other fraction of a period than the half or the quarter, as, for instance, the twelfth: if they be equal in amplitude they both have the same or equal circles of reference. Let the circumference of these be divided into twenty-four equal parts, as in Fig. 35, in which only a part of each circle of reference is shown.

If, now, the particle be at the middle of its course with reference to the S.H.M. along the axis BA, and if it be at the same time $\frac{1}{12}$ of a period behind (so as not yet to have arrived at the central point) in its execution of the S.H.M. referred to the axis CD, the point at which it must be situated in order to satisfy both these conditions must be N, G. When $\frac{1}{12}$ period has elapsed, it will have advanced to the middle of its horizontal course, but will have moved vertically as far as the point M, H; at the end of another $\frac{1}{12}$ it will be at the point L, J, then at

A, K; then it still advances horizontally to the limit of its course, but returns along AB to L, thus reaching the point L, D; then it returns to M, K, and so on, and in this way it describes an **ellipse.** When the difference of phase is less, the point N, G is nearer to O, and the ellipse is narrower; when there is no difference of phase, the point N, G coincides with O, and this ellipse is a straight line, as has been already learned (Fig. 32). When, on the other hand, the difference of phase is greater, the point N, G is farther from O, and the ellipse widens out until, when

Fig. 35.



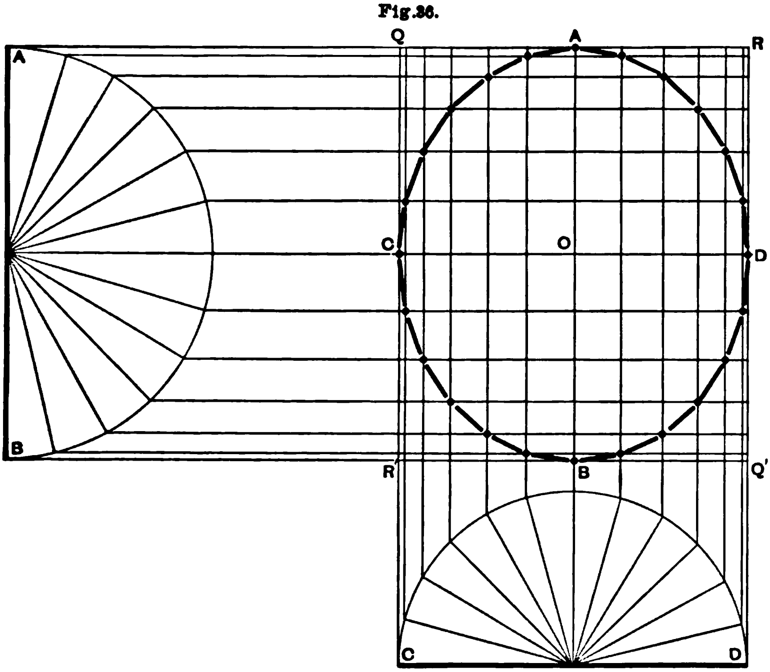
the difference of phase is $\frac{1}{4}$ period, the point N, G is opposite to C, and the ellipse is a perfect circle.

When the amplitudes are not equal, the circles of reference will not be equal. If the two S.H.M.'s be in AB and CD, the corresponding construction is shown in Fig. 36. If they be in the same phase, the resultant is S.H.M. in the line R′OR; if they differ in phase by half a period, the resultant is S.H.M. in the line QOQ′; if the difference of phase be $\frac{1}{4}$ period, the path is the ellipse ADBC, traversed in the direction BC if the S.H.M. in AB be $\frac{1}{4}$ period in advance, and in the direction BD if it be $\frac{1}{4}$ period in arrear. If the difference of phase be any other fraction of a period, the resultant will be motion in some other

ellipse contained within the same bounding rectangle QRQ′R′. The construction is the same as that of Fig. 35.

Composition of S.H.M.'s of different period. — The same method with little modification may be here employed. The respective circles of reference are drawn and are divided into arcs corresponding to equal intervals of time. The lines representing the S.H.M.'s are divided in accordance with the now well-known construction, and the positions of the body traced out accordingly.

Fig. 36.

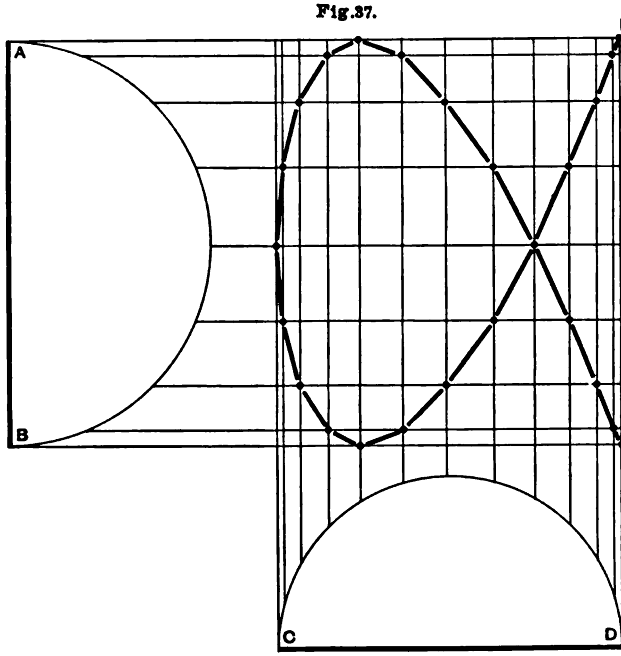


In Fig. 37 the periods are as two to three, the period of the vertical S.H.M. being the shorter: the ranges of oscillation are represented by the lengths of AB, CD respectively. The respective circles of reference are drawn: they are equally divided into arcs corresponding to intervals of time arbitrarily chosen, say the sixteenth part of the period of the more rapid oscillation in AB, this being the $\frac{1}{24}$ of the period of the slower oscillation in CD. The arcs AB and CD having been thus divided into segments corresponding to equal intervals of time, the usual construction enables us to trace out, point by point, the path of the body, which — if we assume the body to be in the position

of greatest positive elongation with respect to both S.H.M.'s simultaneously, and therefore to touch the point R, there being there no difference of phase — we find to be a looped curve over which the body travels backwards and forwards without quitting it.

In Fig. 38 the period of the S.H.M. in CD is $\frac{4}{5}$ of that in AB. The construction is essentially the same. The arcs cut off on the circle AB must subtend angles at their centre $\frac{4}{5}$ of those similarly subtended by the arcs on CD. Hence the circle AB

Fig.37.



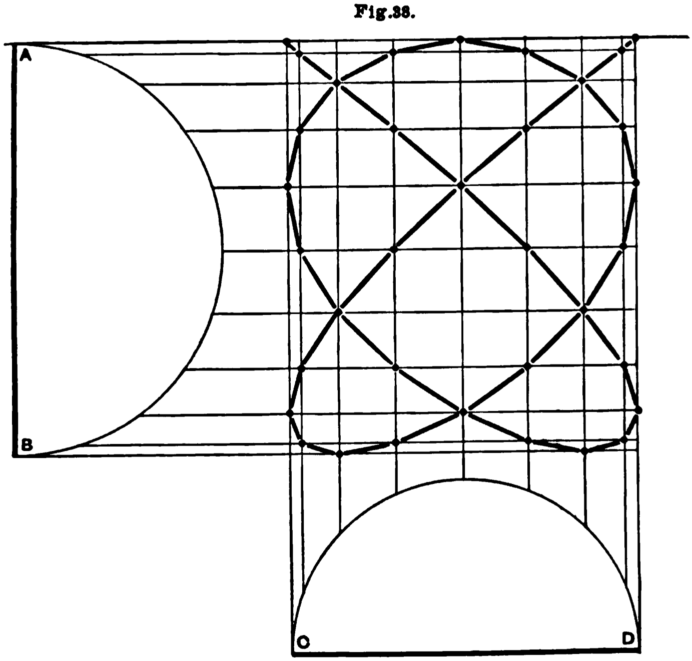
has been divided into arcs, each of which represents one-twentieth of the circumference, while the circle CD has been divided into 16. These curves may assume a variety of forms depending on variations in the relative lengths of AB and of CD.

Composition of S.H.M.'s differing in Phase. — If the two S.H.M.'s differ in phase, and if those chosen as illustrations be, for convenience of reference, the same as those of Figs. 37 and 38, the same circles may be drawn and divided in the same way. If the difference of phase between the two S.H.M.'s correspond to such a fraction of the period of the more rapid oscillation as may be represented by the arc DE

(whether this be an aliquot part of the circumference or not), the body cannot be at the extremity of both its S.H.M.'s at one time, and when it is opposite the point A it will simultaneously be opposite not the point D but the point E. Figs. 39 and 40 indicate the modifications undergone by the resultant curves of Figs. 37 and 38 in consequence of such differences of phase.

These resultant curves vary considerably in form, according to the amount of difference of phase of the component S.H.M.'s. Fig. 41 shows, for example, a series of modifications of the curve

Fig. 38.



of the ratio 1 : 2, in which the more rapid oscillation is in advance by periods which successively differ from one another by one-eighth of the period of the more rapid oscillation.

A S.H.M. in a third dimension may be compounded with two in a plane.

Composition of non-commensurable S.H.M.'s.—The periods in all the cases already considered have been commensurable, *i.e.* they have borne to one another ratios expressible in whole numbers, and consequently, after a certain number of oscillations, the moving body has returned to the starting-point, and the path has been a closed curve which the body has traversed repeatedly. If, however, the periods be not commensur-

able, the body cannot return to the starting-point after any definite number of oscillations, and the path never becomes a closed curve.

Composition of S.H.M.'s whose periods approximate to an aliquot ratio. — If the periods of the two component S.H.M.'s

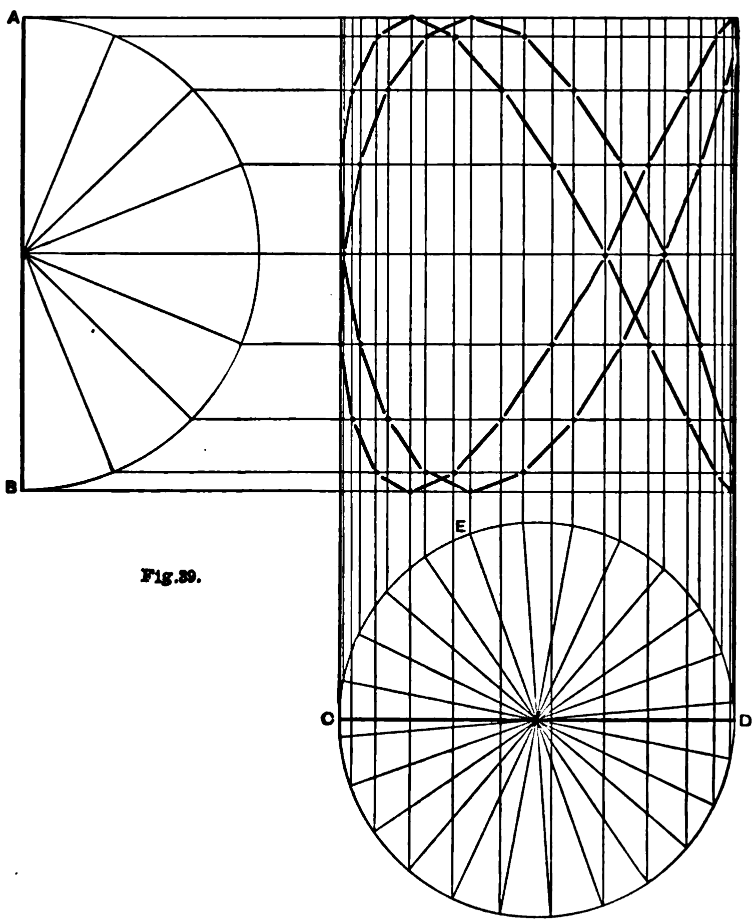


Fig. 39.

be, for example, very nearly as one to two, but not exactly so, the resultant curve may be, at a given moment, practically the same as that of (*a*) in Fig. 41. The moving body cannot, however, continue to maintain this parabolic path, for the want of exact aliquot proportion of the two periods causes one of the two S.H.M.'s to pass in advance of the other, which, as it were, lags behind, and thus it establishes an increasing difference of

phase. When this accumulated difference of phase amounts to $\frac{1}{8}$ the period of the more rapid oscillation, the path described is approximately that of (*b*) in Fig. 41. In this way, by continuous modification, the curve passes successively through all the forms shown in Fig. 41.

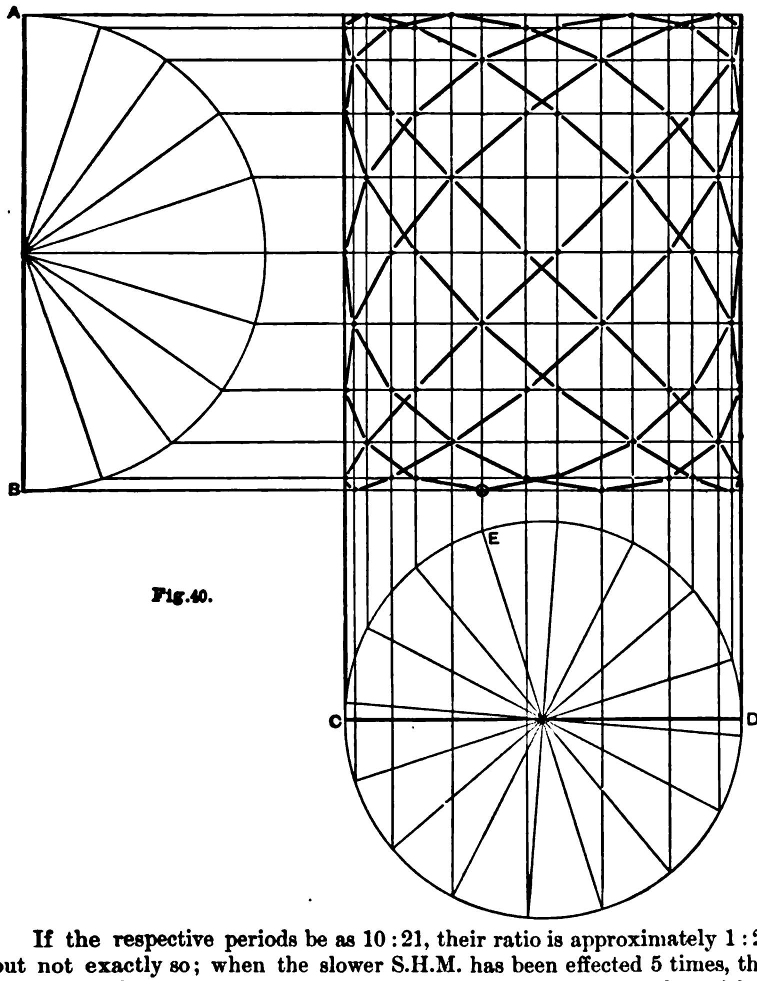


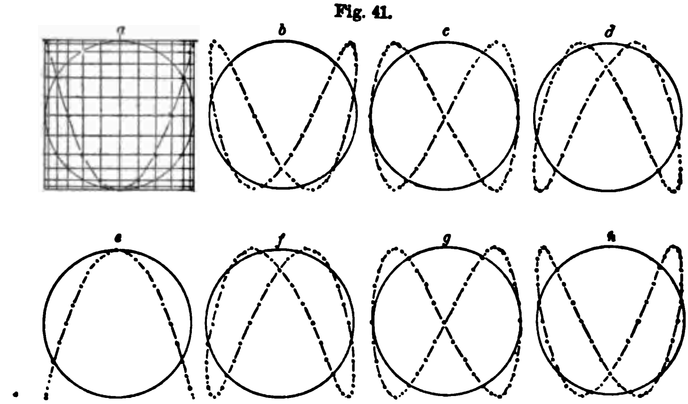
Fig. 40.

If the respective periods be as 10 : 21, their ratio is approximately 1 : 2, but not exactly so; when the slower S.H.M. has been effected 5 times, the quicker has been effected not 10 but $10\frac{1}{2}$ times, and consequently the quicker motion is a $\frac{1}{2}$ period in advance, and the form of the path has been modified from nearly that of (*a*) to nearly that of (*e*) in Fig. 41. When 10 of the slower S.H.M.'s have been effected, and of course in the same time 21 of the more rapid ones, the path resumes for an instant its original form (*a*).

If the periods were as 1000 : 2001, in the same way it will be seen that the path regains its original form, when 1000 of the more slowly-performed S.H.M.'s have been executed.

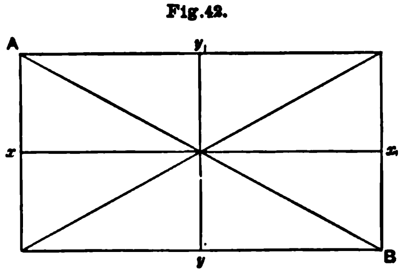
Hence the less the proportionate divergence from the simple aliquot ratio to which the actual ratio approximates, the greater the number of oscillations that must be performed, and hence the longer the time that must elapse before the original form of the path recurs, as it will do, approximately if the periods be non-commensurable, perfectly if they be commensurable.

Fig. 41.



Resolution of S.H.M. into two rectangular components.— We have seen that two S.H.M.'s at right angles to one another, and having the same period and phase, may be compounded into a single S.H.M. by a construction precisely the same as that of the rectangular parallelogram of velocities. Conversely, just as a velocity may be resolved into two component velocities in any two directions at right angles to one another, so may any S.H.M. be resolved into two S.H.M.'s in any two directions at right angles to one another. If in Fig. 42 the S.H.M. be in AB, it may be resolved into two, in $x_{,}x$ and $y_{,}y$ respectively.

Fig. 42.

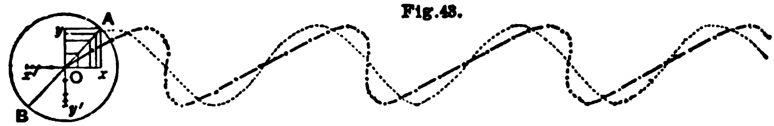


Any number of S.H.M.'s, in any directions, may be resolved into their components in three rectangular axes, and these may then be compounded.

Composition of any S.H.M. with a uniform movement in any direction.—If we wish to compound a S.H.M., which is effected in a line neither parallel nor at right angles to the axis Ox, with a uniform motion in the direction Ox, we must first break the S.H.M. up into its components in the respective directions Ox and $yy_{,}$. We may then compound the component S.H.M. in the direction $yy_{,}$ with the uniform movement in the axis of

x, thus producing (as was done in Fig. 31) the Harmonic Curve or Curve of Sines, indicated by a dotted line in Fig. 43. We may then from point to point compound this harmonic curve with the component S.H.M. in $x_{\prime}x$ by determining point after point in advance of or behind the dotted harmonic curve to an extent corresponding to the displacement produced by that com-

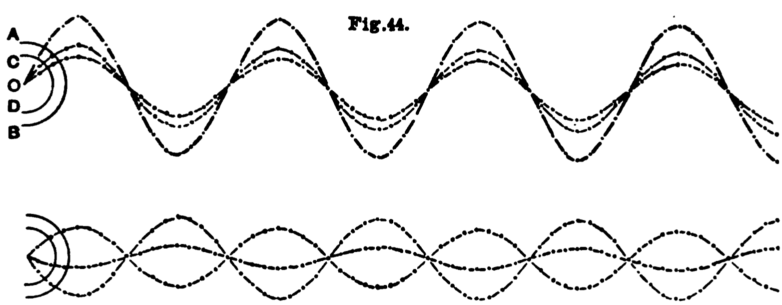
Fig. 43.



ponent. The resultant path, indicated by the thick dotted line, is compounded, then, of a uniform motion in the axis of x, a S.H.M. in the same axis, and another S.H.M. of the same period and phase in a line at right angles to that axis. The form of the resultant curve varies according to the speed of that uniform motion which is compounded with the oblique S.H.M.

Composition of two S.H.M.'s in the same line.—If two S.H.M.'s in the same line be compounded, the resultant motion will also be in the same line, and it is best studied by reference to the harmonic curve. Let two S.H.M.'s, which have the same periods and phases, and which are in the same straight line AB, have the amplitudes OA and OC, and let a corresponding Harmonic Curve be traced for each. Then the corresponding curve produced by the superposition of these two motions may be traced from point to point by adding the displacements separately indicated by the harmonic curves. This resultant is found to be a Harmonic Curve, and on careful drawing to

Fig. 44.



scale it may be shown absolutely to coincide with the Curve of Sines derived from a S.H.M. in a line whose direction is the same as that of AB, and whose amplitude is equal to the sum of OA and OC. On the other hand, when these two S.H.M.'s are in opposite phases, differing by half a period, so that while one raises the body above the point O, the other depresses it below that point, the resultant curve is also found to be a Harmonic Curve corresponding to a S.H.M. whose amplitude is equal to

the difference between the amplitudes of the two components. Hence two S.H.M.'s of the same period and in the same straight line will, when compounded, produce a single S.H.M. of the same period and in the same line, whose amplitude is the sum of the amplitudes of the components if they agree in phase, and their difference if their phases be opposed. Manifestly, if the two component S.H.M.'s be equal to one another, the resultant will be, in amplitude, double of either of them if they agree in phase, and will be zero — that is, the body will be at rest — if they be opposed in phase, the corresponding harmonic curve being in this latter case reduced to a straight line.

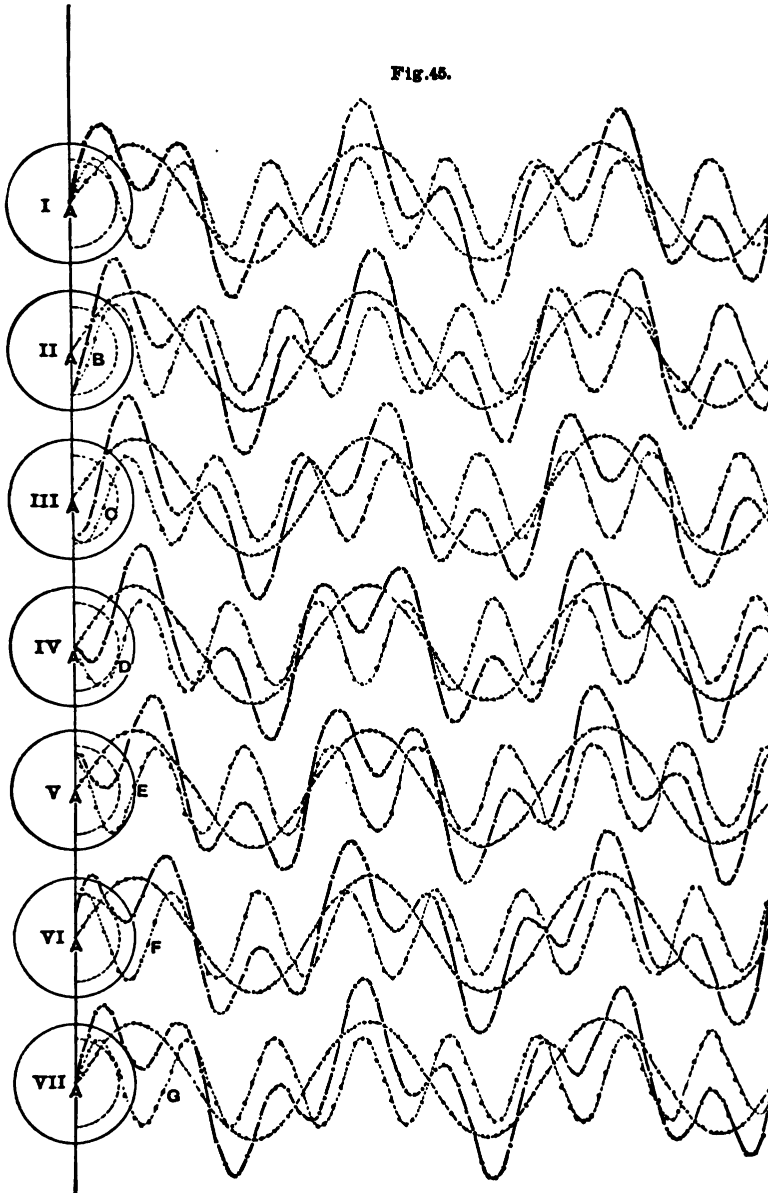
If the phases be neither in exact accord nor in exact opposition, the resultant curve is still Harmonic, but the amplitude is found by a construction the same as that of the parallelogram of velocities: lines representing the two component amplitudes are laid down at an angle representing the difference of phase, and the diagonal thus found represents, in length, the amplitude sought. Any number of S.H.M.'s of the same period and in the same straight line, but differing to any extent in amplitude and phase, may be similarly compounded, by a construction like that of the Polygon of Velocities.

If the two S.H.M.'s have different Periods, the result is more complex. Let the periods of the S.H.M.'s bear the ratio 3 : 8. Then the Harmonic Curve corresponding to the more rapid S.H.M. will present eight undulations for every three of the less rapid ones. These are drawn in Fig. 45 (I). On adding the displacements represented by these curves, the resultant may be traced from point to point, and is found to form a comparatively complex curve. Obviously there may be an indefinite number of forms of this resulting curve, for the ratio of the amplitudes may vary indefinitely.

In Fig. 45 the curves show the change produced in a compound harmonic curve by a **difference of phase** in the component S.H.M.'s. The curve (1) is that corresponding to the composition of two S.H.M.'s whose periods are as 3 : 8, whose amplitudes are as there shown, and whose phases at the point A coincide. In the next, the more rapid S.H.M. is seen to be, in respect of its phase, in arrear by an interval of time represented by the length of the line AB, and the superposition of the two curves now produces a resultant slightly differing from its predecessor, but, on the whole, similar to it. A similar construction produces the succeeding curves, in which the differences of phase correspond to the respective intervals AC, AD, etc. It will be plain that if the differences of phase be intermediate between

those chosen in the figure, there may be drawn any number of resultant curves intermediate in form between those shown.

Fig. 45.



The time taken by the moving body to go once through its periodic movement, or, in other words, the period of the result-

ant complex harmonic motion, is unaltered by variations in the amplitude and phase of the component S.H.M.'s, and depends only on their relative periods.

If in Fig. 45 the difference of phase were not constant but continuously increased, the curve would successively assume all the forms there shown, and it would naturally pass through all the possible intermediate forms, returning at intervals to the form (I).

Beats. — If two harmonic curves be compounded, of which one corresponds to a more rapid vibration than the other, the periods being approximately equal, the resultant curve will be one which at any one spot approximates in form to the curve of sines, but alternately waxes and wanes in amplitude. If the respective periods be as 2000 : 2001, the quicker oscillation gains $\frac{1}{2000}$ period on the slower at each complete S.H. movement, and at the end of 1000 of the slower S.H.M.'s the quicker is in complete disaccord with the slower; then, if the amplitudes of the two oscillations be equal, the particle affected is at rest; thereafter the quicker oscillation comes more and more completely into renewed accord with the slower, and at the end of 2000 of the slower oscillations or 2001 of the more rapid, the amplitude of the compound vibration is equal to the sum of those of the components. This is the cause of beats in music. If the periods approximate to any other whole-number ratio than that of equality, similar phenomena occur; at any one instant the curve resembles the corresponding compound harmonic curve, but alternately waxes and wanes; the deficiency of amplitude occurring once for each complete oscillation gained by the more rapidly vibrating body. Thus oscillations whose frequencies are 500 and 751 per second give one beat or period of relatively small amplitude during each 751 of the more rapid vibrations — that is, for every occasion on which it gains one oscillation on that number, 750, which would make the ratio of frequencies exactly the ratio 2 : 3. If the oscillations had been 500 and $750\frac{1}{2}$ per second, there would have been a beat every two seconds.

If the periods of the S.H.M.'s be non-commensurable, the resultant curve approximates in form to that of the nearest commensurable ratio, and successively assumes forms nearly resembling those assumed by that curve when the difference between the phases of its components gradually changes.

As a particular case of the composition of harmonic motions we may take the following problem, which is of importance in the Theory of Light.

A particle is acted upon by two simultaneous circular vibrations. These, considered singly, act in opposite directions, as in Fig. 46. Let them be identical in period and in amplitude. Let them be resolved into components at right angles to one another, which lie respectively in the lines AB and CD. Let the S.H.M.'s in AB coincide in phase, while those in CD differ in phase by half a period. In such a case the one circular vibration resembles the other, as an object resembles its image in a mirror situated in a plane parallel to AB; and the S.H.M.'s in CD will neutralise one another, while those in AB will reinforce one another; so that the result will be a S.H.M. in the plane AB, and of double amplitude.

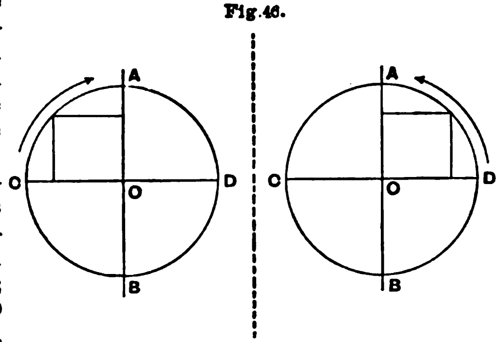


Fig. 46.

If, however, the components in CD do not differ in phase by half a period, the same considerations of symmetry do not apply in reference to the plane of AB; and the resultant motion cannot be a S.H.M. in that plane. If the circular motions be identical in period and amplitude, there must be some plane in respect to which the circular motions are symmetrical, as in Fig. 47; the resultant motion being a S.H.M. of identical period parallel to that plane and of double amplitude.

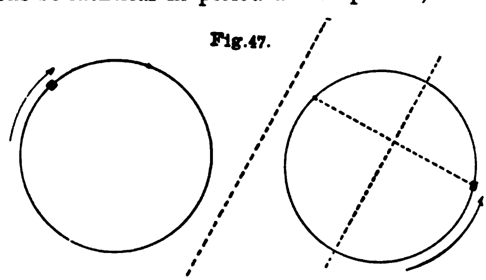


Fig. 47.

If the two circular motions differ in period, they will continuously differ in relative phase, and the resultant will be a S.H.M. of double amplitude effected in a plane which is constantly changing — a plane at right angles to that of the paper, and rotating round O, the point of rest in Figs. 46 and 47.

Hence a S.H.M. can always be considered as compounded of two circular oscillations; and if one of these be retarded or accelerated, whether suddenly or continuously, the plane of the S.H.M. will be rotated, suddenly or continuously.

Composition of several S.H.M.'s in the same Line. — This may be geometrically effected by the same method as that employed in the construction of the curves of Fig. 45, viz., by drawing separately the Harmonic Curves corresponding to each S.H.M., and adding from point to point all the respective displacements indicated by each of these curves. Fig. 48 shows the harmonic curves corresponding to five S.H.M.'s, each of which is drawn so as to represent its proper phase, period, and

amplitude relatively to the others. The resultant curve is periodic — that is, the complex form is repeated at regular intervals — if the periods of the component S.H.M.'s be commensurable; it cannot be if they are not so.

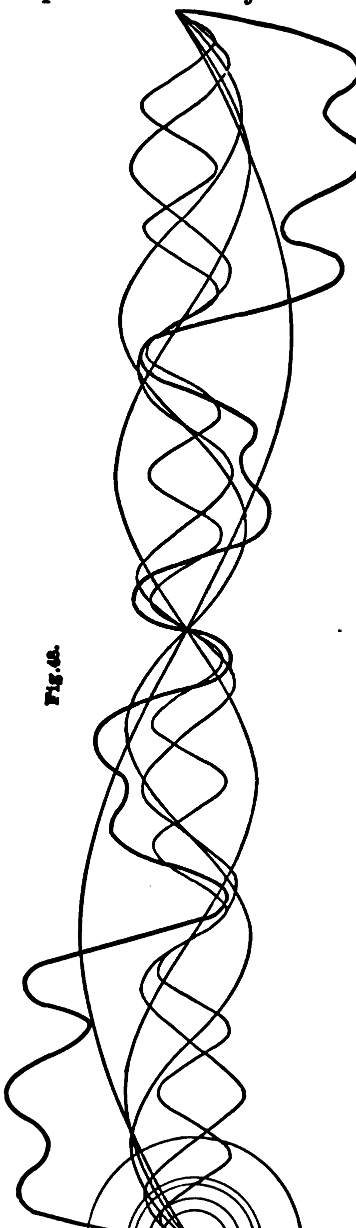


Fig. 48.

In all cases of a body affected by several simultaneous S.H.M.'s, in which the component S.H.M.'s have been in the same line, the **real resultant motion** of the particle may be studied by finding the complex harmonic curve produced by compounding these S.H.M.'s with a uniform movement, and on this curve laying a card in which a slit is cut which is laid at right angles to the axis of the curve. The card is then moved uniformly along this axis, the slit being kept at right angles to it. At any one moment only one point of the resultant curve can be seen in the slit, if that slit be made narrow enough. As the card is moved along, this point appears to move up and down in the slit with greater or less regularity, and the way in which it so moves is the way in which the body really moves when affected with the given simultaneous S.H.M.'s in the same line.

Fourier's Theorem. — The great variety in the forms of the resultant curves drawn to illustrate the previous discussion will prepare the reader to accept the positive statement that by properly choosing a number of harmonic curves, their amplitudes, their periods, and their phases,

and by compounding these, any Periodic Curve of any complexity may be built up, provided that the curve required never goes off to an infinite distance from the axis. Conversely, any complex periodic motion must be compounded of and may be resolved into a definite number of S.H.M.'s of definite periods, definite amplitudes, and definite phases. In order, however, that such motion may be periodic — that is, that the complex resultant motion may accurately repeat itself at regular intervals — it is necessary that the periods of the component S.H.M.'s should be exactly commensurate; for if they were not so, the resultant motion could not exactly repeat itself, and would not be periodic. Granted, however, the periodicity of the complex motion and the limitation mentioned above as to the form of its curve, Fourier's Theorem states that any such motion is compounded of a definite number of *commensurate* S.H.M.'s; and this is true not only of motion represented by the curve, but also, with wider interpretation, of any phenomenon which the curve may represent.

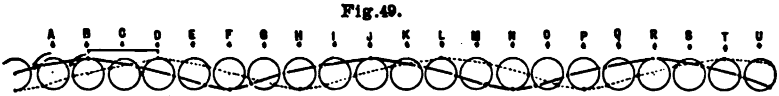
Tide Calculating Machine. — A number of wheels may have prearranged velocities imparted to them by being separately connected with cranks, of which a number are actuated by the same clockwork, through the intervention of toothed wheels. Thus the mechanism of Fig. 33 may be multiplied and any number of S.H.M.'s simultaneously produced. If the extremities of the oscillating rods be connected by a tense and flexible cord, that cord will be drawn upon or tightened to an extent depending at each instant upon the position of the wheels. One extremity of this cord being fixed to an immovable point, the other may be connected with a spring, and the varying distortions of that spring will indicate the varying tensions of the cord. To some point of the spring a pencil or pen may be attached, and under this writing-point a piece of paper may be unrolled at a rate proportioned to the velocity of the clockwork. When the mechanism is set in motion there is recorded upon the unrolling paper a curve which represents the summation of all the S.H.M.'s which are being executed. The preliminary adjustment consists in adjusting for each wheel the position of the pin which works in the slot — this regulating the amplitude, and also in modifying the angular position of each crank so as to represent the appropriate epoch of each S.H.M. at the moment of starting. A machine of this kind, after preliminary adjustment, enables a curve to be drawn which represents the height of the tide for every moment of a considerable period, such as a year, without the aid of further calculation than that involved in deducing from astronomical considerations, and from the tidal record of a place, a knowledge of the component S.H.M.'s, their respective periods, amplitudes, and epochs.

Oscillatory Movement of Systems of Particles. — In Fig. 49, A, B, C, D, etc., represent a number of particles in a linear series at equal distances. In the lower part of the figure

the same particles are seen, each describing a circular path in the plane of the paper.

The point B is represented as being $\frac{1}{8}$ period behind A, C an equal amount behind B, and so on. If a line be drawn through the positions of A, B, C, D, etc., at a given instant, it will be seen that the system of particles has assumed for that instant the form of a line more or less resembling the curve of sines. If such an interval of time be now supposed to elapse that each particle has moved forward through, say, $\frac{1}{4}$ of the circumference of its circular path, a line similarly drawn through the then position of the particles will present exactly the same form, but it will lie, as shown by the dotted line in Fig. 49, at a little distance (= BD) from its previous position. A similar result is obtained after the lapse of any other interval of time. So long as the particles continue to move in their respective circular paths, so long will there apparently be a Travelling of a Wave-Form along the system of particles. Particles whose

Fig. 49.



position is intermediate to those shown as equidistant will be found to occupy intermediate positions on the same line, which presents no abrupt angles but is a continuous curve; and so, if we roughly represent a linear system of particles by a chain or cord, we can understand how it is possible for a wave-form to run along such a cord while its component material particles never separate themselves by more than a certain distance from the mean positions round which they oscillate.

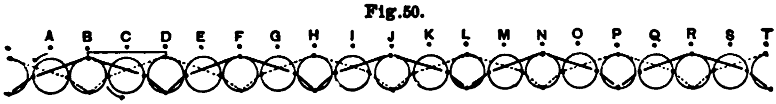
Wave-Length. — Such a wave-form is seen to consist of a successive series of parts which resemble one another. In Fig. 49 the particle B and the particle J are seen to be at the same time in their position of maximum displacement in the same direction, and the form assumed by the system between B and J is repeated beyond J and behind B. This distance between B and J is called the Wave-length, the distance between a point on one wave and a similarly situated point on the next wave. The point of maximum displacement in one direction may be called, from the analogy of waves on the surface of the sea, the Crest of the wave; the point of maximum displacement in the opposite direction may in the same way be called the Trough,

and the wave-length may be defined as the distance between crest and crest, or that between trough and trough. Each wave is in this case like its successor and its predecessor, and an observer stationed at a fixed point near the cord would perceive a succession of similar waves passing him. When the wave-length is great, a smaller number of waves will pass him in a given time than when the wave-length is small; twice the wave-length, half as many waves pass; half the wave-length, twice as many waves; thus the number of waves arriving at a given point in a given time is inversely proportional to the wave-length.

Velocity of Propagation.—If the particles perform their individual revolutions in, say, half the time supposed to be taken by those represented in Fig. 49, and if the amount by which B lags behind A be still the same fraction ($\frac{1}{8}$) of the time taken by each particle to perform a complete revolution, shortened though that be, the form assumed by the cord will be the same as in Fig. 49; the wave-length will be the same, but the wave will travel twice as fast. The velocity of propagation of the wave would vary directly as the velocity of oscillation of each particle.

If, however, the retardation of B behind A, of C behind B, and so forth, be independent of the rapidity of motion of these particles,—if, that is to say, the retardation be for a period of time which depends only on the distance between the particles, then, whatever the rate of oscillation of the particles, the wave will travel at the same rate, but the wave-length may vary, and the form of the wave may vary with it. This is illustrated by Fig. 50, in which the particles are represented as moving, for

Fig. 50.



example's sake, twice as fast as those shown in Fig. 49; while the interval of time by which B is delayed in its path as compared with A is exactly the same in the two figures, and therefore in Fig. 50 twice as large a fraction of the circumference expresses the difference of phase between A and B. In Fig. 49 the difference of phase between A and B is assumed to be represented by a difference of 45° in their positions on their respective circles; in Fig. 50, the movement being twice as rapid, and

B being retarded by the same interval of time as in Fig. 49, A must assume a position twice as far in advance of B, *i.e.* 90°.

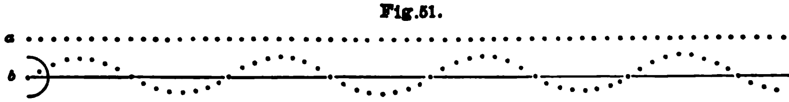
The results shown in this case by drawing the wave in angular outline, are — (1) that there are twice as many waves, the wave-length being half as great as in the previous figure; and (2) that the speed of travelling of the wave-form is the same in both cases; for if a sufficient time be supposed to have elapsed to permit each particle to have performed half a revolution (this corresponding to the time allowed in Fig. 49 for the accomplishment of a quarter revolution), and if the then position of the chain of particles be investigated, the wave-form will be found to have travelled forward through a space which is the same as in Fig. 49. If, then, the relative retardation of the particles be independent of the speed of the particles, the rate of propagation of the wave-form will be constant, and the only effect of a change in the rate of the oscillation of the particles will be a change in the wave-length and in the corresponding curved form assumed by the chain, and in the number of waves which pass any given point during a given interval of time.

The circular form is not a necessary attribute of the path of each particle: the path may be elliptical with a similar result, the difference being one in the form of the resultant wave.

The two limiting cases are of great interest. These are (1) the case in which the ellipse is reduced to a straight line at right angles to the chain of particles; and (2) that in which it is reduced to a straight line in the same direction as that chain. The former gives rise to Transversal vibrations; the latter to Longitudinal vibrations.

Transversal Vibrations. — Each particle effects a S.H.M. in a direction at right angles to the chain of particles. In Fig. 51 are shown (*a*) the series of particles unaffected by vibration; (*b*) the same particles affected by transversal vibrations,

Fig. 51.



executing S.H.M.'s, the phases of which differ to an equal extent in equidistant particles. The form assumed by the system of particles in this case is **exactly** the **curve of sines.** It is an easy matter to show in this case, as in those previously discussed, that as the particles perform their several S.H.M.'s the wave-form travels along the cord.

Composition of Transversal Vibrations. — It is quite possible for an indefinite cord or series of particles, such as the one considered in these paragraphs, to have several wave-motions running along it simultaneously, each producing its own effect, and the total effect of their united action should be traceable by some process of composition analogous to our previous composition of simultaneous movements. There are two main cases to be considered — (1) that in which the vibrations are in the same plane, and (2) that in which they are not in the same plane.

Transversal Vibrations in the same plane: their Composition. — Since the effect of transversal vibrations on an indefinite straight cord is to cause it to assume the form of the curve of sines, and since each vibration acts independently in this sense, the effect of compounding such movements is reduced to exactly the same problem as has been considered on pages 97–103, and there illustrated. If, for instance, we refer to Fig. 48, the resultant wave on a perfectly flexible and extensible cord on which the five wave-systems there represented were simultaneously travelling, would, for the instant at which the phases happened to be as there shown, assume the form there drawn; but when that wave had travelled a little way along the cord, the relative phases of the component transversal vibrations would have altered, and the wave would thus continuously alter its form from instant to instant, returning, however, nearly to the form shown in Fig. 48, as often as the same coincidence of phases recurred.

Their Resolution. — If a changing wave-form run in this way along a cord, and if the same form recur at regular intervals, the wave-form passing at every recurrence through the same changes, then Fourier's theorem applies, and the most complex phenomenon of this kind may be analysed or resolved into a number of separate waves, whose periods are commensurable, running simultaneously along the cord.

Transversal Vibrations not in the same plane: their Composition. — Here we have to consider two cases — (1) that in which the vibrations are at right angles to one another, and (2) that in which they are not so. The latter case differs from the former only in the form of the curve described by each particle.

Let the simultaneous vibrations be in planes at right angles to one another. The motion of each particle is confined to a plane at right angles to the line of the cord. In the plane in

which it moves, each particle describes paths such as those exemplified in Figs. 35–40. In these figures OA may be taken as representing the amplitude and direction of the S.H.M. in one plane, and OD as representing the amplitude and direction of the S.H.M. in the plane at right angles to it. If the periods of the component vibrations be not the same, the different particles of the cord will be in different phases of S.H.M., and the form of their respective paths will differ; and as the compound wave runs along the cord, the path of each particle will pass successively, and in the way exemplified in Fig. 41, through all those forms which are possible as the result of the rectangular composition of S.H.M.'s whose periods are those belonging to the wave-motions which are to be compounded.

If the periods of the vibrations to be compounded be the same, the points at which the respective curves of sines cross the line of mean positions will be the same for each vibration. Hence, if one particle describe an ellipse or circle, all the particles which are in motion describe circles or ellipses similar in form, though differing in size or in the direction of motion; while, as the wave runs along, any given particle will describe an ellipse or circle which alternately enlarges, diminishes, vanishes, and reappears, but in the reversed direction, first enlarging and then diminishing, and then vanishes to reappear, recommencing the cycle in the original direction.

This kind of movement may be roughly realised by taking a rope, fixing it at one end, and rapidly rotating the hand which holds the free end. The rope assumes, and may by practice and dexterity be caused to retain, a form in which there are a certain number of fixed points, the number of which may increase with the rapidity of movement of the hand. On each side of these steady points the particles of the rope are describing circles or ellipses in opposite directions. If this *condition* — instead of being steady in its position — *travelled* along a rope of indefinite length, it will, on consideration, become plain how a particle would rotate first in one direction and then in the other, and how, while rotating in each direction, the extent of motion is at first small, increases to a maximum, and then wanes away till it vanishes, again to reappear.

If the rectangular vibrations which have to be compounded be more than two in number, the problem of finding the approximate path at any instant is precisely that of compounding several S.H.M.'s. Point by point the independent displacements produced by each S.H.M., whatever the plane of that S.H.M., must be added together and the resultant points joined.

Resolution of Transversal Vibrations in general. — In the paragraph illustrated by Fig. 42, it has been seen that any S.H.M.

may be resolved into any two others in directions at any angles to each other. The only case of other than theoretical interest in its application to the doctrine of transversal vibrations is that in which the S.H.M. is resolved into two components at right angles to each other, these components then being of the same period and phase, and their amplitudes and directions being represented by the sides of a rectangle of which the diagonal may, in the same respects, represent the original S.H.M.

If a simple transversal vibration in one plane were prevented from taking place in a direction at right angles to that plane, such prevention would be superfluous, and the vibration would not be affected. If it were prevented from taking place in the plane in which it is actually occurring, obviously the vibration would cease. If, again, it were hindered by some cause which prevented any movement in a given plane, inclined to the plane of vibration at an angle intermediate to these extremes, then a reference to Fig. 42 will enable us to see that if a vibration in a plane passing through AB be prevented from effecting any vibratory movement in a plane passing through $yy_{,}$, the result is as if the vibration in the plane AB had been broken up into two components of the same phase and period as the original one, and executed in the planes $xx_{,}$ and $yy_{,}$ respectively, and the latter of these then extinguished; there is thus left only a vibration in the plane $xx_{,}$, the amplitude of which, as compared with that of the original vibration in the plane passing through AB, depends on the angle between AB and $xx_{,}$, being proportional to the cosine of that angle.

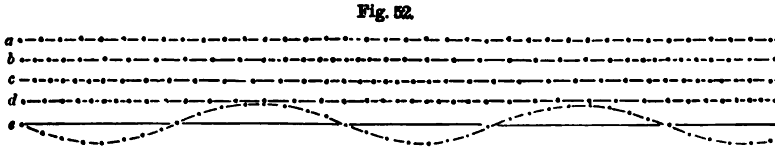
Let now the plane $yy_{,}$ rotate round the centre O; when its position is at right angles to AB, $xx_{,}$ sweeps round so as to coincide with AB, and there is neither diminution of the vibration in amplitude nor change in its direction; when $yy_{,}$ coincides with AB, $xx_{,}$ is reduced to nothing, and the vibration is completely stopped: between these limits there is an indefinite number of positions of $yy_{,}$, and an indefinite number of corresponding values of amplitude of the resultant vibration in $xx_{,}$, as that plane sweeps round at right angles to $yy_{,}$.

What is thus true of one transversal vibration is true of each of all those which may be running simultaneously along the cord; and as the effect of inhibiting vibration parallel to the plane of $yy_{,}$ (Fig. 42) is to restrict it to that of $xx_{,}$, whatever the original direction of AB, so, if a number of wave-motions affect the same cord simultaneously, and if the cord be restricted from executing any vibrations whatsoever parallel to a certain plane $yy_{,}$, the result will be a more or less complex

vibration restricted to the plane $xx_{\prime}$ at right angles to $yy_{\prime}$. If we can suppose such a cord, along which a number of waves are running, to be passed through a slot in a thick wall, so that all vibrations in a direction at right angles to this slot are completely prevented, and if then the same cord be passed through another such slot placed at right angles to the first, all vibration whatsoever will be prevented in the part of the cord which lies beyond the second slot, and that part of the cord will be at rest. If, however, the second slot be inclined at any other than a right angle to the first, a certain amount of vibration will pass through, which will be executed in the plane of the second slot, with an amplitude which will be proportional to the cosine of the angle between the two slots; in this way the more nearly the two slots coincide in direction, the greater the amplitude of that vibration which affects the cord beyond the double obstruction.

Longitudinal Vibrations.—The other limiting case of vibrations of a linear series of particles, spoken of on page 106, was that in which each particle performed a S.H.M. in the direction of the line of particles. The result is represented in Fig. 52, in

Fig. 52.



which *a* indicates the primitive position of the particles when at rest; and *b*, *c*, *d*, their positions after equal intervals of time, when oscillating in this way. Each particle has its amplitude of vibration, may be in a certain phase and have a certain period of oscillation: the wave-motion runs along the vibrating cord: several wave-motions of this kind may simultaneously affect it; and a complex longitudinal wave-motion may be analysed into simple wave-motions, as in the preceding paragraphs we have seen that we may analyse a complex transversal wave into its components. The study of this kind of vibration is, however, greatly facilitated by giving an arbitrary representation to the form of the wave. If the disturbance represented in Fig. 52 *b* be indicated by drawing lines at right angles to the line of primitive position, each of these lines having a vertical height, positive or negative, equal to the horizontal displacement of the corresponding particle, forwards or backwards in the line

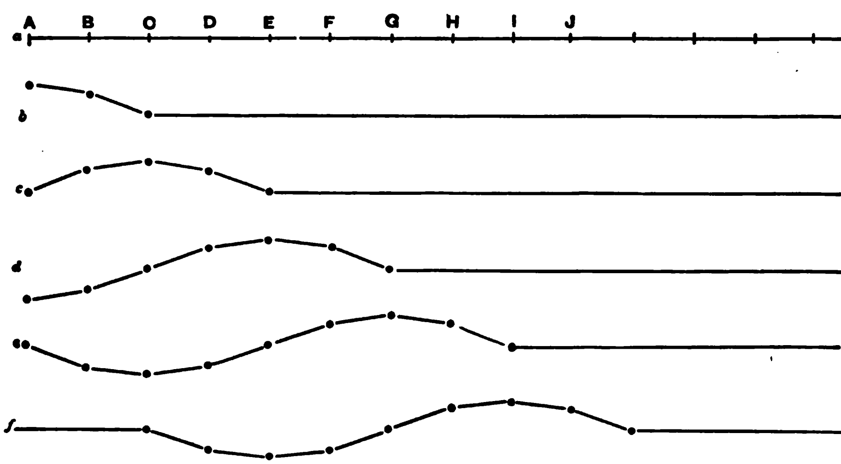
of the cord; by joining the extremities of these ordinates we shall produce, as has been done in Fig. 52 *e*, a curved line which is the curve of sines, or **harmonic curve,** simple or compound, with which we are already familiar. The interpretation of such a curved line would, however, be different in the case of a longitudinal vibration from that of the same form in the case of transversal vibrations. In the latter case, the curve represents the actual form assumed by the cord: in the former case, that of the longitudinal vibrations, it only indicates from point to point the extent of the departure of the corresponding particle from its primitive position. In a longitudinal wave, there are, at the successive points of zero displacement, where particles occupy their original positions, alternate maxima of crowding together and of separation of the particles, or alternate maxima and minima of density; while midway between these are points of maximum displacement, at which the density remains unchanged.

We have hitherto supposed the vibration to be permanent, and to be kept up by a continuous and periodic movement of each of the particles of a linear body. Let it be now supposed that there is some relation between the particles of this linear body, such that, when one particle is displaced, it executes a S.H.M., and, in some way exerting Force upon them, induces its neighbours to commence executing similar S.H.M.'s, following its own at intervals of time, and therefore with differences of phase, corresponding to their respective distances from it. The result will be as shown in Fig. 53. There the cord is first seen undisturbed; then the particle A being disturbed moves in S.H.M., and sets the following particles B, C, etc., in motion. On comparing *b* and *c* it will be seen that the Wave-Form travels along or is propagated, the "**Wave-front**" travelling onwards with a velocity equal to that of the permanent wave described in the preceding paragraphs.

If the disturbed particle A do not oscillate continuously, but travel once merely through a complete S.H.M., the figure 53 *f* shows that a single wave is propagated, leaving at rest the part of the cord which it has traversed, and continually displacing fresh particles if the cord be of indefinite length. If the motion of each particle be not completely extinguished after the execution of one exact S.H.M., but dwindle away with diminishing amplitude, the wave is not single, but is followed by a certain number of shallower waves, which presently die

away. If the wave-motion be kept up by a continuous supply of energy, there may be a continuous succession of waves, equidistant and following one another at equal intervals of time. The distance between any point in one wave and a precisely similar point in its predecessor or successor is the *wave-length* of the wave; the distance traversed by a given wave during one second is the *velocity of propagation* of the wave; and this velocity, divided by the length of each wave, plainly gives the *number* of waves which pass a given spot during a second of time, or, in other words, the **Frequency** of the undulation; while the reciprocal of this number gives the length of time

Fig.53.



taken by a single complete wave in passing a given spot, and therefore denotes the *period* of one complete oscillation.

If λ be the wave-length, v the velocity of propagation, and n the frequency or number of waves per second, T the period of each wave,

$$v = n\lambda = \lambda/T.$$

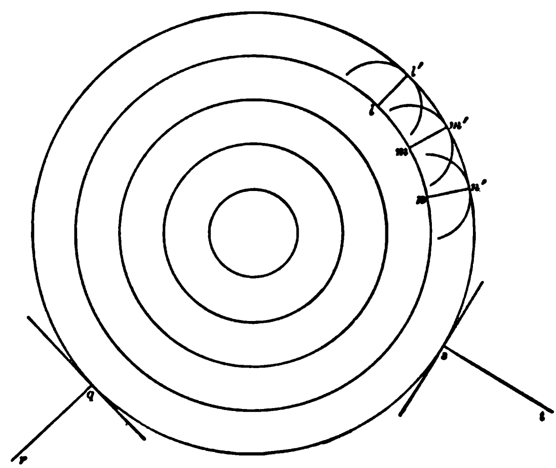
In these cases of propagation of wave-motion along a linear body the wave-front implicates only one particle, and its form is accordingly a single point. The amplitude of the wave as it travels along will be constant, if no energy be lost by the way.

Waves on a surface.—On a surface there may run, from a starting-point, waves of compression and rarefaction in the plane of the surface, or waves of vibration transverse to that plane. If one point be disturbed the disturbance is propagated

in all directions. If it be equally so in all directions, the wave-front will be circular; if the material be such that the velocity of propagation in one direction differs from that at right angles to it, the result will be an elliptical wave-front. In Fig. 54 is shown a wave in a membrane (an ideal solid which has length and breadth, but indefinitely small thickness), whose structure is such that the disturbance is propagated equally in all directions; and at *l*, *m*, *n*, three points are shown which themselves act as centres of disturbance, and the wave-front is propagated to *l′*, *m′*, *n′*, the whole still retaining its circular form. In the same way each point on an elliptical wave-front may act as the centre of an elliptical disturbance; the propagation of an elliptical wave is thus kept up.

In Fig. 54 it will be seen that the lines *ll′*, *mm′*, *nn′*, are parts of radii of the circles; and these lines are hence at right angles to the circles at *l*, *m*, *n*, and also at *l′*, *m′*, *n′*. The **normal** (*i.e.* a line perpendicular to the tangent) **to the wave-front** at any point at any instant is also normal to the corresponding part of the wave-front at any succeeding instant, if the medium be *isotropic*, *i.e.* if the velocity of propagation be equal in all directions. If, knowing the form of the wave-front at any instant, we desire to learn its form after any given interval of time, this may be found by drawing normals to the original wave-front equal to each other, and of lengths corresponding to the time indicated, and by joining their extremities. In this case we simply obtain a circle surrounding a circle, but we shall soon come upon cases in which the result is not so extremely simple.

Fig. 54.



When the initial disturbance is single, the wave which is produced is also single. The energy imparted to the system by

the single disturbance remains, a fixed quantity. As the circular wave progressively increases, it acts upon material whose mass increases with the radius; the energy imparted to each particle varies inversely as the radius; and the amplitude of movement of each particle in its S.H.M. varies inversely as the square root of the radius. In this way, the farther the wave has travelled from its centre of disturbance, the shallower it becomes.

At two instants of time, the respective radii of the circular wave are r and $r_{\prime}$, and the maximum velocities of the particles affected v and $v_{\prime}$. Let m be the amount of mass affected per unit of length of the wave-front. The whole mass set in motion is, at the two successive instants, $2\pi r \cdot m$ and $2\pi r_{\prime} \cdot m$. But the energy is constant, and $2\pi rm \cdot v^2/2 = 2\pi r_{\prime}m \cdot v_{\prime}^2/2$. Whence $r/r_{\prime} = v_{\prime}^2/v^2$, and $v_{\prime}/v = \sqrt{r/r_{\prime}}$.

But the amplitude of a S.H.M. is proportional to the velocity with which the particle passes its mean position; for the velocity in the circle of reference, which is equal to the velocity at the middle point of the S.H.M., is ($2\pi \times$ radius of circle $\div$ Period) = ($2\pi/\mathrm{T} \times$ amplitude); whence the amplitude $= v \cdot \mathrm{T}/2\pi$, and therefore varies as that velocity. The amplitudes therefore vary inversely as the square roots of the radius of the wave.

Waves propagated in a tridimensional substance.—The solid figure whose surface is everywhere at equal distances from its centre is a globe or sphere. If a disturbance at a point be propagated with equal velocities in all directions in space, the form of the wave-front will be spherical. If the velocities be unequal, the wave-front will be ellipsoidal or spheroidal. As the distance from the centre increases, the amplitudes of oscillation of the particles will diminish, for they vary inversely as the distance from the centre of disturbance.

When the radius of the spherical wave changes from r to $r_{\prime}$, the mass set in motion becomes greater in the proportion of r^2 to $r_{\prime}^2$; but the energy, proportional to $2\pi r^2 v^2$, or to $2\pi r_{\prime}^2 v_{\prime}^2$, remains constant. Therefore $r^2 v^2 = r_{\prime}^2 v_{\prime}^2$, and $v_{\prime}/v = r/r_{\prime}$, or the velocity of the particles varies inversely as the radius of the wave-front. But the amplitude varies as the velocity, and therefore varies inversely as the radius or the distance of the wave-front from the centre of disturbance; and the energy of motion of each particle—that is, the **Intensity** of its vibration—varies as the square of the amplitude, and therefore inversely as the square of the radius. This corresponds to the statement that the intensity of Light varies inversely as the square of the distance from the illuminating point.

Concentric Waves.—In all these cases, if the primitive disturbance be repeated at regular intervals, the wave system will assume the form of equidistant and concentric waves of circular, elliptical, spherical, ellipsoidal, spheroidal form, as the case may be. If the primitive disturbance be repeated at irreg-

ular intervals, the waves will still be concentric but not equidistant, and they will arrive at any point in an order of irregular sequence exactly reproducing the irregularity of the central disturbance.

If the central particle be affected by complex periodic disturbances, these will be, as regards the period and the phase as well as the relative, but not the absolute, amplitude of every component motion, faithfully reproduced in the motion of any particle affected by the resultant complex wave-motion; and this motion of such a particle may in many experimental instances be taken cognisance of by an observer.

Direction of the wave-front. — When a wave is said to be at a certain time and place travelling in a certain Direction, in an isotropic medium, it is meant that the normal to the wave-front, a straight line drawn at right angles to the wave-front—*i.e.* at right angles to its tangent or tangent-plane—takes the direction said to be that of the wave itself. In Fig. 54 the lines *qr* and *st* indicate the directions of the circular wave at the points *q* and *s*.

Flat wave-front.—The nearer the centre of disturbance, the more marked the convexity of the wave-front; the farther

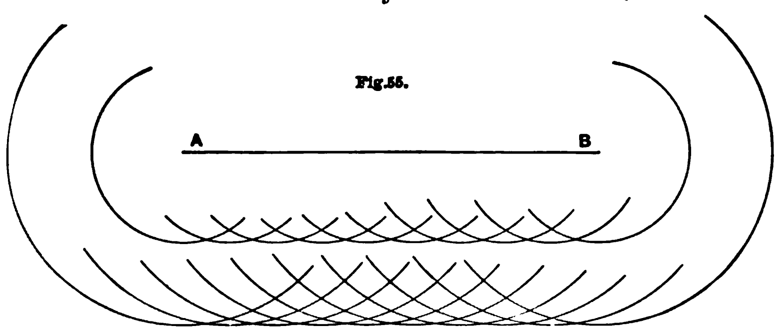


Fig. 55.

the centre, the flatter the wave-front: when the centre of disturbance is very far, the wave-front may for any small area be regarded as approximately plane, just as any small portion of the surface of a very large sphere may be.

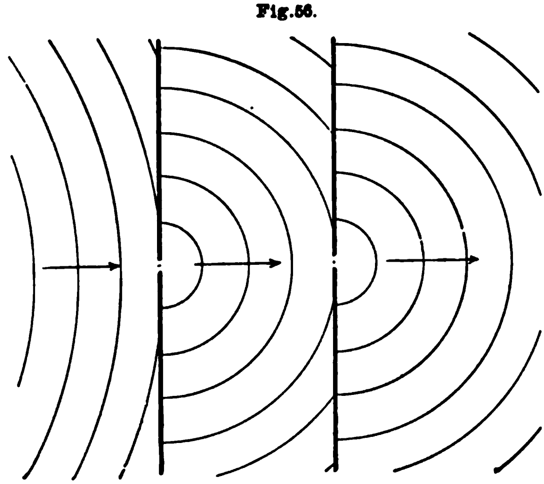
If, again, all the points of a plane surface act as centres of disturbance, then in the immediate proximity there may be a flat-fronted wave. If the disturbed surface be represented in section by AB in Fig. 55, the wave-front will be flat opposite its centre.

Modifications in the Form of a flat Wave-Front. — Through the upper and cooler layers of the atmosphere, waves travel with less rapidity

than they do through the lower. A flat wave-front is thus distorted; its upper part travels with the least velocity, and the wave-front comes to converge upwards. Points on a level with the point of disturbance may remain unaffected, for the wave-front is, in the main, restricted to its normals, and the sound ascends. If, however, there be a movement (such as that due to wind) in which the upper strata move more rapidly than the lower, it is not difficult to see that the wave-front may come to bear down, and that sound-waves may thus appear to travel with the wind, and to be best heard at certain distances, which depend upon the speed of the wind.

Wave passing through an aperture. — If a flat wave impinge upon an obstacle containing an orifice, it will, in part, be propagated through that orifice. If the orifice lead into the lumen of a cylindrical tube whose diameter is the same as that of the orifice, no lateral expansion of the wave-front is possible in that tube. If there be no such tube, there may or there may not be expansion of the wave beyond the orifice. Such expansion, if it take place at all, will take place in the way shown in Fig. 56. The disturbed particles in the aperture act as centres of disturbance to those lying beyond.

Fig. 56.



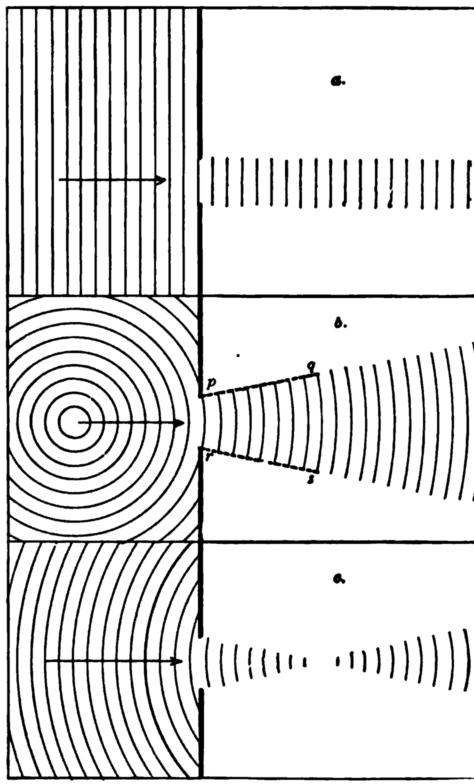
There is a curious proposition, the nature of the proof of which will be indicated farther on, that if the *aperture* through which a wave passes be *small in comparison with the wave-length*, there will be expansion of the wave-front, such as that shown in Fig. 56; but that if the aperture be wide in comparison with the wave-length, the wave will only travel in the direction of all the lines drawn normal to that part of it which passes through the aperture, the wave therefore travelling with a correspondingly limited amount of expansion or none at all; while for conditions intermediate, there will be a certain amount of expansion beyond the limitation indicated by the normals.

When a wave-motion passes through an aperture relatively wide, then the cases are three: —

(*a.*) The wave-front may be *flat*, as in Fig. 57 *a*, in which case it does not expand: this is the condition of a "parallel beam" of light.

(*b.*) It may be *convex*, as in Fig. 57 *b*, in which case the wave-front expands, being limited by the normals *pq*, *rs*. This is the condition of a "divergent beam" of light.

Fig. 57.



(*c.*) It may in some cases be *concave*, in which case the wave-front first contracts and then expands: this is the condition of a "convergent beam" or "convergent pencil of rays" passing through a "Focus."

When a wave-front passes through a focus, exactly or approximately, the energy, constant in amount, is distributed over a comparatively small field, and the intensity of disturbance is, at the focus, correspondingly great.

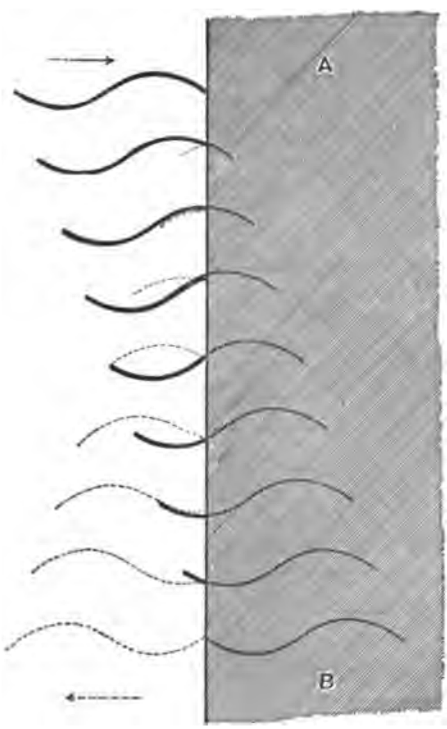
Reflexion of Linear Waves.—If a linear longitudinal wave of compression be incident, or impinge, on an obstacle so firm that the first particle of it at which the wave arrives does not move, there is then produced between this first particle of the obstacle and the nearest particle of the vibrating cord (which we shall represent as particle *i*) a Compression, which results in particle *i* rebounding at a rate equal to that with which it struck the obstacle—that is, with velocity $v_{\prime}$—but in the opposite direction, and in its then meeting the next particle, *ii*, as it comes up with velocity $v_{\prime\prime}$.

We may here borrow a proposition from the theory of Elasticity, which shows that if two equal elastic bodies meet one another and rebound, they will do so with exchanged velocities.

Particle *i*, meeting particle *ii*, exchanges velocities with it; *i* acquires velocity $v_{\prime\prime}$ and returns towards the obstacle; *ii* acquires $v_{\prime}$. Particle *i* strikes the obstacle with velocity $v_{\prime\prime}$ and

rebounds; in the meantime particle *ii* has acquired velocity $v_{,,,}$ by exchange with particle *iii*. When particles *i* and *ii* again meet, *i* is impelled towards the obstacle with velocity $v_{,,,}$, and the backward velocity $v_{,,}$ is imparted to particle *ii*. So on: the particle *i* successively strikes and rebounds from the obstacle with each successive velocity, $v_{,}$, $v_{,,}$, $v_{,,,}$, $v_{,,,,}$, etc.; at the same time the backward speed $v_{,}$ is transferred successively to all the particles *ii*, *iii*, *iv*, etc., and is followed by the successive velocities $v_{,,}$, $v_{,,,}$, $v_{,,,,}$, etc. The consequence is, that just as the end of the wave is being dashed against the obstacle, a wave-front exactly like the original one is travelling away from the obstacle, at the distance of one wave-length. The "reflected wave" has travelled through the incident wave, and then, becoming clear of it, travels alone, equal to the incident wave in wave-length, in period, in phase, and in amplitude, but opposed in direction.

Fig. 58



In Fig. 58 a single wave is shown, running along a cord against a fixed obstacle AB. Within the obstacle thin lines show the course which the wave would have taken, had the cord not been interrupted. To the left of AB, light dotted lines indicate the course of the reflected wave. The light dotted lines are seen to be of exactly the same form as the thin lines within AB, but turned sharply in the reverse direction at the surface of impact. The reflected wave is then a direct continuation of the incident wave in all but direction.

If of incident waves there be a succession, simple, complex, regular, irregular, all these peculiarities will be faithfully reproduced in the reflected waves.

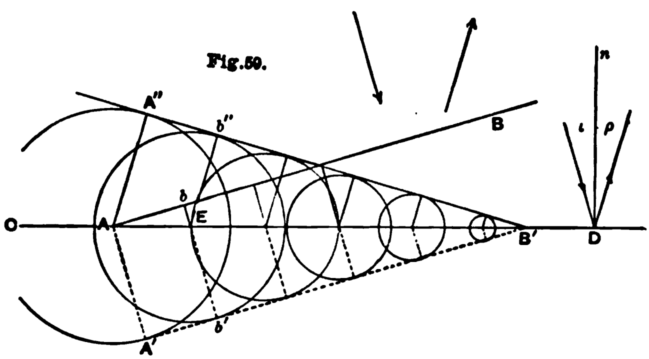
What has been said of a wave longitudinal and commencing with a compression may be easily modified so as to become

generally applicable to the explanation of any kind of linear undulatory disturbance. If the obstacle stand fast, it is a matter of indifference whether it be composed of matter whose particles lie more or less closely together.

Special consideration of the reflexion of waves traversing space of two dimensions may be omitted.

Reflexion of a plane wave-front at a plane surface.—If a plane wave-front meet a plane surface, any section through the wave and surface will present a condition such as that shown in Fig. 59. The line AB represents the wave-front advancing; CD represents the surface on which it impinges. Every point in the wave-front acts as a centre of disturbance. Thus the wave-front advances, parallel to its former plane forms. After the lapse of a certain time the whole of the wave-front has

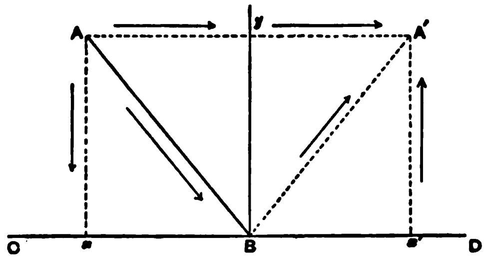
Fig. 59.



impinged on the obstacle: what is then its condition? The part of the wave corresponding to the particle A would have travelled as far as A′, if there had been no obstacle. After reflexion it has travelled to a corresponding extent in some direction tending away from the surface—that is, to some point on the circumference of a circle, the centre of which is at A, and the radius of which is AA′. So the part of the wave indicated by *b* would have reached *b*′; the line *bb*′ crosses the surface at E; that part of the wave is reflected to a distance limited by a circle whose centre is the point E on the surface, and whose radius is the distance, E*b*′, between the surface and the position at which the wave-front would have arrived if there had been no obstacle. By drawing a sufficient number of circles in this way, we see that the aggregate disturbance produces a plane wave-front A″B′, receding from the surface CD. If, as it approached the plane surface, it had been

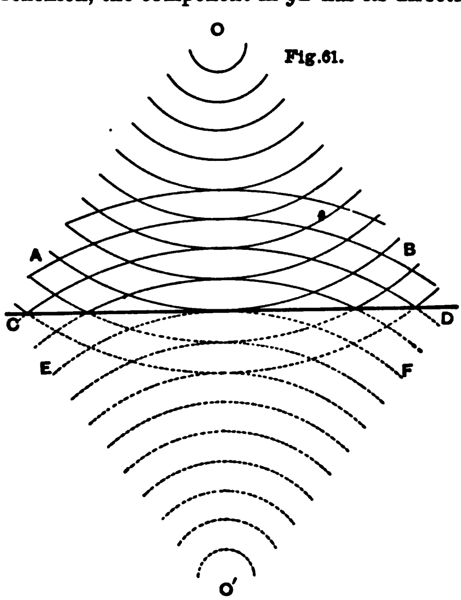
parallel to that surface, it would retrace its path. If it had approached the surface obliquely, so that the direction of the wave makes an angle ι with the normal to the surface, the direction of the receding wave will make an equal angle ι with the normal, but on the other side of it. This is expressed by saying that the Angle of Incidence, ι, is equal to the Angle of Reflexion, ρ; these being understood to be angles made between the direction of the wave and the normal to the surface, or, what amounts to the same thing, between the plane of the wave and the plane of the surface.

Fig. 60.



The same proposition may be otherwise demonstrated. In Fig. 60 let AB represent the direction of a wave, and CD the reflecting plane surface. Every movement of the vibrating body with reference to the direction AB may be resolved into two, these being referred to the axes By and Bx. On reflexion, the component in yB has its direction reversed; that in the direction xB is not thus interfered with. After reflexion, on recompounding the components, the resultant is found to be a disturbance similar to the original one, but in the direction BA$'$, while the angle ABy is equal to the angle yBA$'$.

Fig. 61.



Reflexion of a curved wave-front at a plane surface.—If a wave-front be curved, it may be considered as consisting of a very large number of very small plane surfaces. To each of these a normal may be drawn; each such normal indicates the direction of the corresponding part of the wave-front; the angle which each such normal makes with the reflecting surface, when its own part of the wave-front strikes the obstacle,

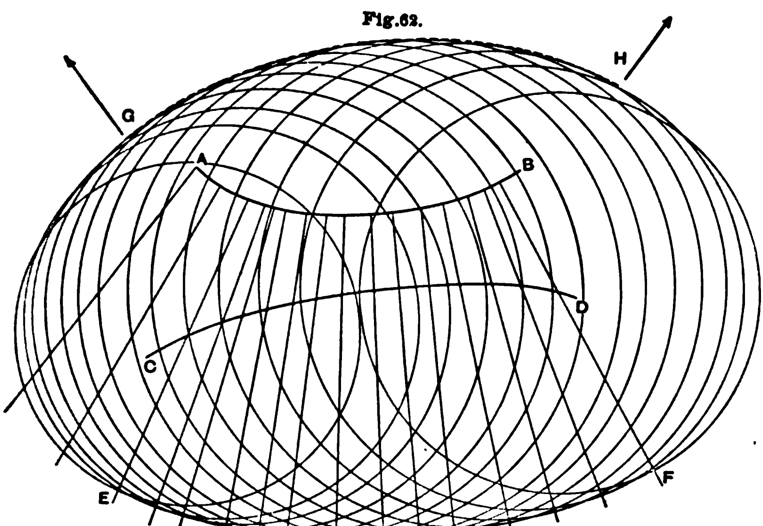
is the angle of incidence for that part of the wave-front; to this angle the angle of reflexion for that part of the wave-front must be equal.

Let the convex spherical wave-front AB strike the surface CD. Each part of the wave-front is reflected at its own angle. The result is the reflexion of a convex wave, which is of the same form as AB would have assumed in the time, but which travels in the opposite direction. Such a wave is, in effect, exactly such a wave as would have travelled from the point O′, as far behind the reflecting surface as O is in front of it.

In the same figure, if the directions be reversed, so that a concave wave-front travels towards the reflecting surface, converging upon O′, it will, when reflected, become reversed, and on receding from the reflector, it will converge upon O, which is as far on the one side of CD as the point O′, upon which the wave had originally been converging, is on the other.

General construction of a reflected wave.—Let AB (Fig. 62) be a wave-front, and CD a reflecting surface, both of any

Fig. 62.

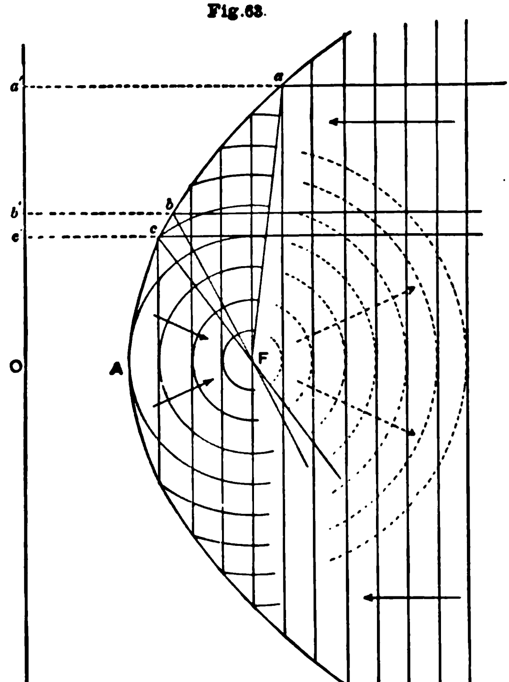


form. Draw normals to AB of such lengths that they may all cut CD; from these normals cut off equal portions, and join the extremities of these portions; the line EF is thus obtained, which represents the form that the wave would have assumed but for the reflecting obstacle. Draw a number of circles; the centre of each of these is a point at which one of the normals

to AB cuts CD; the radius is the distance along the normal in question from the surface CD to the surface EF. These circles have a common tangent, the curved line GH, which indicates the form of the reflected wave. Normals drawn to this, of equal length, may indicate the form of the reflected wave at any subsequent instant; if these be drawn backwards, all the previous positions, real or apparent, may be investigated.

This is the general construction; but it very frequently leads to difficulties where different parts of the wave cross one another. In most cases, however, the following method is effective. Consider a part of an incident wave and the point at which it impinges; from the point of incidence draw a line indicating the direction in which that part of the incident wave will be reflected. Find to what distance behind the reflecting surface the incident wave would have travelled in a given time if there had been no obstacle; measure off, along the direction of the reflected wave, a distance equal to this; repeat this operation for several parts of the wave-front; join all the points thus obtained. This gives the form of the reflected wave at the time chosen. Equal distances, measured forwards or backwards along the normals to this wave, will give the form of the wave at instants subsequent, and its true or hypothetical form at instants previous. The following are examples:—

Fig. 63.



Problems.

1. Let the reflecting surface be a paraboloid: let the advancing wave-front be plane. The "focus" of the parabola is at F (Fig. 63). We may choose three instants for consideration.

a. That at which the whole wave-front would have arrived at O. Each part of the wave-front is reflected as shown in the diagram. The part which would have taken the course aa' is turned into the direction aF; bb' into bF; cc' to cF; and so on. $aF = aa'$; $bb' = bF$; $cc' = cF$; AO = AF. The wave-front is reduced to a point; it is at that instant passing through the focus F.

b. Any instant at which the wave-front, having passed the point A, would not yet have reached the point O; the reflected wave is spherical and concave, converging on F.

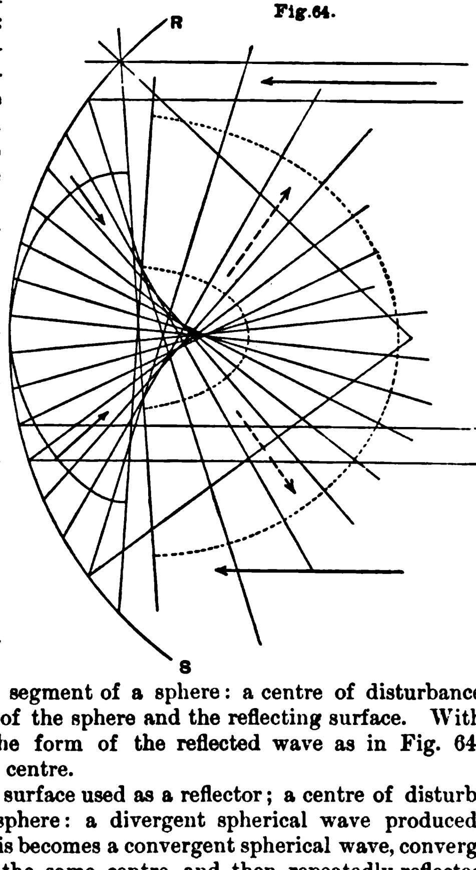
c. Any instant at which the wave would have reached a plane farther away from the reflecting surface than $a'O$; the wave is spherical, divergent from the focus F.

2. In the same figure, the wave-front is one which starts from F as a centre; it meets the paraboloid reflector; it is reflected with a plane wave-front.

3. The reflecting surface is spherical and concave, the incident wave-front flat.

In Fig. 64 the reflecting surface is represented by the line RS. The wave travels from right to left and meets RS. Each part of the front of the wave is turned back at its own angle of reflexion: the wave-front becomes convergent. It does not, however, converge on any one point; it is not spherical. The figure shows that there is a curved line, a "**caustic by reflexion,**" in which lie all the foci of all the separately considered parts or elements of the wave-front. In each wave the element reflected from the outer part of the surface RS will sooner come to focus than that reflected from the centre of that surface; hence if the wave be single, a spot of maximum disturbance will appear to run along each limb of the caustic, and to disappear in a diverging wave at its apex. A succession of waves will keep the whole of the caustic in a state of maximum disturbance.*

Fig. 64.



4. A reflecting mirror a segment of a sphere: a centre of disturbance midway between the centre of the sphere and the reflecting surface. With ruler and compass draw the form of the reflected wave as in Fig. 64. Approximately plane at its centre.

5. A complete spherical surface used as a reflector; a centre of disturbance at the centre of the sphere: a divergent spherical wave produced. Prove that after reflexion this becomes a convergent spherical wave, converging on and passing through the same centre, and then repeatedly reflected and alternately converging on and diverging from the original point of disturbance.

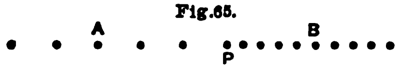
* **Take a long strip of bright tinplate; bend it into a semicircle; place it on its side on a sheet of paper in the sunlight, exposing the concavity to the sun: a brilliantly-illuminated Caustic Curve will be seen on the paper. The form of this curve may be varied by altering that given to the tinplate.**

6. Prove that a spherical wave starting from one focus of an ellipse converges after reflexion on the other focus. Hence prove that if the centre of disturbance be at one focus, and if it be surrounded by a complete ellipsoidal reflecting surface, a wave passes back and fore between the foci, alternately converging and diverging, first at one, then at the other focus.

7. A reflecting mirror a small segment of a circle. Draw another circle, whose diameter is the radius of curvature of the mirror. Waves proceeding from any point of this new circle will, after reflexion from the mirror, converge upon some other point of the same circle.

Transmission of a linear wave into a denser medium. — If, as in Fig. 65, the particles be more closely placed in B than in A, B is the denser medium. A linear wave-motion travels to the right in A; it arrives at P. It meets a relative obstruction. P, the first particle of the denser substance, is more resisted than the preceding particles set in motion by the wave. The wave is not entirely obstructed, and goes on into B; but there is, to some extent, the production of a reflected wave in A. This reflected wave, like that of Fig. 58, is of the same phase and period, and of the same wave-length as the original wave. It cannot be of the same amplitude, for some of the energy of the wave-motion has been spent in setting up a wave in B. The wave-motion propagated along B must necessarily be of the same period as that in A, for the particles in B must move in unison with those of A, which impel them; it must be of the same phase, for it is the direct continuation of the wave in A. Since the particles are more crowded together in B (a less distance corresponding to the same number of particles), a wave cannot propagate itself in B so far in a given time as it can in A, for its doing so, still retaining the same wave-length, would imply its setting a greater mass in motion. This would, however, require a greater amount of energy. If the latter be definite in amount, as it must be, the wave-length and the speed of propagation must be less in the denser medium.*

Fig. 65.



As to the relative amplitudes of the respective vibrations, the original, the reflected, and the transmitted, the amplitudes of the two latter taken together are not necessarily equal to that of the first; but in every case the energy of vibration of the original wave is equal to the sum of the energies of the reflected and the transmitted waves.

* To avoid misconception it may be remarked here that in concrete cases, while its density is a powerful factor in determining the velocity of propagation of vibration in a given body, this also depends greatly on the peculiar molecular properties — the elasticity — special to each substance.

If in Fig. 65 a wave beginning with compression be supposed to run through B towards the left, when it comes to the particle P — which may be considered as the last of B or the first of A — that particle, meeting less resistance than its predecessors in B had encountered, plunges into the rarer medium and sets up in A a wave depending on the original wave in B for its period and phase, but of greater amplitude; and the wave in A will also have a greater wave-length than that in B, for a reason the converse of that stated in the last paragraph. The effect on the denser body is, however, singular. The particle P, plunging away from the rest of the particles of B, produces in that part of B a dilatation which is propagated backwards, and there then travels in B a reflected wave, agreeing with the incident wave in period and in wave-length, necessarily not in amplitude, and opposed in phase. A maximum compression arriving at P causes that particle to yield to the greatest extent, and to produce a maximum dilatation in B; hence, when the incident wave produces a maximum compression among all the particles of A in the neighbourhood of P, P itself starts a wave in B, commencing with a maximum dilatation, and the incident and reflected waves are not continuations of one another as in Fig. 58, but there is *loss of half a wave-length.**

A comparison of the diagrams in the preceding discussion shows that, in every case where the medium in which the wave has travelled is the same, the space traversed by every part of the wave, reflected or not reflected, or sooner or later so affected, must necessarily be the same in a given time; and hence, counting from any initial condition to any final wave-front form, the space traversed before reflexion + that traversed after it = a constant quantity for a given time, and that for every element of the wave-front; or, as it is often expressed, the

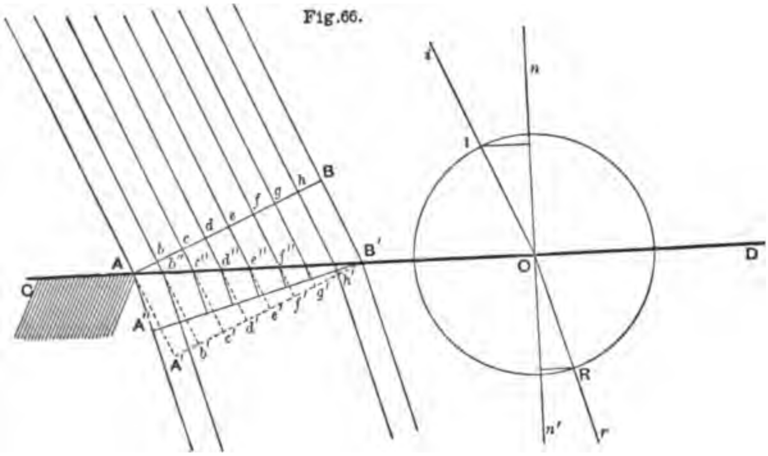
* This curious result has an interesting bearing on the Conservation of Energy. Both the amplitude and the length of the wave in the rarer medium are greater than in the denser. The body B, if the metaphor may be allowed, finds itself to have done more work on the body A, and therefore to have transmitted more energy to it than it had intended: the particle P has compromised the body B by giving a greater dash forward than was expected. Under the circumstances, matters are adjusted by the propagation of a wave of opposite phase in the body B. If each of these waves, the reflected and the transmitted, be regarded separately as containing so much energy, the sum of their energies may appear to exceed that of the original wave. The whole vibrating matter must, however, be regarded as forming one system. In this system, a compression in one wave, and a dilatation in another, produce a relative motion amounting only to their difference; and this is the true motion of the system, the energy corresponding to which is equal to the energy of the original vibration.

"incident ray" + the "reflected ray" = constant for the whole wave.

Refraction of a plane wave at a plane surface. — If the incident wave strike the plane surface simultaneously at all parts of its own front, it will simply pass more slowly through the denser medium, while a reflected wave is sent back; but if it strike it obliquely, there are some changes in the wave, which result from one part of it being hampered in the retarding substance while the rest is still moving with comparative rapidity in the rarer medium.

In Fig. 66 let AB be the wave-front in the rarer medium; CD the surface separating the denser from the rarer medium; A′B′ a position at which the wave-front would have arrived

Fig. 66.



if it had not encountered the denser substance. The lines *bb′*, *cc′*, *dd′*, etc., are normals to the incident wave-front, meeting the line CD in *b″*, *c″*, *d″*, etc. The angle BAB′ between the wave-front and the surface, or *i*O*n* between the direction of the incident wave and the normal to the surface, is called the *angle of incidence.*

Let us suppose that the velocity of propagation in the denser medium is $\frac{2}{3}$ of that in the rarer. Then with centres A, *b″*, *c″*, *d″*, etc., and radii = $\frac{2}{3}$ AA′, $\frac{2}{3}$ *b″b′*, $\frac{2}{3}$ *c″c′*, etc., draw circles. The line A″B′, which is their common tangent, indicates the position of the wave-front at the end of the time during which it would have advanced to A′B′. The wave has been rendered somewhat broader, and has changed its direction. The angle AB′A″ or *n′*O*r* is called the "**angle of refraction.**"

In the figure, AA″ : AA′ : : 2 : 3; but in the two triangles AB′A′, AB′A″, we see that AA″ : AA′ : : sin AB′A″ : sin AB′A′.

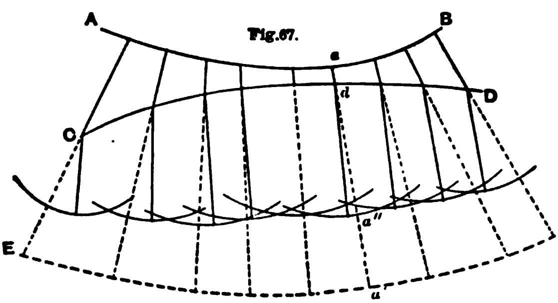
$$\therefore \frac{\text{Sin AB}'\text{A}''}{\text{Sin AB}'\text{A}'} = \frac{\text{sin ang. refr.}}{\text{sin ang. incid.}} = \frac{\sin \varrho}{\sin \iota} = \tfrac{2}{3} = \frac{\text{velocity in denser medium}}{\text{velocity in rarer medium}}.$$

Generally, if $\frac{\text{vel. in denser med.}}{\text{vel. in rarer med.}} = \frac{1}{\beta}$, a fraction; then sin ang. incidence = $\beta \times$ sin ang. refraction; $\sin \iota = \beta \sin \varrho$, and β is called the "**index of refraction**" of the denser substance as compared with the rarer one.

This formula shows that in Fig. 66 if *i*O indicate the direction of the incident, O*r* that of the refracted wave; *n*O*n*′ the normal to the plane refracting surface; if a circle be drawn with centre O and any radius, — the lines *i*O and O*r* will cut it in I and R; from I and R draw lines at right angles to *nn*′, as in the figure. These lines always bear to one another, whatever the angle of incidence, the same ratio as the Velocities in the respective media, and this law defines the relation of the angle of refraction to the angle of incidence. An equivalent construction is given in Fig. 182.

Refraction of a wave at a surface: General construction. — Let AB be an advancing wave-front, CD the bounding surface

Fig. 67.



of a denser medium. Let the assumption be made that each several element of the wave-front, so long as it is in the same medium, travels mainly in the direction of the normal drawn to it. In this way the whole wave-front is always simply related to all its previous forms, all the parts of it having at any instant travelled along their respective normals to an equal extent during any given interval of time; and a line once normal to the wave-front is, if produced, always normal to it so long as it travels in the same isotropic medium.

Then a number of these normals to the incident wave, such as *aa*′ in the figure, are drawn equal to each other, and of length sufficiently great to enable the surface to cut them all. The extremities of these equal normals are joined; in this way a

curve EF is produced, which indicates the form that the incident wave would have assumed had it travelled thus far in the original medium. Lines normal to AB are also normal to EF. We see segments of these normals, such as da', cut off between CD and EF. The fraction $1/\beta$ (= the velocity of propagation in the denser medium ÷ that in the rarer) must now be known. From da' cut off da'', which is to da' as $1/\beta : 1$; with d as a centre, and da'' as radius, draw an arc of a circle through a''. Treat similarly all da''s fellow-normals. A number of arcs are thus obtained, to which the common tangential curve must be drawn. This gives the form of the refracted wave-front.

This having been obtained, normals may now be drawn to it; these will not in general coincide with aa' and its fellows. By measuring off equal distances along these normals to the refracted wave-front, all the future forms of the refracted wave and all its apparent past forms may be ascertained.

Refraction of a spherical wave at a plane surface. — Let a spherical wave whose centre is at F strike the plane surface RS and enter a denser medium. The wave would at a certain

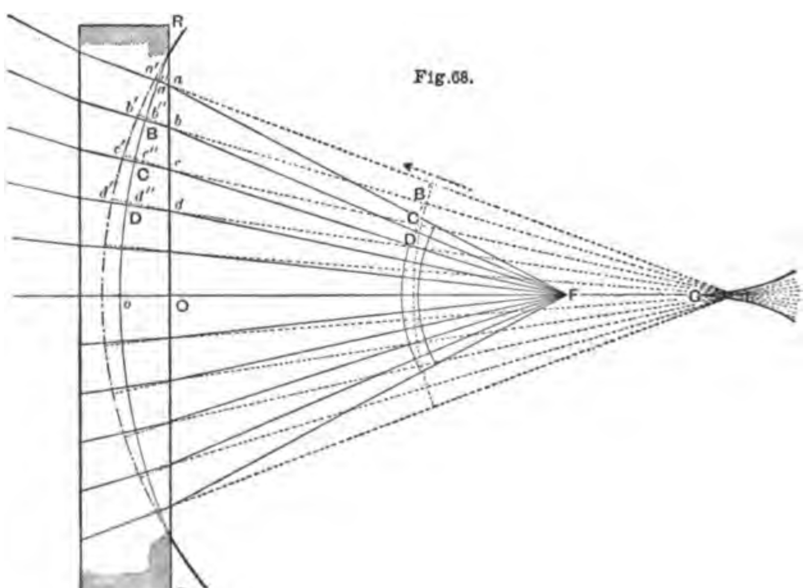


Fig. 68.

instant have arrived, say at $a'b'c'd'$. According to the preceding construction, lines aa'', bb'', etc., are cut from aa', bb', etc., to which they respectively bear the constant ratio $1 : \beta$. Arcs

are drawn with centres a, b, c, d, etc., and radii aa'', bb'', cc'', dd'', etc.: the common tangent BCD—a curved line—is found: this gives the form of the refracted wave in the denser medium; it is hyperbolic.

Normals may now be drawn to this, by means of which the future and the apparent past forms of the wave may be traced out. In the denser substance, as it travels onward it retains the hyperboloid form; and if the normals bB, cC, etc., be traced backwards, and such equal lines as BB′, CC′, etc., measured off along them, all those hypothetical wave-fronts may be drawn from which the wave-front, as it travels through the denser medium, presents the appearance of having been developed. On tracing back far enough, we find that the wave appears as if developed from a wave-front convergent not upon the centre F, but through a caustic the apex of which is at G, where OG is to OF as $\beta:1$.

Problem.

A spherical convergent-wave meets the plane surface of a refracting substance. By construction show that the wave converges through a caustic, the distance of whose apex from the surface is less than that of the original centre in the ratio of $1:\beta$.

Passage of a spherical wave through a parallel-sided sheet of a denser substance.—At its entrance into the denser substance, the wave-front becomes hyperbolic; at exit every part of the wave resumes the direction which had pertained at the instant of the first refraction to that part of the wave-front from which it had been developed, and thus the wave-front again approximately, but not exactly,* resumes its original spherical form. In Fig. 68 the part of the wave-front which passes through a is, when the wave approximately resumes its spherical form, again refracted, so that that element assumes a direction parallel to its original direction Fa.

Approximate Foci.—In Fig. 68, if of the hyperbolic wave in the denser medium only a limited portion in the centre be considered, it will be found to approximate very closely to a small arc of a circle whose radius is Go. We have already learned to express this by saying that its radius of curvature is Go. If accordingly the incident wave-front be narrow, the refracted wave-front appears to have diverged from a group of points in the immediate neighbourhood of G, the apex of the

* An object seen through a pane of glass is never as distinct as the same seen through the intervening air alone.

caustic; and the narrower the incident wave, or the less its divergence, the more nearly will it appear to come from a single point, the very apex of the caustic itself. Similarly in Fig. 64, the narrower the incident wave is in comparison with the breadth of the curved reflecting-surface, the more nearly will the reflected wave converge on the very apex of the caustic, midway between the surface and the centre of the sphere. Consequently these points, the apices of the caustics, are approximate foci for comparatively narrow-fronted waves.

Refraction of a plane wave at a spherical surface. — Constructions somewhat similar to that of Fig. 64 will show that if a plane-fronted wave be made to strike the convex spherical surface of a denser or the concave surface of a rarer medium, it will be made to converge: if it strike the concave spherical surface of a denser, or the convex spherical surface of a rarer medium it will diverge: in all these cases it will form a Caustic, actual or virtual.

The tip of the Caustic (the approximate focus for central rays) is at a distance (measured along the Axis, a line passing through the sphere-centre at right angles to the wave-front) from the refracting surface equal to $r\beta_l/(\beta_l - \beta_0)$, where r is the radius of the surface, and β_0, β_l the refractive indices of the original and the refracting media respectively.

Refraction of a spherical wave at a spherical surface. — Again there will be convergence or divergence produced: and in all cases except that in which the wave-front is made to become approximately plane-fronted, there will be a Caustic, actual or virtual.

The centre of the incident wave and the tip of the caustic are respectively at distances d' and d'' from the refracting surface; then $\beta_0/d' - \beta_l/d'' = (\beta_0 - \beta_l)/r$; d', d'', and r (the radius of the refracting surface) being all reckoned positively, that is towards the *right*, and being therefore negative if they lie towards the left of the refracting surface. The centre of the incident wave and the point upon which the wave converges after refraction are, in respect to one another, called **conjugate points**.

If the waves travel in the rarer medium from right to left and are plane-fronted, so that $d' = +\infty$, then $d'' = +\beta_l r/(\beta_l - \beta_0) =$ "f'": if they travel in the denser medium from left to right, $d_l' = -\infty$, but "β_0" is now the density of the more refracting medium and is equal to the former β_l, and similarly "β_l" is the former "β_0": therefore $d_l'' =$ "f''" $= \beta_0 r/(\beta_0 - \beta_l) = -(\beta_0/\beta_l)f'$; the distances between the tip of the caustic and the vertex of the refracting surface in these two cases are called the **Principal Focal Distances**, f' and f'', for the refracting surface between the two media in question.

If the wave, after entering the second medium, again return into the first through a second spherical bounding surface, the point upon which it

now converges may be found in precisely the same way by finding what point is, with respect to the second refracting surface, the point conjugate to the centre, real or virtual, of the wave approaching the second surface. By this method, keeping in view the requisite exchange of numerical values of β_0 and β_I at the second surface, the ordinary lens-formulae (p. 540) are arrived at; and the process may be repeated for any number of media and surfaces, at any mutual distances.

Utility of the idea of "Rays" in geometrical construction. — All the preceding discussions have been grounded on consideration of the various forms assumed by the wave-front: we have shown that any form may be developed from any of its predecessors by taking each point in that predecessor as a centre of disturbance, and drawing equal circles of appropriate radii which indicate the extent to which the disturbance has travelled; then of these circles the common tangential line denotes the developed form of the wave-front: we found that in simple cases every line normal to any wave was normal to all those developed from it, and to all those forms through which it had passed; in all this it being supposed that the medium was one in which the velocity of transmission was the same in all directions.

We also made an assumption that when the form of the original wave was complex and the medium isotropic, the same law applied; that the maxim once a normal always a normal was true; that there was no lateral expansion of the wave-front beyond the limits indicated in a diagram by lines set down to represent such normals. The assumption is approximately true only in special cases — in general there is lateral expansion of the wave-front beyond such limits; but on reference to what was said in connection with Figs. 56 and 57, we are reminded that it is possible for us to conceive a point of disturbance on a wave-front as one in a *wide aperture;* and hence if the wave-length be very small, the wave-front propagated from each little element of the wave-surface travels along the normal to that element. In any case this is never an absolute statement, and there is always more or less lateral divergence; but as a first approximation of sufficient value for most purposes, it may be said that in an isotropic medium (where the velocity of propagation is equal in all directions) the wave-front is developed from all its predecessors along their common normals; that this is nearly the case when the wave-length is comparatively short: but the greater the proportionate length of the wave, the more lateral expansion there is, and the less able would we be to find the form of the

wave-front at any moment by exclusively considering the normals to its previous forms. If, however, the wave-length be comparatively short, we may, by considering the normals only, erect in a diagram the scaffolding on which the form of the wave-front may be constructed. In reflexion, for instance, as in Fig. 64, normals may be drawn to the front of the incident wave; the reflected wave is of such a form that each normal to it makes, with the corresponding normal to the reflecting surface, an angle equal to that made by a normal to the incident wave. But the wave-front itself might have been omitted from the diagram, and the same results, as regards focus and caustic, would have been obtained. Then attention might be fixed on the normal lines, and on the way in which these change their direction on reflexion or refraction. They might be treated as if they were physical entities, and might receive special names. This has actually happened. The imaginary straight line drawn at right angles to the wave-front at any point has been called a "*ray*" passing through that point. Each ray is straight, for the normals preserve always the same direction; and of all wave-motion — such as Light — which does not expand perceptibly beyond the limits laid down for it by its normals, as in Fig. 57, the progress is described by saying that its rays travel in straight lines so long as it is in the same medium. This mode of expression has both advantages and disadvantages. It leads to the assumption that a divergent wave-front is a divergent "pencil" of rays, each of which is somehow distinct from its fellows; it leads to these rays being conceived as themselves reflected, refracted, etc.; it isolates the physics of those phenomena—those of Light—in which waves approximately follow their normals only, from those in which this approximation is much less complete, as in the case of Sound. On the other hand, it presents certain advantages; it simplifies diagrams; it enables any problem to be reduced to its simplest elements by an absolute rejection of all lateral disturbances, and of the effects produced by any parts of the wave other than those at the points of the fronts crossed by the normals; and it gives results which in the theory of Light are, up to a certain point, of sufficient accuracy. This advantage persists, however, up to a certain point only, and, on the whole, a habit of referring the phenomena of wave-motion to the form of the wave-front is to be preferred, though hereafter we shall make free use of the device of reference to rays whenever it is found convenient to do so.

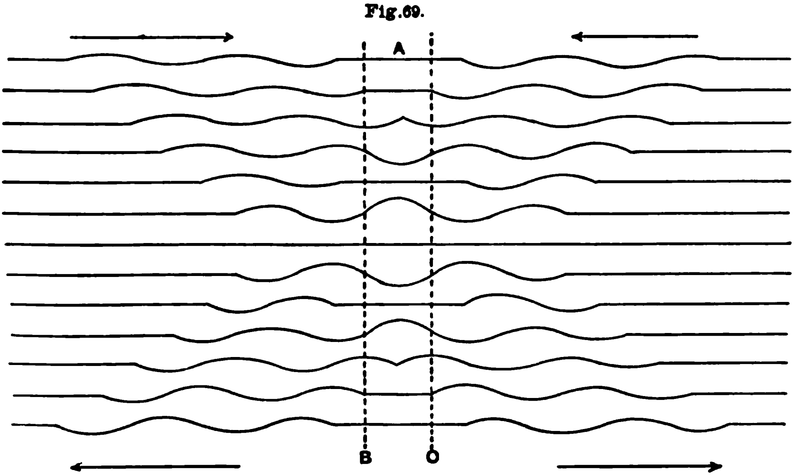
The law can easily be verified that the path traversed by every element of the wave in the rarer medium, together with $\beta \times$ that traversed in the denser, is a constant quantity.

Ptolemy's Law.—If a ray pass from A to B, striking some point of a plane reflecting surface in its course, and being thence reflected to B, there is no path from A to B *viâ* any point of the mirror, so short as that actually traversed by the ray, under the law of reflexion.

Fermat's Law.—If a ray pass from A in one medium to B in another, there is between these points no path which, the relative velocities in the two media being taken into account, could be traversed in so short a time as that actually traversed under the law β sin ang. refr. = sin ang. incid. If the construction be attempted, it will be seen that the actual law allows the ray the greatest possible proportion of time in the rarer medium.

Superposition of simultaneous wave-motions on an indefinite cord.—We shall here simply discuss the single case in

Fig. 69.



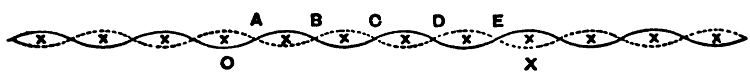
which two equal waves travel in opposite directions on the same cord. In Fig. 69 are seen two waves approaching one another on a cord of indefinite length; they meet at A, and pass through one another. Where crest meets crest and trough trough, as at A, the amplitude is doubled; where crest meets trough, as at B and C, there is no movement so long as the waves are passing through one another: B and C are, during this mutual interference of the waves, **non-vibrating** or "**nodal**" **points,** between which the cord vibrates.

If the waves had been equal, but opposite in phase, so that crest met trough at A, then A, the point of meeting, would have been a nodal point.

The two nodal points, B and C, are seen to be at a distance of *half a wave-length* from each other.

If the waves meeting each other had been indefinitely numerous, the interference would occur in every region of the indefinite cord, and there would be an indefinite number of

Fig. 70.



nodal points, half a wave-length distant from each other. In Fig. 70 such waves are seen running on an indefinite cord, and the nodal points, where crest meets trough, are marked with crosses.

Cord of definite length.—In Fig. 70 let us limit our attention to a part of the string comprised between two nodal points, say between O and X. Within this limited part of the string we observe two waves running in opposite directions, the crest of one meeting the trough of the other at four points: there are five loops, each of which is equal to half a wave-length; the centres of the loops, the points of greatest vibration, are the points A, B, C, D, E; the wave-length is here equal to $\frac{2}{5}$ of OX.

Nodes and Loops.—If the nodal points O and X in Fig. 70 had been the fixed ends of the string, and if it had been possible to establish in OX two waves, each of wave-length = $\frac{2}{5}$ OX, these meeting one another so that trough always coincided with crest, the necessary result would be a continuous vibration of the cord in five segments, marked off by four non-vibrating points, and these segments would always be in opposite phases of vibration. The vibration would in such a case be said to be **Stationary Vibration.**

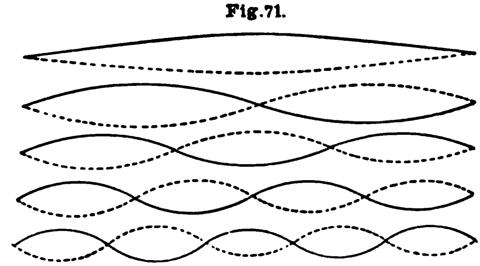
This is precisely the case where a wave running from O to X meets its own reflexion returning from X to O, the point X not being free to move, but being held fixed. If the wave-length be $\frac{2}{1}$, $\frac{2}{2}$, $\frac{2}{3}$, $\frac{2}{4}$, $\frac{2}{5}$, etc., of the length of the cord OX, the result of the composition of the wave and its reflexion will be a stationary vibration, in which the string will vibrate in the first case as a whole, or in the others in two, three, four, five, etc., vibrating segments or Loops, separated by non-vibrating points or Nodes.

The nodes are points where there is no displacement, and a maximum variation of density: at the centres of the loops, on the other hand, there are maximal displacements and velocities.

If the end at which reflexion occurs be not fixed — as in the case of water-waves running up against a cliff — the first reflexional node is at a distance $= \lambda/4$ from the reflecting obstacle, and the succeeding nodes at distances $= \lambda/2$ from the first node and from each other successively.

Vibrations of a cord whose extremities are fixed. — A string acted on by any force tending to bring the particles back to their mean position, and varying as the displacement — a criterion, as we have seen, of harmonic motion — will enter into vibrations of a type obeying Fourier's law, and in the general sense any periodic disturbance of such a cord will thus be compounded of vibrations such as those shown in Fig. 71. These simultaneous vibrations will, as regards amplitude, be independent of one another, and will also from moment to moment necessarily differ in their relative phases. The whole motion is, however, periodic.

Fig. 71.



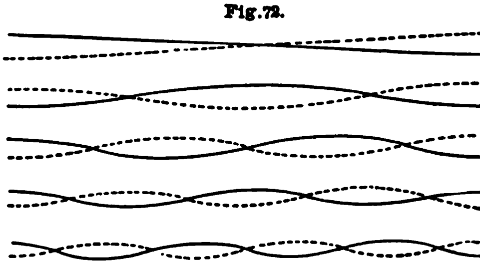
If a point situated in the loop of any one of these harmonic components be held fixed, the corresponding oscillation is prevented. If the centre of a vibrating string be touched, the oscillations corresponding to the whole string, to one-third, to one-fifth, etc.—all the odd components—are suppressed, and only the even components—those, namely, which already have a node at the point fixed—are allowed to go on. If the string be touched at $\frac{1}{3}$ of its length from the end, all vibrations except those corresponding to $\frac{\text{string}}{3}, \frac{\text{string}}{6}, \frac{\text{string}}{9}$, etc., cease; those still continue, for the effective fixing of their nodes does not affect them. Similarly, if the string be held steady at a point $\frac{1}{4}$ of the string-length from the end, the 4th, 8th, 12th, 16th, etc., components remain unaffected, while all the rest are stopped.

Longitudinal vibrations of a string or rod—for a rod acts in this case like a bundle of parallel strings—whose ends are held fixed obey the same principles as transverse vibrations. Fourier's law holds good; and if any point be held steady,

those component vibrations which have a node at the point held steady, and those components only, will remain unaffected. In longitudinal vibration, where, as at the centres of the loops, there is the greatest velocity and displacement, there is least actual change of density; and at the extremities of a rod fixed at both ends, and at the nodes, while there is no displacement, there are maximum variations of density.

If longitudinal vibrations occur in a string or rod, or in a cylindrical mass of gas — such as the air in an open organ-pipe — which is free at both extremities, it is plain that at the free ends there can be no change of density, but that there is freedom of movement; hence each extremity must, as regards displacement, be the centre of a loop. The component vibrations which make up the Fourier-motion in such a case are such as those shown in Fig. 72. In this case, as well as in the preceding, all the components, even and odd, are possible, and the wave-length of the slowest or fundamental vibration is equal to twice the length of the rod or string vibrating longitudinally.

Fig. 72.

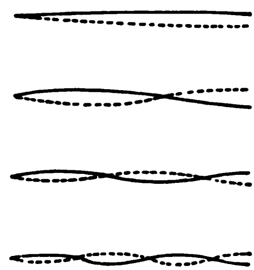


In these cases we see that the wave-length of the fundamental as well as of the concomitant vibrations is determined by the length of the vibrating string or rod itself. We have seen that $\nu = \lambda/T$; λ, the wave-length, may easily be found from l the length of the rod, for $\lambda = 2l$; T, the period, may be measured by acoustical or graphic methods; these being experimentally known, we may find the quotient $\lambda/T = \nu$, the velocity of propagation of an undulatory disturbance in a vibrating string or rod.

If the central particle of the system of Fig. 72, vibrating longitudinally, be held fixed, those vibrations (2, 4, 6, etc.) are suppressed which have not their nodes at the centre of the rod. Thus only the odd components are left; but the rate of these is unaffected. If now one half of the rod were removed altogether, we would have remaining a rod fixed at the one end, free at the other. This rod would have component vibrations, as shown in Fig. 73. A rod thus vibrating longitudinally will have a fundamental vibration whose wave-length will be four times the length of the rod: the concomitant components will have wave-lengths equal to $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, etc., of that length.

If the same rod be supposed to be set in longitudinal vibration, first with both ends free, and next with one end fixed, the fundamental wave-length will in the latter case be doubled, and the period of vibration will also be doubled.

Fig. 73.



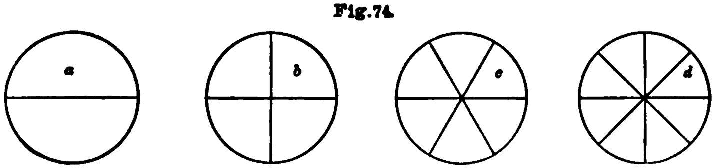
Nodes and Loops in a vibrating membrane. — A membrane may vibrate in such a way that certain lines may be at rest. The number of these lines, if they extend from the centre to the circumference, must be even, for on each side of a node the directions of movement are opposite, and there cannot be an uneven number of nodes.

The forms of these lines vary according to the shape of the membrane and the mode of disturbance. In a square membrane, for example, the nodal lines may be one diagonal — two diagonals — lines joining the centres of opposite sides — lines more numerous parallel to these — curved lines symmetrical with reference to the centre — complex lines obtained by the superposition of these. In a circular membrane we may have concentric circles, or radial lines even in number.

In a circular membrane of which the centre and one point of the circumference are held fixed, the frequency of the fundamental vibration varies inversely as the radius.

The frequency of vibration of a circular membrane vibrating as in Fig. 74 (*a*) being taken as 1, that of the same mem-

Fig. 74.



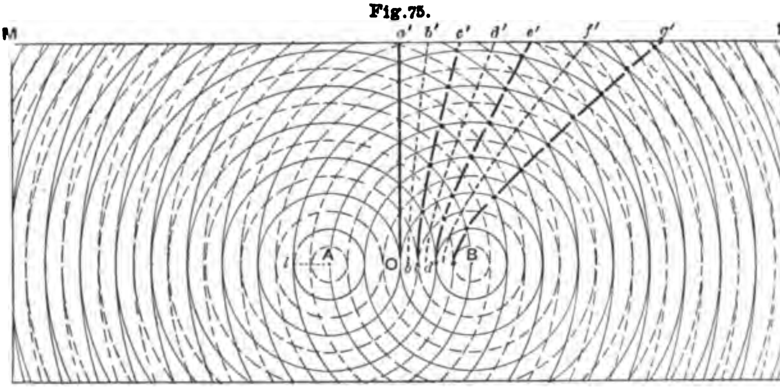
brane vibrating as in (*b*) is $\frac{4}{3}$ nearly;* as in (*c*) $\frac{5}{3}$ nearly; as in (*d*) 2 nearly.

Waves from two different centres — Interference. — In Fig. 75 let A and B be the centres of disturbance; A*l* the wave-length: the dotted circles indicate troughs, the plain circles crests of waves. Where crest coincides with crest, the elevation or compression produced will be the sum of those produced by the two waves; where trough meets trough, the converse will hold; but where the trough of one wave coincides with the crest of another, if that crest be equal, the resultant motion at that

* Lord Rayleigh, *Theory of Sound*, i. 275.

point is null. This is the result of the mutual interference of waves. Join the points at which there is maximum movement, whether of crest or trough; join also those at which crest and trough coincide: we thus obtain a series of hyperbolas indicated in the figure. Along Oa' there is motion due to the concurrent effects of the disturbances at A and B; along

Fig. 75.



bb', or a line very closely approximating to it, there is rest; along cc', concurrence; along dd', approximate rest; and so on.

The hyperbolic lines bb' and dd' would be lines of perfect rest if it were not that the one wave is half a wave-length, one-and-a-half wave-length, etc., behind the other, and hence the amplitudes are not equal. The divergence of the true lines of rest from the true hyperbola, occasioned by this, could not be indicated in the diagram, is less the greater the distance from the centre, and, if the wave-length be very small, will approximately vanish.

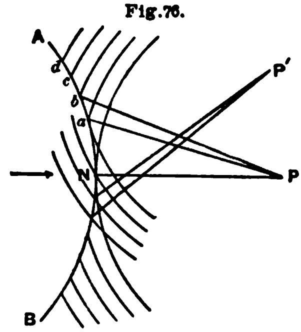
If these two points were the only centres of disturbance, and if a screen MN were placed in the field of the wave, there would be movement at a', c', e', rest in the neighbourhood of b', d', f'.

Propagation of waves along normals in isotropic media. — The principle has been already stated (see Fig. 54) that the propagation of any wave-front is due to the sum of the effects produced by all the points of it acting as centres of disturbance.

Let AB in Fig. 76 be a wave-front whose normal at the point N is NP. Trace the effect of the wave-front when the wave-length is comparatively small. The point P is situated on the normal; P$'$ is situated laterally. From P as a centre draw circular arcs, whose radii, PN, Pa, Pb, Pc, Pd, etc., differ from one another successively by half a wave-length. The disturbance caused at P by the movement of Na is to some extent counteracted by that derived from ab, aided by that from bc,

counteracted by that from *cd*, and so on. But N*a* is greater than *ab*, *ab* than *bc*, *bc* than *cd*, and so on. On the other side of N the circumstances are similar; and thus, on the whole, P is disturbed by the wave-front on both sides of N.

Fig. 76.



If on the other hand the point P′ be considered, it will be seen that if the wave-front be wide enough in comparison with the wave-length, the disturbances radiating from it interfere with one another; for points on the wave-front can always be chosen and set off in pairs, differing in distance from P′ by half a wave-length; and consequently there is no disturbance produced at any such lateral point as P′ *by the wave-motion at N*, and the wave-front travels along the normals without any lateral expansion.

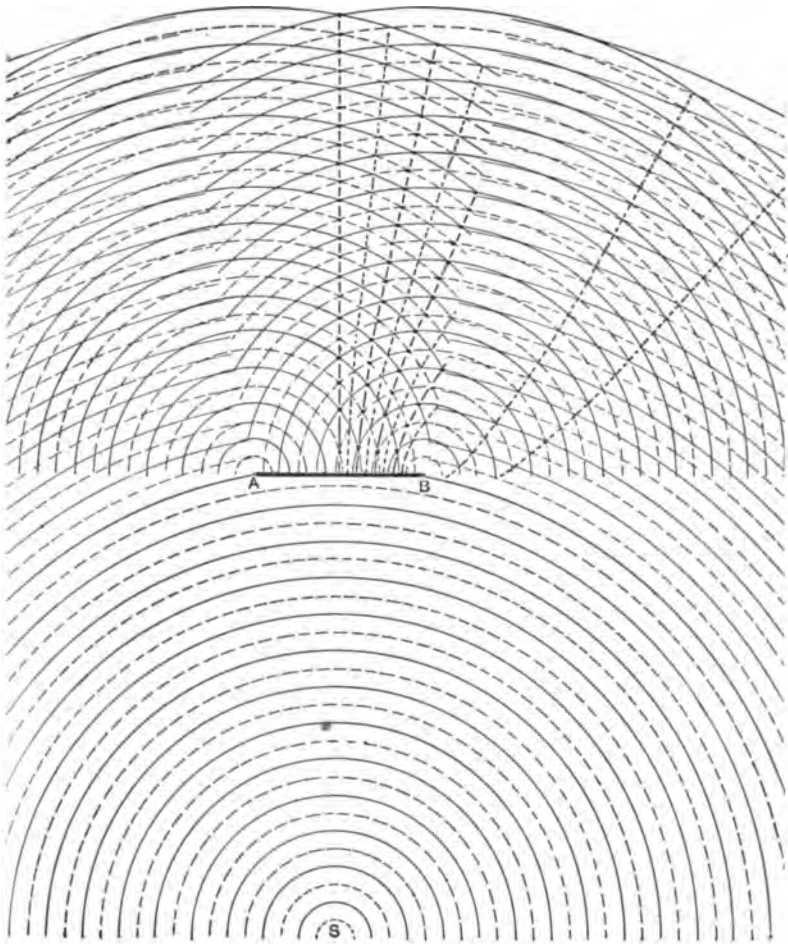
The narrower the wave-front or the greater the wave-length, the greater will be the difficulty in this construction, and the greater will be the lateral divergence of the wave. The wave-front must be at least one wave-length in breadth before this construction begins to become possible.

Effect of a screen.—In Fig. 76 we may neglect the influence on P of the part of the wave-front lying beyond *d*, for the extent to which it disturbs P is very small. If a screen were thrust between the wave-front and the point P, so as to cut off the influence of the part *cd*, the disturbance of P would be *increased:* if the screen come to *b*, the motion of P will be *less* than at first: if it come to *a*, it will be greater than if there were no screen: if to N, it will be less, being about one half of the original amount. If the screen be pushed still farther, P will, in the same way, be in more or less active motion according to the position of the screen. The waves therefore pass round the edge of the screen, producing fringes of alternate maximum and minimum motion. The possibility of this result depends on the smallness of the wave-length.

Effect of a very small screen.—If a wave-front be interrupted by a very small screen placed so as to allow the wave-motion to pass it all round its edge, Fig. 77 shows that disturbances passing round this obstacle produce, even behind the screen, hyperbolic fringes of maximum disturbance, between which there may be traced hyperbolic fringes of minimum dis-

turbance where crests coincide with troughs; and that, further, even beyond the shadow of the screen, there are hyperbolic fringes of maximum and minimum disturbance. The centre of the shadow of the screen is a spot of maximum disturbance.

Fig. 77.



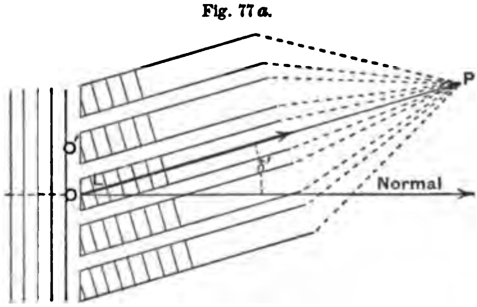
Wave traversing an aperture. — A wave whose wave-length is small, passing through an aperture, gives similar fringes of maximum and minimum disturbance, beyond the edge of the aperture. A very curious result is, that a wave-front passing through an aperture may produce no movement at a point situated immediately opposite its centre, for there may be complete interference between the waves proceeding

from the edges and those from the central regions of the aperture.

Relation of the wave-lengths to Fringes.—The position of the fringes depends on the wave-length. The smaller the wave-length, the nearer to one another will the fringes be. If the incident wave, diverging from a point, be compound, each component will form its own set of fringes without reference to the others; and any particle within the field of fringes may be stationary as regards one of the component vibrations, while at the same time it is affected by the others.

Broken wave-front.—In Fig. 77 *a*, let a plane wave-front be represented only by equal and equidistant portions, O, O′. Then in the first place, there is propagation in the original direction along the normal. Next, there are directions laterally situated, along which there are maxima of disturbance. To find the first of these, construct the right-angled triangle OO′L, in which OL is one wave-length. Disturbances at O′ and those at L from O are in the same phase. Plane fronts are thus developed, parallel to O′L; the wave travels towards an infinitely distant point, or may by refraction be made to converge upon a point P. The deflection is the angle δ'; let the distance OO′ be $1/N$ cm.; then since $OL = OO' \cdot \sin \delta'$, $\sin \delta' = N\lambda$. Hence the greater the wave-length the greater the deflection; and the more finely the wave-front is broken up, again the greater the deflection δ'. On the other side of the normal there is, symmetrically situated, another such deflection δ'. If the construction be repeated, making OL equal to two wave-lengths, we find another system of deflected wave-fronts, whose deflection δ'' is such that $\sin \delta'' = 2N\lambda$; and with consecutive similar constructions we find $\sin \delta''' = 3N\lambda$, $\sin \delta'''' = 4N\lambda$, and so on. This principle is utilised in the use of Diffraction-gratings in Optics.

Fig. 77 *a*.



If the incident wave-front be not parallel to the screen OO′, the construction is similar; but it will be found that the deviation is a minimum when the angle of incidence and the angle of diffraction (the angle between OP and the normal at O) are equal. When the angle of incidence is such that the angle of diffraction is nearly zero, the ratio between a difference of wave-length and the corresponding difference in the diffraction-angle is approximately constant (Normal Spectrum, p. 550).

Energy of S.H.M.—The Energy of a S.H.M. is proportional to the Square of its Amplitude. The angular velocity is constant, for S.H.M.'s are isochronous; the velocity in the circle of reference varies as the radius, *i.e.* as the amplitude; the Energy varies as the square of the velocity with which the body executing the S.H.M. passes the midpoint; this velocity is the same as the velocity in the circle of reference; therefore the Energy varies as the square of the amplitude.

If a S.H.M. be wholly in one line, its energy is wholly kinetic as the moving body passes the midpoint with velocity $\mathbf{v}$; it is equal to $\frac{1}{2}m\mathbf{v}^2$.

If another S.H.M. of equal amplitude and period, and in the same line with the former, be compounded with it, the amplitude is doubled, and the energy therefore quadrupled; there must therefore be a draft of energy from elsewhere before this superposition can actually occur.

Energy of Conical Pendulum.—If with one S.H.M. there be compounded another S.H.M. of equal amplitude and period, but in a line at right angles to the former, the energy of the compounded movement is simply double that of either of the components. In circular motion these two S.H.M.'s differ by $\frac{1}{4}$ period; when the energy of the one is wholly kinetic that of the other is wholly potential, and *vice versâ;* while in intermediate positions the one has gained as much potential or kinetic energy as the other has lost; so that the amount of kinetic energy is continuously equal to the amount of potential energy, each of these being $\frac{1}{2}mv^2$. The whole energy of a circular movement of velocity v, when the moving body is attracted towards the centre by a force which varies directly as the distance from the centre, is thus mv^2; of which half is kinetic, half potential.

Energy of wave-motion.—The energy of a wave-motion is equally divided between the potential and the kinetic forms. Let us first consider a linear wave: at the crest and at the trough the whole energy is potential; midway between crest and trough the energy of the particles, as they pass through their mean position, is wholly kinetic; elsewhere all the particles are symmetrically and continuously losing potential and gaining kinetic energy, or gaining potential while losing kinetic energy; the gains must be equal to the losses, for there is no change in the type of vibration, and no change in the amount either of potential or of kinetic energy (friction being imagined absent). Hence the whole energy of the wave is divided into two equal moieties, kinetic and potential in their respective forms. In a circular wave the kinetic energy is, under the same supposition, invariable in its absolute amount, and the potential energy bears to it the same symmetrical fixed ratio of equality. So for tridimensional waves.

The Energy of tridimensional waves per cubic cm. is numerically equal to the pressure per square cm., which is exerted on the bounding surface in consequence of the continued propagation of wave-motion. This is equal, if ρ be the density and v the maximum velocity of vibration, to $\frac{1}{2}\rho v^2$ dynes per square cm., or ergs per cubic cm.; or, if a be the amplitude, and n the frequency, to $2\pi^2a^2\rho n^2$ dynes or ergs, as the case may be. It is therefore proportional, for waves of a given frequency, to $a^2\rho$.

In interference-bands, due to two equal sources, the amplitudes are alternately double and zero; the energies are accordingly alternately quadruple and zero; the average energy is double of the energy coming from one source. There is thus a redistribution of the energy.

Rate of propagation of groups of waves.—In every case where the rate of propagation of an individual wave does not depend on the wave-length,—*e.g.*, sound-waves in air, and, so far as yet known, ether-waves—a group of waves advances with the same speed as the individual waves do. The case is, however, different where the velocity of an individual wave depends on the wave-length. For example, in waves in deep water, where the restitution of form is due to gravity, the group only travels half as fast as the individual waves, and the individual waves appear to travel through the group, dying away towards its anterior margin.

CHAPTER VI.

KINETICS.

GENERAL PROPOSITIONS relating to the possible forms of Motion find their parallel in those relating to Forces. The formula $F = ma$, already established, shows that every force is measured by the Quantity of Motion produced in unit of time. In this way we see that the truth of the propositions entitled the Parallelogram of Velocities and that of Accelerations involves that of a similar proposition in regard to Forces. Two forces acting on a single particle produce the same result as would be produced if a single force were acting on it, represented in magnitude and direction by the diagonal of a parallelogram, the adjacent sides of which represent, in the same respects, the two simultaneous forces. This is the proposition of the "**Parallelogram of Forces.**"

This proposition shows us that if we have two component forces at right angles to one another, the square of the resultant force will be equal to the sum of their squares. Refer to Fig. 11(a); let it be desired to apply force to a particle lying at A, so as to make it move in the direction AC: the available force is represented in magnitude and direction by the line AD: then obviously this force cannot exert its full effect in the direction AC; it is only its effective component in that direction that can produce any such effect: this component is represented by the line AC. The other component AB, at right angles to AC, can produce no such effect. This is an example of the **Resolution of Forces.** Plainly, we may resolve any Force into two components at any angle to one another, just as we may so resolve a Velocity. In tridimensional space we may resolve a force into three components. Suppose, then, such a question as the following:— A pull is made in the direction AD; this pull is designed to draw an object through a tube whose direction is AC: what proportions do the component effective in pulling down the

object, and the lateral pressure on the walls of the tube, respectively bear to the force applied? If Fig. 11 (a) were drawn with the proper angle CAD $= \xi$ between the direction of application of the force and the line along which the object is to be drawn: then the effective component would be represented by AC, and the component producing lateral pressure by AB. But AB $=$ AD $\sin \xi$; AC $=$ AD $\cos \xi$: or—

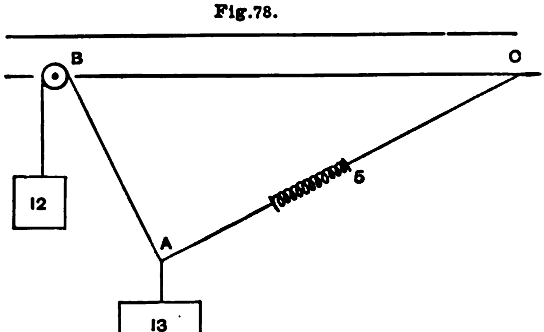
$$AB : AD :: \sin \xi : 1$$
$$AC : AD :: \cos \xi : 1.$$

Hence, if AD be taken as unity, AB and AC may easily be found, if the angle ξ be known, by finding the value of $\sin \xi$ and $\cos \xi$ in a table of trigonometrical ratios. If AD have any other value than unity, the values of $\sin \xi$ or $\cos \xi$ derived from the tables must be multiplied proportionately.

Problem. — A force of 50 lbs. is applied to a solid body drawn down a fixed canal which the solid body exactly fits; the angle ξ which the direction of traction makes with the axis of the canal is 29°. What is the effective component available in pulling the solid body? and what is the pressure produced by the traction-force on the walls of the canal? In Fig. 11 (a), if ξ be 29°, AD represents the force applied, equivalent to the weight of a 50-lb. mass; AC represents the effective component $=$ AD $\times \cos \xi$; AB represents the component at right angles to AC — that is, the detrimental pressure $=$ AD $\times \sin \xi$. But $\cos 29° = \cdot 8746197$; $\sin 29° = \cdot 4848096$. The effective component is thus ·8746197 of the force applied — *i.e.* it is equal to the weight of 43·731 lbs.; the detrimental pressure is 24·2405 lbs.

It seems rather surprising that the effective and lateral components together should appear to be so much greater than the original force applied. But geometrically this is the same thing as to say that two sides of a triangle are greater than the third side; and further, we have already seen that there is no law of the Conservation of Force, though there is a law of the Conservation of Energy. The principle of the Conservation of Energy is maintained, for the work done in enforcing the detrimental pressure, together with the work done in pulling the body down the canal, is equal to the energy imparted.

Fig. 78.



Experimental proof of the Parallelogram of Forces. — A cord, two masses of 12 and 13 lbs. respectively, a pulley, and a dynamometer are arranged on a beam as shown in Fig. 78. The string can be adjusted so that the angle BAC may have a wide range of values: for every position there is a corresponding stress on the dynamometer.

When AB and AC are fixed at right angles to one another, the spring of the dynamometer is pulled out to the same extent as it would have been by the weight of a 5-lb. mass. The dynamometer may be replaced by a 5-lb. mass suspended over a pulley; in that case the cord and the three suspended masses would so adjust themselves that the angle A would be a right angle. Here $5^2+12^2=13^2$, or $25+144=169$; and the law is confirmed.

In a similar way, the propositions known as the Triangle of Velocities, the Polygon, etc., are replaced in Kinetics by the **Triangle,** the **Polygon,** etc., of **Forces**; and the resultant force is the missing side of the triangle or polygon, of which all the sides except the missing one represent the various forces acting, in precisely the fashion already studied under Kinematics.

In tridimensional space we have the propositions of the parallelepipedon, the skew-polygon, etc., of forces, parallel to similar propositions under Velocity and Acceleration.

Equilibrium of Forces.—There here emerges an important and difficult question of nomenclature. It may appear not strictly consistent with the definition of Force to speak of two forces, equal and opposite, balancing one another and producing no effect, as being still two distinct Forces; for the essence of Force is acceleration observed. But it is not more inconsistent than it is to speak of two equal and opposite simultaneous velocities being equivalent to rest, or of two equal and opposite simultaneous accelerations resulting in no change of velocity. There is no doubt that we are entitled so to speak; and if so, then we are entitled to speak of equal and opposite simultaneous forces balancing one another, for propositions concerning forces are strictly parallel to those concerning velocities or accelerations.

The inconsistency finally vanishes when we observe that the Forces which apparently destroy one another are not physical entities, but mental artifices, "paper bullets of the brain;" and that we do not really think of them as simultaneous: we look first to the one and then to the other, and see that what the one does the other undoes; in the end there is thus no result. The imaginary Forces are in equilibrium; the actual body is in a condition of Stress of some kind. But "Force" is a compendious phrase; its use saves many words; and bearing this in mind, we may admit that it is often more convenient to study the balanced forces, each in its turn, than it is to consider the actual stressed condition of the body: and thus while we cannot speak of a single force unless there be actual motion

produced or checked, we may allow ourselves to speak of the equilibrium of the several forces acting upon a body at rest.

If we suppose a stone perched upon a height, we say that the earth attracts it with a definite and measurable Force. Yet there is no movement. Here, however, we have really a downward Action of the earth, an upward Reaction of the support; and the phenomenon is an equilibrium of two "forces," of which we may confine our attention to one only, the downward Force of Gravitation.

So we may say that an electrified body, freely suspended at a certain distance from another electrified body, is attracted or repelled with a definite Force; and so it is, as we may see if it be free to move; but if it be not free to move, the attraction or repulsion is still the same, but is now balanced by an equal and opposite Reaction of the support; and this Force and Reaction together correspond to a Stress, a mutual Pressure or Tension, between the body attracted or repelled and its support. Hence we may say that the body is in every such case impelled to move (repelled or attracted) with or by a certain definite Force.

Centre of Figure, Centre of Mass, or Centre of Inertia.— We have already seen that a rigid body, of which the several particles are subject to accelerations which are equal and parallel to one another, moves as if concentrated at its centre of figure, and as if this were subject to a single acceleration. A body may thus be acted upon by parallel forces affecting its particles, the result being the same as if a single force had acted at the centre of figure; while conversely, if a single force act at the centre of figure, the result is to impart parallel and equal accelerations to all the particles, and thereby to effect a Translation of the body. Hence most of the propositions of Kinematics, which describe the motion of a single point, may be transferred to Kinetics, not only as relating to the movements of single particles, but also as relating to translation of material bodies.

If, however, the body be not uniform in density, then, as in the case of the moon, the centre of mass may not coincide with the centre of figure.

Inertia of Matter.—If in any system of bodies there be no force acting, the formula $F = ma = 0$ shows that $a = 0$, that there is no acceleration: hence if there be no force acting, there is no change in the speed with which a body is moving, or in its state of rest, as the case may be; in other words, Matter has Inertia. This is Newton's first law of motion, and it appears to be here derived from the formula; but it will be remembered that the formula was itself derived by implication from that law.

Coefficient of Inertia.—If a body have translational momentum $m'\mathbf{v}'$, then, in order to bring it to rest in time t', the mean negative acceleration required is $-(\mathbf{v}'/t')$, and the mean retarding force required is $\mathbf{F}=(m'\mathbf{v}'/t')$. The body offers a mean Resistance to Stoppage equal to $(m'\mathbf{v}'/t')$. If the required acceleration $(\mathbf{v}''/t''=\mathbf{v}'/t')$ be the same in another body, but the momentum $m''\mathbf{v}''$ different, the resistance to stoppage is now $m''\mathbf{v}''/t''=m''\mathbf{v}'/t'$. Generally, the resistance to change of momentum is thus equal to the product of m into the necessary acceleration; and m is the Coefficient of Translational Inertia for a body whose Mass is m.

Examples of Inertia.—Examples of this abound. Collisions between ships and between trains, which do not stop if there be not sufficient retarding-force at command; trains passing stations when their speed is great and the rails are slippery; a person falling off the stern of a boat or the back of a car, when the vehicle makes a sudden movement forwards in which his body does not participate; the onward motion retained by a rider when his horse stops under him; the jerk received by a horse suddenly starting in order to set in motion a heavy waggon; when the waggon is running, if the horse suddenly stop, he is bruised, for the massive waggon does not stop at once; a greyhound chasing a hare is carried forward and cannot stop or turn his path instantly at the spot where the hare doubled or turned abruptly from her course; the inertia of the dust of a carpet, when the carpet is beaten—the carpet moves forwards at each blow, but the dust remains, and is thus separated from the carpet and blown away by the wind; the inertia of dust when it is shaken off a book—the book and the dust are made to describe together a rapid movement in the air—the book is suddenly arrested by a smart blow, while the dust does not stop but moves onwards; the inertia of the snow which in the same way is kicked off one's boots—the boot is suddenly stopped, but the snow goes on, and is thus shaken off; the inertia of loose grain cargo in a ship—it acquires a certain velocity when the ship rolls, and does not stop when the ship arrives at its normal limit, but pours on so as sometimes to make the ship roll beyond the limits of safety; the oscillations of mercury in an ordinary barometer at sea, the mercury being jerked up by each roll of the ship; the jerking of the blood against the valves of the blood-vessels of a bad rider; the inertia of the mercury in a mercury manometer used to investigate fluid-pressure—the variations in the height of the mercurial column being greater than the real variations in

the pressure, for the mercury does not stop moving when the fluid-pressure ceases to rise or fall; the inertia of a mass suspended on a spring-balance, by reason of which the weight is apparently increased when the balance is suddenly raised, and lessened when it is suddenly lowered; the inertia of water in house water-pipes if it be set to run and then suddenly stopped—the water is compressed against itself and a violent jerk is produced, which is utilised in the hydraulic ram; the inertia of water in the case of the water-supply of the locomotive engines of passenger express trains on the L. and N.W. Railway system—the engine puts down a tube, the lower end of which acts as a scoop for the water, which tends to remain in its trough on the ground between the rails and at rest relatively to the ground; but this being equivalent to a backward movement relatively to the engine, the water slips up the inclined tube into the tender if the train be moving at sufficient speed.

There are some further remarkable consequences of the inertia of matter. A body may be struck or pressed so suddenly that it stands practically at rest during the time that the blow is being spent on it, and it may be crushed or broken by such a blow. A candle can thus be fired through a board; a bullet may be fired through glass without cracking it; and a cannon-ball through a half-open door without opening it. Water will reflect a cannon-ball or flatten a bullet. A dynamite-charge, exploded upon a stone, developes pressure between the stone and the air, so suddenly that the stone is shattered before the air has time to move away. A grain of corn or a granule of gold-quartz, if thrown up into the air and struck a blow by an iron bar moving at the rate of about 180 feet a second, will be crushed by compression, and will be, by a succession of such blows, very effectively pulverised. Milling machinery has been constructed on this principle. A bullet in a gun, though free to move onwards, is crushed against itself before it fairly starts, so that the soft lead is moulded into the grooves of the rifle-barrel by the rapidly-applied pressure, due to the explosion of quick-burning gunpowder.

Another example is afforded by that instrument with the aid of which M. Rosapelly* investigated the movements of the larynx during the emission of sounds. A heavy mass of metal is suspended in a light frame-work which is tied over the larynx: as this mass cannot at once participate in

* Trav. du Laboratoire de M. Marey, 1876.

the rapid movements which the vibration of the larynx communicates to the light framework, it forms a kind of fixed point, and the light framework, as it vibrates in contact with the skin over the larynx, may strike the heavy mass a series of blows; these may cause an electric current to be alternately made and broken; the number and frequency of these interruptions may be registered on an appropriate recording-instrument.

Further, the inertia of matter is a property of retaining whatever motion an object has, and that in a plane fixed in space, without reference to the movements of surrounding objects, unless these are so connected with it as to be able to affect its motion. A hammock retains its position in space independently, in the main, of the pitching and rolling of the ship. The statement would be approximately accurate that the hammock does not swing in the ship, but that the ship swings enclosing the hammock, which may for any short period of time be regarded as moving onward in space with the average velocity of the ship, but independently of it. A long and heavy pendulum set to swing in one plane, and connected by a very slender attachment to the roof of the building in which it is suspended, will swing in the same plane in absolute space though the earth rotate under it: the apparent result is, that the plane in which the pendulum swings gradually alters its aspect, so that the pendulum swings successively in every possible direction. The real state of the case is not that the heavy pendulum alters its direction of oscillation, but that the earth rotates or has a component of rotation under the pendulum, except at the equator. If a heavy wheel be set in motion, it will in the same way, if it can rotate for a sufficiently long time, show the same phenomenon, for it tends to continue to rotate in the same plane in space.

Momentum. — The product $m\mathbf{v}$ of m, the mass of a moving body, into $\mathbf{v}$, its velocity in any given direction, is called the *Momentum* of the body in that direction.

If a shell explode, its fragments form a system of bodies moving at different velocities. The average velocity of the centre of mass of the whole system is, however, unchanged: some fragments travel with a greater, some at a less velocity than that with which the shell had travelled before the explosion; but the mass m is unchanged though differently arranged, the mean velocity of the system is the same as that of the original shell, and thus the momentum of the whole system, in the direction of movement before the explosion, is the same after explosion as before it.

Impact. — If there be two *inelastic* bodies, of masses $m_{\prime}$ and $m_{\prime\prime}$ respectively, of which the first moves with velocity $\mathbf{v}_{\prime}$, while the second is at rest: if the moving one, whose momentum is $m_{\prime}\mathbf{v}_{\prime}$, strike the other, it will divide its momentum with that other; it itself will travel more slowly, while the other is set in motion; but the two will travel together with a common velocity $\mathbf{v}$, in the original direction of the mass $m_{\prime}$.

The whole mass moving with this new velocity $\mathbf{v}$ is $(m_{\prime} + m_{\prime\prime})$; its momentum is equal to the original $m_{\prime}\mathbf{v}_{\prime}$; hence the velocity $\mathbf{v}$ may be found by stating this equality of momenta in the form of the equation —

$$(m_{\prime} + m_{\prime\prime})\mathbf{v} = m_{\prime}\mathbf{v}_{\prime}$$
$$\therefore \mathbf{v} = \frac{m_{\prime}\mathbf{v}_{\prime}}{m_{\prime} + m_{\prime\prime}}.$$

If the mass $m_{\prime\prime}$ be large in comparison with $m_{\prime}$, the new velocity $\mathbf{v}$ is much less than $\mathbf{v}_{\prime}$. If a man lie with an anvil on his chest, and if the anvil be struck a blow with a hammer relatively not too heavy, the person lying down, if he can support the anvil, will not be much affected by the blow, for the movement imparted to the anvil will be slow as compared with that of the hammer.

Let the two inelastic masses be $m_{\prime}$ and $m_{\prime\prime}$, moving with the respective velocities $\mathbf{v}_{\prime}$ and $\mathbf{v}_{\prime\prime}$, and together moving after impact with the velocity $\mathbf{v}$; the respective momenta of the masses before impact were $m_{\prime}\mathbf{v}_{\prime}$ and $m_{\prime\prime}\mathbf{v}_{\prime\prime}$; that of the conjoined mass after impact is $(m_{\prime} + m_{\prime\prime})\,\mathbf{v}$. Hence

$$m_{\prime}\mathbf{v}_{\prime} + m_{\prime\prime}\,\mathbf{v}_{\prime\prime} = (m_{\prime} + m_{\prime\prime})\,\mathbf{v}.$$
$$\mathbf{v} = \frac{m_{\prime}\mathbf{v}_{\prime} + m_{\prime\prime}\mathbf{v}_{\prime\prime}}{m_{\prime} + m_{\prime\prime}}. \quad (1.)$$

It was found experimentally by Newton that, in such a case, the momentum lost by one body was equal to that gained by the other. To express this algebraically, if M represent the momentum gained by one and lost by the other,

$$m_{\prime}\,\mathbf{v} - m_{\prime}\mathbf{v}_{\prime} \;\; = \mathrm{M}. \quad (2.)$$
$$m_{\prime\prime}\,\mathbf{v}_{\prime\prime} - m_{\prime\prime}\mathbf{v} = \mathrm{M}. \quad (3.)$$

From either of these, with the aid of equation (1) we find

$$\mathrm{M} = \frac{m_{\prime}m_{\prime\prime}\;(\mathbf{v}_{\prime\prime} - \mathbf{v}_{\prime})}{m_{\prime} + m_{\prime\prime}}.$$

Apparent loss of Energy. — In this case the kinetic energy after impact

$$\left(i.e.\ \tfrac{1}{2}\ \text{mass} \times \mathbf{v}^2 = \tfrac{1}{2}\ (m_{\prime} + m_{\prime\prime})\left(\frac{m_{\prime}\mathbf{v}_{\prime} + m_{\prime\prime}\mathbf{v}_{\prime\prime}}{m_{\prime} + m_{\prime\prime}}\right)^2 = \tfrac{1}{2}\,\frac{(m_{\prime}\mathbf{v}_{\prime} + m_{\prime\prime}\mathbf{v}_{\prime\prime})^2}{m_{\prime} + m_{\prime\prime}}\right)$$

is less than the sum of the kinetic energies before impact (which were $\frac{1}{2}m_{\prime}\mathbf{v}_{\prime}^2$ and $\frac{1}{2}m_{\prime\prime}\mathbf{v}_{\prime\prime}^2$ respectively). This is not true if $\mathbf{v}_{\prime}$ be equal to $\mathbf{v}_{\prime\prime}$, but in that case the two bodies would be travelling in the same direction with

equal speed, and the one could not overtake and strike the other. The energy which has apparently disappeared has assumed the form of Heat.

Impact of Elastic Bodies. — We may here anticipate a statement of the nature of Elasticity so far as to say that a perfectly elastic body, possessed of a certain amount of kinetic energy, and striking a perfectly rigid body, will rebound, and will possess as much kinetic energy after the impact as before it; for it leaves the rigid body with a velocity equal to that with which it had approached it.* The mass and the velocity being numerically unchanged, the momentum is numerically equal after impact to that before it; but as it is no longer $m\mathbf{v}$ but $m \times (-\mathbf{v}) = -m\mathbf{v}$, it has become negative, and has therefore altered by an amount equal to $2m\mathbf{v}$. If the body be imperfectly elastic, so that the velocity is not completely regained, it is found experimentally that it returns with a certain fraction, λ, of its original velocity (this fraction, λ, being called the **coefficient of restitution**), and the change of velocity is not $2\mathbf{v}$ but $(1 + \lambda)\ \mathbf{v}$; and its momentum has become negative and $= -\lambda \cdot m\mathbf{v}$, so that it has changed by the amount $(1 + \lambda)\ m\mathbf{v}$.

If two masses $m_{,}$ and $m_{,,}$, moving with velocities $\mathbf{v}_{,}$ and $\mathbf{v}_{,,}$ in the same direction, and formed of such material that the coefficient of restitution between them is λ, strike one another, they will, after impact, travel with velocities, say, $\mathbf{v}_{,}'$ and $\mathbf{v}_{,,}'$. The momentum gained by the one is equal to that lost by the other; but it is not equal, as it is in the case of inelastic bodies where $\lambda = 0$, simply to $\dfrac{m_{,}m_{,,}(\mathbf{v}_{,,} - \mathbf{v}_{,})}{m_{,} + m_{,,}}$, but to $(1 + \lambda) \times$ that quantity. This equality of momenta is expressed by the equations —

$$(1)\ \text{Gained by } m_{,};\ m_{,}\mathbf{v}_{,}' - m_{,}\mathbf{v}_{,} = (1+\lambda)\ \frac{m_{,}m_{,,}(\mathbf{v}_{,,} - \mathbf{v}_{,})}{m_{,} + m_{,,}}.$$

$$(2)\ \text{Lost by } m_{,,};\ m_{,,}\mathbf{v}_{,,} - m_{,,}\mathbf{v}_{,,}' = (1+\lambda)\ \frac{m_{,}m_{,,}(\mathbf{v}_{,,} - \mathbf{v}_{,})}{m_{,} + m_{,,}}.$$

Whence

$$\mathbf{v}_{,}' = \mathbf{v}_{,} + (1+\lambda)\frac{m_{,,}(\mathbf{v}_{,,} - \mathbf{v}_{,})}{m_{,} + m_{,,}}.$$

$$\mathbf{v}_{,,}' = \mathbf{v}_{,,} - (1+\lambda)\frac{m_{,}(\mathbf{v}_{,,} - \mathbf{v}_{,})}{m_{,} + m_{,,}}.$$

Take, as a particular instance, the case in which the elasticity is perfect or the restitution complete (*i.e.* $\lambda = 1$), and the balls which strike one another are of equal weight, so that $m_{,} = m_{,,}$; then $\mathbf{v}_{,}' = \mathbf{v}_{,,}$ and $\mathbf{v}_{,,}' = \mathbf{v}_{,}$; *i.e.*:—

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* Such is the elementary theory. There is, however, no perfectly elastic body, and even if there were, a part of its energy would necessarily be spent, upon impact, in setting up vibrations in it, and the speed of rebound could never come up to the theoretical limit.

Two equal and perfectly elastic balls, striking one another directly in the line joining their centres, exchange their velocities, and that whether they meet or overtake one another.

Oblique Impact. — If a ball strike a rigid surface obliquely, its motion relative to that surface may be resolved into two components: one parallel to it, which is not affected by the impact; the other at right angles to it, which, after impact, will be wholly or in part restored in the reverse direction. Reference to Fig. 60 will show that if $\lambda = 1$, the angle of reflexion will be equal to the angle of incidence; while if λ be less than unity, the angle of reflexion will be proportionately less acute, to an extent easily determined by construction.

If the oblique impact be between two balls, the investigation is based on similar principles. Take a line joining their centres of figure; in a direction at right angles to this line the motion is unaffected by the impact, and the component in this direction will be the same after impact as before it; in the line joining the centres of figure, the velocities $\mathbf{v}_{\prime}'$ and $\mathbf{v}_{\prime\prime}'$ may be found as in the preceding discussion. By compounding the component velocities after impact, the resultant velocities and their directions may be found. In practice, as the two balls are passing one another in contact, friction between their surfaces causes a relative delay of one aspect of each, and causes rotation of the balls; the energy necessary for this is taken from that theoretically available for the direct translational movements of the balls as wholes.

Energy in impact of Elastic Bodies. — In the case of perfectly elastic bodies, the energy after impact is equal to that before it; $\frac{1}{2}m_{\prime}\mathbf{v}_{\prime}'^2 + \frac{1}{2}m_{\prime\prime}\mathbf{v}_{\prime\prime}'^2 = \frac{1}{2}m_{\prime}\mathbf{v}_{\prime}^2 + \frac{1}{2}m_{\prime\prime}\mathbf{v}_{\prime\prime}^2$. If λ be less than 1, there is apparent loss of Energy, which has assumed some other form than that of motion of the mass. If a horse with loose traces rush forward and jolt a car, the energy which disappears is wasted in the form of heat, or deleteriously spent in disintegration of the materials of the car, or in bruising the animal.

Accelerated Motion. — The discussion of the accelerated motion of a particle moving with constant acceleration — *i.e.* under the continuous influence of a constant force in the line of the existing motion — has already (pp. 69–70) led us to the formulæ —

$$\mathbf{v}_t = \mathbf{v}_0 \pm \mathbf{a}t, \qquad \text{(i.)}$$

$$\mathbf{s} = \tfrac{1}{2}(\mathbf{v}_0 + \mathbf{v}_t)\,t = \mathbf{v}_0 t \pm \tfrac{1}{2}\mathbf{a}t^2, \qquad \text{(ii.)}$$

$$\mathbf{v}_t^2 = \mathbf{v}_0^2 \pm 2\mathbf{a}\mathbf{s}, \qquad \text{(iii.)}$$

$$\mathbf{s} = \mathbf{v}_t t \mp \mathbf{a}t^2/2 = (\mathbf{v}_t^2 - \mathbf{v}_0^2) + 2\mathbf{a}, \qquad \text{(iv.)}$$

where $\mathbf{v}_0$ represents the velocity of a particle at the beginning, and $\mathbf{v}_t$ that at the end of time t, $\mathbf{s}$ the space traversed during that time, and $\mathbf{a}$ the acceleration (positive or negative, as the case may be) per unit of time. The most familiar examples of this kind of movement are those in which bodies exposed to the constant influence of gravity fall with constantly-increasing speed.

Problems.

1. A train travelling at 50 miles an hour comes into collision with a fixed obstacle and is abruptly stopped: the passengers receive a blow. What height must one fall in order to receive a similar blow?—*Ans.* 50 miles an hour = $73\cdot\dot{3}$ feet per second. A body falling from rest ($\mathbf{v}_0 = 0$) acquires a speed of $73\cdot\dot{3}$ feet per sec. in falling 83·51 feet, for

$$\mathbf{v}_t^2 = \mathbf{v}_0^2 \pm 2\mathbf{as} = \mathbf{v}_0^2 + 2gh. \text{ (iii.)};$$
$$(73\cdot\dot{3})^2 = 0 + (2 \times 32\cdot2 \times h)\,;\ \therefore h = 83\cdot51.$$

A blow in a collision at 50 miles an hour is equivalent to a blow received in consequence of a fall of 83·51 feet; for a body which has fallen 83·51 feet is in consequence travelling at 50 miles an hour at the instant.

2. A ball weighing 5 ounces is hurled upwards. It is supposed that while it is in the hand it is swung through 4 feet; the thrower during this swing continuously exerts an accelerating pressure on it. This pressure must be equivalent to the effort which would be put forth in raising some definite mass in the same position of the body. What is this mass if the ball rise 100 feet?

The ball leaves the hand with an unknown upward velocity $\mathbf{v}_0$; it rises through a vertical height $h = 100$ feet against a force (gravity), the negative or downward acceleration produced by which is 32·2 ft.-per-sec. per second; it comes for an instant to rest ($\mathbf{v}_t = 0$) at the top of its course.

$$\mathbf{v}_t^2 = \mathbf{v}_0^2 \pm 2\mathbf{as} = \mathbf{v}_0^2 - 2gh. \text{ (iii.)}$$
$$0 = \mathbf{v}_0^2 - (2 \times 32\cdot2 \times 100).$$
$$\mathbf{v}_0^2 = 6440. \quad \mathbf{v}_0 = 80\cdot25.$$

The question thus becomes—Under the influence of a force $\mathbf{F}$ acting upwards through 4 feet, an upward velocity 80·25 feet per second is imparted to a mass $m = \frac{5}{16}$ lb. Find $\mathbf{F}$; or, since $\mathbf{F} = m\mathbf{a}$ and $m = \frac{5}{16}$, find $\mathbf{a}$.

$\mathbf{v}_t^2 = \mathbf{v}_0^2 \pm 2\mathbf{as}$ (iii.); $\mathbf{v}_0 = 0$; $\mathbf{a}$ is positive.

$\therefore \mathbf{v}_t^2 = 2\mathbf{as}$; but $\mathbf{v}_t = 80\cdot25$, the velocity with which the ball leaves the hand.

$6440 = 2\mathbf{a} \times 4 = 8\mathbf{a}$. $\quad \mathbf{a} = 805$ ft.-per-sec. per second.

$\mathbf{F} = m\mathbf{a} = 226\frac{9}{16}$ British units of force.

But wt. of 1 lb.-mass = 32·2 Brit. units of force.

$$\mathbf{F} = \text{wt. of } \frac{226\frac{9}{16}}{32\cdot2} \text{ lb.-masses} = \text{wt. of } 7\tfrac{13}{16} \text{ lbs.}$$

The effort then is the same as that put forth in lifting a mass of 7 lbs. 13 oz. against gravity; the time is 0·0997 sec., for $t = 2\mathbf{s} \div (\mathbf{v}_0 + \mathbf{v}_t)$; and the activity $= \mathbf{Fs}/t = 7\cdot8125$ lbs. $\times$ 4 ft. $\div$ 0·0997 sec. = 313·475 ft.-lbs. per sec. = 0·57 horse-power.

3. A shot is fired vertically from a gun whose barrel is 30 inches long: it rises half a mile. Compare the acceleration of a body falling under the influence of gravity with that under which the bullet acquires such velocity in the space of 30 inches.

First find the velocity of the shot as it leaves the gun. In its course in the air (friction being entirely neglected) it commences with the unknown velocity $\mathbf{v}_0$, traverses height $h = 2640$ feet against gravity which produces a vertically downward acceleration $\mathbf{a} = g = -32{\cdot}2$, and comes to rest ($\mathbf{v}_t = 0$) for an instant.

$$\mathbf{v}_t^2 = \mathbf{v}_0^2 \pm 2\mathbf{a}\mathbf{s} = \mathbf{v}_0^2 - 2gh. \quad \text{(iii.)}$$
$$0 = \mathbf{v}_0^2 - (2 \times 32{\cdot}2 \times 2640). \quad \mathbf{v}_0 = \sqrt{170016} = 412{\cdot}3.$$

In the gun-barrel a velocity of 412·3 feet per second is acquired within a space $\mathbf{s} = 2\frac{1}{2}$ feet. We want to know the acceleration. Now 412·3 ft. per sec. $= \mathbf{v}_t$; $\mathbf{v}_0 = 0$; $\mathbf{s} = 2{\cdot}5$; $\mathbf{a}$ is unknown, but is positive.

$$\mathbf{v}_t^2 = \mathbf{v}_0^2 \pm 2\mathbf{a}\mathbf{s}. \quad \text{(iii.)}$$
$$(412{\cdot}3)^2 = 0 + (2\mathbf{a} \times 2\tfrac{1}{2}).$$
$$170016 = 5\mathbf{a}. \quad \mathbf{a} = 34003{\cdot}2 \text{ ft.-per-sec. per second.}$$

If the barrel were long enough to expose the projectile to the influence of such an accelerating force for a whole second, this velocity would be acquired, and the barrel would be 17001·6 feet long; but the time in the barrel is $t = \mathbf{v}_t/\mathbf{a} = 0{\cdot}012$ second only.

But gravity produces an acceleration of 32·2 ft.-per-sec. per second. Hence the acceleration due to the gunpowder is greater than that of gravity in the ratio of $\frac{34003{\cdot}2}{32{\cdot}2}$, or 1056 : 1. The *force* exerted by the powder $= m\mathbf{a} = 34003{\cdot}2m$, and in the case of an ounce-bullet would be equal to 2125·2 British units of force.

4. With what "force" will a 10-lb. mass falling 100 feet strike at the end of its course? As it stands, this question is devoid of sense, for it does not specify the time during which the momentum is changed on impact. If the body struck were rigid, and the falling mass perfectly elastic, it would, apart from vibrations, rise on rebounding to an equal height of 100 feet. During the impact it must have come to rest. Let us arbitrarily assume it to have taken $\frac{1}{2000}$ sec. to come to rest, and an equal period, $\frac{1}{2000}$ sec., to gain its upward initial velocity: this upward initial velocity is 80·25 feet per second, for

$$\mathbf{v}_t^2 = \mathbf{v}_0^2 \pm 2\mathbf{a}\mathbf{s} = \mathbf{v}_0^2 - 2gh.$$
$$0 = \mathbf{v}_0^2 - (2 \times 32{\cdot}2 \times 100).$$
$$\mathbf{v}_0 = 80{\cdot}25.$$

The question thus becomes—What is the mean pressure between the body which has fallen and that on which it falls, if a speed of 80·25 feet per second can be arrested or developed by it in $\frac{1}{2000}$ sec.? The answer is—Since $\mathbf{v}_t = \mathbf{a}t : \mathbf{v}_t = 80{\cdot}25$; $t = \frac{1}{2000}$; $\therefore \mathbf{a} = 160{,}500$; and $\mathbf{F} = m\mathbf{a} = 1{,}605{,}000$ British units of force = the wt. of $\frac{1{,}605{,}000}{32{\cdot}2} = 49844{\cdot}7$ lbs.

5. A ball weighing 10 lbs. falls from a height of 100 feet on a rigid floor. It is flattened to the extent of $\frac{1}{30}$ inch, measured in the direction of its motion: it recovers its form and rebounds. What is the time taken to bring the ball to rest, and what is the mean total pressure between the ball

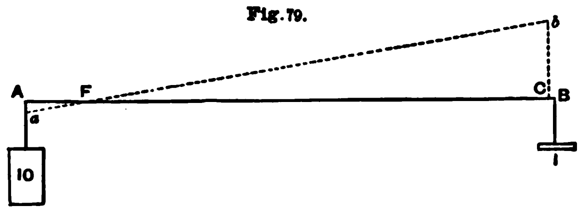
and the floor on which it falls? Here a velocity of 80·24954 ft. per sec. is arrested in the space of $\frac{1}{10}$ inch: what is the retarding acceleration $\mathbf{a}$? what is the corresponding pressure **P**? what is the time t? It is again assumed that the ball is perfectly elastic, and that there are no vibrations.

$$\mathbf{v}_t^2 = \mathbf{v}_0^2 - 2\mathbf{as}. \quad \text{(iii.)}$$
$$0 = (80{\cdot}24954)^2 - (2\mathbf{a} \times \tfrac{1}{10}\text{ inch}) = 6440 - (2\mathbf{a} \times \tfrac{1}{120}\text{ foot.})$$
$$\mathbf{a} = \tfrac{1}{2}\,(6440 \times 360) = 1{,}159{,}200 \text{ ft.-per-sec. per second.}$$

Again, $\mathbf{P} = \mathbf{F} = m\mathbf{a} = 1{,}159{,}200 \times 10 = 11{,}592{,}000$ British units of force,
= the weight of $(11{,}592{,}000 \div 32{\cdot}2) = 360{,}000$ lbs. = *mean* pressure.
Lastly, $\mathbf{v}_t = \mathbf{a}t$ (i.); $80{\cdot}24954 = 1{,}159{,}200t$; $\therefore t = \frac{1}{14444}$ sec.

The Principle of Moments.—In Fig. 79 a linear body is poised at the point F; at A suppose a force **F** equal to 10 units, at B a parallel force equal to 1 unit. The former, acting alone, would turn the bar round F through an angle θ, and the work done at A by the force **F** is equal to $\mathbf{Fs} = 10$ units $\times$ Aa =

Fig. 79.



10 AF sin θ. The latter force applied at B would turn the bar round F through an angle, say ϕ, and would in producing rotation do work = 1 × FB sin ϕ. The work done by the force applied at A during any small displacement is opposite in sense to that done by the force applied at B. Together they may balance one another, and produce equilibrium. If the bar be rigid, $\phi = \theta$. If the one force raise as much as the other depresses every point of the bar, there is equilibrium. As regards rotation round F, the forces are of opposite sign; they are accordingly + 10 at A, tending to produce **positive** rotation in the direction opposed to that of the hands of a watch, and − 1 at B, tending to produce a **negative** rotation. Hence, if there be equilibrium, the work done by force = + 10 acting at A and that done by force − 1 acting at B are together = 0.

$$(10 \times \text{AF} \sin \theta) + (-1 \times \text{BF} \sin \theta) = 0; \quad (1.)$$
$$(10 \times \text{AF}) + (-1 \times \text{BF}) = 0;$$

or, generally, if the parallel forces be **P** and **Q**,—

$$(\mathbf{P} \times \text{AF}) + (\mathbf{Q} \times \text{BF}) = 0;$$
$$\text{or } \mathbf{P} : \mathbf{Q} :: \text{BF} : \text{AF}.$$

The parallel forces which balance one another are inversely proportional to their distances from the fixed point F.

Thus a smaller force acting at a greater distance can balance a greater force acting at a less distance. The Importance of the greater force with reference to the point F is exactly the same as that of the smaller force, which has the advantage of greater distance, or greater "leverage" or "purchase."

This Importance of a force not passing through a point is called the **Moment** of that force round that point. It is equal to the amount of the force × the shortest distance from the point *to the line of application* of the force. The shortest distance from a point to a line is well known to be a line drawn from the point to the line in question, at right angles to the latter. In Fig. 79 the moments of the forces round the point F are respectively 10 × AF and − 1 × BF. In Fig. 80 the forces acting at A and B are not parallel; their lines of application are AE and BE; the distances of these lines from F are FC and FD at right angles to AE and BE: the moments of the respective forces round F are Force A × distance CF, and Force B × distance FD; and if the forces are to produce equal and contrary rotational effects round F, so that there may be rest and statical equilibrium, their moments must be equal and of opposite sign, so that their sum = 0. This is the ***Principle of Moments.***

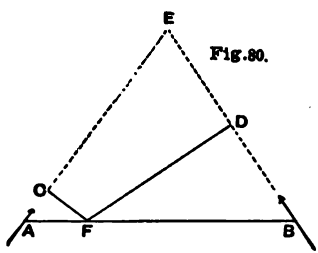


Fig. 80.

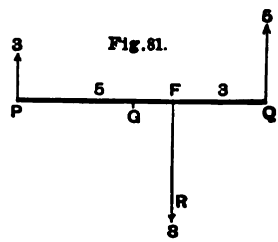
Moments should be specified in terms of dyne-centimetres, or of poundal-feet; or, it may be, in pound-feet, or in ton-feet, if the engineers' gravitational units be employed.

If in Fig. 79 the forces at A and B acted at the ends of an immovable rod, there would be a reaction = 11 units spread over the whole extent of the rod, but more intense near the point A: if the rod be held fast only at one fixed point F, all the reaction (= 11 units) is concentrated at that point, if it be such a point that there is no tendency to rotation round it — *i.e.* if the moments round the point of resistance = 0; if these be not = 0, the pressure on it is still 11 lbs., but the energy is partly spent in producing rotation round that point. If the reaction pass through the point round which the moments = 0, there is neither translation nor rotation, and hence the three forces are in equilibrium: these are, 10 units at A, 1 unit at B,

parallel to the former, and 11 units at F, parallel but in the opposite direction.

Thus Fig. 81 is established as indicating the conditions of equilibrium of two parallel forces, **P** and **Q**; a third, **R**, equal to their sum, must act in the opposite direction at F, a point round which their moments vanish or are together equal to zero. If the two conditions be satisfied (1) that **P** + **Q** = − **R**, and (2) that the moments of **P** and **Q** round F be equal and opposite, there will be statical equilibrium: if the former be not satisfied there will be translation; if the latter, there will be rotation; if both be violated there will be both translation and rotation.

Fig. 81.



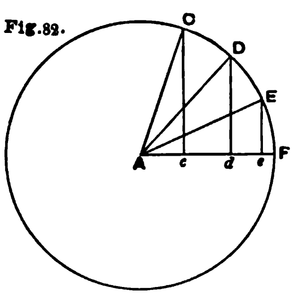
Now let the point of application of the force **Q** be shifted to the right: the force **P** must increase in order that its moment may remain equal to that of **Q**. If **Q** be transferred to an indefinite distance the force **P** would have to become indefinitely great in order to balance it. Two unequal forces, tending to produce rotation, may be balanced by a single force: **P** and **R** are balanced by **Q**. In this case **P** and **Q** have opposite and equal moments round F; **R** has no moment round F, its own point of action. There is equilibrium here between **P**, **Q**, and **R**; their moments round F are together =0. So are their moments round any other point, as may be easily proved. In general, whatever point is considered, if there is to be no rotation round that point, the sum of all the moments of all the forces acting round that point, each taken with its proper sign, must be equal to 0.

Example.—Suppose a slab AB, where AB = 100 cm., and the mass of the slab is 100 kilogrammes, to be supported upon two feet, D and E, each at a distance of 10 cm. from A and B respectively: and let a mass of 40 kg. be hung over the end B by means of a cord: what will be the pressures between the slab and its two supports D and E? The Weight of the slab acts as if at its midpoint C, 40 cm. from D and from E; the forces involved are 100 kg. wt. at C, and 40 at B; the upward reactions, $\mathbf{R}_D$ and $\mathbf{R}_E$, at D and E are required. There is no rotation round E: therefore the sum of the moments round E = 0; *i.e.* $(\mathbf{R}_D \cdot ED) + (100\text{ kg.} \times EC) + (40\text{kg.} \times EB) = 0$; or $(\mathbf{R}_D \times (-80)) + (100 \times (-40)) + (40 \times 10) = 0$; whence $\mathbf{R}_D = -45$ kg., a negative or upward Reaction of the support D against the slab. Similarly, round D, $(100 \times DC) + (\mathbf{R}_E \times DE) + (40 \times DB) = 0$; and DC = 40, DE = 80, DB = 90; whence $\mathbf{R}_E = -95$ kg. These upward reactions are equal and opposite to the downward pressures of the slab on its supports D and E. If the mass hung over B be raised to 800 kg., $\mathbf{R}_D$ becomes equal to zero, and the slab is about to tilt over

Torque.—The Torque or Turning Power of a Force round a Point is measured by its Moment round that point. In rotatory movements, Torques are analogous to Forces in translatory movements. For example, a body tends to remain at rest or in a state of uniform rotation round its centre of mass except in so far as it may be acted upon by an external Torque; and the Angular Acceleration is proportional to the applied Torque.

Force causing rotation constant in direction.—If a body be caused to rotate by force whose direction is the same or nearly so throughout the movement, the effect of the force varies greatly. In Fig. 82 let AC, AD, AE, AF, be successive positions of a rod rotating round A, and acted upon by a force applied at the extremity remote from A, and always parallel to the lines C*c*, D*d*, E*e*. In the position AF the effect produced by the force is a maximum, because the force is there applied with the greatest "leverage," or so as to have the greatest possible Moment or Torque. In this way the forearm moves with the greatest swiftness at the middle of flexion.

Fig. 82.



Couples.—Two forces not directly opposed, and concurring in producing rotation, may sometimes, as in some of the Examples of Couples below, be called a Couple, whether these forces be equal and parallel or not. In another sense the word Couple is restricted to two equal and parallel forces causing rotation of a symmetrical body round its centre of mass, when that centre is situated midway between these forces and in the straight line joining their points of application. The standard definition of a Couple is, however, a generalised one, is independent of any fixed point of rotation, and is based upon the following considerations. Two forces always have a resultant and may be balanced by a third except in one case, viz. that in which the two forces are equal and opposed in their direction, but not opposed in the same straight line. This pair of equal forces constitutes a **Couple,** strictly so-called; and the Standard Definition of a Couple is—two equal and parallel forces opposed in direction, but not in the same straight line.

A Couple, as thus defined, has the following properties:—

1. It cannot be balanced by any one force at any finite distance. If **P** become equal to **R** in Fig. 83, **Q** vanishes.

2. It can be balanced by an opposed couple.

3. It produces rotation round any point which may happen to be fixed, whether within the same plane or not.

If, on the other hand, two unequal forces (**P** and **R** of Fig. 81, in the absence of **Q**) tend to produce rotation, there is always at least one point such that, if this point be held fixed, there can be no rotation.

4. It produces no pressure upon the point fixed, wherever that point may be situated.

If, on the other hand, two unequal forces, opposed in direction but not opposed in the same straight line, act upon a body, there is a tendency to translation. If the handles of a copying-press be equally acted upon by the two hands, there will be rotation simply; if the hands act unequally, the copying-press, with the table on which it is fixed, may be pulled or pushed over.

5. The algebraic sum of the Moments or Torques of the components of the couple is the same round all points in space.

6. Applied to a freely moving mass, a Couple produces no translation of that mass, wherever it may be applied; the centre of mass remains unmoved; the mass is set in rotation round the centre of mass. The consequent angular acceleration is determined by the algebraic sum of the two torques of the couple; but, by (5), this is the same for all points; hence, the angular acceleration round the centre of mass is the same, wherever the couple may be applied.

These special properties have earned for this pair of equal forces the specific name of Couple; and an example of a Couple, as thus defined, is furnished by every case in which Reaction, though equal and opposite to Action, is not in the same straight line with it; and that whether the reaction be due to a support, to friction, or to the inertia of the body acted upon.

If a couple be applied to a body of which no particle is held fixed, there will accordingly be rotation round the centre of mass; but the direction of the axis of rotation round the centre of mass will depend upon the relative unwieldiness of the body in respect of the various possible axes passing through the centre of mass.

A single force, or resultant of forces, on the other hand, applied in a line which does not pass through the centre of mass, produces rotation round that centre *plus* translation of that centre parallel to the line of application of the force. In Fig. 83*d*, the concurrence of these two movements causes the point A to be at rest when C is struck a sudden blow.

Moment of a Couple. — The sum of the moments round all points in space is the same. Take the midpoint; the whole

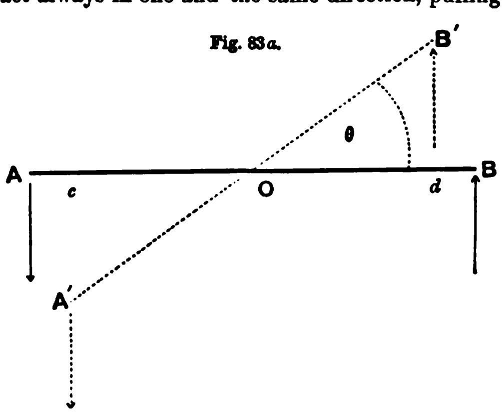
distance between the forces is l; the moment of each force round the midpoint is $\mathbf{F}\cdot\frac{1}{2}l$, where $\mathbf{F}$ is either force: both forces concur in producing rotation; the joint moment is $\mathbf{F}l = \mathrm{M}$, the Moment of the Couple, the product of either of the equal forces into the distance between them. The turning power or **Torque** of a Couple, like that of a Force, is equal to its Moment.

Examples of Couples. — The action of the two hands on the handles of a copying-press is that of a couple: one pulls, the other pushes.

Examples abound in the muscular and osseous system.* Such are — the elbow joint, where the triceps pulls the olecranon process backwards, and the reaction of the articular surface of the humerus against the sigmoid cavity of the ulna constitutes the other member of the couple; the jaw in lateral chewing, where the external pterygoid muscle may pull one side of the jaw forward while the result of the action of the hinder fibres of the opposite temporal muscle, together with the corresponding muscles below the jaw, is to pull the opposite side of the jaw backwards; the weight of the head when a person stands in a very erect position is equivalent to a force acting along a line passing through a point a little behind the occipital condyles, and this, together with the reaction between the atlas and the occipital condyles, forms a couple which is equilibrated by an opposing couple due to the contraction of the muscles of the front of the neck, together with the additional reaction between the atlas and the occipital condyles which is produced thereby; the same weight of the head, when this bends forward a little, passes along a line a little in front of the condyles, and it forms with the reaction of the atlas a couple, which is balanced in the same way by the contraction of the muscles of the back of the neck: when these contractions slacken, as when a person is falling asleep, the head is rotated by the couple on a transverse axis, and it drops forwards or backwards according to the position in which it happens to be at the time when muscular contraction ceases to balance its weight.

Equilibrium of Couples. — Let a couple, consisting of two equal forces, act always in one and the same direction, pulling the particle A (Fig. 83 *a*) and pushing the particle B, and let A and B be so connected as to form a system capable of rotation round the point O midway between them. When AB is at right angles to the couple, the Moment of the Couple is equal to twice the product of either force into the arm OA or OB; it is therefore equal to either force $\mathbf{F} \times$ the length AB. Let the system rotate into the position A′B′ making an angle θ with its previous direction; the couple acts upon a rod whose virtual length is

Fig. 83*a*.

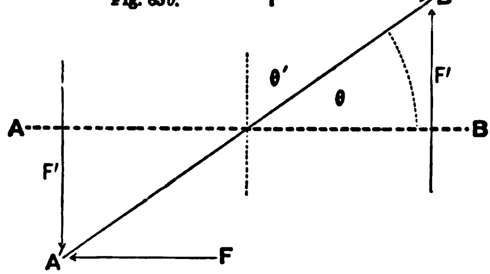


* Numerous examples may be found discussed in Hermann Meyer: *Die Statik u. Mechanik des menschl. Knochengerüstes*; Leipzig, Engelmann.

reduced, by way of projection, to cd or AB $\cos\theta$. The moment of the couple is now $\mathbf{F}\cdot$ AB $\cos\theta$, and when θ is 90° the moment of the couple is reduced to zero, and there is no further effect.

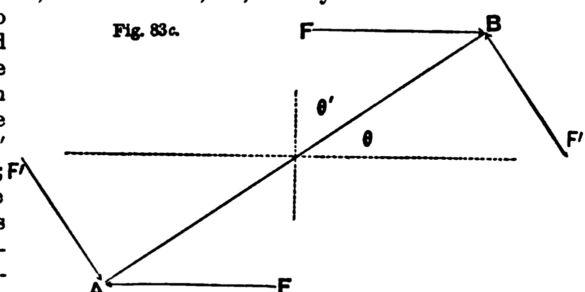
Now let two similar couples act upon the same system AB, and let their directions be at right angles to one another and their actions opposed. There will be equilibrium when $\mathbf{F}$ (Fig. 83 b) pushes B so as to diminish θ just as much as $\mathbf{F}'$ pushes it so as to increase θ. At that moment, and in that position of AB, the effective moments of the two couples are equal. The one is $\mathbf{F}'\cdot$ AB $\cos\theta$; the other is $\mathbf{F}\cdot$ AB $\cos\theta'$. Expressing this equality by means of an equation, we have $\mathbf{F}'\cdot \text{AB}\cos\theta = \mathbf{F}\cdot\text{AB}\cos\theta' = \mathbf{F}\cdot\text{AB}\sin\theta$. Hence $\mathbf{F}'\cos\theta = \mathbf{F}\sin\theta$ or $\mathbf{F}' : \mathbf{F} :: \tan\theta : 1$, where θ is the deflection from a position parallel to $\mathbf{FF}$. This proposition is applied in the construction of the Tangent Galvanometer.

Fig. 83 b.



Again, let the one couple $\mathbf{F}'\mathbf{F}'$ have a direction always at right angles to the direction of AB, while the other, $\mathbf{FF}$, has any direction whatsoever not at right angles to AB. AB is deflected through an angle θ from a position parallel to $\mathbf{FF}$. The moment of the $\mathbf{F}'$ couple is $\mathbf{F}'\cdot$ AB; that of the $\mathbf{F}$ couple is $\mathbf{F}\cdot$ AB $\sin\theta$ as before. These couples being in equilibrium, we have $\mathbf{F}'\cdot\text{AB} = \mathbf{F}\cdot\text{AB}\sin\theta$ or $\mathbf{F}' : \mathbf{F} :: \sin\theta : 1$. This proposition is applied in the Sine Galvanometer.

Fig. 83 c.



Rotation. — Propositions concerning rotational movement run parallel to those concerning translational movement.

Rotations are produced by accelerations which are radial, directed towards a point or line, which is or becomes the instantaneous or the permanent centre or axis of rotation. In the most general case, a force applied in any direction to a moving body may have a component in the direction of the existing motion, and a component radial towards some point, and producing rotation round that point.

If a particle move along a circular path, of radius r, with uniform angular velocity ω radians per second (see p. 75), the angle swept round in time t will be $\theta = \omega t$. If the motion be accelerated, the angular acceleration being such as to increase or diminish the angular velocity ω by $\dot\omega$ radians-per-sec. each second, we have, corresponding to the equations of p. 152, the following equations: $\omega_t = \omega_0 \pm \dot\omega t$ (i.); $\theta = \frac{1}{2}(\omega_0 + \omega_t)\,t = \omega_0 t \pm \frac{1}{2}\,\dot\omega t^2$ (ii.);

and $\omega_t^2 = \omega_0^2 \pm 2\dot{\omega}\theta$ (iii.). When the angular velocity is ω radians per second, the actual linear velocity along the circumference is $v = r\omega$ cm. per second; the Energy of movement is $\frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2$. The Force required to impart this velocity v in one second, starting from rest, is mv or $mr\omega$; that required to do so in t seconds is $F = ma = mv/t = mr\omega/t = mr\dot{\omega}$. The Torque required is $Fr = mr^2\dot{\omega}$. When torque is considered as the cause of rotation, we have, parallel to the expression $F = ma$ in linear translation, the expression Torque $= (mr^2)\,\dot{\omega} = (mr^2) \times$ Angular Acceleration. Hence mr^2 is, in rotational kinetics, the analogue of m, the coefficient of inertia, in translational; the inertia opposed to setting a mass m in rotational movement under a given torque depends not only on the quantity m of the mass to be moved, but also on its position with regard to the axis of rotation.

Moment of Inertia.—This product, mr^2, is called the Moment of Inertia. In a mass m whose several particles are respectively at different distances from the proposed centre or axis of rotation (which may be either internal or external to the mass itself), the Moment of Inertia is found by summing up in appropriate units the products of the mass $\bar{m}$ of each particle of the mass into the square of its corresponding distance $\bar{r}$ from the axis of rotation. This operation generally requires the aid of the Integral Calculus, but the resultant sum, $\Sigma\bar{m}\bar{r}^2 = N$, is a numerical quantity, and is always positive. Then, Torque $= N\dot{\omega}$.

Radius of Gyration, or Radius of Inertia.—Suppose a uniform disc of radius r to rotate round its centre, with a given quantity of rotational energy; it rotates with less angular velocity than it would have assumed if the same matter had been gathered nearer the centre; for the energy of rotation of each particle is $\frac{1}{2}\bar{m}\bar{r}^2\omega^2$, and if the mean value of $\bar{r}$ be greater, the value of ω must be less, if the energy of rotation be constant. Such a disc rotates, on the other hand, with greater angular velocity than that which it would have assumed if the matter had been gathered near the circumference. Between these two limits there must be a mean distance from the centre, such that if the whole mass had been concentrated there, the angular velocity would have been the same as that actually assumed by the disc. This mean distance is the Radius of Gyration or the Radius of Inertia, with reference to that point. If ι be the radius of inertia, $\iota^2 = \Sigma\bar{m}\bar{r}^2/m = N/m$; and if the whole mass m were placed at the distance ι from the point of suspension, it would have, with reference to that point, the same Moment of Inertia as that actually possessed by the physical mass in question. Hence $m\iota^2 = N = \Sigma\bar{m}\bar{r}^2$.

If the body do not continuously rotate round the same point, the radius of inertia may continuously change in value.

Radii of Inertia and Moments of Inertia in particular cases.—(1.) A uniform rod of length l, suspended at its extremity, rotates with the same angular velocity as if its mass were accumulated at a distance $l/\sqrt{3}$ from the point of suspension. The rad. of inertia $\iota = l/\sqrt{3}$; the mom. of inertia, $N = ml^2/3$.

(2.) A uniform rod of length l, poised on its centre; $\iota = l/\overline{2\sqrt{3}}$; $N = ml^2/12$.

(3.) A rectangular lamina of sides a and b, poised at its centre; rotation round an axis at right angles to the lamina; $\iota = \sqrt{(a^2 + b^2)}/\overline{2\sqrt{3}}$; $N = m\,(a^2 + b^2)/12$.

(4.) A circular disc of radius r, and of uniform thickness; rotation round an axis at right angles to the disc and passing through its centre; $\iota = r/\sqrt{2}$; $N = \frac{1}{2}mr^2$.

(5.) The same, round a diameter; $\iota = r/2$; $N = \frac{1}{4}mr^2$.

(6.) A solid cylinder rotating round its axis; same as (4).

(7.) A solid ring cut out of a uniform disc of any thickness; inner and outer radii $r_{,}$ and $r_{,,}$; rotation round an axis passing through the centre of the ring and at right angles to the plane of the ring; $\iota = \sqrt{(r_{,}^2 + r_{,,}^2)/2}$; $N = m\ (r_{,}^2 + r_{,,}^2)/2$.

(8.) Solid ring, whose cross-section is a circle of radius a; distance between the centre of the ring and the centre of this circle $= b$; rotation round an axis passing through the centre of the ring and at right angles to its plane; $\iota = \sqrt{b^2 + \frac{3}{4}a^2}$; $N = m\ (b^2 + \frac{3}{4}a^2)$.

(9.) Spherical shell of radius r; rotation round any diameter; $\iota = r\sqrt{2/3}$; $N = \frac{2}{3}mr^2$.

(10.) Solid sphere of radius r; rotation round any diameter; $\iota = r\sqrt{2/5}$; $N = \frac{2}{5}mr^2$.

(11.) Let $m\iota_0^2$ be the moment of inertia round an axis passing through the centre of gravity of a mass of any form; round any other axis, parallel to the former and at a distance h from it, the moment of inertia $m\iota_{,}^2$ is $m(\iota_0^2 + h^2)$ * where m is the whole mass. For example, if the disc of (4) be rotated round a point in its edge, $\iota_{,} = r \cdot \sqrt{\frac{3}{2}}$. At what point is $\iota_{,} = r$?

Angular Momentum.—The analogue of linear momentum mv is angular momentum $N\omega$. This, like ordinary momentum, cannot be increased in one body without an equal negative momentum being developed in another. Further, in respect of one and the same body, any change in the value of N or of ω causes a corresponding change in the value of ω or of N, for $N\omega$ is constant. Thus the sun or the earth, shrinking as it cools, acquires a smaller moment of inertia, and ω, the angular velocity, tends to increase. Similarly, the water in a basin, when the central portions are withdrawn, begins to swirl; the circumferential portions, coming towards the centre, acquire smaller moments of inertia, and any existing rotation, however small at first, becomes increasingly more rapid.

Energy of a rotating body.—The energy of a particle in rotational movement is $\frac{1}{2}\overline{m}\overline{r}^2\omega^2$; that of a system of particles, each at its own distance $\overline{r}$ and with its own mass $\overline{m}$, must be $\Sigma(\frac{1}{2}\overline{m}\overline{r}^2\omega^2) = \frac{1}{2}\omega^2\Sigma(\overline{m}\overline{r}^2)$; ω, the angular velocity, being the same in all particles of a rotating body. But $\Sigma\overline{m}\overline{r}^2 = N$, the moment of inertia: therefore the energy of a rotating body is $\frac{1}{2}\omega^2 N$, or, where ι is the radius of inertia, $= \frac{1}{2}\omega^2 m\iota^2$. The energy of the ring of example (7) above is therefore $m\ (r_{,}^2 + r_{,,}^2)\omega^2/4$.

* Draw a triangle ABC; A represents the centre of gravity of the object spun, through which the central axis of rotation passes, perpendicular to the paper; B is the position of the other axis parallel to the former; C *any* point whatsoever in the mass rotated. Draw a line CD from C at right angles to AB or to AB produced. Then $BC^2 = AC^2 + AB^2 \pm 2\ AB \cdot AD$. AB is the distance h between the two axes; BC the distance of the particle C from the new axis, AC its distance from the centre-of-gravity axis. If the particle at C have mass $\overline{m}$, $\overline{m} \cdot BC^2 = \overline{m} \cdot AC^2 + \overline{m} \cdot h^2 \pm 2\overline{m} \cdot h \cdot AD$. Now sum up for all such particles as C, and we have $\Sigma(\overline{m} \cdot BC^2) = \Sigma(\overline{m} \cdot AC^2) + \Sigma(\overline{m} \cdot h^2) \pm 2h \cdot \Sigma(\overline{m} \cdot AD)$. The last term disappears, for all round the centre of gravity AD has as many positive as negative values; $\Sigma(\overline{m} \cdot BC^2)$ is $m\iota_{,}^2$, the moment of inertia round B; $\Sigma(\overline{m} \cdot AC^2)$ is $m\iota_0^2$, the moment of inertia round A; $\Sigma(\overline{m} \cdot h^2)$ is mh^2. Whence $m\iota_{,}^2 = m(\iota_0^2 + h^2)$.

A flywheel in motion possesses a large amount of kinetic energy; and if an obstacle be placed in the way of the engine, the engine cannot be stopped by it unless the flywheel can be arrested also: this would involve the sudden exercise of a very great force; hence an engine with a heavy flywheel rapidly rotating can overcome a very great resistance, and in this way, for ordinary resistances, it is prevented from manifesting any very great irregularity of motion.

If a flywheel whose energy is $\frac{1}{2}\omega^2 m\iota^2$ were called upon to expend W units of energy in overcoming a certain resistance, the energy in it after doing so would be $(\frac{1}{2}\omega^2 m\iota^2 - W)$, and a new angular velocity $\omega_{\prime}$ would be assumed, such that $\frac{1}{2}\omega_{\prime}^2 m\iota^2 = (\frac{1}{2}\omega^2 m\iota^2 - W)$. The amount of kinetic energy in a flywheel thus fluctuates. If a very large flywheel have a heavy rim, and if the spokes be relatively thin, the radius of inertia is practically the distance between the centre of the wheel and the middle of the thickness of the rim; and the energy is, approximately, $\{\frac{1}{2}\omega^2 m \cdot (\text{mean radius})^2\}$.

Minimum Angular Velocity.—In examples (4) and (5) above, the respective energies of rotation for a given angle of velocity ω are: axial rotation, $\frac{1}{2}\omega^2 \cdot mr^2/2$; diametral, $\frac{1}{2}\omega^2 \cdot mr^2/4$. The former is twice the latter. For a given amount of energy W, the respective angular velocities are: axial, $\omega_a = 2/r \cdot \sqrt{W/m}$; diametral, $\omega_d = 2/r \cdot \sqrt{2W/m}$. Rotation in a freely suspended body, if effected round its shortest axis of symmetry, involves the smallest angular velocity for a given supply of kinetic energy; and this is the most stable form of rotation. The Earth rotates round its shortest axis. A hard-boiled egg or an egg-shell, freely suspended, does the same; though if spun on a table it rises, as explained on p. 76. A mass of liquid set in rotation always tends to spread out or to set itself so as to have a maximum moment of inertia and a minimum angular velocity; and in the spinning of an unboiled egg on a table this tendency overcomes that of the egg-shell itself to rise on the table: an unboiled egg does not rise.

Suspended Body.—Suppose a heavy body of mass m to be suspended at a point, and then, that point of suspension retaining its fixed position, to swing down so far that its centre of figure sinks through a vertical height h, acquiring an angular velocity ω. The kinetic energy acquired by the body considered as rotating round the point of suspension is $\frac{1}{2}\omega^2 N$; the potential energy lost by descent of the mass m through height h is mgh. These are equal. Hence $\frac{1}{2}\omega^2 N = mgh$; or $\omega^2 = 2mgh/N$. But $N = m\iota^2$, where ι is the radius of inertia; whence $\omega^2 = 2gh/\iota^2$.

Centre of Oscillation.—In Fig. 83 *d* let A be the point of suspension of a body, B its centre of figure or of mass (centre of gravity), ι the length of the radius of inertia of the mass with reference to the point of suspension A: then there is in the same straight line with A and B, and on the opposite side of B from A, a point C, called the *Centre of Oscillation*, which has the following properties:—

(1.) The body may be swung upon A or upon C indifferently, and in either case it will oscillate pendulum-wise with equal rapidity.

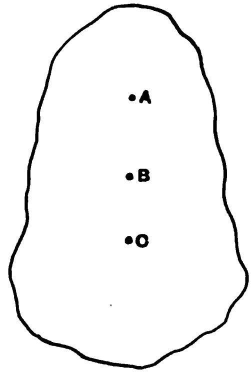
(2.) The body thus suspended at A or at C will oscillate at the same rate as an ideal simple pendulum of the length AC. (Proved at p. 214.)

(3.) This body will, if struck at C, oscillate round A without producing

any pressure on the supporting axis at A. In batting at cricket, A represents the shoulder-joint and C the proper point of impact on the bat.

(4.) Though the support at A were withdrawn — as, for instance, if the body float submerged in water — yet if the point C were struck by a properly directed blow, the point A would remain at rest, and all the part of the body lying above A would move in a direction opposite to that in which C is struck. For every point C at which a body may be struck, or for every **centre of percussion**, there is a corresponding point A on the other side of the centre of figure, through which passes an instantaneous **axis of spontaneous rotation** round which the body impulsively rotates. If the lower part of any object be suddenly pulled forwards, the upper part will move backwards. This property is found applied in the jaw of echinoidei; the upper end of each of the five jaws is suddenly tilted outwards, and the lower, the tooth-bearing ends, are tilted together.

Fig. 83 d.



(5.) The distance AC is equal to ι^2/AB when the body is suspended at A, ι being the radius of inertia in this case; or to $\iota_,^2/CB$ when suspended at C, $\iota_,$ being the radius of inertia in this case. These radii of inertia are so related that $\iota^2/AB = \iota_,^2/CB$. (See p. 214; there written $\iota^2/h = \iota_,^2/h_,$.)

(6.) $AB : \iota_0 :: \iota_0 : BC$, where ι_0 is the radius of inertia round B.

The table on p. 166 gives a comparative conspectus of certain quantities in Translational and Rotational Kinetics.

"Centrifugal Force," so-called. — It has been already shown (p. 79) that when a body describes a curved path, there is an acceleration towards the centre $= v^2/r$, where v is the tangential velocity in the curve, and r the instantaneous radius of curvature. This centripetal acceleration changes the direction of motion without changing the velocity. If the path be a circle of radius r, this acceleration is constantly $= v^2/r$. The component force drawing the body from the tangential path is therefore one which produces an **acceleration towards the centre** $= v^2/r$; and it is itself $\mathbf{F} = mv^2/r$.

Since $v/r = \omega$, the centripetal acceleration $v^2/r = \omega^2 r$, and the centripetal Force $= m\omega^2 r$.

Suppose a stone of mass m to be whirled round like a sling-stone by a string, but in a perfectly circular path. This circular path may be supposed to be made up of very numerous short straight lines or elements (p. 58), each of which is tangential. The actual velocity along any one of these tangential elements during any one instant may be hypothetically resolved during the next instant into two components; one along the circle $= v$;

Translational.			Rotational.		
Quantities.	Symbols, etc.	Dimensions.	Quantities.	Symbols, etc.	Dimensions.
Displacement, Linear .	s	[L]	Displacement, Angular, measured in radians .	$\theta = \omega t = \frac{1}{2}\dot{\omega}t^2 = s/r$	[Number]
Velocity	$v = s/t$	[L/T]	Angular Velocity . . .	$\omega = v/r$	[1/T]
Acceleration	$a = v/t$	[L/T²]	Angular Acceleration .	$\dot{\omega} = a/r$	[1/T²]
Force	$F = ma = mv/t$	[ML/T²]	Torque . . . . . . / Moment of Force . . . / Moment of Couple . .	$Fr = mar = mr\dot{\omega}\cdot r = mr^2\dot{\omega} = mr^2\omega/t$ / Fl	[ML²/T²]
Mass . . . . / Coefficient of Inertia .	m / $F/a = m$	[M]	Moment of Inertia . .	$\text{Torque}/\dot{\omega} = mr^2$	[ML²]
. . . .			Radius of Inertia . . .	N/m or $N/\Sigma\overline{m}$	[L]
Momentum . . . . . / Impulse	mv / $Ft = mv$	[ML/T]	Angular Momentum . . / Moment of Momentum . / Moment of Impulse . .	$N\omega = mr^2\cdot\omega$ / $mv\cdot r = mr\omega\cdot r$ / $Ft\cdot r = mr\omega/t\cdot tr$	[ML²/T]
Work . . . . / Energy, Kinetic . . . / Energy, Potential . .	$Fs = F\cdot\frac{1}{2}at^2$ / $\frac{1}{2}mv^2$ / $mas = ma\cdot\frac{1}{2}at^2$	[ML²/T²]	Work = Torque × Angle / Energy, Kinetic . . . / Energy, Potential . . .	$mr^2\dot{\omega}\times\frac{1}{2}\dot{\omega}t^2 = \frac{1}{2}mr^2\omega^2$ / $\frac{1}{2}N\omega^2 = \frac{1}{2}mr^2\cdot\omega^2$ / $N\omega\theta = \frac{1}{2}mr^2\omega^2$	[ML²/T²]
Activity or Power . .	Energy/Time = Force × Velocity	[ML²/T³]	Activity or Power = Torque × Angular Velocity	$mr^2\dot{\omega}\omega = mr^2\omega^2/t$	[ML²/T³]

one away from it, in the line of the radius, and corresponding to an outward acceleration $\mathbf{a} = - v^2/r$. Of these two components the former freely manifests itself as velocity v along every successive element or, practically, as a continuous velocity v in the circular path; the latter, the outward component, never manifests itself, for it is, at every instant, counteracted by tension in the string. This tension is a stress set up in the string by the action of its molecular forces, when the whirling ball tends to pull the outer end of the string outwards; and numerically it is, across every complete cross-section of the string, a Total Tension equal to that which would have been established by the application of mv^2/r units of force. If the string snapped or were suddenly cut, this tension would cease; there would then be nothing to hinder the actual tangential velocity at the instant of snapping from persisting during the next and succeeding instants; the motion of the stone would therefore be continuous motion in a straight line, the tangent to the curved path at the point where the stone had happened to be at the instant of snapping, and the stone, thus liberated and flying off at a tangent, would then obey Newton's First Law of Motion.

As the ball flies off in its tangential path, it will spin: for in its circular path, its outer particles had travelled with greater velocity than its inner.

The stone flies off at a tangent, and not straight from the centre; there is therefore no counteraction, on the part of the string, of any tendency on the part of the stone to fly off in some direction straight away from the centre; there is therefore no so-called "centrifugal force," in the old sense of the term, counteracted by the tension of the string. The tension of the string is, however, equivalent to a force $\mathbf{F} = mv^2/r$, acting upon the stone, directed inwards along the string, and the inward acceleration of which, $\mathbf{a} = v^2/r$, balances at every instant the opposite tendency on the part of the whirling stone to increase its actual distance from the centre by pursuing a tangential path.

Any string will snap if force be applied to it beyond a certain limit. If a string be just so strong that x grammes of matter may be suspended on it without its snapping, it can survive the application of force equal to $981x$ dynes. If this string be used to whirl a slingstone of mass m, it will snap unless the velocity v be such that mv^2/r is less than $981x$—that is, v must be less than $\sqrt{981xr/m}$. If the velocity exceed this limit, the string will snap. As the velocity increases, its

centrifugal component increases, and requires a greater force or reaction to be exerted in a direction towards the centre in order to bend the path into the same curve in a shorter time. In the same way, if a fly stand on the rim of a rotating wheel, the adhesion between the foot of the fly and the rim of the wheel necessary in order to enable the fly to retain its footing may become so great that the fly cannot hold on, and is hurled off at a tangent.

When a grindstone or flywheel is rotated too rapidly, the molecular forces of cohesion cannot keep the particles together against their tendency to fly off at a tangent.

If the earth rotated on its axis seventeen times as fast as it does, the attraction of gravitation, the effect of which is even now masked to some extent by the rotation of the earth, would only just be able, at the equator, to keep bodies from flying off its surface at a tangent.

The greater the velocity of a railway train the greater is its tendency to fly off the track as it is rounding curves.

If a drop of oil be suspended in a mixture of spirit and water, so that it is free to assume any form, and if a motion of rotation be communicated to it, the globular drop assumes the form of an oblate spheroid, and bulges at its equator; for particles at its original equator have, when set in motion, a greater velocity than those nearer its poles. For the same reason the earth itself has assumed the form of an oblate spheroid.

In the trundling of a wet mop, when the drops fly off because they do not adhere firmly enough to enable them to retain their position — in the rotation of a steam governor, the balls of which fly asunder as the speed of the engine increases, thereby actuating an appropriate train of mechanism which to a greater or less extent shuts off the steam — we find examples of this phenomenon. If a man were placed on a revolving table, with his feet towards the centre, the blood in his body would be urged towards his head; and this has actually been proposed as treatment in bloodlessness of the brain.

When a circular cylindrical vessel containing water is rotated on its axis, the water is heaped up towards the sides of the vessel. If the speed exceed a certain limit, the water will be hurled over the sides of the vessel, and if the supply of water and the rotation be continuous, an engine may expend its energy in thus continuously lifting water against gravity. This principle is applied in Siemens's governor for machinery; when the engine goes too fast it begins to spend energy in producing

this current of water. The form of the surface of water thus produced is always parabolic.

When light and heavy particles in mixture are whirled, the heavier fly outwards; thus milk, if rotated, separates into heavier milk externally and lighter cream internally.

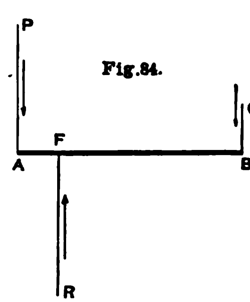
A badly-balanced flywheel exercises a fluctuating pressure on its bearings, which varies as ω^2.

The Mechanical Powers.

The principle of moments or — what is essentially the same thing — the principle that the work done by or on a machine = 0, or that, on the whole, there is no accumulation of work in a machine, is the key to the explanation of the action of many of the Simple Machines or so-called Mechanical Powers. The work done by a simple machine is equal to that done upon it, and upon the machine itself there is no work done. This is, of course, not strictly accurate; but simple machines are supposed, in the first instance and for the sake of theory, to be themselves without weight, and to work without friction.

The Lever. — This is a bar of any substance, rigid enough to retain its form under the forces applied to it. We consider it at the moment when the forces or pressures applied to it are all in equilibrium, so that there is no movement. If the point A be pressed down with force **P**, if the fixed point or Fulcrum be at F, and if the point B be pressed down with a parallel force **Q**, then round F the moments $(\mathbf{P} \cdot AF) + (-\mathbf{Q} \cdot BF) = 0$. If AF be shorter than BF, **Q** is numerically less than **P**; then the smaller **Q** can balance the greater **P**; a practical mechanical advantage. As an example of this take a crowbar: the man's strength is exerted at B; the fixed point is at F, and the weight of the body to be lifted acts downwards at A. Suppose the lever to be 42 inches long, the point F to be 2 inches from A, and the man's strength, which is competent to raise 56 lbs., to be exerted at B: then AF = 2, FB = 40, and $\mathbf{Q} : \mathbf{P} :: 2 : 40$; whence the man can, by exerting at B a force of 56 lbs., keep a mass, weighing 1120 lbs. and resting on A, from moving downwards; an effort a little greater will lift it.

Fig. 84.



On the other hand, a force equal to the weight of 1120 lbs.

applied at A can only balance a weight of 56 lbs. at B: here there is great mechanical disadvantage.

An example of this occurs in the case of forceps, with blades relatively long: the pressure which they can exert at the tip is relatively small, for each blade of the forceps is a lever supported at the hinge. Extremely long scissors do not cut so well at the point as near the joint.

But what is gained or lost in force is lost or gained in range of movement; for the work done ($=\mathbf{Fs}$) by the one arm is always the same as that done upon the other.

Whatever the special arrangement of the two forces and the reaction, the principle is always the same, that the lever is studied at the instant of equilibrium, when round the fulcrum the sum of the moments $= 0$; then an excess either of the Force applied or of the Resistance, beyond their proportions at that instant, will cause rotation round the fulcrum.

There is a popular division of levers into three classes, which it is well to explain; Fig. 84 illustrates them all.

Class I.— The fixed point is at F in Fig. 84. A crowbar, a handspike, a pair of forceps, scissors, or shears, a poker, a common balance,—all these have the fulcrum or fixed point between the point of application of the force and that of the resistance.

Class II.— If the fulcrum be at A in Fig. 84, and the force be applied at B, the resistance overcome at F is numerically greater than the force applied at B, as is found by taking moments round A; $(AF \times \mathbf{R}) = (AB \times \mathbf{Q})$. Examples of this are furnished by nut-crackers, where the resisting nut is nearer the hinge than the hand is; by the oar of a boat, in which the force is applied at the handle, while the tip of the oar is approximately at rest, and the resistance of the boat is overcome between these; by a claw hammer used for extracting nails, where the fulcrum is at the end of the claw, the force is applied through the handle, and the resisting head of the nail is between these points; by a wheelbarrow, in which the fulcrum is at the axle of the wheel, the raising force is applied at the handle, and the resistance to be overcome is the weight of the substance in the barrow between the handle and the wheel.

Class III.— This is the same as the second class, except that the Force and the Resistance have changed places. As an example of this we find that in a pair of tongs for sugar or for coal, in which the fixed point is at the hinge or the flexible end, the resistance is encountered near the other end, and the force is applied between these points. The pressure that can be applied by such an arrangement is comparatively feeble, while to overcome any given resistance the force applied must be proportionately very great. This is seen in opening a gate by pressing on it near the hinges; a considerable force has to be exerted. Such an arrangement, in which force is sacrificed in order to gain amplitude of movement, is of ordinary occurrence in the muscular system. The biceps is inserted into the radius at a point about one-sixth of the distance between the axis of rotation of the elbow joint and the centre of the palm of the hand. In order to raise a pound-mass in the hand, that muscle, if it acted alone, would have to exert a force which

would directly lift 6 lbs.; but, on the other hand, the forearm has a range and rapidity of movement which it would not have had had the muscles been inserted in the position of greatest mechanical advantage, not to mention the inconvenience of having muscles extending from prominence to prominence of the skeleton like the rigging of a ship. The pectoral muscle of a bird, the deltoid muscle of man, his glutei muscles, actuate conspicuous examples of osseous levers of the third order.

Problems. — 1. Two porters bear a burden, 56 lbs. in weight, by means of a bar of such length that the distance between shoulder and shoulder is 70 inches. The weight is suspended from a point 40 inches from the shoulder of one of the porters. What share of the burden is borne by the shoulder of each respectively? — *Ans.* This is a case of Fig. 84, in which the weight of the burden corresponds to **R**, and the upward shoulder-reactions to **P** and **Q**. There is no tendency to rotation round F, which is relatively fixed; hence the reactions at A and B must be such that their moments round F are equal. Hence the two equations, $\mathbf{P} + \mathbf{Q} = 56$ and $40\ \mathbf{P} = 30\ \mathbf{Q}$, give $\mathbf{P} = 24$, $\mathbf{Q} = 32$. The porter nearer to the burden carries 32 lbs., the one farther from it carries 24 lbs.

2. A nut-cracker 6 inches long has a nut in it an inch from the hinge. The hand exerts a force equal to the weight of 4 lbs.: what is the total stress on the hinge? — *Ans.* The nut, so long as it does not yield, affords a fixed point: the total stress on the hinge = the weight of 20 lbs.

The Wheel and Axle. — The lever, when it has done work and raised a burden against resistance, moves into a position where the leverage and the corresponding Moment or Torque are diminished (see Fig. 82), if the force retain, or nearly retain, its original direction. If by any means matter could be so arranged that a lever would, when it had moved out of its position of greatest advantage, be replaced in the most favourable position by another lever, to which the burden and the force applied were shifted, the apparatus thus constructed would in some respects be more useful than a simple lever.

This criterion is satisfied as regards levers of the first order by the Wheel and Axle. This consists of a large wheel or cylinder and a small one, both on the same axis, and capable of rotating together on that axis.

Each wheel may, if solid, be regarded as consisting of an infinite number of spokes. One of these spokes in the larger wheel, and one running in the opposite direction from the centre in the smaller wheel, together make up, when they are for an instant at right angles to the lines of application of the force and the resistance, a lever in the most favourable position. As soon as this has left the position of greatest advantage, by reason of rotation of the system, its place is at once taken by another.

The weight of a large mass hung on the smaller wheel and that of a smaller mass hung on the larger wheel will balance one

another, if their moments round the axis of rotation be equal. The weights may be replaced by a force applied at the circumference of the larger wheel, and a larger resistance balancing this at the margin of the smaller wheel. This is the principle of the **capstan** and the **winch**—the former used on ships for raising the anchor, the latter in use for drawing buckets up wells. In the former the spokes of the larger wheel are few, while the smaller takes the form of a cylinder or drum; in the latter the smaller takes the same cylindrical form, while the larger consists virtually of only one spoke, the handle, which is turned through all successive positions in a circle.

The wheel and axle is a statical instrument so long as its moments round the axis are together $= 0$; but when one of the moments is numerically greater than the other, there is rotation.

Wheelwork.—If a force be applied to the first wheel of a chain of wheelwork, so that it acquires an angular velocity ω, and if the last wheel of the chain have, in consequence of this, an angular velocity $\omega_{\prime}$, the force which the last wheel can exert is, as compared with that which the first wheel alone might exert when running with angular velocity ω, as $\omega : \omega_{\prime}$. The principle holds good, whatever the nature or complication of the mechanism which intervenes between the first and the last wheel. In a crane or in a lathe arranged for metal-cutting, we see the wheelwork so devised that the last axis moves very slowly, and with a correspondingly great power of overcoming resistance.

The Inclined Plane.—The mechanical advantage of this machine depends on the principle of the resolution of a force into its components.

When a body is pushed up an inclined plane by a force or push just sufficient, and no more, to prevent it from moving down the plane in obedience to gravity, there is equilibrium between three forces—viz. this Force, acting along the slope of the incline, the Weight of the body acting vertically, and the Reaction between the body and the surface of the plane, acting at right angles to the latter. These three forces can be represented by the sides of a right-angled triangle, in which the

Hypotenuse : Height : Base
as Weight of body : Push up the plane : Reaction.

If the push be applied horizontally, the three forces in equilibrium—which are the Weight of the body downwards,

the Reaction at right angles to the surface of the plane, the Push up the plane applied horizontally—will have the relation of the sides of a right-angled triangle, in which

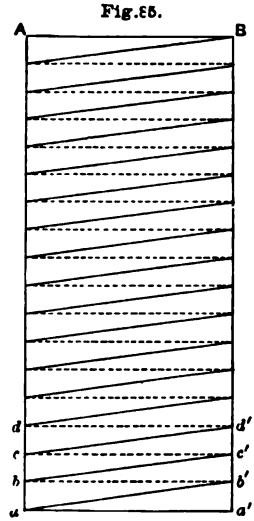
Hypotenuse	: Height	: Base
as Reaction	: Horizontal Force	: Weight of body.

Velocity of fall down a frictionless inclined plane.—If a body slip down an inclined plane, the potential energy lost by it in virtue of its vertical descent h is mgh; there is none gained or lost in virtue of horizontal motion, in which there is no work done by or against gravity. The kinetic energy acquired is $\frac{1}{2}mv^2$. These must be equal; hence $v = \sqrt{2gh}$, the same speed as would have been acquired by a vertical fall. In the latter case, however, the direction of motion would have been directly downwards; in the former it is in the direction of the plane. The reaction of the plane has not modified the speed of the fall; it has modified its direction. The speed at which the body is moving down the plane after effecting a vertical descent h is thus the *same velocity* as that which it would have acquired if it had fallen vertically through that height h. But it has travelled through a greater space in order to attain this speed. The acceleration down the plane is therefore smaller; and a body slipping down a smooth slope of 1 in 20 would take a *greater time* to reach the bottom than it would take to fall vertically through an equivalent height, in the ratio of 20 : 1.

If the body travelled down a succession of inclined planes, or down a curve, the same ultimate velocity would be acquired: the reaction of the curve alters the direction but not the speed. If it *rolled* down the plane or curve, a part of its energy would be rotational; it would acquire a correspondingly smaller velocity of fall.

The Screw.—In Fig. 85, across the rectangular parallelogram $Aaa'B$ are drawn equidistant lines aa', bb', cc', dd', etc., at right angles to Aa and Ba'. The lines ab', bc', cd', etc., are drawn as there shown. If the surface $Aaa'B$ be wrapped round a cylinder whose circumference is equal to AB, the line Aa will coincide with Ba', and the lines ab', bc', cd', etc., will form a continuous spiral line $abcd$ round the cylinder, and will trace out the form of the thread of a screw whose *pitch* is ab, the distance between the equidistant lines ab', bc', etc. Hence the thread of a screw is seen to correspond to a narrow inclined plane wrapped round a cylinder.

Fig. 85.



A screw is usually used as a mechanical power for the sake of moving a body through a small space with great force, as in the copying-press.

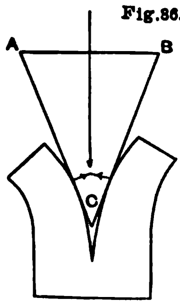
The less the pitch of the screw — the greater the number of turns to the inch—as well as the greater the leverage of the handles, the greater the mechanical advantage that can be derived from its use.

Problem.

What is the mechanical advantage which can be obtained in a copying-press of the following construction : — Effective radius of the arms 12 inches —screw 1½ inches thick—pitch ⅛ inch?—*Ans.* The hands move through 1 inch, while the point of the screw descends $\frac{1}{603}$ inch. $\therefore F^1 = 603\,F$. The thickness of the screw is of no consequence, except as a means of securing structural strength.

The Wedge.—A wedge, as seen in Fig. 86, is practically a double inclined plane movable between resistances. During a blow there are at work (1) the driving force acting downwards through the centre of AB, (2) a reaction at right angles to AC, and (3) one at right angles to BC; these latter being through the point of contact, or, if there be contact over the whole of AC and BC, through the centre of these lines. These must cross in a point if the equilibrium, which subsists the instant before the wedge commences to move, be considered; and they must be represented by the sides of a triangle. Round the point at which they meet, the moments = 0.

Fig. 86.



Pulleys.—These well-known objects are wheels, solid or spoked, mounted in a framework or block, which is either movable or fixed to a beam or other solid attachment. The simplest use of a pulley is to change the direction of application of a force applied to a cord. The total tension of the cord on one side of a pulley would, if there were no friction, be equal to that on the other side of it, while the motion of the cord on the one side of the pulley is in any case equal to that on the other side, whatever be the size of the pulley, and whatever be the amount of the flexure to which the cord is subjected. A single pulley thus produces no mechanical advantage if it simply serve this purpose, except in so far as the change of direction of the cord, produced by the intervention of the pulley, may itself be of advantage; but if this pulley be itself movable against a resistance—if, for instance, a heavy mass be suspended from it, while the other end of the cord is attached, say, to the roof—a movement of the suspended mass through

one inch would correspond to the pulling in of two inches of cord, and the hand exerting the force would move through a space twice as great as that traversed by the pulley. Thus, by the intervention of a cord, one end of which is fixed, and of a single pulley round which the cord is bent through 180°, so as to make it turn back parallel to itself, a resistance 2 may be overcome by a force just greater than 1. This principle of reduplication of a string round a pulley is taken advantage of and practically turned to use in combinations of pulleys, in any of which the mechanical advantage is the numerical ratio of the amount of string pulled out to the corresponding movement of the body pulled upon.

The Bell-crank.—If in Fig. 87 the rigid body ABC, which can rotate round A, have a force applied to it at C, its tendency to rotate round A may cause motion at B against a resistance **R**. The principle of moments shows us that, whatever the ornamental shape of the crank, the relation of the resistance **R** overcome to the force **P** exerted depends on the relative lengths of the effective arms AP and AR.

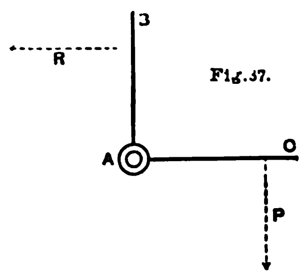


Fig. 87.

The Knee.—In Fig. 88, if two bars be jointed at O, and their ends A and B be confined to a given straight line CD, a movement of the hinge O athwart the direction of the line CD corresponds, especially when AO and OB are nearly in the same straight line, to a relative motion of A and B, which is proportionately very small. Hence A and B are thrust asunder with a force greater than that which acts upon the hinge and presses it into its central position. This contrivance is used in some copying-presses, hand printing-presses, and railway-ticket endorsing machines. It is seen in the human knee: when the leg is straightened out a vigorous thrust upwards and forwards is given to the body, and a corresponding one downwards and backwards to the earth on which the foot presses.

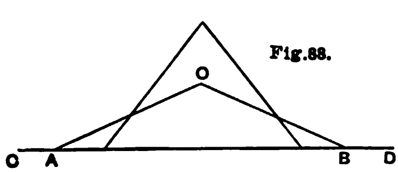


Fig. 88.

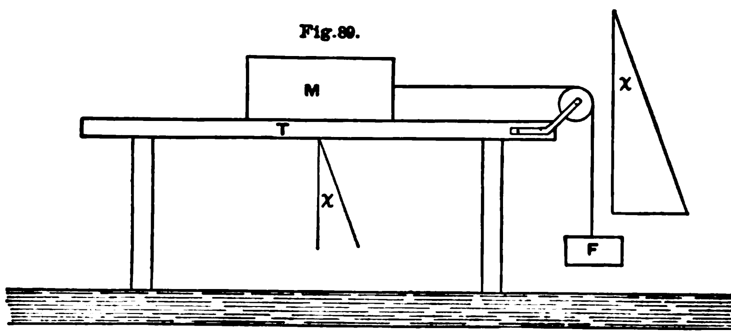
A wire stretched between two points, and loaded by a weight or by the pressure of the wind, is a *knee* whose action is reversed. It tends to pull together the two supports to which it is fixed; and if there were any move-

ment of these supports, it would be small in comparison with the corresponding movement of the centre of the wire. Thus the force acting upon the supports and resisted by them is greater than that acting upon the wire itself.

FRICTION.

Statical Friction between Solids. — Let a body M, of mass m, be supported on a table T; and let the total pressure between M and T be P, which may either be the weight mg of the body M or have any other value or source; and let a force **F**, say the weight of a mass m' suspended over a pulley, be employed to pull it towards the edge; then the body M will not begin to slide

Fig. 89.



along T unless the force **F** bear a certain proportion to the total pressure P. This proportion, a fraction less than unity, is the Coefficient of Statical Friction, is usually represented by the symbol μ, and has to be experimentally found.

The force **F** encounters a Frictional Resistance, F, which has a maximum numerical value μP. When **F** is less than μP, there can be no sliding; when **F** $= \mu$P, sliding is just about to commence; and when **F** is greater than μP, there will be sliding. This Frictional Resistance F is brought into being by the application of the force **F**; and it enters into calculations as if it were an oppositely directed Force, equal to **F**, and preventing movement, until **F** comes to be equal to μP, but unable itself to exceed that value.

Experiment has shown that μ, the coefficient of statical friction, depends upon (1) the nature of the substances of which M and T consist; (2) the smoothness or roughness of their surfaces; (3) the presence or absence of thin films of lubricating material — oil, soap, blacklead — between them.

The coefficient of friction is the same between the same two substances, whatever the value of **F** or that of the pressure P may be; and hence F, the statical resistance to sliding,

being numerically equal to μP, varies directly with the total pressure P between the given surfaces. Again, the mass of M may be distributed in any way, and the contact between M and T may be by a surface large or small. If the area of contact be diminished, the pressure on each unit of area will be increased, and therefore the friction on each unit of area will also be proportionately greater; but the number of units of area over which the resistance is exerted is correspondingly smaller, so that the force which is just competent to pull the body M towards the edge remains the same. Hence the Total Frictional Resistance F is independent of the area of contact between two given masses; but the Frictional Resistance per Unit Area of contact varies directly as the Pressure per Unit-Area.

Limiting Angle.—In Fig. 89 we may consider the equilibrium subsisting at the instant before the body M begins to slide on T. The body M is at rest under (1) the force **F** acting horizontally, (2) its own weight, mg, acting vertically, and (3) the reaction R between M and T. This reaction R is inclined at an angle χ to the vertical. The horizontal component of R is to the vertical as $\tan\chi$ is to 1. But (horiz.-compon./vert.-compon.) $= \mathbf{F}/mg = \mathbf{F}/P = \mu$; $\therefore \mu = \tan\chi =$ the Coefficient of Statical Friction.

Any force applied to M, or any set of forces whose resultant acts on it, in a line making with the normal to the surface between M and T an angle less than χ, will not produce sliding. If there were no friction, any force applied to M in a direction differing in the least degree from the normal would have a component which would produce sliding; but Friction makes it necessary that a Force should be wide of the normal to the surfaces of contact, by something more than the Limiting Angle χ, before sliding can occur.

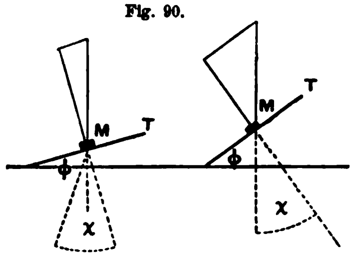
If a flat piece of wood be placed on a table, and pressed against the table by a stick held at right angles to it, it will not slip; the stick may be inclined somewhat, and still it will not slip; when the stick is inclined more than a certain amount, the piece of wood begins to slip on the table.

If M be pressed against T by a force $\mathbf{F}_\phi$, acting in a line which makes any angle ϕ with the normal, we may resolve this force into two components: one at right angles to the surface, $\{\mathbf{F}_\phi \cdot \cos\phi\} = P$, produces pressure between these surfaces; the other, $\{\mathbf{F}_\phi \cdot \sin\phi\} = \mathbf{F}$, is the component which tends to produce sliding. If the sliding-component $\div$ the pressure-component (*i.e.* $\tan\phi$) be less than or merely equal to μ, there will be no sliding; if it be greater than μ (*i.e.* if ϕ be greater than χ), sliding will occur. If there be no sliding, the component which tends to produce it sets up a condition of stress between the particles of the two bodies, and thus a reaction

is set up by molecular forces, equal and opposite in the direction of the sliding-component. The amount of this reaction depends on the molecular conditions of the substances, and is only to be determined by experiment.

Angle of Repose.—Suppose a body to be placed on a table, and the table to be tilted up until the body is just about to begin to slide. At that moment there is equilibrium. To what angle can the table be tilted up? Let μ be the coefficient of statical friction between the body and the table. The one will then slip on the other if a force be communicated between them in the direction of a line making, with the normal to the surface of contact, an angle greater than χ. In Fig. 90 are shown two positions of the table T bearing the body M. In both the dotted lines indicate the limiting angle χ; in both the weight G of the body M acts vertically downwards. The equilibrium, then, is between (1) the Pressure P produced between the body and the table, at right angles to the latter, $= G \cos\phi$; (2) the Sliding-Component, $F = G \sin\phi$; and (3) the Reaction of the table, equal and opposite to G, the weight of the body. In the first case of Fig. 90 the reaction between the body and the table falls within the limiting angle, and there is no sliding: the sliding component is less than $\mu \times$ the pressure. In the second case the limit is reached; the sliding component is just equal to $\mu \times$ the pressure, and is just able to balance the friction. If the table were tilted up any farther, sliding would occur. But in the second case it is easy to show that the angle ϕ, to which the table has been tilted, is itself equal to χ the limiting angle. Hence the angle χ is also called the Angle of Repose. Upon the coefficient of mutual friction depend in this way the angles at which heaps of sand, of grain, and the like, will adjust themselves when poured out and allowed to find their own position.

Fig. 90.



This angle χ has, then, three properties: (1) the Coefficient of Statical Friction, $\mu = \tan\chi$; (2) χ is the Limiting Angle; (3) χ is the Angle of Repose.

Friction of a rope round a post.—This is familiar in the example of a rope passed round a post on a quay in order to hold fast a ship. If any little part or element of the rope be considered, it will be seen that the friction is proportional to the pressure of that part of the rope on the post, and

that to a certain extent it tends to prevent slipping; in this it partly counteracts the tension of the rope; the total tension communicated to the end of the element of the rope farther from the applied force is less in consequence of this than it would have been if there had been no friction. If we trace out in this way, along the rope, the gradual diminution of tension, we find that the tension, after a complete turn of the rope round the post, dwindles down to a constant fraction of the original tension. Between a flexible rope and wood this constant fraction is about $\frac{1}{9}$; hence a force of 1 lb. could prevent a force of 9 lbs. from pulling a flexible rope round a post round which it had been passed so as to form a complete turn. After two turns the tension falls to $\frac{1}{81}$; after ten turns it becomes $1/9^{10} = 1/3486{,}784401$. Hence a man exerting a pull of 1 lb. at the end of a rope wound ten times round a post would be able to resist a pull of about one-and-a-half million tons. Of course this is not attained in practice, because no ropes are thoroughly flexible, and none are strong enough to stand such stresses; but a perfectly flexible rope would diminish tension in this manner without reference to the diameter of the post round which it is wrapt.

Kinetical Friction between Solids.—After slipping has begun, the motion is retarded by a Frictional Resistance, which is still, at low speeds and with moderate values of P, proportional to the pressure P between M and T. This is equivalent to a virtual negatively-directed pulling force $-R = -b \cdot \mathrm{P}$. The effective accelerating force acting upon the mass $(m + m')$ of Fig. 89 is thus $(\mathbf{F} - R) = (m'g - b\mathrm{P})$. If the weight $m'g$ of the pulling mass m' be replaced by an equivalent constant sliding-force of $\mathbf{F}$ dynes (where $\mathbf{F} = m'g$), the mass moved is not $(m + m')$, but m alone; and the acceleration produced is $(\mathbf{F}/m - R/m) = (\mathbf{F}/m - b \cdot \mathrm{P}/m) = (1 - b\mathrm{P}/\mathbf{F}) \cdot \mathbf{F}/m$. If there had been no friction, the acceleration would have been $\mathbf{F}/m$; the negative acceleration $(-b \cdot \mathrm{P}/m)$ is constant, whatever may be the sliding force $\mathbf{F}$—within a pretty wide range of values—and is therefore independent of the velocity; and b is the Coefficient of K i n e t i c a l F r i c t i o n, which is smaller than μ, the coefficient of statical friction.

Influence of Duration of Contact.—There are, however, even for moderate values of the sliding force $\mathbf{F}$ and of the pressure P, slight variations in the values of μ and b. When two bodies have been in contact for a long time, the particles of each develope such relations to one another that μ, the coefficient of statical friction, increases with the duration of contact; it is more difficult to make a body slide on another with which it has been long in contact than on one on which it has been freshly placed. When one surface slides on another, the particles seem to have no time to assume such relations, and the coefficient of kinetical friction is comparatively small; at very great velocities it is even somewhat smaller than at ordinary velocities; but when the velocity of sliding is very small, the condition approximates to one of relative rest, the coefficient of kinetical friction approximates to that of statical friction, a larger proportion of

energy disappears at very low speeds than at high, and a body which has come to travel very slowly soon comes to rest.

Transformation of Energy by Kinetic Friction.—The sliding force F has apparently lost a fraction of its amount; the velocity produced is diminished in proportion; Energy is wasted, but not destroyed, by being transformed into molecular kinetic or potential energy. In the former case the energy absorbed may perhaps assume the form of the energy of electric condition, but ultimately it takes that of Heat, which warms the machinery and the air surrounding it; in the latter case it corresponds to a stress between the particles of the bodies, to pull which asunder requires a certain amount of force.

Kinetic Friction ($-R$) is, accordingly, **not a Force**; it is a Resistance or Reaction: but, like static friction, $-F$, it enters into calculations as if it were a Force, never coming into action unless Force be applied, always tending to prevent or to diminish slipping, and always proportional to the pressure between the rubbing surfaces.

Negative Acceleration in Kinetic Friction.—Problems of loss of momentum through Friction may be dealt with by using the four equations of p. 152, writing $(-b \cdot \mathrm{P}/m)$ for the constant acceleration a; P being the number of dynes of Total Pressure and m the mass, in grammes, of the moving body.

Brakes.—The function of a Brake is to modify the Total Pressure, P dynes, between the moving mass and the surface against which it rubs. This may be done by clamping the brake against the moving mass to any desired extent. This affects the value of the negative acceleration $(-b \cdot \mathrm{P}/m)$. The total pressure may also be affected by multiplying the surfaces of contact. If, for example, two pamphlets be arranged with their leaves alternately interplaced, it is surprising how small a weight superimposed will lock them firmly together.

The Critical Angle in Kinetical Friction.—This angle, corresponding to χ in statical friction, is ψ, where $b = \tan \psi$. There is no work done in pulling a body downhill at this angle, for the frictional resistance is then exactly balanced by the component of the Weight, directed down the slope. If the slope be steeper than this, brakes or ropes are required to prevent acceleration. If $b = \frac{1}{250}$, as in most railway work, with good lubrication, this slope is such that $\tan \psi = \frac{1}{250}$; a slope of 1 in 250.

The mechanical powers, when friction is taken into account, give rise to several problems; but the physical principle underlying the whole subject is the same, that Friction acts in the same way as a Force opposed to sliding, and that it is proportional to the Total Pressure. As an example let us take this question: A copying-press is pressed hard down on the copying-book; the hands are removed; the book remains under pressure;—why does the screw not come up? The reaction of the book has a component up the line of the thread of the screw; this would tend to send up the screw, but

it is counterbalanced by Friction, acting as a Resistance in the opposite direction down the thread. It may be left to the reader to show (1) that the better the screw is oiled the less able will it be to retain its hold; and (2) that a screw of too large a pitch (one the turns of whose thread are too far apart) may fail to hold the book down. The upward pressure is resolved into two components, of which the one along the thread of the screw must not be greater than $\mu \times$ the component at right angles to the thread; if it be greater than this the screw will slip upwards in its nut. When there is actual motion, the forces acting are subject to deductions equal to the respective values of $R = b \cdot P$.

Work done against Friction. — The acceleration $(-b \cdot \mathrm{P}/m)$ being constant, the work done against Friction $= R\mathrm{s} = m \cdot b\mathrm{P}/m \cdot \mathrm{s} = b\mathrm{Ps}$; and this is equal to $b \cdot mg \cdot \mathrm{s}$, when gravity alone determines the pressure P. To this would have to be added any work done against gravity or other external forces while the space s is being traversed.

Resistance to Traction. — The Frictional Resistance, $-R = -b\mathrm{P}$, corresponds numerically, when $\mathrm{P} = mg$, that is, when gravity is the only cause of the pressure P, to $b \cdot mg$, the Weight of a mass bm, where b is a numerical factor less than unity. Suppose $b = \frac{1}{320}$; then so long as b remains constant, which it does within wide limits, the work to be done in making any mass m move horizontally through a space s at any uniform velocity is $\frac{1}{320}mgs$, and is therefore one three-hundred-and-twentieth part of the work required to raise the same mass m vertically against uniform terrestrial gravity through the same space in the same time. Engineers would express this by saying that the frictional resistance is 7 lbs. per ton, or 3⅛ kg. per tonne of 1000 kg. In this way a uniform rate of motion might be maintained in a train weighing 100 tons pulled along a level road by an engine which exerted a pull equal to the weight of 700 lbs., or 7 lbs. per ton of train-load; and then the work to be done by the engine would be the same as it would have been had the same engine been set to pull 700 lbs. vertically upwards with the velocity at which the train is travelling.

The resistance is said to be so many lbs. per ton, nothing being said as to the velocity. This is because, within wide limits, the coefficient b of kinetical friction is practically independent of the velocity; though, when the speed becomes very small, the waste of energy occasioned by friction becomes proportionately large, the converse holding good at high speeds.

If the road be not level, but go uphill, then there is lifting work to be done as well as work done against friction; and if the slope be, say, 1 in 100, then for every 100 feet of horizontal travel, the whole load must also be lifted one foot. Therefore an engine, moving an eighty-ton train along a level road with resistance equal to 7 lbs. per ton, would have to do work equivalent to raising 560 lbs.; whereas when it begins to go up a slope of 1 in 100, it has, in addition to these 560 lbs., to lift 0·80 ton, making 2352 lbs. in all.

When a man walks his knee is straightened, and his body is projected forwards and upwards at each step. The impulse may be resolved into two components: one upward, which may raise the centre of gravity of the body about an inch or an inch-and-a-quarter; one forward, which has to overcome

the intermittent resistances introduced by the stoppage occurring at the end of each step, when the foot of the opposite side strikes the ground. This is an intermittent frictional resistance.

If there were no friction between the wheels of a railway train and the rails of the railroad, there would be slipping but no progress. Friction between the wheels and the rails—friction proportional to the Weight of the vehicles—has the effect of preventing slipping; but (as in the case of belting) this corresponds to the maintenance of a state of rolling adhesion, under which each wheel is turned round and rolls upon the rail.

Rolling Friction.—When a ball is set to roll on smooth ice, it goes farther than it can on a wooden floor; farther on that than on a carpet; farther on a carpet than on grass. The rotation is, however, at length stopped. To produce rotation a torque is needed: to stop rotation a torque is also required. This may be that of a force or resistance acting at any point which is not the centre of mass. The greater the moment of the resistance round the centre of mass the sooner, for a given momentum, will the rotation be stopped. In this manner, the rotation of the ball is stopped by the Resistance of Friction. This is equivalent to a small force acting at the circumference of the ball, and bearing a constant ratio to the pressure produced by its weight. This ratio is very small. The resistance, then, to a wheel rolling along the ground may be very much less than the resistance to the same object when pressed upon by a brake. It is very much easier to move the trunk of a tree by setting it on logs which roll on the ground and under the trunk, than it is to drag it along the ground. There is less friction at a well-oiled hinge or well-lubricated joint than there would be in any other contrivance used for transferring a given mass from one position to another. If a wheel, instead of having its axle supported in bearings, have it supported on a couple of pairs of Friction-wheels which are free to rotate, the axle as it turns does not rub against a fixed bearing, but the friction-wheels yield and rotate, so that the rotating axle is supported by surfaces which travel at the same rate with it, and the friction is accordingly very small. In all these cases the frictional resistance has some definite moment round the axis of rotation.

The friction is affected by the relative softness of the surfaces in contact (Osborne Reynolds). An iron wheel rolling upon an indiarubber plane will raise up before it a little mound of indiarubber; and if it stop, this little mound will recover its form and drive the wheel backwards, thus making it oscillate. The friction of iron upon indiarubber is thus ten times as great as the friction of iron upon iron. Conversely, an indiarubber tire is deformed in the same way against a hard surface. This tendency to thrusting forward the contact-layer of both substances, but particularly that of the softer, results, in the case of iron railway rails, in the wear of the rail by scaling off of successive laminæ of iron. A similar result may influence most cases of ordinary friction, as in the spreading of putty with the thumb, to take an extreme example; or as in the transverse wearing of railway rails by trains rounding a curve.

Belting.—There is a very interesting and familiar case in which friction serves as a means for the transmission of energy—that is, transmission by machine-belting. A rotating wheel has a belt tightly drawn over it, as also over a second wheel, not too near. The belt must be tight, so that there may be more pressure between the leather and the iron. If the wheel be very small or the motion be very rapid, the mutual pressure between the

leather and the iron may be lessened by the inertia of the belt, which tends to pass the wheel and to be carried on. The friction is proportional to the pressure between the wheel and the belt, for the relation of the wheel to the belt is practically one of rest, though the surfaces in contact are changed from instant to instant. There being no slipping, the friction is statical, and is proportional to the pressure. Though the belt and the wheel do not move relatively to one another, they move relatively to surrounding objects, and the belt is set in motion. If the second wheel be free to rotate on an axis, portion after portion of its rim tends to remain at rest relatively to the leather, and the second wheel is set in motion round its axis. The tension of the belt, that is, the Total Tension, is greater nearer the driving power than it is on the other side of the wheel driven. This is because Energy has been taken up in preventing relative motion of the belt and the driven wheel, or—another mode of expressing the same thing—in producing absolute motion of the latter. This difference of tension is equivalent to a force directly applied to the rim of the wheel. Thus the kinetic energy of the driving wheel is in part imparted to the leather belt and the rotating wheel; these come to form a part of the same system with the driving wheel; the latter cannot rotate so fast when it is driving a second wheel as it can when not doing so, the same energy being supplied to it; and thus energy is transmitted.

Such is the theory of belting when there is no slipping; but in practice there is always some slipping. The part of the belt in front of the pulley is under greater tension than the part behind; it is therefore more stretched out and assumes a greater length; and this involves slipping, which causes a loss of energy spent in deforming the belt, and ultimately transformed into heat in the belt; a loss which in the case of leather belting is appreciable, but which in the case of indiarubber belting is very considerable (Osborne Reynolds).

The efficiency of belting is greatly increased when the rim of the wheel is lined with leather, hair side outwards, the hair side of the leather belt being inwards: or when the rim is coated with paper.

Activity in Belting.—The transmission of energy by belting is subject to the law that Activity = $\mathbf{Tv}$; $\mathbf{v}$ being the velocity at which the belt runs, and $\mathbf{T}$ the tension (*i.e.* total tension) along the belt; this tension being, essentially, the difference of tension between the outgoing and the incoming parts of the belt. Let, for instance, the speed of the belt be 400 feet a minute or, say, 200 cm. per second; and let the effective total tension on the belt be equal to the weight of 100 kilogrammes or to 98,100,000 dynes; then the Activity, or rate of transmission of energy $= \mathbf{Tv} =$ Tension × Velocity = 98,100000 × 200 = 19620,000000 ergs per second, or about 2½ horse-power.

If the velocity be very great, the tension may be small. Thus let the velocity be 6000 feet per minute or, say, 3000 cm. per second, a speed which has been attained in practice; and let the desired activity of transmission be 100 horse-power, or 745,948,005000 ergs per second; we have Activity $= \mathbf{Fv}$ $= \mathbf{Tv}$ or 745,948,005000 = 3000 $\mathbf{T}$; whence $\mathbf{T} =$ 248,649335 dynes, and the tension $\mathbf{T}$ is therefore equal to the weight of 248,649335/981 = 253,465 grammes or 253·465 kilogrammes. Such a tension would (since steel has a "breaking-weight" of 33 tons per square inch) be barely able to snap a steel wire of ½ cm. diameter; whence a slender steel wire-rope may, at very high speeds, be safely used to transmit large amounts of energy; a conclusion which experience has confirmed.

Friction-Dynamometers.—Friction may be utilised as a means of measurement of Rate of Doing Work. Suppose a cord passed round a revolving pulley; the two ends of the cord pass away from the pulley, both vertically or otherwise in line with one another; the lower end is stretched by a weight G dynes; the upper end pulls upon a fixed spring and imparts to it a strain which indicates a total tension of $\mathbf{T}'$ dynes. The weight G is so great and tightens the string so much that the whole of the energy of the pulley is spent in overcoming friction, and the pulley stops at once when the driving power is withdrawn. The string wraps round the circumference of the pulley, *i.e.* $2\pi r$ cm.; the velocity of that circumference in passing any point of the string is, if the pulley rotate n times per second, $\mathbf{v} = n \cdot 2\pi r$ cm. per second; the force overcome, $\mathbf{F}$, is the difference between G and $\mathbf{T}'$, *i.e.* $(\mathrm{G} - \mathbf{T}') = \mathbf{T}$ dynes; the product $\mathbf{Tv}$ is therefore equal to $n \cdot 2\pi r \cdot (\mathrm{G} - \mathbf{T}')$. This product $\mathbf{Tv}$ measures in ergs the work done per second by the revolving pulley, the Rate of Doing Work of the pulley (p. 42). Instruments of this class may be graduated so as to indicate, by the amount of distortion of a spring, the working value of a steam-engine in horse-powers: the whole power of the engine is turned on to the dynamometer for a brief period, and the scale-reading of the spring observed, as well as the speed of rotation of the pulley.

Variations in Kinetical Friction.—The coefficient of kinetical friction is found to present, at high values of $\mathbf{F}$ or P, or at high velocities, or with varying lubrications or forms of surface, or with different kinds of movement (continuous or alternating), considerable differences from the simple constant value obtained by making one solid slide upon another under moderate pressures and velocities. For example, with abundant lubrication, the coefficient b actually varies inversely as the pressure P, and also varies directly as the square root of the velocity; so that $-R = -\mathrm{B}\sqrt{\mathbf{v}}$, where B is a coefficient depending on the kind of lubricant, and R becomes independent of the pressure P. The different values of B, this varying coefficient of kinetical friction, or Friction-factor, under different circumstances, can only be ascertained by laborious direct observation.

Between metal and metal, at ordinary working velocities of axles in their bearings, the coefficient of kinetical friction is approximately constant. If castor oil be used as a lubricant, this coefficient is, at low speeds, very small; but it increases rapidly as the speed rises. If water or thin petroleum oil be used as a lubricant, the friction at speeds beyond a certain limit is very small; but at speeds whose average is below that limit, there is alternate "biting" and slipping. Hence for axles at low speeds, thick oils; for high speeds, thin oils or water.

Friction of Moving Solids against Liquids depends directly upon the extent of surface exposed. Further, when the speed is very small, the frictional resistance is nearly constant, and the power required to overcome the resistance there-

fore varies as the velocity, as in the case of kinetical friction between solids; as the speed increases, the frictional resistance itself comes to vary as the velocity, and the power required comes to vary as the square of the velocity.

Perhaps the friction of sharp skates against smooth ice may be found to be in this category, the ice being melted as the skate runs.

Work done against a uniform frictional resistance R through space $\mathbf{s}$ is $R\mathbf{s}$, or $R\mathbf{s}/t$ per second. This value, $R\mathbf{s}/t$ per second, is the Activity or Power required, and is equal to $R\mathbf{v}$. If R be constant, the Power required to overcome the friction $\propto \mathbf{v}$; if R itself vary with $\mathbf{v}$, and if it be, say, $k\mathbf{v}$, the Activity is $k\mathbf{v}\cdot\mathbf{s}/t$ or $k\mathbf{v}^2$.

Friction on a raindrop. — A raindrop falling *in vacuo* through a height h feet would acquire a velocity $v = \sqrt{2gh} = 8{\cdot}249\sqrt{h}$ feet per second. Its starting point might easily be so distant that a blow from a raindrop travelling under these circumstances would be fatal to any living being struck by it. But at every instant of its course it is subject to kinetic friction tending to reduce its velocity at the instant; at the same time it is subject to the accelerating force of gravity: and thus there must be a certain velocity at which the retardation of friction and the acceleration due to gravity will balance one another, and the drop, if it once attained that speed, would retain it, and fall with a constant velocity. This happens in the case of the raindrop, and also in the case of a stone or granule falling in deep water.

Viscosity-resistances. — If a body start free, with initial velocity $\mathbf{v}_0$, in a viscous medium which offers frictional resistance varying as the velocity, its speed will diminish in geometrical progression in successive equal intervals of time, and it will gradually approach a condition of rest.

The retarding acceleration is proportional and opposite to the velocity; $\mathbf{a} = \dot{\mathbf{v}} = -\mathbf{k}\mathbf{v}$; this is a Differential Equation, which gives the result that at the end of time t, the velocity $\mathbf{v}_t = \mathbf{v}_0 \cdot \epsilon^{-\mathbf{k}t}$, where $\mathbf{v}_0$ is the initial velocity, and $\epsilon = 2{\cdot}7183$. At the end of one second, $\log \mathbf{v}_t = (\log \mathbf{v}_0) - \mathbf{k}$; at the end of two seconds it is $(\log \mathbf{v}_0) - 2\mathbf{k}$; at the end of n seconds, it is $(\log \mathbf{v}_0) - n\mathbf{k}$. Thus during each second the (Naperian) logarithm of $\mathbf{v}$ is altered by the numerical quantity $\mathbf{k}$; and this is the Naperian Logarithmic Decrement of the velocity.

Friction in S.H.M. — If a Circular Pendulum be set to oscillate in a viscous medium in which the frictional retardation is proportional to the velocity, the circle described by it will gradually dwindle. It will take a longer time to go round 360° than it would in a frictionless medium, but it will do so, on its consecutive rounds, in equal times; and its path will be the curve known as a logarithmic spiral. The distance between its bob and the point of ultimate rest always diminishes in equal proportions after describing equal angles; so that this distance diminishes in geometrical progression, for equal intervals of

time. If the frictional retardation be small, the bob may go many times round the midpoint before reaching it; but if it exceed a certain limit, the bob will travel with diminishing velocity, by a more or less indirect path, towards the midpoint, taking, theoretically, an infinite time to reach that point.

If this conical pendulum be looked at from one side, the S.H.M. which it describes will be isochronous, but will be slower than it would have been in a frictionless medium; and the amplitude will appear to diminish in geometrical progression; while if the retardation be excessive, the displaced bob will simply appear to return to its median position with diminishing velocity. Actual instances of these kinds of movement are to be seen in vibrating bodies where the retardation is due to the resistance of the surrounding fluid medium or to an equivalent resistance, that of **viscosity,** which has its seat within the vibrating substance itself; and in the **damping** of oscillations of a moving body by increasing the resistance of the surrounding medium.

The acceleration $\mathbf{a} = \ddot{\mathbf{s}}$ consists in these cases of two parts; one, $= -\mathrm{n}^2\mathbf{s}$, proportional to the displacement $\mathbf{s}$ and oppositely directed; the other, $\mathrm{k}\mathbf{v} = -\mathrm{k}\dot{\mathbf{s}}$, proportional and opposite to the velocity. Then $\ddot{\mathbf{s}} = -(\mathrm{k}\dot{\mathbf{s}} + \mathrm{n}^2\mathbf{s})$. This is again a Differential Equation, and it has two orders of solution. First, when $\mathrm{n} > \mathrm{k}/2$, the displacement $\mathbf{s}_t$ (that is, the value of the displacement at the end of time t) $= (a \cdot \epsilon^{-\mathrm{k}t/2})(\cos t\sqrt{\mathrm{n}^2 - \mathrm{k}^2/4})$. On comparing this with the equation $\mathbf{x} = a \cdot \cos \omega t$, on p. 82, we see that it represents a S.H.M. in which the amplitude $a \cdot \epsilon^{-\mathrm{k}t/2}$ diminishes in geometrical progression, with a constant log. dec., from a to 0; the angular velocity is $\omega' = \sqrt{\mathrm{n}^2 - \mathrm{k}^2/4}$, instead of $\omega = \mathrm{n}$, and is constant, so that the motion is isochronous. If on the other hand $\mathrm{n} < \mathrm{k}/2$, the equation is satisfied by the condition that at the end of time t, the displacement is reduced from a to $a \cdot \epsilon^{-\mathrm{n}_{,}t}$, where $\mathrm{n}_{,} = \mathrm{k}/2 + \sqrt{\mathrm{k}^2/4 - \mathrm{n}^2}$; and the displacement accordingly diminishes from a, its maximum, to nothing, without oscillations.

CHAPTER VII.

ATTRACTION AND POTENTIAL.

WHEN one body in contact with others forms with them a system, a Conservative System, which will be put in a condition of stress when the bodies are removed from contact with one another, these bodies are said to be Attracted towards one another; and if they be fixed in such a position that the stress of the system is permanent, the condition is one of statical equilibrium. When a spring is drawn out and fixed by a catch, there is equilibrium between the recoil of the spring and the molecular forces within the catch, which resist its deformation; when a heavy stone is placed on a wooden table, if the table be strong enough to support the stone, there will be equilibrium between the weight of the stone and the resistance to crushing offered by the wooden support. If the spring be released it will fly back: if the supporting table be removed the stone will fall. In the former case there is a visible medium, the spring, the elasticity of which comes into play; in the latter case there is no such elastic medium visible.

There is no direct analogy between the two cases. In the former, the greater the displacement the greater the stress; in the latter, the greater the mutual distance the less the mutual attraction.

Let us consider attraction, which, whatever may be its cause, obeys the particular law (the so-called "Law of Inverse Squares") that a mass m and a mass $m_{\prime}$ are attracted by each other, along the line joining them, with a force which depends on each mass directly, and on the square of the distance between them inversely. Then $\mathbf{F} \propto mm_{\prime}/d^2$; that is, $\mathbf{F} = k \cdot mm_{\prime}/d^2$.

In a system of particles of this kind we must further assume — and experience warrants us in so doing — that every particle is connected with every other particle by an independent attraction; then the total attraction of one set of particles for another set of particles has to be found by a process

of summation. To effect this summation the aid of the Integral Calculus has in general to be called in; the process is, however, of this kind: — The mass and the distance of each particle from every other being taken into account, the attraction between each particle and every other particle is to be separately found, and the whole attractions are then to be summed up. In simpler cases, mutually attracting masses may be considered as acting at their centres of figure; then the mean distance between two such masses is the distance between their centres of figure, and each mass may be supposed to be concentrated at its centre of figure.

Attraction in particular cases. — (1.) A hollow spherical shell, whose thickness is infinitesimal, attracts an external particle as if all its mass were gathered at its centre. Its area is $4\pi r^2$; the amount of mass per unit of surface (its "surface-density") $= \sigma$; its mass m is therefore $4\pi r^2\sigma$. It acts on mass $m_,$ placed at a mean distance d from the centre of the shell as if the whole mass $4\pi r^2\sigma$ were at that centre, and the attraction $\mathbf{F} = k \cdot m_, \cdot 4\pi r^2\sigma / d^2$. If the attracted particle be of unit-mass, $m_, = 1$, and the attraction of the shell on a unit-particle is $k \cdot 4\pi r^2\sigma / d^2$.

If the external particle, of mass $m_,$, be *just outside* the shell, so that its distance d from the centre is practically equal to r the radius of the shell, $d = r$ and $\mathbf{F} = k \cdot m_, \cdot 4\pi\sigma$.

(2.) A solid sphere and an external particle $m_,$ will in the same way act on one another as if all the mass of the sphere were gathered at the centre. The attraction between a sphere whose radius is r and whose amount of mass per unit of volume ("volume-density") is ρ, and a particle $m_,$ at a distance d from the centre of the sphere, is $k \cdot m_, \cdot (\frac{4}{3}\pi r^3)\,\rho / d^2$ (for the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$): if the particle be just on the surface of the sphere the attraction is $k \cdot m_, \cdot (\frac{4}{3}\pi r^3)\,\rho / r^2 = k \cdot m_, \cdot \frac{4}{3}\pi r\rho$.

(3.) An attracting spherical shell of any thickness, if this thickness be uniform, has no action whatsoever on a heavy particle contained within it. For every area of the shell on one side of the particle which may attract it in one direction, there is another on the other side attracting it in an opposite direction; and the one exactly balances the other, for what advantage the one area may have in size the other exactly makes up for in proximity. Thus it is not possible to find any area of the sphere, the attracting effect of which on the particle within the sphere is not exactly counterbalanced by the opposed attracting effect of an opposite area. The particle, attracted equally in every direction, remains at rest.

No other law than that of the inverse square of the distance will give this entire absence of effect within a hollow spherical shell of uniform thickness, as will easily be found on trial.

If the shell have any other form than the spherical, it must, in order to retain this absence of interior effect, have a thickness which is other than a uniform one. For example: an ellipsoidal shell whose inner ellipsoidal surface is concentric and confocal with the exterior ellipsoidal surface has a thickness which at any point is proportional to the shortest distance between the centre and the tangent to the ellipsoid touching the point in question (see Fig. 197); and such a shell has, under the law of inverse squares, no

action at any point within it. Such a shell is thickest at the extremities of its major axis.

If a particle $m_{,}$ be just outside such a shell, at a point where the surface-density is σ, the attraction just outside is $\mathbf{F} = k \cdot m_{,} \cdot 4\pi\sigma$; just inside, $\mathbf{F} = 0$; and thus the particle, if it pass through the shell, passes from a field where the attraction is $k \cdot m_{,} \cdot 4\pi\sigma$ to another where the attraction differs from its former value by $k \cdot m_{,} \cdot 4\pi\sigma$.

(4.) Between a particle $m_{,}$ and an arc of a circle of radius r, at the centre of which the particle stands, the attraction is $k \cdot m_{,} \cdot (r\sigma/r^2) \times 2 \sin \frac{1}{2}$ angle subtended $= k \cdot m_{,} \cdot (\sigma/r) \cdot 2 \sin \frac{1}{2}\theta$; as if a mass equal to that of a chord of the arc, with the same density as the arc, were concentrated at the midpoint of the arc.

(5.) Between a semicircle and a particle $m_{,}$ at its centre; the angle subtended is 180°; the force is $k \cdot m_{,} \cdot (\sigma/r) \times 2 \sin 90° = k \cdot m_{,} \cdot 2\sigma/r$, towards the midpoint.

(6.) Between a definite line AB and a particle at D opposite the centre C of the line AB. Draw AD and BD. With centre D and radius DC draw a circular arc limited by the lines AD and BD. The line AB and this circular arc exert the same attraction on a particle at D: and the attraction of the circular arc we know from (4).

(7.) Between an indefinite line and a particle at a distance r from it; the angle subtended is 180°; the force is, as in (5), equal to $k \cdot m_{,} \cdot 2\sigma/r$.

(8.) Between a hemispherical shell and a particle $m_{,}$ at its centre, the attraction is $k \cdot m_{,} \cdot 2\pi\sigma$, and is independent of the radius.

(9.) Between an indefinite plane and a body of mass $m_{,}$ at a finite distance d from it, the attraction is the same as in case 8, and is $k \cdot m_{,} \cdot 2\pi\sigma$, at right angles to the plane. If the particle pass through the infinite plane it passes to a region where the attraction is $- k \cdot m_{,} \cdot 2\pi\sigma$, because it acts in the opposite direction, and therefore differs from its former amount by $k \cdot m_{,} \cdot 4\pi\sigma$.

Since the force acting is independent of the distance d we may make $d = 0$. If $m_{,} = 1$, we now have, as the body acted upon, a unit-mass of the substance of the plane itself, and it is acted upon by a force $k \cdot 2\pi\sigma$. The matter distributed over a sq. cm. is σ, and this is acted upon by a force $(k \cdot 2\pi\sigma) \cdot \sigma = k \cdot 2\pi\sigma^2$. The force acting on the substance of the plane itself is therefore $k \cdot 2\pi\sigma^2$ per sq. cm., and is at right angles to the surface.

(10.) In a spherical shell, the inward force on the outmost film is $k \cdot 4\pi\sigma$ per unit quantity (see above, (1.)); on the inmost film it is zero (see above, (3.)); the average inward attraction for the substance of the shell itself is, per unit quantity, $k \cdot 2\pi\sigma$, or *half* that for an external unit-mass; the attraction per unit of area is $k \cdot 2\pi\sigma^2$, at right angles to the surface, the same as in (9.) above.

In the case of gravitation, the constant k in the above examples is the gravitation-constant γ, p. 202.

Convention as to Attraction and Repulsion. — A force of this kind is conventionally said to be Positive when its effect is to separate (or to increase the distance between) the bodies by whose relative motion it is manifested. Thus a repulsive force is positive; an attractive, which diminishes the distance between two masses, is negative. This convention is opposed to the ordinary use of speech.

Potential Energy in case of Repulsion. — If, as the phrase goes, two bodies repel one another, and if one or both of them be free to move, their mutual separation may be carried on to an infinite distance. So long as it is still possible under any specified circumstances for the bodies to become still farther separated by their mutual repulsion, there is still some potential energy in that system which consists of the two masses (together with the intervening medium, if there be any such); the mutually repelling bodies must therefore be separated to an infinite distance from one another before their repulsion can cease to act, before the Potential Energy of the system becomes reduced to zero. When, on the other hand, the bodies which repel one another are in contact, the Potential Energy of the system is as great as it can possibly be.

Work done by Repulsion. — If two masses, m and $m_{\prime}$, situated at a distance d from one another and repelling one another with a force $= k \cdot mm_{\prime}/d^2$, be allowed to separate through a little distance δd, the system has exchanged a configuration in which the mutual force was $k \cdot mm_{\prime}/d^2$ for one in which it has been diminished to $k \cdot mm_{\prime}/\overline{(d + \delta d)^2}$; and the work done by the repulsion is the product of the mean force into the space δd. If δd be taken small enough this product becomes, with an indefinitely close approximation to accuracy, equal to $(k \cdot mm_{\prime}/d^2) \times \delta d$. If the bodies increase their distance, making the distance $(d + \delta d)$ grow to $(d + 2\delta d)$, the work done in this stage is $(k \cdot mm_{\prime}/\overline{(d + \delta d)^2}) \times \delta d$. Summing up by means of the Integral Calculus the work done by the repelling force in separating the bodies, stage by stage, from a mutual distance d to a distance d', we find that it is $k \cdot mm_{\prime}(1/d - 1/d')$.

This proposition may be otherwise presented in the form of a positive statement. The work done is the product of the Space traversed, $(d' - d)$, into the Mean Force; but the mean force in question is not the arithmetical but the geometrical mean between the extreme values; that is, these extreme values being $k \cdot mm_{\prime}/d^2$ and $k \cdot mm_{\prime}/d'^2$, the mean force is the square root of their product, or $k \cdot mm_{\prime}/dd'$; the latter being multiplied by the space traversed, $d' - d$, gives the product $k \cdot mm'(d' - d)/dd'$ or $k \cdot mm_{\prime}(1/d - 1/d')$ as the work done.

Hence the work done by a repulsion (which at any distance d is equal to $k \cdot mm_{\prime}/d^2$) in separating two masses from a distance d to a distance $2d$, is $k \cdot mm_{\prime}(1/d - 1/2d) = k \cdot mm_{\prime}/2d$; from distance d to an infinite distance ∞, the work done is equal

to $k \cdot mm_{,}(1/d - 1/\infty) = k \cdot mm_{,}(1/d - 0) = k \cdot mm_{,}/d$. Hence if a certain amount of work be done by the repulsion in doubling the distance between two mutually repelling bodies, the repulsion would do exactly twice as many units of work, and no more, in separating the two bodies to an infinite distance from one another.

The Potential Energy unexhausted at any given distance.—To remove a mass $m_{,}$ from a point at a distance d from a fixed repelling mass m to an infinite distance would involve expenditure (by the repulsion) of work $= kmm_{,}(1/d - 1/\infty) = kmm_{,}/d$: this work not having been done when the distance between the bodies is finite, the potential energy of the system is so far unexhausted. At an infinite distance, $d = \infty$, and the unexhausted potential energy is $kmm_{,}/\infty = 0$.

The unexhausted Potential Energy of two bodies of masses m and $m_{,}$, repelling one another and situated at a mutual distance d, is $k \cdot mm_{,}/d$; this is called the Mutual Potential of the two masses.

If $m_{,}$ be a unit, the Mutual Potential is $k \cdot m/d$. This is numerically equal to the Potential as defined in the next paragraph.

Direction of Movement.—At an infinite distance, where the potential energy attributed to a body there placed would be zero, there would be no force impelling to any further separation. At any place where the potential energy has a positive value, it will tend to exhaust itself, and a body there placed will, if free to do so, move away towards some place where it would have less potential energy. But the Potential Energy which a Unit-mass would have if placed at a particular Point in Space,—the work which would be done by the repelling force in removing the unit-mass from that point to a place of zero-repulsion, or would have to be done against repulsion in conveying the unit-mass from such a place to that point, —may be stated as an attribute of that Point in Space and may be called its **Potential.** This may be numerically high or low. Then, under a repelling force, a body tends to move from a place of high potential to a more distant place of low potential, and if the body be free to move in that sense, the force will do work; while if the body be moved from a place where the potential is low to one where it is high, the movement is effected against repulsion or resistance, and work is done against the repelling force.

The Direction of the Force is opposed to the direction in which the potential increases most rapidly; and its amount at any point is (per unit-mass acted upon, and in any given direction) equal to the mean **decrease** of potential per unit distance traversed (in that direction), that is, to the **potential-gradient** or **potential-slope** (in that direction). The product

of this force per unit-mass into the space traversed, **Fs**, the Work Done on a unit-mass, is numerically equal to the whole **diminution** of potential in the whole distance traversed.

Potential a condition at a point in space. — We must distinguish between the *Potential Energy* which a *mass* may be said to have in virtue of its position at a certain point, and of its consequent relation to neighbouring masses; and the *Potential* of that *point* in space. The condition at that point of space is such that *if* a body of mass $m_{\prime}$ were placed there, the forces acting on it would do $\mathrm{V}m_{\prime}$ units of work in conveying it to an infinite distance, or would, on the other hand, have $\mathrm{V}m_{\prime}$ units of work done against them if the mass $m_{\prime}$ were forced against them from an infinite distance to that point: and this is a property numerically expressible by the numerical value of V (the value of $\mathrm{V}m_{\prime}$ when $m_{\prime}$ is a unit-mass), but independent of the actual presence or absence of any mass at that point.

At a point situated at a distance d from a mass m the "Potential" V is equal to km/d; and $\mathrm{V}m_{\prime} = k \cdot mm_{\prime}/d$.

Work done against Attraction. — If a body $m_{\prime}$ be at a given distance d from an attracting mass, the action between the two bodies is a force tending to approximate them: work is done by the attracting force in doing this: but "the work done *by* the attracting force in separating the bodies to an infinite distance" is a negative quantity, for work ($= \mathrm{V}m_{\prime}$ units) would have to be done *against* the attraction in producing this movement; and the **potential** at a distance d from the attracting mass is $-\mathrm{V}$, a **negative** quantity.

Potential in the special case of Gravitation. — It would, if the Earth were a sphere of radius 637,000000 cm., require the expenditure of $k \cdot mm_{\prime}/d = (kmm_{\prime}/d^2) \times d = m_{\prime}g \cdot d = m_{\prime} \times 981 \times 637{,}000000 = 624897{,}000000 m_{\prime}$ ergs, and no more, to remove a mass $m_{\prime}$ from the earth to an infinite distance *against* gravitation; and therefore any point on the earth's surface would be at a negative potential $\mathrm{V} = -624897{,}000000$, while the potential of any point at an indefinitely great distance would be zero. By a special exception, however, the Gravitation-Potential of a point at the earth's surface is considered to be zero, and a body lying on the earth's surface has no potential energy; while a mass $m_{\prime}$, removed to an infinite distance, could have no more than $624897{,}000000 m_{\prime}$ ergs of potential energy stored up in it; and the Gravitation-Potential of a point at an infinite distance is $+624897{,}000000$ units $= \mathrm{V}$.

Absolute Zero of Potential. — A point is at zero potential when a body placed there would have no potential energy. This is the condition of a point at an infinite distance from all repelling masses.

Fields of Space in opposite conditions. — If there be two bodies, the one attracting, the other repelling: a unit-mass brought near the former will on the whole be attracted; from

the other mass, it will on the whole be repelled. The space in the neighbourhood of the attracting mass will be a field of space in which the potential is negative; round the repelling body there will be a field of force of positive potential.

Continuity of Potential through Zero value. — A particle passing from a region of positive potential into one of negative potential must pass through a point where the potential is zero; for if it were possible for it to do otherwise there would be physical discontinuity. As it thus moves, the positive potential energy of the body is gradually exhausted, becomes zero, and then becomes a negative quantity.

Arbitrary Zero of Potential. — We may arbitrarily assume any point or surface in the neighbourhood of attracting or repelling masses as one whose $V = 0$; then those places which have a greater potential are said to be localities of positive potential, and those at which the potential is less are said to be localities of negative potential. This is convenient, for absolute zero we know no more than we know absolute rest.

Analogy of Sea-level. — Let us assume that the surface of the earth is the sea-level taken at high-water mark. This is an arbitrary assumption, for low-water mark might just as well have been chosen. If a body be placed at a certain height above sea-level, gravitation may do a certain amount of work in bringing it down to that level, for the mass placed at that height has a certain amount of potential energy: at a less height it has less potential energy; at the sea-level it has none; if placed below the sea-level, its potential energy is, on this assumption, a negative quantity. Hence the gravitation-potential above sea-level is of opposite sign to that below it, if the gravitation-potential at sea-level be taken as zero.

Obviously it would be possible, instead of saying that a point is so many feet above or below sea-level, to say that a mass $m_{,}$ there placed would have $Vm_{,}$ units, + or −, of potential energy if there placed, and thus to define the distance between that point and sea-level by its gravitation-potential $\pm V$.

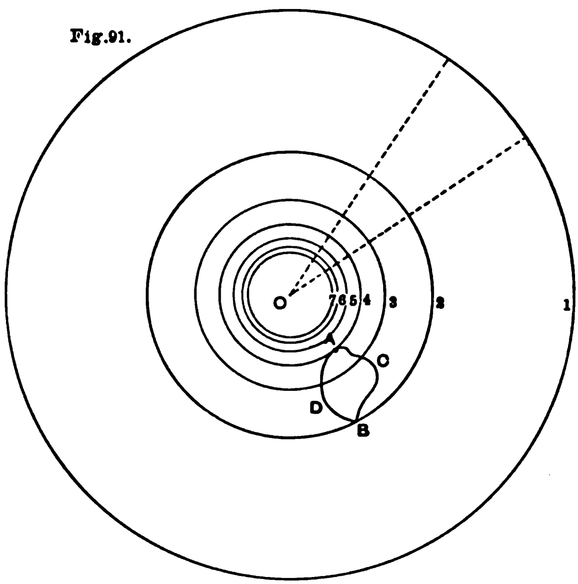
Equipotential Surfaces. — In Fig. 91, O is a repelling particle. All points at equal distances from it are at the same potential. If these be joined they form a sphere. The potential at every point of the surface of one of these imaginary spheres is the same, and may be represented by V_1. This sphere is an *equipotential surface* for potential V_1. Within this, and concentric with it, lies another sphere, the potential at every point of which is V_2. Within this lie successive shells or imaginary spherical surfaces, over each of which the potential is equal. If these surfaces be chosen such that their potentials have a common difference — that is, that $V_2 - V_1 = V_3 - V_2 = V_4 - V_3$, etc. — and if these differences each repre-

sent one unit of work done per unit-mass moved from one to the next, a set of equipotential surfaces thus obtained is called a "*System of Equipotential Surfaces.*"

Motion parallel to Equipotential Surfaces does not involve work done either by or against the attracting or repelling force.

Motion across Equipotential Surfaces, from one surface to another, implies movement from a place where the potential has one value to a spot where it has another. A unit-mass moving away from the second to the first surface in Fig. 91, loses potential energy $=V_2-V_1$: on a mass m_I the repelling force would do work $= m_I \times (V_2-V_1)$. A mass m_I moved up from

Fig. 91.



equipotential surface No. 10 to surface No. 15 in a system of such surfaces, whatever be their form, would have work $= 5m_I$ units done upon it against the repelling force.

The **work done** would be the same *whatever be the points* of the respective surfaces between which the motion is effected. Any transference of a particle from one equipotential surface to another may be effected by a vertical translation from the one to the other, which involves work, compounded with a translation along the second equipotential surface, which involves none.

The **work done** by a transference of a particle from a point A on one equipotential surface to a point B on another is also always the same, by *whatever path* the transference be effected,

provided always that there be no friction. The most complex path may be resolved into so much movement at right angles to the equipotential surfaces, which implies work done by or against the forces, and so much parallel to them, which consumes or liberates no energy.

This may also be proved by a *reductio ad absurdum.* If in Fig. 91 there were two possible paths between A and B, one of which, ACB, corresponded to W units of work done by a unit-mass of matter traversing it, while the other, ADB, corresponded to a greater amount, W′ units, of work; then it would be possible to cause a body to fall from A to B down the path ADB, corresponding to the greater work, and by falling to pull directly or indirectly a mass equal to its own up the easier path BCA: it would itself acquire kinetic energy corresponding to energy $= W' - W$; the body thus pulled up along BCA might in its turn fall down the path ADB, and raise along the path BCA the mass which had previously traversed the path ADB, again with gain of energy equal to $W' - W$. Thus the circuit might be kept up with continuous gain of energy, and this contrivance might be utilised as a perpetual motor; but this is an impossibility; therefore there is an equal expenditure or liberation of energy, so far as the attracting or repelling forces are concerned, in effecting a transference along every possible path between any two given points in space.

Analogy of Surfaces of equal level.—Obviously the same propositions apply if we read the word level for potential.

Distances between Concentric Equipotential Surfaces.— In a system of concentric spherical equipotential surfaces, the distance between every pair of these surfaces is proportional to the square of their mean distance (*i.e.* of the geometrical mean) from the centre of the single attracting or repelling mass.

Two concentric spherical equipotential surfaces whose potentials are V and $(V + 1)$, and whose respective radii are r and r'; we wish to find the value of $r - r'$. Then $V = k \cdot m/r$, and $(V + 1) = k \cdot m/r'$; whence $r - r' = k \cdot m \div V(V + 1) = rr'/km$; $\therefore (r - r') \propto \{\sqrt{rr'}\}^2$.

Thus, if the equipotential surfaces be those surrounding the earth, over which the potential due to **gravitation** is constant, and if the distances between the surfaces be such that transfer of a gramme-mass from any one surface to the next one represents one erg of work done: then, at the distance of one earth's-radius from the centre of the earth — that is to say, on the surface of the earth — the distance between two equipotential surfaces is $\frac{1}{981}$ cm.; twice as far from the centre — that is, 4000 miles (nearly) from the surface of the earth — the distance is $\frac{4}{981}$ cm., and the same amount of work which would raise a gramme-mass through $\frac{1}{981}$ cm. near the surface of the earth would, at a height of 4000 miles, raise it $\frac{4}{981}$ cm.; and similarly, at a height

of 8000 miles, it would raise it $\frac{8}{981}$ cm., and so on. Thus at a very great distance exceedingly long paths would be traversed by a gramme-mass as the result of doing a single erg of work on it.

A mass at a distance of 240,000 miles (= 60 radii nearly) from the earth's centre would be attracted by the earth with a force which bears to the attraction at the earth's surface the proportion of $(1/60)^2 : (1)^2 = 1 : 3600$. Hence, to move a gramme-mass through one cm. — that is, from an equipotential surface by any path to any point on an equipotential surface one cm. distant from it — at a distance of 240,000 miles, or, roughly, at the distance of the moon, would involve the expenditure of approximately $\frac{981}{3600}$ erg of work.

It follows that if the equipotential surfaces be chosen *at equal distances* from one another, the amount of work corresponding to the removal of a mass from one surface to the next is in the inverse ratio of the square of the mean distance of the two surfaces from the attracting mass.

Two concentric spherical equipotential surfaces whose potentials are V and $V_{,}$, and whose radii are r and $(r+1)$; $V = k \cdot m/r$: $V' = k \cdot m/(r+1)$; $V' - V = k \cdot m \div r(r+1)$; $\therefore (V' - V)m_{,} \propto \{\sqrt{r(r+1)}\}^{-2}$.

Equipotential Surfaces of Complex Form. — If A and B be two equal particles, A attracting and B repelling external particles, the space surrounding A will be a region of negative potential, while the potential of the neighbourhood of B is positive. Over a plane symmetrically situated with respect to A and B the potential will be zero, and the equipotential surfaces will present the form indicated by the lines marked "Lines of Force" in Fig. 234. If A and B be not equal, or if there be more than two masses concerned, the form will be still more complex.

Free movement always at right angles to Equipotential Surfaces. — Whatever be the form of any equipotential surface, it always happens that a body placed on such a surface, and free to move, will tend to move, under the influence of the attracting or repelling forces, in a direction at right angles to that surface. This is because the forces of attraction or repulsion can have no component tending to produce motion in any direction along a surface of equal potential, or parallel to it.

Lines of Force. — Thus, if the equipotential surfaces be concentric spheres, as those of Fig. 91, a body repelled from O will travel along radial lines such as are exemplified by the dotted lines in that figure. When the equipotential surfaces have a more complex form, the lines along which a body tends

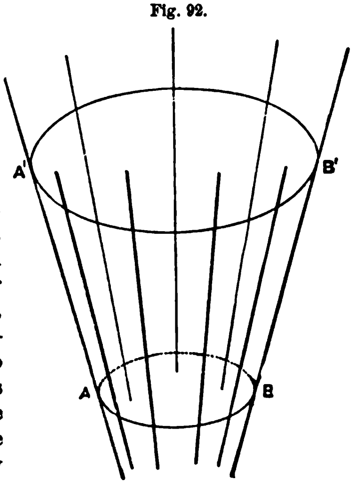
to travel are more complex, as is shown in Figs. 234 and 235. These lines, always at right angles to the equipotential surfaces which they cross, are called Lines of Force.

Space in the neighbourhood of an attracting or repelling body may be conceived to be pervaded by a system of Lines of Force, along which bodies will move if free to do so. The work done on a particle thus set in motion by an attraction or repulsion is the product of the mean force into the space traversed; the latter must be measured along the line of force which is the body's actual path.

Lines of Force are analogous to lines of steepest fall in topography; water poured out will at any spot run in the direction of steepest fall; and a body acted upon in a field of force will tend to fall away from a spot of higher potential to one of lower potential, following the direction of most rapid potential-fall, the line of force.

Tubes of Force.—Suppose AB to be a portion of an equipotential surface: lines of force pass through the equipotential surface: some of these lines graze the edge of the area AB; these cut off an area A′B′ from another given equipotential surface. The space comprised between these equipotential areas and the marginal lines of force is called a Tube of Force. This space may be supposed to be filled with a bundle of lines of force, extending from AB to A′B′. Such a tube may be curved in form.

Fig. 92.



Tubes of force have this property, that so far as the area A′B′ cut by them from one equipotential surface is greater than the area AB cut off from another, so does the intensity of the force acting across any unit of area diminish; so that if A and A′ be the respective areas of AB and A′B′, and f, f' the respective forces per unit of area acting across these equipotential areas, the product, intensity of force × area, is constant, or $fA = f'A'$. Thus the force f per sq. cm. at the level A′B′ is less than that at AB, in inverse proportion to the relative magnitude of the area A′B′ cut off by the tube of force.

Tubes of Force drawn in such fashion as each to contain one line of force are called Unit Tubes of Force.

Number of Lines of Force.—The forces at any two points may be compared by stating the relative numbers of the lines of force which pass through units of area of those equipotential surfaces which pass respectively through each of the points compared; the fewer these lines, the less is f, the local intensity of

the force, in the direction, at any point, of the local Lines of Force. Thus in Fig. 91 the lines of force which cross the outer spheres are less numerous, per unit of area of the sphere, than those which cross the inner spheres, and the force per unit of area is there correspondingly less.

Systems of Surfaces and Lines. — The space in the neighbourhood of an attracting or repelling mass or system of masses may thus be mapped out by a system of equipotential surfaces and lines of force, and such a region of space is called a **Field of Force.** The system of surfaces and lines may be so constructed that (1) the work done in passing a unit-mass from one equipotential surface to the next is always the same, one unit of work; and (2) the lines of force are drawn in just such numbers that at a place where the force on a unit-mass is equal to unity, one line of force passes through the corresponding equipotential surface in each unit of area of that surface, and $f = 1$. This secures the following advantages: —

(1.) The potential at any point in the field of space surrounding the repelling or attracting mass or masses is found by determining on which imaginary equipotential surface that point stands.

(2.) If unit-length of a line of force cross n equipotential surfaces, the mean force (on a unit-particle) along that line, along the course of that part of it, is equal to n units; for the difference of potential of the two ends of that part of the line of force $= n$; it is also equal to fs, because it represents numerically a certain amount of work done on a unit-particle made to travel from one end to the other of that part of the line of force; but $s = 1$; whence $n = f$, where f is the mean force acting on a unit-particle, along the line of force.

(3.) The force at any point of the field corresponds to the extent to which the lines of force are crowded together; and thence it may be determined by the number $(=f)$ of lines of force which pass through a unit of area of the corresponding equipotential surface, that area being so chosen as to comprise the point in question.

Uniform Field of Force. — If the equipotential surfaces be plane, parallel, and equidistant, the lines of force are equally distributed, parallel to one another, and at right angles to the equipotential surfaces, and the Field is **uniform.** The force per unit of area, in the direction of the lines of force, is the same in all parts of such a field.

Variations in Difference of Potential. — Any movement of a body across the surfaces of equal potential, if these surfaces be not equidistant, alters the relative difference of potential between its two extremities, because a body approaching a repelling or attracting mass meets and cuts more equipotential surfaces than it quits, as may be seen from Fig. 91; and, *vice versâ*, a receding body meets fewer surfaces than it quits. In the former case the movement tends to cause an increase in the difference between the potentials of the extremities of the body moved in the non-uniform field of force; in the latter it tends to diminish it.

An increase in the central attraction or repulsion has the same effect as an approach; a diminution the same as a recession.

Theorem. — If a closed surface be drawn round a system of attracting or repelling masses, the number of lines or unit tubes of force traversing the surface is numerically equal to $k \cdot 4\pi Q$, where Q is the algebraical sum of the whole matter within the closed surface, and when the law of force is that $\mathbf{F} = k \cdot mm_{,}/\text{distance}^2$.

Take first a single particle q at the centre of a spherical surface. The force per unit of area is $f = k \cdot q/r^2$; the whole surface is $4\pi r^2$; the force over the whole surface, *i.e.* the number of lines of force crossing the surface, is $F = k \cdot q/r^2 \times 4\pi r^2 = k \cdot 4\pi q$.

Take next any particle q at any point within a closed surface of any form. Any small area δA is taken, which subtends at the particle a solid angle ω, and the normal to which is inclined at an angle ξ to a line drawn from the particle to the centre of δA. The area δA is equal to $d^2\omega/\cos\xi$, where d is its mean distance from the particle; the normal force per unit of area is $k \cdot q \cdot \cos\xi./d^2$; the product $(d^2\omega/\cos\xi) \cdot k \cdot (q\cos\xi./d^2) = k \cdot q\omega$ represents the number of lines of force passing through the element of surface, a number which is seen to be independent of the obliquity or the distance of the element of surface considered. When the surface completely surrounds the particle, the solid angle subtended by the surface is $\omega = 4\pi$, and the force due to this particle and acting through the whole surface is $k \cdot 4\pi q$. So for every particle, wherever situated, within the closed surface, and if the sum of the q's be Q, the number of lines of force traversing the whole surface is $F = k \cdot 4\pi Q$.

If some of the q's be attracting, some repelling, they must be affected with their proper signs, and Q is their algebraic sum.

If the surface be one which is repeatedly indented so as to be repeatedly traversed by lines of force, the exits must be more numerous by one than the entrances; the exits and entrances in any region of the surface subtending the solid angle ω must compensate one another, with the exception of the last exit; this alone contributes to the aggregate number of lines of force finally issuing from the surface. The closed surface may thus be of any degree of complexity, without affecting the numerical value of the whole system of lines, as enunciated by the above theorem.

Potential of a Double Sheet.—Take a very small sheet of **repelling** matter, whose quantity is Q, uniformly distributed over the area δA. A similar and equal sheet, this time of **attracting** matter $-Q$, is brought up parallel to the former, so as to be separated from it only by the very small mutual distance l. Take any point P; then the distance between that point and the middle of the small area δA will be d, and its direction will make some angle ξ with the normal to that area. At P the solid angle subtended by that area will be $\omega = \delta A \cdot \cos \xi./d^2$. Now, the potential due to the charge on the one of these two faces, say at distance d, will be Q/d; that due to the other, at distance $d \pm \delta d$, will be $-Q/(d \pm \delta d)$; the algebraic sum of these is the effective potential at P, and it is $Q \cdot \{1/d - 1/(d \pm \delta d)\}$ $= \pm Q \cdot \delta d/\overline{d \cdot (d \pm \delta d)}$, or, when the mutual distance l is extremely small, it is $\pm Q \cdot \delta d/d^2$. But δd is, under the same conditions, equal to $l \cdot \cos \xi$; hence the potential at P is $(\pm Q/\delta A) \cdot l \cdot (\delta A \cdot \cos \xi./d^2) = (\pm Q/\delta A) \cdot l \cdot \omega = \sigma l \omega$, the product of the Superficial Density σ, or the quantity of matter per sq. cm. of either of the two opposed sheets, into the Distance l between them, into the Solid Angle ω subtended by the double sheet at the point P. On the repelling side of such a double sheet, the potential in the surrounding field of force is positive; on the other side it is negative: and the lines of force and equipotential surfaces in the surrounding field are disposed as indicated in Fig. 234. The direction of each line of force is outward from the repelling or positive, and back (by a more or less ample sweep) to the attracting or negative side of the double sheet.

Isodynamic Surfaces and "Lines of Slope."—If all those points in a field of Force be connected, at which the force is equal, we have a set of **isodynamic** surfaces. These may coincide with equipotential surfaces, as in the case of gravitation; but they may have a totally different lie. For example, the field of electromagnetic force surrounding a long straight wire, bearing a current of electricity, is permeated by equipotential surfaces, plane and radiating from the straight wire; the lines of force, cutting these at right angles, are concentric circles round the wire; the isodynamic surfaces are concentric cylindrical surfaces surrounding the wire. At right angles to the isodynamic surfaces we may imagine lines, the so-called "**Lines of Slope,**" or of Intensity-Slope, which trend in the directions in which the intensity of the force in the field falls away most rapidly. In the case of gravitation these trend in the same directions as the lines of force; in the case of the straight current they radiate from the wire at right angles to it, and are therefore at right angles to the lines of force.

In a uniform field of force there are no such lines or surfaces, for the whole region is isodynamic.

$\frac{Fd^2}{M}$

CHAPTER VIII.

GRAVITATION AND THE PENDULUM.

Law of Gravitation. — Every particle of matter in the Universe is attracted directly towards every other particle with a force varying directly as the mass of each particle, and inversely as the square of the distance between them.

We have already seen that the Weight of a body is a synonym for the Force with which it is attracted by the earth. The law just enunciated indicates that the weight of a double mass is twice that of a single mass, and so on. This seems a truism; but it is an experimental result, not a truism, that the weight of a mass of lead is equal to that of an equal mass of wood. This might have been otherwise. The mass of a given piece of wood is known to be equal to that of a certain piece of lead by the experimental fact that equal forces acting on each for equal times produce equal velocities: $F = ma$; these velocities being those of short horizontal trajectories, which are independent of gravitation. Now a piece of iron and a piece of cork, whose masses are thus found to be equal, will, if placed in the neighbourhood of a magnet, be found to be by no means affected by equal accelerations towards the magnet; yet they are both equally attracted by the earth, have both the same weight in the balance, and, if caused to fall through a vacuum (the friction of the air being thus removed), are found to fall with concurrently equal velocities.

It is remarked that horizontal trajectories are independent of gravitation. A cannon ball at the moon, if it had weighed 60 lbs. on the earth, would weigh less than 10 lbs. there: and so, as has been said, it might be dropped on the toes of the observer there without serious consequences; but it would be found exactly as difficult there as here to heave the ball horizontally, for it would still contain 60 pounds mass.

Problem. — What would a pound-mass weigh, half-way between the Earth's surface and centre? *Ans.* — $\frac{1}{2}$ lb. See prop. 3, p. 188.

Again, a heavy and a light mass of any substance fall at the same rate through a vacuum. It was long believed that the heaviest bodies fall fastest; but Galileo experimentally disproved this. The attraction of the earth for a large mass is greater than for a small one, but the mass to be moved increases in the same proportion as the attraction; and thus the acceleration produced is the same in all cases, and is independent of the amount as well as of the substance of the falling mass.

Cavendish's Experiment. — This was a direct measurement of the attraction of masses for one another. Small balls of lead were poised on a rod and their position carefully noted: large balls of lead were carefully brought near them: the light balls were attracted by the heavy masses, and their displacement measured. Great experimental precautions were necessary, such as the observation of the position of the balls with a telescope placed at a distance, the avoidance of draughts of air and of vibrations, etc.; the result showed that if lead balls had been employed as large as the earth, the attraction of such balls would have been greater than the actual attraction of the earth in the ratio of 11·35 to 5·67: but lead is 11·35 times as heavy as water; hence the earth as a whole is 5·67 times as heavy as an equal bulk of water, or the **density of the earth** is 5·67.

If two masses be respectively m and $m_{,}$, and their distance d, the gravitation-attraction between these masses $\propto mm_{,}/d^2$, or $G = \gamma \cdot mm_{,}/d^2$, where γ is a constant, the Gravitation-Constant. The earth being approximately spherical attracts falling bodies of mass $m_{,}$ as if its own mass, m, were gathered together at the centre, about 637,000,000 cm. from the surface. Its mass m is 6140,000000,000000,000000,000000 grammes. A mass $m_{,} = 1$ gramme is attracted by the earth with a force equal to 981 dynes: hence, in this case, $G = 981$ dynes $= \gamma \cdot mm_{,}/d^2 = \gamma \cdot 614 \cdot 10^{25} \times 1 \div (637{,}000)^2$: and $\gamma = 981 \times (637{,}000)^2 \div (614 \cdot 10^{25}) = \frac{1}{15430000}$. Hence $G = \frac{1}{15430000} mm_{,}/d^2$: and the gravitation-attraction between two gramme-masses whose centres are 1 cm. apart is $\frac{1}{15430000}$ dyne.

The astronomical unit of mass is 15,430000 grammes, and the corresponding unit of force 15,430000 dynes. With such units the constant $\gamma = 1$, and $G = mm_{,}/d^2$, simply.

Prof. C. Vernon Boys, by means of Cavendish's experiment conducted according to methods described in *Nature*, Aug. 2, 9, and 23, 1894, finds $\gamma = 15{,}020400$, and the density of the Earth = 5·5270.

Accelerated Motion under Gravity. — A body free to fall *in vacuo* would be subject to constant downward acceleration of about 981 cm.-per-sec. or 32·2 ft.-per-sec. — that is, of $\mathbf{a} = g$ units of velocity — per second, and its movement would be described by the four formulæ of page 152; the + sign being used when

the attraction of gravity acts in the same sense as the original velocity v_0; the $-$ sign when it acts in an opposite sense.

When bodies fall through the air there is **friction** between the air and the falling body. This is found to vary as the radius of the sphere if the falling body be an exceedingly small sphere; and generally it increases with, but is not proportionate to, the surface exposed. Thus a feather, which presents much surface, falls more slowly than a similar feather rolled into a ball.

In a cloud, the minute droplets fall extremely slowly, for the weight of each is proportional to the cube of the radius, while the resistance is proportional to the radius itself; and thus, as the size of the droplets diminishes, their Weight diminishes more rapidly than the Resistance.

Path of a Projectile. — We have already seen that combination of a uniform rectilinear movement with a uniformly accelerated movement, not in the same direction, results in movement in a parabolic path. This is the theoretical course of a bullet flying *in vacuo;* but the actual course of a shell or bullet in the air differs widely from this on account of friction, its path being at first somewhat straight and ending with a somewhat sudden fall.

If a shot were fired horizontally *in vacuo* at such a rate v (about 26,077 feet per second), that $\mathbf{a} = g$, the acceleration earthwards ($= 32{\cdot}2$ ft.-per-sec. per second) would be v^2/r, r being the distance of the earth's centre from the bullet's path, the shot would never fall to the ground, but would travel round the earth at the level of the gun's mouth.

The most general case of continuous motion of a body round a point, under the influence of an attraction towards that point, varying inversely as the square of the distance, is motion in an Ellipse. When this occurs, the following three propositions hold good: — (1) The point towards which the acceleration is directed is at one of the two "foci" of the ellipse; (2) a line ("radius-vector") drawn from that point to the moving body sweeps over equal areas in equal times as the body moves; and (3) the time taken to perform a complete revolution in the elliptical path is proportional to the square root of the cube of the mean distance from the central point. These propositions had been empirically established by Kepler as a statement of the actual relative movements of the planets with reference to the Sun; and they are known, in that connection, as **Kepler's Laws**. Sir Isaac Newton showed that they are all a consequence of the law of inverse squares, and that the same law of Gravitation is accordingly adhered to throughout the Solar System.

Some Comets move in ellipses, these being sometimes so long that thousands of years must be taken in completing one revolution; others sweep round the sun, are deflected by its attraction into a hyperbolic path, and again disappear into the depths of space.

Universal Gravitation. — The fact of terrestrial gravitation and many of its laws were well known before Newton's time;

he stated the law of gravitation as a universal one: "Gravitatem in corpora universa fieri," etc.—*Principia*, Bk. III. Prop. vii. and Corol. 2.

The moon makes a revolution round the earth* in about 2,360,000 seconds, in an orbit whose mean radius is 59·964 times the earth's equatorial radius. The formula $\mathbf{a} = v^2/r$ shows that this corresponds to an actual fall of the moon towards the earth of $\frac{1}{2}\mathbf{a} = 0{\cdot}136$ cm. per second: this, compounded with the tangential velocity at every instant, keeps the moon in its orbit. This acceleration, due to the attraction of the earth, is $1/(59{\cdot}964)^2$ of 981; thus the moon is under the influence of terrestrial attraction which obeys the law that $F \propto d^{-2}$. Newton made similar deductions from other astronomical phenomena, particularly those of the satellites of Jupiter, and ultimately asserted the universality of the law of gravitation.

The average attraction of the sun for a gramme-mass of the earth's substance is $G = \{\gamma \times (\text{Sun's mass} \times 1 \text{ gramme}) \div (\text{Sun's distance})^2\} = \{g \times (\text{Sun's mass} \div \text{Earth's mass}) \div (\text{Sun's distance in earth-radii})^2\} = \{g \times 316000 \div 23200^2\} = 0{\cdot}0005871g$.

At the equator, at an equinox, this would, if the earth and the sun were kept at a fixed distance apart, cause a loose unit-mass on the earth's surface to have an apparent weight of $0{\cdot}9994129g$ at midday, and one of $1{\cdot}0005871g$ at midnight; a variation, during the 24 hours, of $0{\cdot}0011742g$, or 3·63 lbs. per ton: while the rising or setting sun would displace a plumb-line or mercury-mirror to an extent reaching a maximum of $0° \, 2' \, 1''$ east or west; and the pendulum would oscillate at correspondingly varying rates. No such effects are observed. The earth and a stone lying loose on its surface both fall freely towards the sun: if the stone is the nearer, the Earth follows it closely, and there is apparently no tendency to separation between the two bodies; but yet there is a certain small difference. The stone, if it face the sun, is at 23199 earth-radii from its centre; the earth is at 23200 radii: the solar gravitation on the stone is $m_i g \times 316000 \div 23199^2$: the difference between this and the average solar gravitation on the earth is $0{\cdot}000000{,}0515g$ per gramme. In the case of the moon, though the lunar gravitation is much smaller than that of the sun, this difference is greater, being $0{\cdot}000000{,}13g$. To these differences, small though they be, the **Tides** are due: the water nearer the sun, or moon, is more attracted than the bulk of the earth, and is heaped up: but the bulk of the earth is more attracted than the more remote water, and that water is left in a heap at the farther side of the earth. The tides are highest when the sun and moon co-operate in their effects; lowest when the sun and moon lie at right angles to each other. The actual tides are modified by friction, so as to be always belated. If the sun and earth (or the moon and earth) had been fixed in their relative position, there would have been but one heap of

* This is only a rough statement of the fact. The moon, as it runs in its "orbit round the earth," never touching it, describes a sinuous path in space along the course of the earth's orbit. It happens that Fig. 49 may serve to give an approximate notion of this sinuous path. The earth is also equally attracted by the moon, and moves towards it, in its varying positions, with smaller accelerations corresponding to its own greater mass (1:0·0114); its path in its orbit is therefore also affected with small sinuosities, of opposite phase to those of the moon's motion; but its mean distance from the moon, and the moon's mean acceleration towards the earth, remain on the whole unaltered; and the result is as before.

water, on the side nearer the sun (or moon), towards which the water would flow; but the free fall of the earth towards the sun (or moon) prevents this: and then this free fall, compounded with the tangential velocity, results in the orbital motion.

Gravity acts **instantaneously**. Laplace showed that if it were propagated through space with a speed less than 9300,000,000000 miles a second, which is itself a minimum limit, there would be a rapid shortening of the year. It must travel, therefore, at least 50,000000 times as fast as Light does. There is also, so far as appears, no loss by diffusion in Gravitation.

Variations of the acceleration of gravity on the earth's surface. — At the equator the earthward acceleration of gravity is, at the sea-level, 978·1028 kines per second: at the pole it will be 983·1084, if the law of variation in accessible regions of the earth's surface be obeyed there. This law is, that at any place whose latitude is λ, the local acceleration of gravity is, in kines, $g = (980{\cdot}6056 - 2{\cdot}5028 \cos 2\lambda - {\cdot}000003h)$, where h is, in cm., the height of the observing station above the sea-level. This diminution of gravity — equal masses weighing less, and therefore distorting spring-balances less, in regions nearer the equator — is due to two concurrent causes: (1) That the mean **equatorial radius is greater** than the polar; the polar radius is 635,639,000 cm.; the longest equatorial radius (from lat. 14° 23′ E. to lat. 165° 37′ W.) is 637,839,000 cm.; the shortest equatorial radius, at right angles to the former, is 637,792,000 cm.* (2) The **rotation of the earth**. If the earth came to rest, the earthward acceleration of gravity would, at the equator, be increased by $g/289$ or 3·3908 kines per second, and the weight of bodies would be increased in the ratio of 289 to 290. If the earth rotated 17 $(= \sqrt{289})$ times as fast as it does, loose objects would, at the equator, have no weight; and if it rotated faster, they would fly off its surface at a tangent.

The acceleration due to gravity is in Paris 980·94, at Greenwich 981·17, at Manchester 981·30, at Edinburgh 981·54 kines per second.

The velocity of rotation at the equator is 456,510 cm. per second; whence $v^2/r = 3{\cdot}3908$: at Greenwich it is 2·100.

Local Variations. — In the neighbourhood of a high range of mountains a plumb-line inclines towards the mountains. The ebb and flow of water in the Firth of Forth affects the apparent latitude of Edinburgh by about $\frac{1}{1400}$ degree, for when the water is at high tide, plumb-lines are inclined towards it, and the mercury used as a means of producing perfectly level mirrors is, in the vessels containing it, heaped up towards the mass of sea-water. The water of oceans is slightly heaped up towards the continents.

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* Col. A R. Clarke's values are 635,638,756 cm., 637,837,929 cm., and 637,791,478; the longest equatorial axis being from 8° 15′ W. to 171° 45′ E. of Greenwich (*Phil. Maga.*, 1878).

At sea the effect of gravity is less than it is on land, because the mass of water under the spring-balance is lighter than a corresponding amount of rock would have been. The depth of the sea may be determined by a graduated instrument of the nature of a spring-balance, sufficiently sensitive to take account of these variations.

Measurement of the Local Force of Gravity. — The force of gravity must, like all other forces, be measured by its acceleration. This may be done directly by **Attwood's machine,** already described. Observation of a single fall cannot, however, give accurate results, and the value of g is best determined by the oscillations of a **pendulum.**

There are at the basis of this determination four main facts: (1) that a pendulum of a given length will oscillate through small arcs in equal times, of whatever substance it be made — this last experimental result being due to Newton; (2) that the relation between l the length of a simple pendulum, T its time of complete to-and-fro oscillation, and g the local acceleration of gravitation, is given by the formula $g = 4\pi^2 l/T^2$ presently to be proved (p. 212); (3) that the length l of a simple pendulum may be very accurately observed, for in practice it is equal (p. 213) to the distance between two points on a solid rod, called a compound pendulum; and (4) that the time of one oscillation may be very accurately observed by counting the number of oscillations in a sufficiently long period of time. Hence g can be found to any nicety.

Centre of Gravity. — The earth is approximately spherical, and bodies on the surface have all their particles drawn approximately towards its centre. But the centre is so distant that, within the limits of ordinary terrestrial objects, the gravitation forces acting on the several particles of a body are nearly parallel to one another, and their resultant acts on the Centre of Gravity, or Centre of Mass. This centre is the Centre of Figure of a uniform body attracted by the earth.

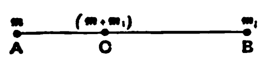
The centre of gravity of any plane figure may be found by cutting that figure out in cardboard, and suspending the card first from any one point and then from any other. A line drawn vertically downwards from the first point of suspension when the body is suspended from it, and another line drawn in the same way from the second point of suspension, will cross one another at the centre of gravity.

Whatever be the form or the arrangement of matter in a body, if it be suspended from any point arbitrarily chosen, the

centre of gravity is in a line vertically drawn through the point of suspension — vertically here meaning at right angles to the free horizontal surface of liquid at that place. If the centre of gravity be found by two suspensions, the vertical lines drawn from any other points of suspension will all pass through the same centre.

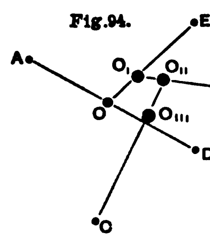
Centre of Gravity of Two Masses. — In Fig. 93 the two bodies, A and B, whose masses are m and $m_{,}$, will have their centre of gravity at a point C, which is determined by the equation $m \times AC = m_{,} \times BC$. The whole mass $m + m_{,}$ may, as regards other bodies, be considered as if it were aggregated at the point C.

Fig. 93.



Centre of Gravity of a System of Masses. — This is found by taking account of each *seriatim*. In Fig. 94 let the bodies be A, B, C, D, E, whose respective masses are $m_{,}$, $m_{,,}$, $m_{,,,}$, $m_{,,,,}$, and m_v. First the centre of gravity of any two, say A and D, is found at O; A and D are supposed to be replaced by a mass $m_{,} + m_{,,,,}$ at O. Next the centre of gravity between another of the masses, say E, and the imaginary mass $m_{,} + m_{,,,,}$ at O is found to be at $O_{,}$; at this point $O_{,}$, there is supposed to be placed a body whose mass is $m_{,} + m_{,,,,} + m_v$. In the same way, the centre between this imaginary mass and another, say $m_{,,}$ at B, may be found at $O_{,,}$. Finally the centre between this and the mass $m_{,,,}$ is found at $O_{,,,}$, and this is the last operation, for, as regards external masses, the system ABCDE acts as if it were a mass $(m_{,} + m_{,,} + m_{,,,} + m_{,,,,} + m_v)$ concentrated at the point $O_{,,,}$; this point is therefore the centre of gravity of the system. The same point will be found whatever the order in which the masses are considered.

Fig. 94.



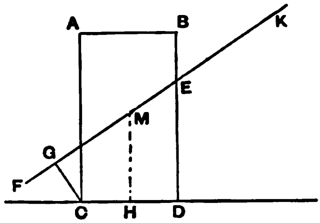
Since the resultant of parallel external forces acting upon the several particles of a body acts upon the centre of gravity, that centre moves as if it were a single particle under the action of a single translatory force. But there may, in addition, be rotations round the centre of gravity, and rotatory oscillations which can not displace that centre; and these are independent of the former.

Thus a ball of unsymmetrical density, thrown through the air, will swerve and gyrate in a puzzling way; but its centre of gravity describes a smooth trajectory. The moon and the earth have sinuous paths in space; but their common centre of mass, which lies between their respective centres (see Fig. 93, C being about 2700 miles from the earth's centre, and therefore within the earth's mass), describes a smooth path.

Overturning a body. — Let ABCD be a block of material supported on a base CD. How great a force applied at E, in the direction EF, is necessary to overturn the block? The

question is really one of moments round C, for if the force along EF prevail over the weight of the block, it will do so by turning it over the point C. From that point C, CG is the shortest line drawn to meet the line EF, and CH is the shortest distance to the line MH, along which the force of gravity may be considered to act. At the instant when overturning is just going to commence, the moments round C must be equal, and CG × force along EF = CH × wt. of body. Therefore, if the force along EF be greater than weight of body × CH/CG, the body will be overturned. The greater CH is, the greater must be the force exerted along EF in order to overturn the body; the smaller CH is, the less need that force be. When CH = 0 — *i.e.* when the centre of gravity M is vertically above C — any force, however small, will upset the block ABCD; while, if H be on the other side of C, the block cannot stand unless propped up. In this way a body resting on a wide base is less easily upset than one standing on a narrow; one in which a vertical line drawn from the centre of gravity falls outside the base of support cannot stand unsupported; while one in which the centre of gravity stands over the very edge of the base of support is upset by the least disturbance.

Fig. 95.



A microscope, then, ought for the sake of steadiness to have a wide base; and since a tripod stand is the most steady form of support, for reasons already stated (see Spherometer), instruments of this class should be supported on broad tripod stands. An old man using a staff widens his basis of support by virtually converting his two legs and the staff into a broad tripod stand.

It amounts to the same thing whether the base of an object be relatively broad or its centre of gravity be relatively low. If the centre of gravity be relatively high or the base relatively narrow, — as in the case of young animals learning to walk, children learning to walk, persons learning to move on skates, or on stilts, or on a narrow rail, or rope, or wire, or a bicycle, or a person standing on one foot or on his heels, — a relatively small displacement of the object will readily cause the centre of gravity to be placed vertically over a point beyond the base of support; then the object, if it be not propped up, or if the centre of gravity be not brought over the base of sup-

port, or the base of support not brought up under the centre of gravity, will topple over.

If, on the other hand, the base be relatively wide, or the centre of gravity be relatively low, as in the case of a lampstand loaded at its base, the task of upsetting such an object is greater, since the centre of gravity is in such a case less easily induced to pass to a position vertically beyond the base.

Curious positions may be assumed by objects when they are so balanced that the centre of gravity is vertically over some point in the basis of support. A man's centre of gravity is at a point about the front of his last lumbar vertebra. If he carry a burden, then, in order to bring the centre of gravity of the conjoined mass of his body and the burden borne by him into a position vertically over some part of the narrow basis of support furnished by his feet (heels, and balls of great toes, and lines joining these), he must stoop; if the burden be towards the front of the body, as in the case of obese persons, the gait becomes very erect.

When a body is suspended from any point in its own substance and set a-swinging, its centre of gravity ultimately finds its way into the lowest position possible.

If a disc roll round a curve, its centre of mass will have an outward horizontal Acceleration v^2/r, and the disc will be overturned unless it can be made to incline towards the centre of curvature so that the Weight of the disc, acting vertically through the centre of mass, will also have an overturning moment round the point of support, equal and opposite to that of the horizontal Force mv^2/r. This takes place at an angle ξ of inclination, such that $\tan \xi = v^2/rg$. On this principle, skaters, circus-riders, etc., rounding a curve, incline inwards: and the permanent way of a railway is adjusted by superelevating the outer rail of a curve, so as to incline the train at the angle ξ.

If the superelevation of the outer rail be e, and the width between the rails be w, $e/w = \tan \xi$; and $e = w \cdot \tan \xi = wv^2/rg$. If e be measured in inches, w and r in feet, and v in miles per hour, this becomes $E = 0 \cdot 173\ WV^2/R$. On English railways, the practice is to make $E = 0 \cdot 8\ WV^2/R$, and thus to allow for a speed of a little more than twice the expected maximum.

Work done in overturning a body.—If there be rotation round the point C of Fig. 95, so far that M, the centre of gravity of the body, comes to be immediately over that point and overturning is effected, the centre of gravity is raised through a certain height h. The weight of the body, mg, $\times$ that height, h, is the work which must be done before overturning can be accomplished.

Angle of Overturning.—If the force applied at E in Fig. 95 be applied in a direction too nearly parallel to BD, its moment may be too

small (its arm CG being too short) to produce overturning. At a certain definite angle, BEK, there will be equilibrium, the arm CG being of exactly such length as will make the moment of EF equal to that of the weight of the body. If, then, this angle BEK be less than χ, the angle of repose, the body will overturn before sliding; if BEK be greater than χ, the body will slide before overturning.

Equilibrium, Stable, Unstable, and Neutral. — If the centre of gravity be so situated in a body that work has to be done in disturbing it — that is to say, if the centre of gravity be already at its lowest possible position — the equilibrium is stable. If a ball lie in a bowl, work must be done in order to effect any displacement of it, for no displacement can be effected without raising the centre of gravity of the ball, and thus imparting potential energy to it. When the ball is let go, it rolls back and oscillates in the bowl until it comes to rest. The same thing is seen in a swing, a cradle, a rocking-horse, a pendulum, or a ship well ballasted, which are all in Stable Equilibrium; in the last case the oscillations somewhat resemble those of a pendulum whose point of suspension and whose length both vary.

If a body have its centre of gravity placed above its point of support, so that any displacement lowers its centre of gravity, then the body already has potential energy, which it is disposed to convert into kinetic by the fall of its centre of gravity to the lowest possible point. Hence in bodies thus in Unstable Equilibrium, a very slight disturbance may cause a very great displacement, disproportionate to the disturbance, but depending on the potential energy stored in the system. In this case are boats in which people stand, high chairs in which children are seated, cars which are heavily loaded atop, deck-loaded ships, and, in short, everything which is "topheavy."

Where no work is done upon or by an object, so far as the attracting forces are concerned, when it is displaced, the Equilibrium is Neutral. A uniform sphere may be displaced and assume a new position without either raising or depressing its centre of gravity.

A sphere floating in water is in neutral equilibrium; a plank floating in the usual way is in stable, while a plank floating with its edges vertical is in unstable equilibrium.

Simple Pendulum. — This is an ideal. It is a heavy particle suspended by a weightless cord. An approximation to a simple pendulum is obtained by suspending a small bullet by a very thin wire. The length of this pendulum is the dis-

tance between the point of suspension and the centre of the bullet.

If in Fig. 96 a simple pendulum of length $l = AC$ be represented as displaced from the vertical position through the angle θ, m being the mass of the bob, and mg consequently its Weight, when the bob is at C the force of gravity may be resolved into two components: one $= mg \sin\theta$ in the direction of the tangent at C, and tending to bring the bob towards the middle line with acceleration $= g\sin\theta$; and a radial component $= mg\cos\theta$, which renders the cord tense. The displacement of the bob, the distance between B and C measured along the arc, is equal to $l\theta$. Now, so long as θ is small, some 2° or 3° at most, $\sin\theta$ and the angle θ are nearly equal, and the tangential acceleration, which is equal to $g\sin\theta$, bears to the displacement $l\theta$ an almost constant ratio, for $g\sin\theta./l\theta =$ (approximately) g/l. But we know that when a body after displacement is subject to a force tending to bring it back, which produces an acceleration proportional to the displacement, the result is a S.H.M.; and thus a pendulum very slightly displaced oscillates in S.H.M.

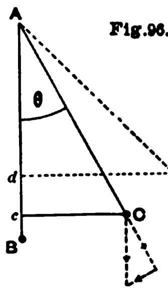
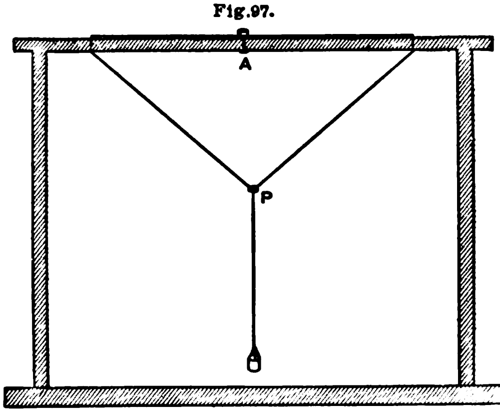


Fig. 96.

Simple Harmonic Motions experimentally performed.—A simple pendulum approximately describes a S.H.M. A pendulum whose bob consists of a flask containing coloured fluid or ink, which pours from a narrow orifice as the flask oscillates, will record the path of the pendulum. A quantity of sand may be used instead of ink. The apparatus shown in Fig. 97 (Blackburn's pendulum) may be used for the description of the compounded H.M.'s exemplified in Figs. 35–40. If the bob, whose path is recorded by sand, be displaced in a line making an angle of, say, 45° with the line of the cross-bar, there will be two simultaneous and independent oscillations

Fig. 97.


set up when it is liberated: one from the cross-bar and at right angles to it; one from the point P and at right angles to the other oscillation. By adjusting the relative lengths of the effective long and short pendulum, by means of a peg A, round which a certain quantity of cord is wound, or by shifting a ring at P, an indefinite variety of such figures may be produced.

The **Time** taken by a single pendulum to effect one complete oscillation — one "*swing-swang*" — depends on the square root of its length l and varies inversely as the square root of g, the local acceleration of gravity. It is equal to $2\pi\sqrt{l/g}$. Thus a clock will go slower at the equator than in polar regions; a clock will go slower when its pendulum has been lengthened by heat: a clock with a ten-inch pendulum will tick twice as often as one with a forty-inch pendulum.

In S.H.M. the angular velocity ω in the circle of reference is equal to $\sqrt{\dfrac{\text{acceleration at any point of the S.H.M.}}{\text{displacement at that point}}}$.

That is, $\omega = \sqrt{g \sin\theta./l\theta} = \sqrt{g/l}$.

But ω = (the angular path traversed in time t ÷ the time t); and if the time be so chosen that in it the body describing the S.H.M. would perform exactly one revolution in the circle of reference — that is, if $t = T$, the period of one complete oscillation back and fore, or *swing-swang* of the pendulum, $\omega = 2\pi/T$; whence $T = 2\pi\sqrt{l/g}$, and $g = 4\pi^2 l/T^2$.

The value of T may be otherwise written. The moment of inertia $N = mr^2 = ml^2$; the weight $G = mg$; whence $T = 2\pi\sqrt{l/g} = 2\pi\sqrt{N/Gl}$.

We see, from the equation $g = (4\pi^2/T^2)l$, that of those pendulums which oscillate at equal rates in different places, the lengths are proportional to the local intensities of gravity.

The leg acts partly as a pendulum, and in natural locomotion a person with short legs has a tendency to take shorter and quicker steps than a person with longer limbs.

Isochronous Oscillations of a Simple Pendulum. — The equation $T = 2\pi\sqrt{l/g}$ shows that so long as $(g \cdot \sin\theta./l\theta)$ may be considered to be equal to g/l, — that is, so long as the angle θ is not too wide, — the period of oscillation does not depend on the amplitude of oscillation, for θ does not enter into that equation; and thus within certain limits a pendulum swings in equal periods through comparatively large or comparatively small arcs.

Length of the Ideal Simple Seconds-Pendulum. — The seconds pendulum performs one "complete oscillation" in two seconds. The equation $g = 4\pi^2 l/T^2$, when $T = 2$ sec., gives $l = g/\pi^2 = 3{\cdot}261608\dot{3}$ feet or $39{\cdot}1393$ inches at the latitude of

London, at sea-level, and when the barometer and thermometer are at standard heights (30 inches of mercury, 60° F.).

Work done in moving a Simple Pendulum. — In Fig. 96 suppose the bob to be displaced from C to D; its angular displacement θ is increased to $\theta_{,}$. When at C its vertical height above B is Bc — that is, AB − Ac, or $\mathfrak{l} - \mathfrak{l}\cos\theta$. When at D its vertical height above B is $\mathfrak{l} - \mathfrak{l}\cos\theta_{,}$; and its vertical height above C is $cd = Ac - Ad = \mathfrak{l}\cos\theta - \mathfrak{l}\cos\theta_{,}$. When the bob, whose weight is mg, is raised from C to D, the work done against gravity is $mg \times \mathfrak{l}\,(\cos\theta - \cos\theta_{,})$.

Compound Pendulum. — Let a body of any shape be poised on a point or axis of suspension, as from the point A, or an axis passing through the point A in Fig. 83d or Fig. 98; let the radius of inertia of the body with respect to this point A be ι; and h the distance of the centre of gravity B below A. If two positions be taken, as in Fig. 98, respectively corresponding to displacements through angles θ and $\theta_{,}$, the vertical distances between the centre of gravity and the point of suspension are respectively $h\cos\theta$ and $h\cos\theta_{,}$. The work done by the body during a movement from the higher position to the lower is $mg \times (h\cos\theta - h\cos\theta_{,}) = mgh(\cos\theta - \cos\theta_{,}) =$ (by page 164) $\frac{1}{2}N\omega^2 = \frac{1}{2}m\iota^2\omega^2$.

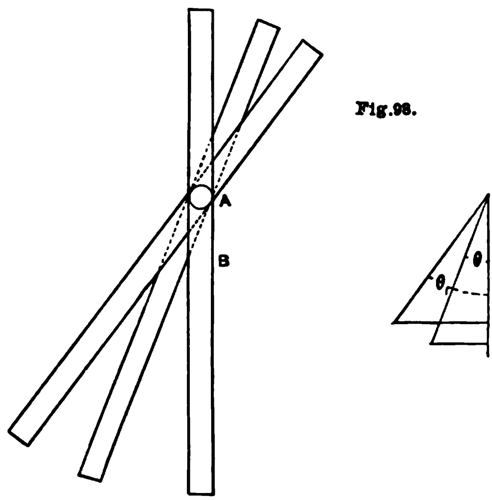


Fig. 98.

$$\therefore\ \omega^2 = 2gh\,(\cos\theta - \cos\theta_{,})/\iota^2. \qquad (1.)$$

The work done by the bob of a simple pendulum during a similar displacement is $mg\mathfrak{l}\,(\cos\theta - \cos\theta_{,})$. In this fall it acquires angular velocity $\omega_{,}$, and kinetic energy $= \frac{1}{2}N\omega_{,}^2 = \frac{1}{2}m\mathfrak{l}^2\cdot\omega_{,}^2$. The kinetic energy acquired is equal to the potential energy lost, whence —

$$\tfrac{1}{2}m\mathfrak{l}^2\cdot\omega_{,}^2 = mg\mathfrak{l}\,(\cos\theta - \cos\theta_{,}).$$

$$\omega_{,}^2 = 2g\,(\cos\theta - \cos\theta_{,})/\mathfrak{l}. \qquad (2.)$$

If the simple pendulum were of such a length as to oscillate at the same rate as the compound one — that is, if ω, the angular velocity of the one $= \omega_{,}$, that of the other — we find from (1) and (2) that

$$2gh\,(\cos\theta - \cos\theta_{,})/\iota^2 = 2g\,(\cos\theta - \cos\theta_{,})/\mathfrak{l}.$$

$$\therefore\ \mathfrak{l} = \iota^2/h.$$

Hence the compound pendulum oscillates at the same rate as a theoretical simple pendulum of length $\mathfrak{l} = \iota^2/h$; that is, its period is $T = 2\pi\sqrt{\iota^2/hg}$.

This length, ι^2/h, the length $\mathfrak{l}$ of the equivalent simple pendulum, is the distance between the centre of suspension and the centre of oscillation. Let $m\iota_0^2$ be the moment of inertia of the compound pendulum round its centre

of mass. Then round the centre of suspension the moment of inertia is $m(\iota_0^2 + h^2)$, and the length $\mathfrak{l}$ of the equivalent simple pendulum is $\iota^2 / h = (\iota_0^2 + h^2) \div h = (\iota_0^2/h) + h$. Similarly, if the body be suspended from the centre of oscillation, the distance between this new point of suspension and the centre of mass being $h_{\prime}$, the length of the equivalent pendulum is now $\mathfrak{l}_{\prime} = \iota_{\prime}^2/h_{\prime} = (\iota_0^2/h_{\prime}) + h_{\prime}$. But (Fig. 83*d*) the centre of suspension A and the centre of oscillation C are interchangeable; whence $\mathfrak{l} = \mathfrak{l}_{\prime}$. Therefore $(\iota_0^2/h) + h = (\iota_0^2/h_{\prime}) + h_{\prime}$; whence $\iota_0^2 = hh_{\prime}$, and $\mathfrak{l} = \mathfrak{l}_{\prime} = h + h_{\prime} = AB + BC$ (Fig. 83*d*) $= AC$; that is, the distance AC between the interchangeable centres of suspension and of oscillation is equal to the length $\mathfrak{l}$ of the equivalent simple pendulum. Therefore —

If a body of any form be suspended at a certain point and be found to oscillate at a certain rate; if, after trial, another point in the body be found, situated on the other side of the centre of gravity, and such that the body will, when suspended from it, oscillate at the same rate, — then the distance between these interchangeable points of suspension is the true length of the ideal simple pendulum oscillating at the observed rate; it being observed, however, that in the case of a symmetrical bar, these two points must not be at equal distances from the centre of gravity, for in that case any two such equidistant points are interchangeable.

Problem. — Prove that a cylindrical rod will swing at the same rate, whether it be suspended from its extremity or from a point one-third of the length from the extremity. — *Ans.* $\mathfrak{l}_{\prime} = \iota_{\prime}^2/h_{\prime} = l^2/3 \div l/2$ (see p. 162, No. 1) $= \frac{2}{3}l$: $\mathfrak{l}_{\prime\prime} = \iota_{\prime\prime}^2/h_{\prime\prime}$; $\iota_{\prime\prime}^2 = \iota_0^2 + h_{\prime\prime}^2$; $\iota_0^2 = l^2/12$ (see p. 162, No. 2); $h_{\prime\prime} = l/6$; $\therefore \mathfrak{l}_{\prime\prime} = \iota_{\prime\prime}^2/h_{\prime\prime} = \{(l^2/12 + l^2/36) \div l/6\} = \frac{2}{3}l = \mathfrak{l}_{\prime}$.

Ballistic Pendulum. — Suppose a heavy mass M to be suspended from a point. Into this heavy mass let a bullet of mass m be horizontally fired, striking the centre of percussion with velocity v, and let the bullet sink into it so as to form a conjoint mass $M + m$, whose centre of mass is at a distance h below the point of suspension. The energy of the striking bullet is $mv^2/2$; this energy is wholly imparted to the conjoint mass before that mass has had time to become appreciably displaced. In virtue of this energy imparted the whole is displaced so far that the suspending cord comes to make an angle θ with its original position, work being thus done against gravity, equal to $(M + m)gh(1 - \cos\theta)$ or $(M + m)gh \cdot 2\sin^2(\theta/2)$; the whole then falls back and thereafter oscillates in ordinary pendulum fashion. The energy imparted and the work done against gravity are equal; whence

$$mv^2/2 = (M + m)gh \cdot 2\sin^2(\theta/2),$$

or

$$\sin(\theta/2) = \sqrt{\frac{m}{M + m}} \cdot \sqrt{\frac{1}{gh}} \cdot \tfrac{1}{2} \cdot v,$$

$$i.e. \sin(\theta/2) \propto v.$$

The throw is such that the sine of half the angle of deflection is proportional to the velocity of the impinging bullet, and therefore to the square root of its energy.

If ω be the angular velocity imparted to the swinging mass, the energy imparted may also be written as $\frac{1}{2}N\omega^2$; whence

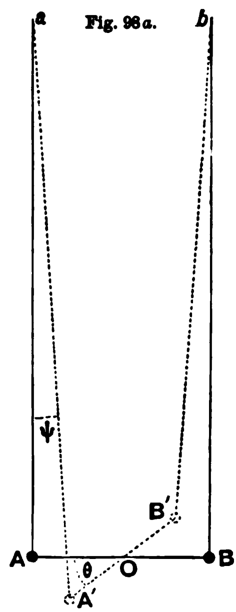
$$N\omega^2 = 2(M + m)gh \cdot 2 \sin^2 \overline{\theta/2}$$

and
$$\omega = 2\sqrt{(M + m)gh/N} \cdot \sin \overline{\theta/2}.$$

Stability of a Ballistic Pendulum. — The amounts of energy which must be imparted in order to jerk such a pendulum through 15°, 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 150°, 165°, or 180°, are the products of $(M + m)gh$ into 0·0341, 0·134, 0·2929, 0·5, 0·7412, 1·0000, 1·2588, 1·5, 1·7071, 1·866, 1·9659, or 2·0000 respectively. From these figures we see that the Stability of a pendulum in any position — the amount of energy which must be imparted to it in order to throw it *farther* through one degree — is greatest when the throw is already about 90°. Thereafter it diminishes; and when the energy imparted exceeds $2(M + m)gh$, or when the velocity of the impinging bullet exceeds a certain limit ($v = 2\sqrt{(M+m)gh/m}$), the pendulum is thrown right over, and describes a somersault. Problems of this nature are of great importance in connection with the stability of ships.

Bifilar Suspension. — If two masses, A, B, at the extremities of a weightless rod AB, be suspended by the parallel cords aA and bB, and if these be displaced so that round O, the central part of AB, there is rotation through an angle θ, there is necessarily a lifting up of both A and B, and there will be components of gravitation tending to stretch each suspending string, and components tending to restore A and B to their original positions. We need only consider the cord aA. Its lower extremity is swung horizontally through an angle θ with respect to O; the whole cord is deflected from its vertical position through an angle ψ such that $a\text{A} \cdot \tan \psi =$ the chord of the arc of $\theta = \text{OA} \cdot 2 \sin \overline{\theta/2}$. The component of gravitation tending to restore A′ to A, acting towards A, is equal to $mg \tan \psi$. Its moment round O is $(mg \tan \psi) \cdot (\text{OA} \cdot \cos \overline{\theta/2})$. The whole moment of the couple is $2mg \tan \psi \cdot \text{OA} \cdot \cos \overline{\theta/2} = 2mg\,(\text{OA}^2/a\text{A})\, 2 \sin \overline{\theta/2} \cdot \cos \overline{\theta/2} = 2mg\,(\text{OA}^2/a\text{A}) \sin \theta$. The moment of the restoring force is thus proportional to the sine of the angle of deflection, and the oscillations of such a system are approximately simple harmonic.

Fig. 98a.



CHAPTER IX.

MATTER.

THE essential nature of matter — its substratum — is unknown to us; we only know matter by those of its properties which we perceive by our senses. These properties are subject to our direct observation and to our study, and from them we may infer as to the constitution of matter much which we cannot directly perceive.

THE PROPERTIES OF MATTER.

Some of these properties of matter are general, so that if they were other than they actually are, the nature of our universe would be totally different; and thus, in relation to the matter of the existent universe, these properties are with sufficient appropriateness said to be **essential.** For instance, all experience leads us to say that matter must necessarily exist in definite or measurable Quantity; and, since quantity of matter is expressed briefly by the word **Mass,** we say that every body must have a definite mass, for it is to us, with our range of ideas, impossible to conceive of a definite body having a physical existence, but consisting of an indefinite quantity of matter. There are many bodies of which we do not definitely know the mass, but every body must have some definite mass, great or small. If the mass of a body be great, the body is said to be **massive;** if its mass be small, it is usually said to be **light,** though that adjective is properly antithetical to *heavy*, a perfectly distinct idea. A massive gate is difficult to move, not because it is heavy, for gravity does not affect the horizontal swing of a gate on its hinges, except indeed by affecting the friction at the hinges; it is difficult to move because its mass m is great; and, since $F = ma$, in order to produce a given acceleration a, if the mass m be large the force applied must be great. It would, in theory, be equally considerable were the

gate and its hinges removed to a region where the effect of gravity vanished, and the gate had therefore not even a feather's Weight.

As regards Quality of Matter, experience shows us that every substance with which we are acquainted is made up of one or other or more or fewer of about seventy different kinds of matter. These kinds of matter are called **elements.** They are considered to be distinct kinds of matter, and are called separate elements, simply as a confession of our relative experimental impotence, and of our complete failure up to this time to break up any one of them into simpler substances, or to build any one up by any synthetic process. A piece of brass may be by analytic processes resolved into its component copper and zinc, and when copper and zinc are fused together in proper proportions, brass of a similar quality may be made; but no one has broken up either zinc or copper into any simpler components, neither have these metals been made by causing any simpler substances to combine. These metals are, then, Elements; and the substances which they form by entering into combination with other elements, as well as the circumstances under which these combinations are effected, form the subject-matter of the Science of Chemistry. The description of any given substance — chlorine, nitrogen, calcium — as an element is thus seen to be entirely provisional. The experience of 1807 may possibly be repeated when least expected. Before that date lime, soda, and potash were enumerated in the list of elements, though, from their strong likeness to metallic oxides, it was vehemently suspected that they were really not elements at all, but oxides. When Sir Humphry Davy brought the galvanic battery of the Royal Institution to bear upon masses of these substances, he resolved them into oxygen and into metals never seen till then; and thus the list of elements suffered a profound modification. Now evidence of a speculative character, on the one hand, based (de Chancourtois, Newlands, and Mendelejeff) upon the remarkable relations existing between the chemical properties of the elements and their atomic weights, and also (Gladstone) upon the relations between these chemical properties and the extent to which such of the elements as are transparent or form transparent compounds possess, either when pure or in combination, the power of refracting a beam of incident light; and evidence, on the other hand, of a directly observational character, based (Lockyer) upon the results of spectrum analysis as

applied to the stellar bodies, results which seem to show that many elements are decomposed by intense heat into simpler elements; this mass of evidence lends cumulative support to a belief, which is rapidly gaining ground, that all the elements differ from one another only in their intimate structure, and have a common basis which may not impossibly be the element helium which exists in the Sun; or in other words, that all the elements are structural modifications of one form of Matter. Thus even the alchemist's dream of the transmutation of metals cannot now be treated with such unmitigated contempt as it received forty or even twenty years ago, though it may continue to be a dream to the realisation of which no approach is possible, on account of the necessary limitations of our experimental appliances.

In reference to Space: every mass of matter must at any instant occupy a definite **volume** of space: it must have some **Extension** in tridimensional space — it must have dimensions expressible in terms of length, breadth, and thickness. As a natural consequence, every mass of matter must have some definite **form,** whether that form be imposed on it by surrounding matter or not — whether, like a solid rock, it have a form of its own, or, like water in a basin, it have a form which depends partly on the form of the vessel containing it, or, like gas confined in a gas-holder, its form as well as its volume depend on that of the vessel in which it is enclosed.

Among the properties of matter which are said to be essential we usually find mentioned that known as **impenetrability.** This means that two masses of matter cannot occupy the same space. In view of the peculiar phenomena attending the solution of substances in water — a very large quantity of different salts being soluble in water without materially increasing its bulk — we cannot state this absolutely. But matter is believed to be composed of minute masses called Molecules, and of these it is held to be true that two cannot coincide in position. But these are not in contact even in solids, and so a body is always free to shrink in size — as, for instance, when it is cooled down or compressed — because the distance between its molecules is capable of diminution; and thus a quantity of water, which is not a continuous substance, may receive between its own molecules a number of molecules of other substances, and so form a solution, without entirely sacrificing that freedom of movement past one another which its own molecules possess

— that is, without entirely losing its fluidity. The **impenetrability** of matter is, then, a property of **molecules, not** necessarily of **masses.**

If a certain bulk of metallic potassium contain 200 atoms or half-molecules of potassium, an equal bulk of caustic potash will contain 331 atoms of potassium, 331 of hydrogen, and 331 of oxygen.

In respect of Time: Lapse of time brings about no change either in the quantity of matter or in its quality — that is, matter is **indestructible** both in regard to its total quantity and to the quantity of each element. Such is the ordinary belief; the former statement is in accord with the universal experience of Chemistry; but he would be bold who, from the experience of mankind on the surface of the earth, should venture to deny that in the interior of this planet there may even now, as the earth is cooling, be an increase taking place in the quantity of the more complex at the expense of the more simple elements; not to speak of the positive probability which spectrum analysis lends to a belief that this kind of action is actually going on in the fixed stars. Be that as it may, within our experimental range of knowledge there is no transmutation of elements and no destruction or creation of matter. Matter changes its forms and its combinations incessantly, but it can always be traced up by chemical analysis. A closed glass tube containing oxygen and powdered charcoal weighs exactly the same after the charcoal has been induced to burn in the oxygen, and thus to disappear, as it did before that action; the gaseous carbonic acid produced is, though invisible, equal in total weight, and therefore in mass, to the sum of the carbon and oxygen which composed it.

The general properties of matter. — By a distinction which is somewhat arbitrary, the preceding properties are said to be essential, while those of inertia, weight, divisibility, and porosity are said to be general, because found to be possessed by all matter. The statement that **inertia** is a general property of matter simply means that Newton's first law of motion is a universal result of experiment.

All bodies possess **weight** at the earth's surface and within experimental or observational limits. A mass placed on the earth's surface is attracted by the earth, by the sun, the moon, the planets of the solar system, and in a less degree — the attraction being so small that we have no direct evidence of its existence — by the distant fixed stars; the resultant differs so very

little from the direct attraction of the earth that the latter alone may be considered as pulling the mass downwards towards its centre; but this is only a first approximation, for a more careful discussion of the intensity and direction of the resultant force helps us to explain the phenomena of the Tides.

All masses are **divisible**; the only question which here emerges is that as to indefinite divisibility. Is a given mass—say of chalk—divisible to infinity, or would we after division effected with sufficient frequency obtain a small mass of chalk which, if further divided, would be no longer chalk, but might perhaps be broken up into lime and carbonic acid? The facts of chemical equivalence, as ascertained by the balance, seem to be susceptible of no natural explanation other than that matter is made up of such ultimate particles, and hence matter is concluded not to be indefinitely divisible. We shall recur to this subject.

Nobert engraved parallel lines on glass at a mutual distance of 1/40,000 cm., half the wave-length for bluish-green light. No microscope made can show these as distinct lines.

All matter is porous or possesses **porosity.** Hydrogen gas leaks through white-hot iron under pressure; cold water can be pressed through iron, as may be seen in Bramah's hydraulic press, or through lead, as in Francis Bacon's famous experiment, in which he took a shell of lead filled with water and compressed it; the water oozed through the lead and stood in drops and beads on the surface of the shell.

Contingent properties of matter.—Some of the properties of matter are contingent, and depend on the particular kind of matter considered and on the surrounding circumstances. As examples, we may take the facility with which a body is heated, the rate at which heat can run along it, the ease or difficulty with which light can pass through it, and so on.

The Quantity of matter per unit of volume is defined as the **density** of the mass occupying that volume. Thus a gramme of water occupies a cubic centimetre, and according to the C.G.S. or centimetre-gramme-second system of measurement, the density of water is (1 gramme/1 cub. cm.) = 1. More accurately (Kupffer), 1 cub. cm. water at 3°·9 C. weighs 1·000,013 standard grammes, and its density is 1·000,013. In the same way the density of lead is 11·35, because 1 cub. cm. weighs 11·35 grammes.

In general, if m be the mass contained in volume $\mathfrak{v}$, and ρ the density, $m/\mathfrak{v} = \rho$, or $m = \mathfrak{v}\rho$.

Every kind of matter, simple or compound, has a special density of its own; thus a given bulk of lead contains 11·35 times as much mass as the same bulk of water. Water is taken as the standard of density; its specific density is said to be = 1, though sometimes, in estimating the density of liquids, it is, in order to avoid decimal fractions, reckoned as 1000. In the same way, the specific density of lead is 11·35, and those of all substances may be experimentally found and recorded in a **table of specific densities,** or, as it is more commonly called, a table of Specific Gravities. These are experimentally found by taking advantage of the fact that Weight is proportional to Mass; $G = mg$. The piece of lead which will occupy a given space not only contains 11·35 times as much mass, but also *weighs* 11·35 times as much as the quantity of water which would fill the same space. Thus the density of a body, as compared with that of water, presents a ratio — its *specific density* — which is numerically identical with the ratio — its *specific gravity* — of its weight to that of an equal bulk of water.

If m and $m_{,}$ be the masses of equal volumes of the body and of water, $\mathfrak{v}$ these equal volumes;

$$\text{Sp. density} = \frac{\text{density of the body}}{\text{density of water}} = \frac{m/\mathfrak{v}}{m_{,}/\mathfrak{v}} = \frac{m}{m_{,}};$$

$$\text{Sp. gravity} = \frac{\text{weight of the body}}{\text{weight of equal bulk of water}} = \frac{G}{G_{,}} = \frac{mg}{m_{,}g} = \frac{m}{m_{,}}.$$

Hence sp. density and sp. gravity are numerically identical ratios.

Sketch of the Experimental Methods of finding the Specific Density of Bodies.

(a) **Of Solids.** — 1. Weigh the body in air (properly *in vacuo*); measure its bulk by dipping it (suspended by a thin string) into water contained in vessel A, and observing the rise of level in that vessel; take it out: out of a known quantity of water in vessel B pour enough water into vessel A to produce an equal rise of level in the vessel A; find the weight of the water that has been poured out of vessel B. Then the weight of the body ÷ the wt. of the equal bulk of water poured out of B = the sp. density of the body. The practical objection to this method is, that the body when taken from vessel A removes some of the water.

2. Weigh the body in air; put it in a vessel — a "specific-gravity flask" — marked distinctively at a certain level; fill with water up to the marked level; weigh. Empty the vessel and fill with water alone up to the mark; weigh.

Then Weight of vessel, body and water up to level, $= V + B + w$.
" vessel and water alone up to level $= V + W$.

$\therefore$ the Weight of the water which replaces the body is $W - w$,
and the Weight of the body is B.

$$\therefore \text{Sp. gr.} = \frac{\text{Weight of the body}}{\text{Weight of an equal bulk of water}} = \frac{B}{W - w}.$$

3. Take advantage of the following proposition in Hydrostatics: — A body suspended in a liquid is buoyed up by that liquid to such an extent as to diminish in apparent weight by an amount equal to the weight of the bulk of the liquid which it may be considered as displacing. If a body of exactly the same density as water be suspended in water, it will neither sink nor rise; its apparent weight will $= 0$; its sp. gr. $=$ (wt. in air $\div$ loss in water) $= 1/1 = 1$. If it be more dense it will sink, but slowly, for while its mass is unaltered, the force acting on that mass is apparently diminished. If it be less dense than water, it will rise; its weight will appear to be less than nothing, negative.

A cork of volume $\mathfrak{v}$ and density $\cdot 8$ will have a mass $\mathfrak{v} \times \cdot 8 = \cdot 8\mathfrak{v}$. An equal bulk of water would have a mass $\mathfrak{v} \times 1 = \mathfrak{v}$. The weight of this mass of water would be mass $\times\, g = \mathfrak{v}g$; the weight of the mass of cork is similarly $\cdot 8\mathfrak{v}g$. The apparent weight of the mass of cork will be $\cdot 8\mathfrak{v}g - \mathfrak{v}g = -\cdot 2\mathfrak{v}g$; *i.e.*, its downward acceleration will be negative, or the cork will move upwards. This upward accelerating force, $= \cdot 2\mathfrak{v}g$, acting on a mass $\cdot 8\mathfrak{v}$, will produce an acceleration (since $a = F/m$) $= \cdot 2\mathfrak{v}g/\cdot 8\mathfrak{v} = \frac{1}{4}g$, and but for friction in the water the cork would rise with an upward acceleration of 8·05 ft.-per-sec. per second.

Therefore weigh a body in air; suspend it by a fine thread from the pan of a balance, so that it just sinks wholly into water. It will appear to weigh less. Then the sp. gr. $=$ (weight in air $\div$ apparent loss of weight in water), or, accurately, (weight *in vacuo* $\div$ apparent loss of weight in water). If the body be lighter than water, attach to it a piece of heavy substance — say lead — of known weight and known density (lead $= 11\cdot 35$). Then the weight of the lead is $11\cdot 35\mathfrak{v}g$; that of the light substance is $\rho\mathfrak{v}_{,}g$; together they weigh $11\cdot 35\mathfrak{v}g + \rho\mathfrak{v}_{,}g$. In water they weigh $10\cdot 35\mathfrak{v}g + (\rho - 1)\,\mathfrak{v}_{,}g$. The loss of the lead alone must be $\mathfrak{v}g$, or $1/11\cdot 35$th of its weight; whence the loss of the light body in water is easily determined by difference; and its density, the fraction (weight in air $\div$ apparent loss in water), found.

4. If the solid be soluble or be otherwise acted on by water, some other fluid is made use of, such as naphtha or turpentine, the specific density of which is known. If the specific density of the solid with reference to the liquid, found by any of the above-mentioned methods, be s/l, and the sp. d. of the liquid with reference to water be l/w, then the product of these specific densities $= s/l \times l/w = s/w$ is the sp. d. of the solid as compared with water.

(b) **Of Liquids.** — 1. Fill a weighed vessel up to a certain mark with the liquid: the whole weighs so much: by difference find the weight of the liquid alone. Empty the vessel and repeat the process with water; the water which fills the vessel up to the same mark weighs so much. Then the sp. gr. of the liquid $=$ (wt. of given bulk of liquid $\div$ wt. of same bulk of water).

2. A body immersed in water appears to lose $1/x$th of its weight; immersed in the liquid to be tested it appears to lose $1/y$th of its weight. This body is x times (x being a whole number or a fraction) as dense as water, y times as dense as the liquid. The density of the liquid is to that of water as the apparent loss $1/y$ in the liquid is to the apparent loss $1/x$ in the water — that is, as $x : y$; and the sp. d. of the liquid is x/y.

3. By the use of Hydrometers, Alcoholometers, and the like. The principle of these instruments is the following: — A body which floats in water without being wholly submerged is in equilibrium under the action of two

forces — (1) the weight of the whole body; (2) the buoyancy of the water, equal to the weight of the part of the water displaced. Thus ice floats in water with $\frac{918}{1000}$ of its bulk submerged. If the volume of a mass of ice be $\mathfrak{v}$ and its specific density ρ, the weight of the mass is $\mathfrak{v}\rho g$, and the weight of the mass of water equal in volume to the submerged part of the mass of ice is $\cdot918\mathfrak{v} \times g$. These are equal; $\mathfrak{v}\rho g = \cdot918\mathfrak{v}g$; whence $\rho = \cdot918$, the specific density of ice as compared with water $= 1$. Thus the sp. d. of a body floating in water $=$ (part immersed $\div$ whole volume). This method might be used to determine the sp. d. of solids were it not for practical difficulties of measurement, which, for the sake of explanation, we here suppose overcome. If a mass of ice be placed in chloroform, it will float with $\cdot6125$ of its mass submerged; the sp. gr. of ice in reference to chloroform is $\cdot6125$. The problem comes to be this: Water is $\frac{10000}{9180}$ times as heavy as ice, chloroform is $\frac{10000}{6125}$ times as heavy as ice — What are the relative densities of water and chloroform? Chloroform is heavier than water in the ratio of $\frac{10000}{6125} : \frac{10000}{9180}$, or $9180 : 6125$, or $1\cdot497 : 1$; that is, its sp. d. is $1\cdot497$. We see, then, that the comparative sp. d. of liquids could be ascertained by finding to what depth bodies floating in them will sink, if we could perform the necessary measurements with the requisite accuracy. But instruments which have been graduated by the instrument-maker, who observes and marks on them with more or less care the depth to which they sink in various liquids of known specific gravity, and marks the corresponding positions by proper figures, are in common use for promptly ascertaining the density of various liquids. They are usually made with large bulbs, generally loaded with mercury or lead-shot, in order that they may float vertically, and they have a narrow stem which is graduated. They are caused to float in the liquid whose density is to be found: the graduated stem stands more or less out of the liquid; the figure upon it which corresponds most nearly with the general level of the surface of the liquid is read off and recorded. It is convenient in effecting such readings to arrange a piece of black paper to serve as a background, and to place the liquid to be tested in a glass vessel.

Rousseau's Densimeter bears at its summit a little cavity. In this a cubic centimetre of the fluid is placed: according to the depth to which this makes the instrument sink in water is the density of the fluid determined, according to the graduation performed beforehand by the instrument-maker.

Fahrenheit's Areometer consists of a similar instrument provided at its summit with a little platform or pan. It is placed in water at $3^{\circ}\cdot9$ C., and loaded by small additional masses placed in the pan, until the areometer sinks in the water just so far that the level of the water coincides with a certain mark on the instrument. Then the sum of the known weight of the areometer + that of the additional masses $= G =$ the weight of the bulk of water displaced. It is removed, dried, and placed in the liquid to be tested, and again loaded till it stands *at the same level* as before. The weight of the instrument + the weights now added $= G_{,} =$ the weight of an equal bulk of the liquid to be tested. Then $G_{,}/G$, the ratio of these weights, is the specific density of the liquid in question. If, for example, the instrument weigh 800 grs. and float in water at the proper level when loaded with 200 grains; loaded with 80 grains it floats at the same level in an aqueous solution of ammonia: the total weights are 1000 and 880; the density of ammonia solution is $\cdot880$.

Nicholson's Areometer, which is sometimes used for determining the

density of solids, is a modification of these instruments. It bears two platforms — one at the summit, out of the water, and a lower one in the water. The body whose density is to be found is placed on the upper pan along with masses just sufficient to sink the areometer to a certain mark. The body is transferred to the lower pan, beneath the level of the water. More masses must be placed in the upper pan to cause the areometer to sink to the same level; the weight of these masses is equal to the apparent loss of weight suffered by the body when placed in the water on the lower pan. Then (weight of body ÷ the additional weight) = density of the solid.

4. By Specific-gravity Bulbs. Bulbs are sold which are known to float without rising or sinking in liquids of the sp. gr. marked in numbers upon them. A number of these are thrown into the liquid; those which bear too high a number sink, those which are too light rise; the one exactly corresponding, if there be one, is at rest anywhere in the liquid.

(c) **Of Gases.** — The density of a gas is found by an application of the same principles as those employed in determining that of a liquid. A copper or glass vessel, as light as is consistent with adequate bulk and strength, has the contained air extracted from it by means of a good air-pump; it is then weighed empty; it is very slowly filled with the gas and left for some time in communication with a reservoir of it; its stopcock is closed, and the vessel thus filled with the gas is again weighed: the weight of the volume of gas which fills the vessel is thus ascertained. It is again emptied by means of the air-pump, and then air is allowed to enter it. After standing for some time in order to acquire the temperature of the room, it is again closed and weighed: thus the weight of that volume of air which fills the vessel is found. Then the weight of the gas ÷ wt. of equal vol. of air = density of the gas in reference to air as a standard.

The Dimensions of Density, $[m/\mathfrak{v}] = [M/L^3]$; those of sp. gr. and sp. d. = 0 = [Numbers merely].

It is convenient for chemical purposes to take the rarest gas — that is, the least dense gas, Hydrogen — as a standard of density, and then we say that the specific density or sp. gr. of Hydrogen is 1, that of air 14·439, that of Oxygen 16, and so on.

A cubic centimetre of Hydrogen weighs ·0000895682 grammes; a cub. cm. of air weighs (at Paris) ·0012932 grammes at the freezing point of water and at the barometric pressure of 76 cm. of mercury. A cub. cm. of liquefied marsh-gas (which liquefies at − 73°·5 C., if the pressure be raised to 56·8 atmospheres) weighs, according to Wroblewski, 0·37 gramme; the same volume of water weighs 1 gramme at 3°·9 C., or more accurately, it weighs 1·000013 standard platinum grammes; a cub. cm. of liquefied oxygen weighs 1·124 grammes (Olszewski); the same volume of lithium, the least dense solid metal, weighs ·5936 grammes, and of hammered platinum it weighs 21·25 grammes. Thus we see that, bulk for bulk, solid platinum is nearly 240,000 times as dense as gaseous hydrogen.

For the estimation of Vapour-density, see p. 391.

The States of Matter.

We know that in popular language there are said to be three states of matter, the solid, the liquid, and the gaseous. Closer observation shows us that these merge into one another, and that, in addition to these, matter exists in conditions more recondite; thus we shall have to pass in review the following conditions: — (1) Rigid solid, (2) soft solid, (3) viscous liquid, (4) mobile liquid, (5) vapour, (6) critical state, (7) gas, (8) radiant matter. These may be all classified under the two heads of Solid and Fluid, which are not, however, separated from each other by any distinct line of demarcation.

A **perfect solid** is an ideal body which, when brought into a condition of stress, if it do become deformed, becomes deformed to a definite extent, and then retains its newly-acquired bulk or shape for an indefinite period of time, so long, that is, as the same condition of stress is continuously kept up in it; thus the ratio $\frac{\text{deformation}}{\text{deforming force}}$ or $\frac{\text{strain}}{\text{stress}} = const.$ A steel spring becomes stretched by the weight of a mass hung upon it: it is stretched to a definite extent, and it then retains the same length for any length of time, or rather it appears to do so when we do not use sufficiently accurate measurements; if it did so perfectly, steel would be a perfect solid. A **fluid,** on the other hand, is a substance which, if continuously acted upon by deforming force, continuously yields with continuously increasing deformation; and thus the ratio $\frac{\text{deformation}}{\text{deforming force}}$ is not constant, but increases with the lapse of time.

The force so applied to a fluid must not be one which is kept up equably over its whole surface — in other words, there must not be a "Hydrostatic Stress" — but it must act more effectively on one part of the mass of the fluid than it does on another.

A **rigid solid** is one which, when a stress is applied to it, experiences no deformation, no strain; and therefore in a perfectly rigid solid the fraction (strain/stress) = 0, and continues so for an indefinite period of time. This is an ideal; no substance is absolutely rigid; but we may form a sufficient idea of a rigid solid by considering an anvil on which a nail is placed; the anvil appears undeformed by the weight of the nail — it appears to be absolutely rigid. There is, then, no body abso-

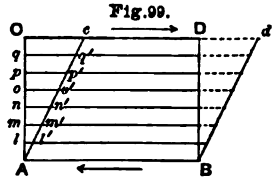
lutely solid, no substance absolutely rigid; the bodies which we call Rigid Solids are found to yield, and to yield continuously if we experiment upon them in sufficiently small masses (such as thin wires), and act upon them with sufficient forces for adequate periods of time. In practice a body is said to be Solid if its deformation remain practically or sensibly constant for a long time; it is said to be Rigid if the deformation associated with a given stress be exceedingly small, or, in other words, if the fraction (stress/strain), the *Coefficient of Rigidity*, be very large. A **soft solid** is one in which the coefficient of rigidity is very small — that is, it is a solid in which a small stress accompanies great deformation. A mass of jelly is very easily deformed; but if a moderate force, such as the weight of a caraway seed or currant, be applied to jelly even of very thin consistency, the deformation produced by it is approximately constant; the jelly does not flow over the small load, and is therefore a true solid, though it is soft.

Very considerable rigidity can be temporarily conferred on a body by rotating it; *e.g.*, a sheet of tissue-paper spun on a wheel becomes quite stiff, the whole paper being under centrifugal tension.

It would seem at first sight rather an abuse of terms to declare that thin jelly is a solid, and that such substances as sealing-wax, pitch, and cobbler's wax are fluids; yet these latter are fluids because they flow, because they suffer continuous deformation under the action of a continuous force. A stick of sealing-wax supported at its ends yields continuously to its own weight; a mass of sealing-wax or pitch will flow down hill; a cake of cobbler's wax of a definite form will soon lose its sharpness of outline; a cake of this material placed in water, with bullets on it and corks under it, will be traversed by the bullets at the rate of about a quarter of an inch per month, and by the corks at a somewhat slower rate; the wax slowly flows round them as they sink or rise under the influence of their relative weights, just as water would much more rapidly do. These substances are, then, Fluids; but their flow is extremely slow, or, in other words, their **viscosity** or resistance to flow is extremely great. Treacle is another example of a viscous fluid; strong syrup, weak syrup, very weak syrup, cold water, alcohol, hot water, ether, are examples of liquids whose flow is successively more rapid, whose viscosity is less; and the last mentioned are said to be **mobile** liquids, though perfect mobility, the perfect absence of viscosity, is an ideal attribute not possessed by any actual fluid.

The Coefficient of Viscosity.—In Fig. 99 let ABCD represent a volume of fluid comprised between two planes passing through AB and CD, and let a shearing force represented by arrows act on the fluid so as to produce continuous deformation, as there shown. The deformation, since the substance is fluid, goes on increasing; let AcdB represent a form assumed through the different layers (imagined separate as in Fig. 25) slipping over one another, the lowest, AB, being relatively at rest. If the velocity imparted to each stratum be proportional to its distance from AB, the ratios of their displacements ll', mm', nn', oo', to their respective distances, Al, Am, etc., will be equal; *i.e.*, $\frac{ll'}{Al} = \frac{mm'}{Am} = \frac{nn'}{An} = \frac{oo'}{Ao} = \frac{Cc}{CA} = \tan CAc$; or, in other words, this is a proper Shear. If the displacement of the stratum CD in time t be Cc, and the angle CA$c = \theta$, where θ is the angle through which a line AC, at right angles to AB and CD, has been rotated, then the total shear is $\tan CAc = \tan\theta$; and the shear per unit of time is $\tan\theta./t$.

Fig. 99.



This slipping of one imaginary stratum over another is retarded as by friction, for each stratum rubs against or is delayed by the one next to it; and thus the retardation acts as a force would do, opposed to that which tends to produce movement—*i.e.*, to the shearing force; and it has to be measured over every unit of surface over which the retardation is effected. The shearing force itself is measured as a force exerted parallel to the planes passing through AB and CD (and therefore, in the general case, tangential to the surfaces of the different strata), and acting on each unit of area of these surfaces (or, equally, on each unit of cross-sectional area of AC or BD).

The ratio $\frac{\text{Shearing Force per unit of Area}}{\text{Shear per unit of Time}} = \frac{\mathbf{f}\cdot t}{\tan\theta} = \eta$, the Coefficient of Viscosity. When $\theta = 0$, there is no deformation in unit of time, and the viscosity is infinite; when $\tan\theta./t$ is very small, the deformation is very small in unit of time—that is, it is very slow, and the viscosity is very great. When the viscosity is very small, the deformation produced in unit of time is very great, or the obstruction offered to flow is very small.

If the distance AC = 1, and the displacement of CD in unit of time (*i.e.*, its velocity) also = 1, then $\tan\theta./t = 1$, and the shearing force per unit of area *is equal to* the coefficient of viscosity; whence the Coefficient of Viscosity may be measured in terms of a force—that is, by "the tangential force on the unit of area of either of two horizontal planes at the unit of distance apart, one of which is (relatively) fixed, while the other moves with the unit of velocity, the space between being filled with the viscous material" (Maxwell). The "kinematical coefficient of viscosity" is η/ρ, and measures the corresponding shearing force required per unit of mass.

Relation between Rigidity and Viscosity.—The measure of Rigidity in the ideal perfect solid, the ratio of the deforming Force to the total Deformation produced by it (that is, using Fig. 99, the quotient $\mathfrak{n}$ = Deforming Force $\mathbf{f}$ per sq. cm. $\div \tan\theta$) remains constant for any period of time during which the deforming force applied remains constant. The viscosity η in a fluid, on the other hand, is the ratio of the deforming force per unit of area to the deformation produced **in a unit of time.** Whether the time of experiment be long or short, the numerical value of the fraction $\frac{\text{Force } \mathbf{f} \text{ applied} \times \text{Time } t}{\text{Shear produced in Time } t} = \eta$ ought to be the same. Yet it is found that

if a very small period of time be taken, the coefficient of viscosity may have some experimental value different from that found when the period of time is longer. If, for instance, Canada balsam be stirred with a spatula, it will be found (Maxwell) to be doubly refracting. This shows that it is for the instant, and locally, in a condition like that of a stretched or a compressed solid, and that, though it is liquid if sufficient time (time of relaxation) be allowed it, yet under the action of impulsive forces its coefficient of viscosity is so high that it practically behaves after the fashion of a solid.

There is no substance perfectly solid, and hence when an apparently solid substance in a suitable form — for convenience, that of a wire — is exposed to a constant distorting force, though its form is found to be affected to a certain extent (say by lengthening or by twisting), and it appears soon to reach the limit of its distortion, which may then be measured, the apparent rigidity of the substance being thus ascertainable; yet in general it will be found that the prolonged application of a constant force induces a slow constantly-increasing additional distortion; and the substance then acts as a fluid of exceedingly great viscosity. Further, we now know by the experience of manufacturing industry that lead and even iron, when exposed to sufficient pressure, can be made to flow slowly; and the operation of wire-drawing involves among the particles of the metal drawn, as they pass through the aperture in the plate, a relative movement which is similar to that of the particles of a flowing fluid. Amorphous solids, such as glass, are particularly apt to shade off into liquids, and to soften gradually by heat instead of presenting a definite melting point.

When the viscosity of a fluid is infinite, there is no difference between that fluid and a rigid solid. It is supposed by some that the matter at the centre of the earth approximates to this condition. If the earth have a crust about 25 or 30 miles thick floating on a melted magma, at increasing depths the pressure (which near the centre must amount to about 45,000000 pounds per square inch) would compress the liquid so much that, though melted, it would have a viscosity so extreme that the mass would have the same relation to extraneous forces as a very rigid solid body would have. The earth would, in consequence of this, comport itself as if it had a solid nucleus floating in and merging by gradations of relative softness into a thin liquid layer on which the crust floats (Osmond Fisher). The crust of the earth is, however, more rigid than is consistent with this theory.

Substances in the solid and liquid form are broadly dis-

tinguished from those in the **gaseous** condition by the two following characteristics. In the first place, the former have a free surface, while the latter cannot permanently retain a free bounding surface independent of the vessel containing them. Secondly, solids and liquids tend (apart from volatilisation) to assume a definite limited bulk and density; while gases always tend to assume an infinite volume and a correspondingly small density. If any definite quantity of a gas be confined within a limited space, it will always fill that space and press against the sides of the containing vessel; and it will subject that vessel to stress or pressure, which the vessel must be strong enough to withstand. Thus a gas, with its **tendency to indefinite expansion,** has Elasticity of Volume: work has to be done in order to compress it, and when compressed it tends to restore the work done upon it. Under ordinary circumstances gases are prevented from expanding to an indefinite extent by the pressure of the mass of air which lies upon the earth's surface. This air is drawn down towards and pressed upon that surface by the downward attraction of gravity, and consequently compresses itself and all objects at the earth's surface with an "atmospheric pressure" of about 15 lbs. per square inch. Hence a closed flask containing air is subject to two equal and opposite pressures, whose resultant is nil; the air in the flask tends to burst it outwards; the air external to it tends to make it collapse: between the two pressures the flask has no need for strength. If, however, the air be in any way extracted from the flask, the external pressure will alone act, and the flask may collapse. In a steam boiler the internal pressure, the tendency of the steam to expand, is greater than the external pressure; and the boiler must be of sufficient strength to provide for the difference.

The pressure exerted by a confined gas is equal over the whole internal surface of the vessel containing it; and a pressure equal to the weight of *a* lbs. per sq. in., exerted on a piston forced into a cylinder of gas, is communicated to every sq. in. of the inner surface of the cylinder, for the face of the in-moving piston is, at any instant, a part of the inner surface of the cavity containing the gas; and if any other part of the wall of the cavity — *e.g.*, the face of a second piston — be at the same time movable outwards, the excess of internal pressure will cause it to yield. If its area be the same as that of the in-moving piston, the out-moving piston would be acted on by an equal

total pressure, and would move to exactly the same extent as the in-moving one.

There is an apparent paradox in the statement that a single-inch piston, pressed in with a pressure equal to the weight of 60 lbs. per square inch, will communicate a pressure of 60 pounds to every square inch of cylinder, however large; but there is no law of Constancy or Conservation of Force: there is one of Conservation of Energy, which in this instance is rigorously respected. The work that can be done by a second piston allowed to move outwards is (friction not being considered) equal to that done by the compressing piston, whatever be their respective areas: the range of movement and the area vary inversely: a smaller second piston may be pushed through a longer stroke than the first, a larger one through a shorter.

In this way machinery may be driven at the distance of a mile: air is driven into a tube; at the other end of the tube there is a cylinder connected with the tube, and by the alternating admission of the pressure to one and the other face of a piston movable in this cylinder, the piston is caused to move, and the energy is thus applied at a distance.

If a uniform pressure p (*i.e.*, p dynes per sq. cm. of the bounding surface) will keep a certain quantity of gas within a space $\mathfrak{v}$, pressure $2p$ is found to be required to confine within an equal space twice as much gas: pressure xp will keep x times the quantity of gas within the same space; that is to say, on the assumption that the temperature does not vary. The densities in these examples are in the ratio $1:2:x$, and the Pressures are proportional to the Densities (the same kind of gas being supposed to be used throughout) or to the quantities of gas forced into a given space.

The pressures being proportional to the densities acquired under their action, $p \propto \rho$; but $m = \mathfrak{v}\rho$; whence p varies as $m/\mathfrak{v}$; or $p\mathfrak{v}$ varies as m: or $p\mathfrak{v}$ is equal to a constant $\times\, m$; or, for a given mass, p varies inversely as the volume, or the volume varies inversely as the pressure applied; and these last are the usual forms of Boyle's law.

This statement may be otherwise expressed in the form of **Boyle's law,** that the pressure exercised by a given mass of gas varies inversely as the volume of the space within which it is confined; or that the volume occupied by a given quantity of gas varies inversely as the pressure. Thus, if a quantity of gas occupying one cubic foot at a pressure of 15 pounds per sq. inch were compressed by a piston forced down with an additional pressure of 15 pounds per sq. inch, the total pressure being doubled the volume would be halved; if, on the other hand,

the pressure were diminished to half by the piston being pulled out with a force equivalent to $7\frac{1}{2}$ lbs. per sq. in., the volume of the gas contained in the cylinder would be doubled. There are no perfect gases which absolutely obey this law at all temperatures and pressures; but a gaseous substance, at a temperature and pressure far removed from those at which it will be condensed into a liquid, approximates to this condition.

Now suppose that a quantity m of a liquid is placed in a vessel which it does not completely fill, every other substance, such as air, being removed from the vessel, which is then closed. The liquid does not fill the whole vessel; it has a free surface. Above this free surface there is a space, which becomes filled with part of the liquid substance, volatilised into a gaseous form. Let the quantity of liquid which has assumed the gaseous form be represented by $m_{\prime}$: the remainder, $m - m_{\prime}$, is still in the liquid form. The proportion volatilised ($m_{\prime}/m$ of the whole) depends on the temperature as well as on the space which has to be filled. At another temperature some different proportion will be volatilised. When the liquid is heated this proportion rapidly increases. But as we have already seen, the pressure exerted by a confined gas on the vessel containing it depends directly on the amount of it. Hence in this case the pressure exerted rises rapidly as the temperature rises. Gas having this relation to the liquid form of the same substance, confined with it in a vessel otherwise empty, and in contact with it, is called the **vapour** of that substance; if it be compressed or cooled, it partly condenses into liquid. Even though not in contact with the liquid, if the gaseous form of a substance be compressed or cooled just so far that any further condensation or cooling will cause the deposition of some of it in the liquid form, it is said to be a Vapour. In some cases a vapour condenses directly into a solid; *e.g.*, arsenious acid.

The term Vapour is often applied in a wider sense to the gaseous form of a liquid or solid substance—as, for instance, ether-vapour, chloroform-vapour, the vapour of arsenious acid; and then those vapours which are on the point of condensation are called **saturated vapours,** while those which can suffer a certain amount of compression or cooling without condensation are called **unsaturated vapours.** In this sense (with the possible exception, as yet, of hydrogen) all gases are unsaturated vapours; for they can all be condensed by the simultaneous application of sufficient cold and sufficient pressure. Oxygen has been con-

densed; at a pressure of 300 atmospheres, and at the temperature of $-39°$ C. (Cailletet), there is no condensation, but when the gas is liberated it becomes foggy; according to Pictet it is liquefied at 320 atmospheres and $-140°$ C.; and then, upon allowing a jet of this liquid to escape into the air, the escaping jet of liquid oxygen becomes extremely cold and is partly solidified, while the remaining oxygen in the vessel becomes cloudy. When oxygen has been compressed into a liquid, this liquid must be in some state of molecular aggregation distinct from that of the gas, for it has no effect on alkali-metals or phosphorus, while it is powerfully magnetisable. Ozone is with comparative readiness condensed by Cailletet's method to a bright-blue liquid. The term Vapour is used in still another sense — that is, a gas at such a temperature that by the application of pressure alone it can be condensed into a liquid. In this sense carbonic acid below 30°·92 C. is a vapour; above that temperature it is properly a gas, for no amount of pressure alone will liquefy it.

The Critical State. — When a liquid and its vapour are together in a tube, otherwise unoccupied, and are exposed to heat, there arrives a temperature at which the singular phenomenon of a blending or **continuity** of the liquid and gaseous (or vaporous) states is observed. If a capillary tube, for instance, be filled with liquid CO_2 and slightly heated, some of the carbonic anhydride will escape: the tube may then be sealed up, and will now contain nothing but liquid CO_2 and saturated vapour of CO_2. If these be heated to 30°·92 C., and if there be sufficient CO_2 present to produce a pressure above 73 atmospheres, the free surface of the liquid becomes blurred and merges into the superjacent gas: above this temperature the tube is full of what appears to be nothing but gas: if cooling be permitted there is a flickering seen in the tube, and the liquid and the gas again separate.

Some say that the liquid and the gas mutually dissolve each other; others (Ramsay) point out that the liquid CO_2 rapidly becomes lighter, while the confined vapour of CO_2 becomes denser, at higher temperatures, and that at the critical temperature and under sufficient pressure these two states meet and become undistinguishable. Both the liquid and the gas would, under these conditions, have the same volume.

If CO_2 gas be exposed to any temperature above 30°·92 C. and be subjected to any pressure above 73 atmospheres, it will still be a gas: allow it to cool, the pressure being kept up, and it will be a liquid after it passes 30°·92 C.; and yet the

transition is unobservable. If pressure and temperature be allowed to fall together, the flickering already mentioned is produced.

If the liquid originally fill the tube and then be heated to the critical temperature, the tube becomes filled with gas, but the precise mode of transition from the one state to the other cannot be observed. If the tube thus containing gaseous CO_2 at a high pressure be locally cooled, there is local condensation and flickering.

This temperature, 30°·92 C. for carbonic acid, is called the Critical Temperature. If the temperature of the gas be above 30°·92 C., no pressure can condense it into a liquid; if it be just below that point, a pressure of 73 to 75 atmospheres is competent to effect its liquefaction: and this pressure is called the Critical Pressure of carbonic acid. (See Van der Waals' Law, p. 375.)

The volume occupied by unit mass of the liquefied gas at the crit. temp. and crit. pr. is called the Critical Volume.

The critical temperature of oxygen, nitrogen, and other gases formerly known as permanent gases, is very low in the thermometric scale (oxygen, – 113° C., Wroblewski), and exceeding cold is a necessary condition of their condensation under pressure.

Water filling a sufficiently strong boiler might be exposed to a low red heat, 720°·6 C., and would then be transformed into a gas exercising such enormous pressure as to make any experiments upon it excessively difficult; yet it is believed (see p. 390) to present this phenomenon at that temperature.

At the critical temperature, matter under sufficient pressure is in the Critical State; if heated a little more it is undoubtedly gaseous; if allowed to cool a little it is undoubtedly liquid, and is far less compressible; and if the pressure be kept up, the transition is unrecognisable. By no optical test can the liquid just below the critical temperature and the gas just above that temperature be distinguished.

If a solid be dissolved in a liquid, and if the whole be heated under sufficient pressure to a temperature above the critical point, the liquid is now gas, and yet the solid remains dissolved in it (Hannay). Iodide of potassium, for instance, or chlorophyll, if dissolved in alcohol and treated in this way, will, it is said, remain in solution in gaseous alcohol at 350° C.

When gaseous matter has been rarefied to a very great degree it assumes remarkable properties, of which the most striking is that such exceedingly rarefied gas or **ultragaseous**

matter can be induced — as we shall see under Electricity, p. 656 — to exercise pressure specially on localised areas of the walls of the containing vessel, and by this concentrated pressure to produce mechanical and luminous effects characteristic of the so-called **Ultragaseous or Radiant Matter** (p. 252).

The Ether. — We have already said that we can know matter only by those of its properties which we perceive by means of our senses. The existence of any form of matter is to us only an inference from the phenomena to which it gives rise; and if a large group of phenomena find their best or their only explanation in the assumed existence of a form of matter of an unfamiliar kind, the evidence for its existence is of exactly the same character as that on the ground of which we believe ourselves entitled to assert the existence of any kind or form of matter whatsoever. The phenomena of Light are best explained as those of undulations; but undulations — even in the most extensive use of the term, as signifying any periodic motion or condition whose periodicity obeys the laws of wave-motion — must be propagated through some medium. Heat while passing through space presents exactly the same undulatory character, and requires a medium for its propagation. Electrical attraction and repulsion are explained in far the most satisfactory way by considering them as due to local stresses in such a medium. Current electricity seems due to a throb or series of throbs in such a medium when released from stress. Magnetic phenomena seem due to local whirlpools set up in such a medium. And the assumption of the existence of a single medium, with properties of great simplicity, will explain these varied phenomena and even co-ordinate them; thus the crest or the trough of a light-wave or a heat-wave is a point of maximum displacement due to transverse tension — exactly the condition of the medium during the persistence of electric attraction or repulsion, that is, the Electrostatic condition; the middle point of the wave is changing its position with rapidity — exactly the condition of the medium during the passage of a current, the Electrokinetic condition; thus Light and Radiant Heat are explicable as electromagnetic disturbances of rapidly-alternating character; and this leads to the conclusion, sustained by experiment, that the velocity of light should be equal to the rate of propagation of an electric disturbance through a medium of this kind. We are led to infer, therefore, that there is such a medium, which we call the Luminiferous Ether, or simply the Ether; that it can

convey energy; that it can present it at any instant partly in the form of kinetic, partly in that of potential energy; that it is therefore capable of displacement and of exercising pressure or tension; and that it must have rigidity and elasticity. Calculation leads us to infer that its density is 936/1000,000000,000000,-000000 that of water (Clerk Maxwell), or equal to that of our atmosphere at a height of about 210 miles, a density vastly greater than that of the same atmosphere in the interstellar spaces; that its rigidity is about 1/1000,000000 that of steel — hence that it is easily displaceable by a moving mass; that it is not discontinuous or granular; and hence that as a whole it may be compared to an impalpable and all-pervading jelly, through which Light and Heat waves are constantly throbbing, which is constantly being set in local strains and released from them, and being whirled in local vortices, thus producing the various phenomena of Electricity and Magnetism; and through which the particles of ordinary matter, with their relatively small translatory velocities, move freely, like bullets through cobbler's wax, encountering but little retardation if any, for the elasticity of the Ether, as it closes up behind each moving particle, is approximately perfect.

Nothing of the nature of an air-pump can remove it from any given space; the most perfect **vacuum** conceivable must be defined as a plenum, a space fully occupied, but occupied by Ether alone.

Change of State. — Work must be done upon a solid in order to convert it into a liquid: energy must in some form be imparted to it. This form may be that of Heat, directly applied so as to fuse the solid. In such a case a definite amount of the energy imparted in the form of Heat apparently disappears (see Latent Heat, p. 361), for it does the mechanical work of liquefying the solid. If the liquid again assume the solid form, as in freezing, the process is reversed: the energy absorbed during liquefaction gradually reappears in the form of heat, which must be dissipated before the freezing can become complete.

If a solid body simply assume the liquid form without having external heat or other energy applied to it, the absorption of some of the heat of the body itself results in a cooling of its substance, as in the case of a freezing mixture, where, on certain chemical salts being dissolved in cold water, the resultant solution is found to be extremely cold. Solid CO_2 and ether sink to $-100°$ C.

Again, where two chemical elements combine, their combination is generally attended with heat, the elements losing their potential energy of separation. The supply of an equivalent amount of energy from without is necessary in order to reverse the process of combination — that is, to effect chemical separation or decomposition. When the processes of chemical combination and liquefaction go on together — the product of the combination of elements of which one or both are solid being itself liquid — the result may be that the cooling effect of the latter action exceeds the heating effect of the former; thus, in the union of carbon and sulphur to form carbon-disulphide, which is a liquid, the absorption of heat due to liquefaction is greater than the evolution of heat due to combination, and the action stops unless heat be supplied from without. On the other hand, in the combination of quicklime with water to form slaked lime, we find much heat evolved — partly due to the chemical combination, partly to the liquid water assuming a solid form.

The transformation of a solid into a gas, in a like manner, involves the expenditure of heat or some other form of energy in performing the mechanical work of volatilisation. Snow evaporates in a cold high wind; arsenic trioxide under ordinary atmospheric pressures is, without melting, volatilised by heat, while, if a sufficient pressure be applied, it melts before volatilising. Dr. Carnelley found that in a similar way ice, if heated under an exceedingly small pressure, may be rendered very hot (180° C.), and will volatilise freely, yet without melting, unless the pressure be allowed to exceed a certain low maximum, which he called the Critical Pressure, this being very low for water, very high for arsenic trioxide. A sheet of metal may be dissipated in vapour by an electric discharge, part of the energy of which becomes spent in producing this mechanical effect. Again, in chemical combination we often see the conversion of solids into gases. Carbon and oxygen combine to form carbon-monoxide; of the heat which is evolved by the union of the elements, a large part is absorbed in rendering the solid carbon gaseous. If the CO produced be in its turn burned so as to form CO_2, none of the heat of combination of oxygen and carbon-monoxide is absorbed in doing mechanical work of this kind, and the amount of heat evolved in the second stage of the production of CO_2 is greater than that evolved in the first. Conversely, where two gases produce a solid, as chlorine and sulphuretted hydrogen do, the amount of heat liberated is determined not only by the

amount of energy absorbed in decomposing H_2S, and by the amount liberated by the union of H_2 and Cl_2, but also by the fact that the sulphur is deposited in the solid form.

If a liquid were exposed to an indefinite and perfect vacuum, it would evaporate at once at any temperature above the absolute zero. If it be exposed to an imperfect vacuum, it will still evaporate readily, but not so readily as before, for its vapour must be able to force its way from the liquid and against the superincumbent pressure. If the pressure be great, the amount of heat which must be supplied to the liquid in order to enable it to overcome this resistance and to enter into ebullition is also greater, and the Boiling-point of a liquid increases with the pressure.

Let a liquid be supposed heated in a vessel provided with a piston, by means of which pressure can be exercised on the contents of the vessel; the vessel being supposed of any sufficient length. The liquid is heated and converted into vapour; the vapour forces out the piston, and the external air pushes it in; the piston rests when the external and internal pressures are equal. If we press home the piston, the vapour is partly condensed: to retain it in the gaseous form we must simultaneously apply a stronger heat. This process may be supposed continued until, at a certain high temperature (the "critical temperature") and great pressure, we have the whole of the liquid evaporated, and its vapour compressed into the same space as the original liquid. If expansion be altogether prevented, this process is continuous, and the temperature at which water can be wholly converted into vapour under such circumstances is 720°·6 C.

Liquids are, as a rule, more bulky than the corresponding solids; hence fusion, which involves expansion, obeys the same law as evaporation, which also involves expansion; it is hindered by pressure, and the fusing-point, like the boiling-point, is raised by pressure. A few liquids — water, cast-iron — are denser than their solids. In such a case, an increase of pressure may be said to predispose the particles to set into the more compact and denser form, the liquid form, and fusion is facilitated by pressure. Thus the melting-point of ice is lowered, that of most other solids raised, by pressure.

Change of state involves, then, either an absorption or a liberation of energy; and the amount of energy which must be supplied to a body in order to enable it to undergo a change of state depends on the pressure which tends to resist or to favour such a change, as well as on the intrinsic energy which it already possesses.

There is no known means of effecting any transformation of matter in any of its ordinary forms into the Ether, or *vice versâ*.

THE CONSTITUTION OF MATTER.

The question as to whether Matter is or is not infinitely divisible has been made the basis of much acute speculation; but it is only within this century that any serious proof has been adduced in favour of an Atomic Theory, or theory according to which matter is considered as made up of indivisible particles. According to this view, matter consists of particles or atoms, each of which it is impossible with our present appliances to divide, and the division of which, if it were possible, would probably result in the subversion of our ideas as to the apparently fundamental nature of some of the properties of matter.

Chemical Views.—The probability of this atomistic view was raised almost to the rank of certainty by the researches of successive chemical investigators. It was first found that every definite chemical substance in a state of purity has always the same constitution; that an analysis effected for one pure sample of, say, oxide of lead, is applicable to every other pure sample of the same substance. Hence the law of **Fixity of Proportions** in chemical compounds.

But it was remarked that the same elements often unite in different proportions to form compounds possessed of essentially different properties. Carbon and oxygen thus unite to form two well-known compounds, of which the percentage compositions by weight are respectively:—

Carbon . . .	42·85	;	Carbon . . .	27·27
Oxygen . . .	57·14		Oxygen . . .	72·72.

Analytical results tabulated in this way are not very instructive; but if the second example be multiplied by 42·85/27·27 we find the respective ratios to become

Carbon . . .	42·85	and	Carbon . . .	42·85
Oxygen . . .	57·14		Oxygen . . .	114·28

or in round numbers,

Carbon	3	;	Carbon	3
Oxygen	4		Oxygen	8.

Here we find that the same quantity of carbon, united in the one compound (carbon-monoxide) with a certain quantity of oxygen, is in the other (carbonic anhydride) united with twice the quantity of that element. From a large number of instances of this

kind was evolved the **Law of Multiple Proportions** — that the same elements may form a series of different compounds by uniting in several fixed proportions which bear a whole-number ratio to each other. Nitrogen and oxygen thus form five compounds, in which the nitrogen and oxygen are present in the respective ratios of 14 : 8, 14 : 16, 14 : 24, 14 : 32, 14 : 40; and in this case the quantities of oxygen, united with a fixed quantity (14) of nitrogen, bear to one another the relative ratios of 1 : 2 : 3 : 4 : 5. Iron has two oxides, in which the iron and the oxygen bear to one another the respective ratios of 28 : 16 and 28 : 24; here the quantities of oxygen, united with the same quantity of iron, bear to one another the ratio 2 : 3.

Then, further, the law of **Chemical Equivalence** was formulated: chemical quantities, which are equivalent to the same thing as regards power of doing chemical work or forming chemical compounds, are equivalent to one another. One part by weight of hydrogen will combine with eight of oxygen (7·98165 ± ·00175); so will 108 parts of silver (107·896). 108 pts. of silver and 1 of hydrogen are mutually equivalent, for they can both do the same chemical work — they can enter into combination with 8 pts. of oxygen; and they are both equivalent to the 8 pts. of oxygen with which they can combine. If now it be found that 1 pt. by wt. of hydrogen can combine with 35·5 (35·478) pts. by wt. of chlorine, then the law asserts that the equivalent quantity, 108 pts., of silver should also, in its turn, be able to combine with an equal quantity, 35·5 pts., of chlorine. This law is a generalisation, based upon facts determined by the aid of the balance and independent of theory; and this law of equivalence, so based, though it be too sweeping a generalisation to be now accepted in its full sense, yet did useful service in its day in enabling tables of Equivalent Numbers or of Combining Proportions to be drawn up, and a system of Chemical Formulæ to be devised, based upon these equivalents. According to this system, the composition of water was symbolised as HO; this symbol might be read in words as — one equivalent of hydrogen and one of oxygen, united to form one equivalent of water. The symbol of hydrogen peroxide was HO_2; one equivalent (1 pt. by wt.) of hydrogen combined with two equivalents ($2 \times 8 = 16$ pts. by wt.) of oxygen.

When such facts as these were known, a reasoned explanation of them was sought. None that offered was so plausible as Dalton's **atomic theory**, a revival of the old hypothesis of Leu-

cippus, Democritus, and Lucretius, that matter consists of atoms, coupled with the proposition that the atoms of the different elements have different relative weights. According to this view the smallest mass of water must consist of an atom of hydrogen and another of oxygen, their relative atomic weights being 1 and 8; and these were connected as one might couple together a ball of wood and one of lead. More complex substances were produced by the union of a greater number of such atoms — as, for instance, HO_2, NO_5, (KO, HO), etc.; and the symbolic formulæ were then used to denote the relative number of such atoms entering into the formation of compound substances.

But it was found that the system of formulæ based upon the facts of equivalence did not work well when made to signify the relative numbers of atoms united to form a compound. The equivalent number for carbon was 6, because that quantity of carbon was equivalent (in carbonic oxide) to 8 of oxygen, which quantity was in its turn equivalent to the standard unity of hydrogen. In marsh gas 6 pts. by wt. of carbon are found to be combined with 2 of hydrogen—*i.e.*, with two equivalents; hence the formula, according to this system, must be CH_2. It is known, however, that one-fourth of the hydrogen can be replaced by half an equivalent ($17\frac{3}{4}$ pts. by wt.) of chlorine, forming $CH_{1\frac{1}{2}}Cl_{\frac{1}{2}}$: an expression intelligible though cumbrous when read in the language of equivalents, but absurd when read in terms of the atomic theory. This last formula had accordingly to be modified to C_2H_3Cl; and then the original marsh gas had to be supposed to enter invariably into reactions as $2CH_2$, or else its formula must be modified to C_2H_4. The latter is the more natural supposition. It was pointed out (Gérhardt) that throughout the whole of the chemistry of the carbon compounds, similar reasoning shows that if the atomic weight of carbon be 6, the atoms always appear in reactions in even numbers; whence the inference is obvious that the atomic weight of carbon must be 12, and the proper formula of marsh gas is CH_4. In a similar way it was shown that the assumption that the atomic weight of oxygen, as well as its equivalent number, is 8, leads to the invariable appearance of O_2, or of an even number of oxygen atoms, in every equation; whence the atomic weight of oxygen must be 16; and the Atomistic formula for water, as distinguished from the Equivalence-formula, must be H_2O. The Atomistic Formulæ now in use do not directly make use of the idea of equivalence: they denote the number of atoms of which the Molecule — a fruitful idea,

due to Avvogadro — is made up. The symbol H_2O, for instance, signifies a molecule of water, made up of two atoms of hydrogen (at. wt. = 1) and one of oxygen (at. wt. = 16). When attention was first directed to this mode of representation, it was found to be entirely in accord with the half-forgotten researches of Gay Lussac on the relative volumes of gases which enter into combination. He had found that one volume (say 1 cub. cm.) of oxygen and two (2 cub. cm.) of hydrogen unite to form two volumes (2 cub. cm.) of water-vapour. The atomistic equation, on the other hand, is $O + 2H = H_2O$; that is, one *atom* of oxygen unites with two atoms of hydrogen to form a molecule of water. These two statements are closely parallel; and the molecule H_2O formed occupies, in water-vapour, the same space as the original two atoms of hydrogen.

Similarly, it had been found that the electric spark decomposed 2 cub. cm. NH_3 into 1 cub. cm. N and 3 cub. cm. H. The equation was

NH_3	=	N	+	H_3
One molec.		One atom.		Three atoms.
2 cub. cm.		1 cub. cm.		3 cub. cm.

Here again the molecule of the compound, NH_3, occupies, in gaseous ammonia, the same space as two atoms of hydrogen.

So forth; the general rule is that the molecule of any compound in the gaseous state occupies the same space as two atoms of free hydrogen.

Hence we may provisionally establish a general rule, subject to exceptions farther to appear: — If in a chemical equation relating to gases we write "2 vols." under every complete molecule, and "1 vol." under every atom of any element entering into or resulting from the reaction in the free state, we learn the relative volumes of the gases concerned in the reaction. Thus, if alcohol-vapour be burned in oxygen,

Alcohol-vapour.		Oxygen.		Carbonic anhydride.		Water-vapour.
C_2H_6O	+	6O	=	$2CO_2$	+	$3H_2O$.
One molecule.		Six atoms.		Two molecules.		Three molecules.
2 vols.		6 vols.		4 vols.		6 vols.

Thus a system of equations based on the atomic theory is found readily to give important information beyond what it was designed to give. This lends probability to the system.

It is not proved, however, that the combining weights of the elements exactly correspond to the relative masses of single atoms or molecules. If they do, then the number of atoms of each kind, in a given quantity of a binary compound, is precisely the same; and similarly for ternary compounds and so on. But there are actual instances which at least point in the

contrary direction. Pentachloride of phosphorus is decomposed by heat into chlorine and terchloride of phosphorus; but in presence of an excess of chlorine, the dissociation is balanced by recombination. In the oxyhydrogen blowpipe, the highest temperature is attained not by means of 2 vols. of hydrogen and 1 of oxygen, but by 2 vols. of hydrogen and about 1¼ of oxygen. There is thus an excess of oxygen required in order to keep down dissociation. For all that appears, it might have been the hydrogen that would have had to be supplied in excess. If we assume that there is a small action of this kind at ordinary temperatures, the combining weights will give numbers only approximately proportional to the atomic masses, or to multiples or sub-multiples of these.

The Molecule of a compound substance is the smallest mass that can exist in the free state. If we could break up a molecule we would sever it into its constituent atoms —as HCl into H and Cl — but we would destroy the substance on which we operated, as such. A molecule of gaseous hydrochloric acid contains 2 atoms; the various hydrates of $CaCl_2$ contain from 21 to 4500 atoms; a molecule of caoutchouc, of gum arabic, or of aluminic hydrate contains about 6000 atoms; while one of egg-albumin has something short of 30,000, and most typical protoplasmic colloids have (Sabanejew) more than 30,000.

What is the condition of elementary substances in the free state? Here such equations as the following come to our aid:

$$CuH + HCl = CuCl + HH,$$
$$Ag_2O + H_2O_2 = Ag_2 + OO + H_2O;$$

and we learn that the molecule, even of an elementary substance, consists of two atoms, and we find by experiment that it occupies, like the compound molecules already discussed, the same space as two atoms — *i.e.*, one molecule — of hydrogen. All molecules, simple as well as compound, are thus seen each to occupy the same space; and conversely, the same space must be occupied by an equal number of molecules of whatever kind they be. This is the extremely important law known by the name of **Avvogadro's Law.** All gases (at the same temperature and pressure) consist, within equal volumes, of equal numbers of molecules.

This is a general law, and its direct consequence is that the specific gravity of every gas, at a given temperature and pressure, as compared with that of hydrogen under the same conditions, is the relative weight of a molecule of the gas as compared with the molecular weight (=2) of hydrogen. Thus the molecular weight of alcohol, C_2H_6O, is $24+6+16=46$; that of hydrogen $= 2$; the single molecule of alcohol is twenty-three

times as heavy as that of hydrogen, and accordingly the density of alcohol-vapour is twenty-three times that of hydrogen, other things — temperature and pressure — being equal.

There are some apparent exceptions. Mercury-vapour which, if two atoms formed its molecule, would have a molecular weight of 400 and a sp. gr. of 200, has a sp. gr. of only 100; hence its molec. wt. (twice its sp. gr.) is only 200, and its molecule contains only one atom. Cadmium, zinc, potassium, sodium, and bismuth have monatomic molecules when in the state of vapour; so has iodine above 500° C. and under a low pressure. Phosphorus and arsenic vapours have, on the other hand, an excessive sp. gr.; that of phosphorus is 62, and its molec. wt. must be 124; but its at. wt. is only 31; hence its molecule must contain four atoms; at 1600° C. it breaks up, however, into diatomic molecules. The molecule of arsenic is also tetratomic, while that of ozone is triatomic. Sulphur at 500° C. is hexatomic; at 800° C. it is diatomic. Chlorine and bromine vapours partially break up into single atoms at high temperatures.

Hence, to provide for these exceptional instances, we must revise the rule provisionally laid down, and adjust it as follows: To find the relative volumes of gases entering or leaving a reaction, modify the equation, so that it represents no free gaseous atoms, but only complete gaseous molecules; then under every complete gaseous molecule write "1 vol." Thus —

	H_2	+	O	=	H_2O	
becomes	$2H_2$	+	O_2	=	$2H_2O$.	
	Two molecs.		One molec.		Two molecs.	.
	2 vols.	+	1 vol.	=	2 vols.	

The equation

	C_2H_6O	+	$6O$	=	$2CO_2$	+	$3H_2O$
becomes	C_2H_6O	+	$3O_2$	=	$2CO_2$	+	$3H_2O$.
	One molec.		Three molecs.		Two molecs.		Three molecs.
	1 vol.	+	3 vols.	=	2 vols.	+	3 vols.

Again, the equation

$$3CaP_2O_6 \quad + \quad 10C \quad = \quad Ca_3P_2O_8 \quad + \quad 10CO \quad + \quad 4P$$

becomes

$3CaP_2O_6$	+	$10C$	=	$Ca_3P_2O_8$	+	$10CO$	+	P_4.
						Ten molecs.		One molec.
						10 vols.		1 vol.

Another order of exceptions is presented in cases of Dissociation or Thermolysis. When NH_4Cl is volatilised, its vapour has half the sp. gr. indicated by the above theory; in other words, it occupies twice the theoretical volume. This is because a molecule of NH_4Cl is in reality split up into separate molecules of NH_3 and HCl (which may be partly separated by diffusion), each of which occupies the whole space that the original single molecule would have been able to occupy had it not been decomposed by the heat applied. Similarly, a molecule of calomel volatilised occupies twice its normal volume; for instead of a single molecule of Hg_2Cl_2 we have, as the result of dissociation, a molecule of $HgCl_2$, and another molecule, complete though monatomic, of mercury, each of these molecules independently taking up as much space as the original Hg_2Cl_2 would have occupied if it

had not been decomposed; and the result is that calomel becomes, on sublimation, contaminated with bichloride of mercury (corrosive sublimate) and darkened by mercury. Sulphuric acid vapour has twice the theoretical volume; $H_2SO_4 = H_2O + SO_3$. Colourless N_2O_4 dissociates more and more completely into dark NO_2 as the temperature rises. By such apparent exceptions Avvogadro's Law is thus confirmed.

Molecules appear in many instances to be able to combine with one another. We thus have water of crystallisation in crystallised salts, coalescence of molecules in acetic acid to form double molecules which are torn asunder by water or by heat, and probably the various allotropic conditions in which many solids, such as sulphur, carbon, phosphorus, present themselves.

Upon this basis has been erected the modern science of Chemistry, one of the leading auxiliary ideas in which is that of the Atomicity of an atom — the number of atoms of hydrogen which an atom of any element in question can combine with or replace. Whether the special manner of thought and expression of particular chemists be or be not adopted, the theoretical chemist can hardly express himself without making some use of the well-known Graphic Formulæ, by means of which the relations of the atoms in a molecule may be indicated or suggested. Yet this mode of representation is sadly deficient, although exceedingly useful and suggestive. It gives a factitious representation, in one plane, of a statical condition of the molecule: it does not account for the energy possessed by a molecule in virtue of any one arrangement of its atoms, as compared with that possessed by a molecule of an isomeric compound in virtue of another disposition of atoms of the same kind and number; and, indeed, it scarcely touches as yet at any point the physical molecule or atom with which perfect knowledge would presumably show it to be identical. Still, the attempt is being made to bridge over the gap — as, for example, by the researches of Le Bel and Van't Hoff, who trace out such relations as those between symmetry of the molecule, as in the case of propionic acid, $\begin{array}{c} CH_3 \\ | \\ H\text{–}C\text{–}H \\ | \\ COOH \end{array}$, and the absence of rotary power as affecting the plane of polarisation, on the one hand, and on the other between graphic asymmetry of the molecule, as in the case of lactic acid, $\begin{array}{c} CH_3 \\ | \\ HO\text{–}C\text{–}H \\ | \\ COOH \end{array}$, and the possession of this rotary power;

and by those of Wislicenus, who has done much good work in showing how the arrangement of the atoms in the molecule may be more comprehensively represented by tridimensional diagrams. But, on the whole, Chemistry and Physics, which should be parts of one dynamical Science of Matter and Energy, are still separated by a wide gap, and one great stride which the Science of the future has to take is that of assimilating the theories of the physical and the chemical molecules, and thereby stepping over this gulf.

Physical Views.—Physicists have been obliged, independently of chemists, to develope mechanical theories of the Molecule or the Atom, as they have indifferently termed it. That such a thing does exist is manifest to them on several grounds. Not to speak of compressibility and porosity of matter as showing that it does not entirely fill space, we learn from Cauchy's investigations that if light be a wave-motion, there would be no dispersion, no prismatic colours of the spectrum, if the glass of the dispersing prism were continuous or were of a granular structure with indefinitely small grains. According to him, matter must be distinctly granular, whether it be discontinuous or not, and its granulations must not be greatly less in diameter than about $\frac{1}{10000}$ of the wave-length of the shortest wave of light—*i.e.*, about $\frac{1}{20,0000000}$ mm., or about $\frac{1}{500,000000}$ inch. Lord Kelvin finds that there must be from 200 to 600 molecules in one wave-length. He also finds by his Electrometer that plates of copper and zinc exert a certain measurable attractive force upon one another. An indefinite number of plates would multiply this attraction to an indefinite amount; and if such plates were allowed to come together, the heat given out and representing their potential energy of separation would be indefinite, and they would combine after the manner of gunpowder. The energy observed to be given out in the form of heat during the formation of brass by the fusion together of copper and zinc is not indefinite: it corresponds to the mutual attraction of a number of plates not more numerous than 100,000000 to the millimetre. Hence copper and zinc could not be made into plates thinner than this, and plates of this tenuity would be only one molecule thick. A soap film could not be stretched beyond a certain thickness without volatilising, if it be maintained at the same temperature, unless it become materially weakened when a certain limit is attained: for the heat which would have to be supplied in order to prevent it from

cooling upon stretching would be more than sufficient to volatilise it. This limit appears to be reached when a thickness of 1/100,000000 mm. has been attained. Further, considerations derived from the kinetic theory of gases lead to the conclusion that a cubic cm. of solid or liquid contains a number of molecules which, though exceedingly large, is limited; and the distance between these is a quantity of the same order as those above mentioned. From these considerations Lord Kelvin concludes — Thomson and Tait, *Natural Philosophy*, vol. i. part 2, App. F, 1883, and *Nature*, July 1883 (which see specially) — that if a globe of water the size of a football (16 cm. diar.) were magnified to the size of the earth, the molecules or granules would each occupy spaces greater than those filled by small shot, less than thóse occupied by footballs.

But this tells us nothing about the nature of the atoms or molecules. It would at first sight be natural to conceive them as hard balls, but this would not explain their elasticity and mutual action; Faraday regarded them as "centres of force;" Macquorn Rankine as nuclei, each surrounded by an atmosphere in which there are whorls and currents of a complicated character.

The most interesting hypothesis is that of Lord Kelvin, who supposes each Atom of matter to be a Vortex-ring in the universal Ether. The Ether itself we do not directly perceive; but this hypothesis would render our perception of matter a phenomenon of exactly the same order as that of light or radiant heat, viz., a perception of Matter as a Mode of Motion of the Ether.

If one look at a smoke-ring blown from a cannon, from a locomotive-engine chimney, from a tobacco-pipe, the lips of a smoker, or from an exploded bubble of phosphuretted hydrogen, it will be seen that the whole of the matter of the ring is in a state of rotation round an axis disposed in a circular form, and having no free ends. This is a Vortex-ring; and such is that motion in the Ether which is supposed to constitute a **vortex-atom.** A rotating ring of this kind in an imperfect fluid, such as air, must be the result of friction; but in a perfect fluid it could only originate by a special creation of some kind. Such a vortex-atom in a perfect fluid would have the following properties: it could move about in the fluid; its volume would be invariable; it would be indestructible; if struck by another it would be indivisible, but would present perfect elasticity, due

to its motion, and though for the moment distorted, it would recoil and oscillate through its mean form: it would thus be capable of harmonic vibration, as the spectroscope shows the particles of matter to be; it would be capable of changes of form, becoming narrow and thick, or wide and thin; and it is practically the only form of Motion in the Ether which could remain in or near the same mean position, and at the same time be capable of being compounded with movements of translation. This kind of atomic structure would also account for what Tolver Preston calls the open structure of matter, which allows light, electric and magnetic stresses, and the action of gravity, to be transmitted through it. These properties of the vortex-ring explain well many of the observed properties of matter; but knowledge falls short, for we have not only the chemical atom and atomicity, but also physical mass and gravitation to explain before we can form any full theory of the inner structure of the Molecule.

In this view, the atom would consist of a certain quantity of the Ether, possessed of a certain amount of energy. If that be so, it is then conceivable that if we were able to arrest the vortical motion, and thus to destroy the atom, the corresponding energy might be liberated.

The Kinetic Theory.—The next question is, Do these molecules remain at the same spot, rotating round it, or oscillating in its vicinity? or have they, in addition to whatever intrinsic motion they may be possessed of, a motion of Translation? The phenomena of Diffusion help us to arrive at a conclusion on this subject. If a solution of a coloured salt be placed in a vessel, and a layer of a colourless solution be laid upon the coloured stratum, the whole being left at rest for some weeks and protected from all disturbance, the plane of demarcation between the strata becomes blurred, the strata ultimately mix, and the solution becomes uniform. This can only occur through a gradual travelling of the coloured solution into the colourless one, and *vice versâ*.

Again, if a jar of hydrogen and a jar of oxygen be brought into communication with one another, even though the former be uppermost, the gases will perfectly mix in a short time. This shows that the hydrogen rapidly travels into the oxygen, and *vice versâ*. The particles of matter, therefore, cannot be at rest, but must be in perpetual relative motion; and this is the Kinetic Theory of Matter.

Chemical analogy also illustrates this position. If steam be passed over red-hot iron filings, the iron takes the oxygen, and hydrogen passes off; if,

on the other hand, hydrogen be passed over oxide of iron, it forms water-vapour, and reduced metallic iron is left behind. These actions, apparently so contradictory, are explained thus: There is a molecular agitation and a continued process of decomposition and recomposition of chemical molecules; the chemical atoms of iron, oxygen, and hydrogen are constantly changing their partners and forming new molecules; and in the first instance any molecules of hydrogen, in the second any molecules of steam, that happen to be formed are carried off in the current of gas which passes through the apparatus. The particles even of one and the same substance appear to be in this ceaselessly restless state of decomposition and recomposition: when the substance is heated, the molecules are easily broken up, but are not so easily formed again, whence we have the phenomena of Thermolysis or Dissociation; but even at ordinary temperatures the atoms associated within the molecules break asunder, and must but seldom happen to meet each other again. Agitation and break-up thus occurring within the molecules are incompatible with rest, and must necessarily be associated with violent translatory movements of the whole molecules.

Dissociation also takes place within a solution. Sal ammoniac dissolved in water gives up ammonia on boiling.

In a gas, then, we must figure to ourselves a very large number of physical atoms, moving about with great velocity, striking one another and the sides of the containing vessel. Then the energy of any given quantity of gas, so far as that is due to movements of Translation, will depend on the aggregate mass m and on the mean velocity $\bar{v}$ of the particles; and it will be $\frac{1}{2}m \cdot \bar{v}^2$.

This mean velocity is the geometrical mean of all the individual velocities.

If we consider a cube of unit-volume, filled with any gas, and take any one internal face of it; that face, whose area must be unity, is struck by particles travelling with an average velocity $\bar{u}$ in a direction at right angles to that face, or having an average component of velocity $= \bar{u}$ in that direction, and having therefore a certain aggregate momentum. This momentum, with which the particles strike the wall during a unit of time, must be equal to the counter-pressure p exerted by the wall of the vessel per unit of area;* the pressure p exerted by the gas on unit-area of the walls of the vessel is therefore equal to the wallward momentum of the particles impinging on a unit-area of the wall in the course of a unit of time. But what is the amount of this momentum? It is the product of the number of particles which strike the unit-area wall in a unit of time, into the average momentum of each.

1. The number of the striking particles —

   If the gas contain N particles per unit of volume, and if these move towards the wall struck by them with an average velocity $\bar{u}$ per second, the number of particles which must strike the unit-area wall in a unit of time is $N\bar{u}$.

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* This momentum would be lost to the gas within the cube were the particles conveying it not prevented by the counter-pressure of the wall from escaping: this loss ÷ time during which it would have been effected is the Rate of Change of Momentum (see p. 20) prevented during that time, per unit of area of the wall.

2. The average momentum of each, towards the wall of the vessel—
The mass $\bar{m}$ of each particle is the same; the average velocity is $\bar{\mathbf{u}}$; the average wallward momentum of each particle is $\bar{m}\bar{\mathbf{u}}$.

The momentum with which the wall is struck is thus $N\bar{\mathbf{u}}\cdot\bar{m}\bar{\mathbf{u}}$ per unit of area, per unit of time: and this $=\mathbf{p}$, the pressure on the wall per unit of area. But the cube is one of unit volume; its volume $\mathfrak{v} = 1$; the aggregate mass of the gas is $\mathfrak{v}N\bar{m} = \mathfrak{v}\rho$; $\therefore N\bar{m} = \rho$; whence $\mathbf{p} = \rho\bar{\mathbf{u}}^2$.

Next, what is the average velocity $\bar{\mathbf{u}}$ in any one direction? The average speed $\bar{v}$ is, if resolved into components $\bar{\mathbf{u}}$, $\bar{\mathbf{v}}$, $\bar{\mathbf{w}}$, at right angles to one another, $\bar{v} = \sqrt{\bar{\mathbf{u}}^2 + \bar{\mathbf{v}}^2 + \bar{\mathbf{w}}^2}$.

But $\bar{\mathbf{u}}$, $\bar{\mathbf{v}}$, $\bar{\mathbf{w}}$ are equal to one another, for there is no difference between one direction and another in respect of velocity: whence $\bar{v} = \sqrt{3\bar{\mathbf{u}}^2}$, and $\bar{\mathbf{u}} = \sqrt{\bar{v}^2/3}$.

Therefore the pressure per unit of area on the bounding surface, and at right angles thereto, is $\mathbf{p}$ or, generally, in any direction, the hydrostatic pressure per unit of area is $p = \rho\bar{\mathbf{u}}^2 = \rho\cdot\bar{v}^2/3$; and consequently, whatever be the volume $\mathfrak{v}$, the product $p\mathfrak{v} = \rho\mathfrak{v}\cdot\bar{v}^2/3 = m\bar{v}^2/3 = \frac{2}{3}(\frac{1}{2}m\bar{v}^2) = \frac{2}{3}$ the aggregate molecular-translational kinetic energy* of the gas whose mass m is confined within volume $\mathfrak{v}$.

Since the molecular-translational kinetic energy of a mass m of any gas is equal to $\frac{1}{2}m\cdot\bar{v}^2$ ergs, and this is equal to $\frac{3}{2}p\mathfrak{v}$,† where $\mathfrak{v}$ is the volume occupied by the mass m, it follows that this energy is, per gramme, $\frac{3}{2}p\mathfrak{v}/m = \frac{3}{2}p/\rho$; and also, this energy is, per cub. cm., $\frac{3}{2}p\mathfrak{v}/\mathfrak{v} = \frac{3}{2}p$ ergs. Hence this energy is, at the same temperature, equal in equal volumes of all gases.

If two gases have the same Temperature, the particles have the same mean molecular energy $(\frac{1}{2}\bar{m}\bar{v}^2)$ of translation. This is a hypothesis; but if it were otherwise, two gases at the same temperature would change in temperature when mixed; for their average molecular energy would become equalised throughout.

If the aggregate kinetic energy of translation $(\frac{1}{2}m\bar{v}^2)$ be equal in equal volumes of two gases, and if at the same time the molecular energy of the molecules of each be equal (their temperatures being equal), it follows that the number of molecules must be equal in the equal volumes of the two gases, and hence Avvogadro's Law is true in the physical as well as in the chemical sense, being a direct deduction from the kinetic theory.

If there be two gases whose respective densities at equal temperatures and pressures are ρ and $\rho_{\prime}$, Avvogadro's law shows that their unequal masses are divided among equal numbers of molecules: hence the mass of each single molecule must be proportional to the density of its gas; for if $\bar{m}$ and

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* Let, for example, the gas be one gramme of hydrogen, at 0° C. and 76 cm. barometric pressure, and occupying under these conditions a volume $\mathfrak{v} = 11{,}165$ cub. cm.; then $p = 1{,}013{,}363{\cdot}4$ dynes per sq. cm.; whence $p\mathfrak{v} = 11{,}317{,}207000$; and the **Translational** Kinetic Energy of one gramme of hydrogen at 0° C. and 76 cm. barometric pressure is $\frac{3}{2}p\mathfrak{v} = 16{,}975{,}810000$ ergs $= 1697{\cdot}581$ Joules, or $408{\cdot}14$ *ca* (see p. 353), or $1251{\cdot}65$ ft.-lbs.

† We here assume the absence of intermolecular forces. If there be such, independent of collisions, the molecular Kinetic Energy $= \frac{3}{2}p\mathfrak{v} + \frac{3}{2}\Sigma(Rr)$ (Clausius), where the last expression (the "Virial") is half the sum—a sum which for given values of p and $\mathfrak{v}$ retains an appreciably constant value—of the products of the mutual distances r of every pair of particles into the corresponding mutual attractive force R.

$\bar{m}_{,}$ be the respective masses of single molecules of the respective gases, $\bar{m} = \rho\mathfrak{v}/N\mathfrak{v} = \rho/N$, and $\bar{m}_{,} = \rho_{,}/N$; whence $\bar{m} : \bar{m}_{,} :: \rho : \rho_{,}$.

Thus the Molecular Kinetic Theory of Gases explains the **pressure** on the sides of the vessel containing a gas: it explains the tendency of gases to indefinite **expansion**: it explains **Heat** as the energy of molecular agitation; equality of **temperature** as equality of the mean energy of agitation in the several molecules. It also arrives at **Avvogadro's Law**, and explains the numerical identity of ratio existing between the relative weights of the several kinds of molecules and the specific **densities** of the corresponding aggregate gases.

The equation $p = \rho\bar{v}^2/3$ given above yields $\bar{v} = \sqrt{3p/\rho}$, by means of which $\bar{v}$, the **mean velocity** of movement of the particles of any gas, may be found. Thus for hydrogen p, the pressure per sq. cm., is equal to the Weight of say 76 cm. of mercury (density = 13·596), or of 1033·296 grms.-mass resting on every square cm. But the weight of 1033·296 grms. is mg; 1033·296 grms. × 981 = 1,013663·376 dynes. Again, ρ, the density of hydrogen, is ·0000895682 grammes per cubic cm. Hence $\sqrt{3p/\rho} = 184260$ cm. per second, the average velocity of the particles of hydrogen.

Hence also the mean velocities of gases vary inversely as $\sqrt{\rho}$; or, which is an equivalent statement, the mean velocities of the particles of gases vary inversely as the square root of the molecular weight: whence oxygen-atoms have one-fourth the velocity of hydrogen-atoms, because they are sixteen times as massive. This is the law governing the relative speed with which the different components of a gaseous mixture will travel through a membrane.

The kinetic theory also informs us that when we double the number of molecules which move in a given space with a given mean velocity we double the number of molecules which strike the walls, and accordingly we double the pressure; or in other words, the pressure varies directly as the density of a given quantity of gas, this being another form of **Boyle's Law.**

It also tells us that if we mix a particles of one gas, b particles of another, c of a third, and so on, the average kinetic energy of all the particles being the same, or soon becoming equalised, the pressure (per sq. cm. of bounding surface) produced by the a molecules of the first gas is proportional to their number, the pressure produced by the second gas is proportional to b, and so forth; or in other words, that in a mixture of gases the pressure produced by each component of the mixture is independent of the rest, and depends only on the amount of such component which is present in the mixture (**Dalton's Law**).

Again, when the temperature is increased, the energy of the

particles is increased; each particle strikes both oftener and harder; the pressure experienced by the walls of the vessel therefore varies as the square of the velocity, and is proportional to the molecular energy of the particle — that is, to the absolute amount of heat-energy possessed by it. This if the volume be kept constant; but if the pressure be kept constant and the volume allowed to increase, then the volume varies in the same proportion; that is, as the "absolute temperature" (see p. 364). (**Charles's Law,** often attributed to Gay Lussac.)

The kinetic theory of gases also explains how it is that when a stream of gas passes through air, its progress is retarded by "**viscosity**;" rapidly-moving particles of the gas travel laterally into the air; slowly-moving particles of the air travel into the gas, and thus its progress is hampered. Similarly, the viscosity of a gas will bring to rest a current set up within its own substance. This viscosity is proportional to the absolute temperature, but is independent of the density in any given gas.

The theory also explains the **conduction of heat** in gases; rapidly-moving particles, by collision, part with some of their energy to others, which in their turn enter into collision with those beyond them: and we have already seen it explain the **diffusion** of gases.

The mutual impact of elastic solid particles would necessarily result in the ultimate transformation of the whole translational energy into energy of vibration; that of vortex-rings seems to imply no such result. The latter seems, therefore, the preferable form of the kinetic theory of matter, although it is as yet far from complete.

These molecules, thus travelling with such great velocities and entering into a practically infinite number of collisions with one another (in hydrogen 17750 millions per second), can never travel very far in an undisturbed path. At the ordinary temperature and pressure the *mean free path* of the molecules of hydrogen, which have the longest trajectory, seems to be about $\frac{1}{20000}$ mm., or a tenth part of the average length of a wave of light (Maxwell); $\frac{1}{10000}$ mm. (Crookes). The *diameter of molecules* is not the same in the case of all elements, but is on the average perhaps $\frac{1}{2,500000}$ mm. Thus the smallest visible organic particle ($\frac{1}{4000}$ mm. diar.) may contain about 480,000000 atoms, which may be arranged in as few as 16,000 molecules. The *number of molecules* in a cubic cm. of a gas at the ordinary temperature and pressure is about 19,000000,000000,000000 (Maxwell), 1000,000000,000000,000000 (Crookes), not more than 6000,-000000,000000,000000 (Lord Kelvin).

These numbers are arrived at by using a proposition formulated by Clausius, that

$$\frac{8^{*} \times \text{free path of each molecule}}{\text{Diameter of each molecule}} = \frac{\text{Whole space occupied by the molecules}}{\text{Their real aggregate volume}}.$$

From this the real aggregate volume, which does not differ very widely from that of the corresponding liquid, is found, and, if divided by the cubic space occupied by each molecule, gives the number of molecules.

Ultragaseous Matter.—When gas is rarefied the number of molecules in a given space is diminished. Let us suppose that the rarefaction is carried on so far that only one particle out of every original million is left in the space exhausted. The pressure is one-millionth of its original amount; but any molecule once in motion has one-millionth its former chance of encountering any other molecule, and consequently its average free-path is magnified a millionfold. The mean path would then be (Crookes) $\frac{1}{10000}$ mm. $\times 1{,}000{,}000 = 100$ mm., or about 4 inches. By means of a good Sprengel pump exhaustion to a hundred-millionth of an atmosphere can be attained, and the mean free-path of the gas so rarefied would be about 33 feet. In our atmosphere at a height of 210 miles the single molecules are relatively so few (1000 to the cubic cm.), that each molecule might travel through a uniform atmosphere of that density for 60,000000 miles without entering into collision; beyond a height of 300 miles the atmosphere is so rare (less than one molecule per cubic foot) that the particles might freely travel through such an atmosphere from one fixed star to another; while in the fields of space, at distances practically infinite from the earth or any other star, the number of cubic miles containing a single molecule would be represented by the figure 1 followed by 314 cyphers.

This opens up to us an extraordinary view of the nature of our atmosphere. We must,—though the process cannot be rapid, for each particle rising from the earth is retarded by gravity and falls back towards the earth,—constantly be losing particles of nitrogen and oxygen as we are dragged through space, and we may constantly be picking up new ones. If we entered regions of space in which there were no particles fit to make up our losses, it would be an interesting question how short a time would suffice altogether to deprive us of our atmosphere.

The region of space through which the earth is at present travelling contains much benzene vapour with ethyl-hydride and other alcohol-derivatives (Abney).

Thus our ideas on the subject of the constitution of matter have undergone a profound modification. Matter is discontinu-

* $\sqrt{72} = 8.48$, Clerk Maxwell; 8.86, Tait.

ous in the highest degree, for it consists of separate particles or molecules, which are mutually non-interpenetrable; the special properties of the different states of matter depend on the number of molecules which are contained within a given space, as well as on the energy of movement which is possessed by each; and each particle is susceptible not only of translation as a whole, but also of vibration or rotation, and may besides be in a state of vortex-motion, upon the continuance of which its continued existence may depend.

Molecular Forces.

Hitherto we have conducted our reasoning on the implied assumption that the molecules had no mutual action, and we have arrived at results such as Boyle's law, Dalton's law, and others, which we have deduced from theory. Now we must confirm our theory by reference to facts, and we find this assumption overruled by such material discrepancies as the following. Boyle's law, though obeyed on the whole, is disobeyed by every gas when the pressure is so high or the temperature so low that condensation is not far off: this departure, though not extensive, is significant. All gases just about to become condensed are, except in the single case of hydrogen, more easily compressed than the law would indicate. Dalton's law is departed from by a mixture of gases condensible with difficulty: such a mixture is found to be even less condensible than the component gases, and the critical temperature is lowered. Charles's law is not obeyed throughout the whole range of experimental pressures and temperatures; at a high pressure any increment of heat produces a disproportionately large increment of pressure.

In fact, Gases obey these laws only when their pressure is very feeble and their temperature at the same time high above the critical temperature — that is, when the molecules are comparatively far from one another. At ordinary temperatures and pressures the particles do affect one another not merely by mutual impact or mutual gravitation, but also by mutual actions or molecular forces, effectively coming into play when the particles are at exceedingly small distances from one another. When the pressure is small, the free path is comparatively long, and the molecules are mutually removed from each other's influence: and the higher the rate at which the particles are moving, the less will be the proportionate effect of the molecular forces;

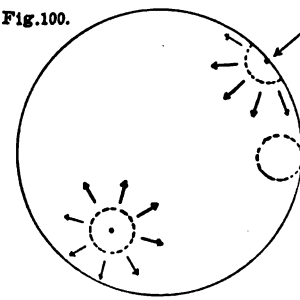
or in other words, the higher the temperature the less appreciable will be the effect of intermolecular action.

When a gas is being compressed into a liquid we know that in the first place, in all gases except hydrogen, the particles are attracted slightly towards one another, and also that there is on the other hand a practical repulsion from one another caused by their energetic movement. We further find, however, that though the particles become approximated with relative ease while liquefaction is approaching, yet when the liquid state has been attained, and even before it has been attained, repulsion takes the place of attraction; the liquid when formed offers a relatively enormous resistance to compression. This is well seen in the case of carbonic anhydride merging insensibly from the gaseous into the liquid state; just before ceasing to be a gas it is very compressible; just after becoming a liquid it is relatively very slightly so.

Air obeys Boyle's law precisely, and the air manometer is therefore correct, at a pressure of 200 atmospheres; below that pressure the volume is in defect; above, it is in excess (Andrews). (See, also, p. 375.)

In Liquids the molecules are within the spheres of one another's action. This accounts for the viscosity of all, even of the most mobile liquids: the particles detain one another by their mutual attraction, and a flowing liquid is thus hindered in its flow by molecular friction. Molecular action also accounts for the fact that a stream of liquid has a certain tenacity and will not readily break: such is the condition of a stream of liquid in a siphon. Again, it explains why, under ordinary circumstances, the effects of molecular attraction are strikingly manifest in liquids only at the surface, and in the form of Surface-Tension. In the interior of a mass of fluid each particle is free to adjust its mean position under the influence of the surrounding molecules; the mean position which it assumes is that in which it is acted on equally on all sides, and there is then nothing to render the mutual attractions manifest. At the surface of a liquid mass, however, if it be a free surface, the particles can only be acted upon by others lying internal to them.

Fig. 100.



The result is, as is shown in Fig. 100, a system of forces acting at right angles to the free surface of the fluid, and tending to

reduce that free surface to the least possible area. We may, indeed, consider a drop of water as consisting of a quantity of water enclosed in a superficial skin of water which is under tension, and whose particles attract one another into the least possible superficial area; and since of all surfaces the sphere has the greatest content for the least area, the superficial film may be said to mould the drop to the spherical form, which in the case of falling raindrops is approximately perfect, as is shown by the rainbow. To these surface-tensions are also due the important phenomena of Capillarity.

Many of the properties of Solids are also due to molecular forces. Such are toughness, hardness, and the like, which may be grouped under the generic name Strength of Materials; these depending probably, in part at least, on the proximity of the particles to one another, and perhaps on their form. The molecular grouping of molecules is also very important, though very little can be said about it; but upon it depend not only the crystalline or amorphous condition of a substance and in part its strength, but also that stable or unstable equilibrium upon which the phenomena of elasticity or the properties of such things as Rupert's drops depend. These last consist of little masses of fused glass dropped into cold water; the outside is suddenly chilled and solidified while the interior is still in a state of fusion. The internal mass has to accommodate itself as it best can to the dimensions of the outer skin; it does so under tension, but the moment that the relations are disturbed by breaking off the narrow end, or even by the slightest scratch, the whole flies to powder: it is in a state of unstable equilibrium, and the slightest displacement precipitates a downfall of the whole arrangement. In the same way the slightest scratch in the interior of a large glass tube, especially if it have local variations of thickness, — even such a scratch as is produced by the fall of a crystal of quartz or a rub with the end of an iron wire, — will often shiver the tube; for which reason no rough treatment should be internally applied to such tubes with any metal harder than copper. This state of internal tension accounts for the danger in the use of cast-iron in structures.

Many solid masses have, however, their particles so arranged as to form Conservative Systems, which tend to restore any work done on them, and consequently are in stable molecular equilibrium; the details of the molecular grouping are unknown, but in a perfectly elastic body, or practically in any solid body

within its Limits of Elasticity, any displacement among the molecules produces a restitution-pressure equal and opposite to the distorting force or stress; and it is observed that, as a general rule, the distortion is proportional to the distorting force (*Ut tensio sicut vis;* "Hooke's Law"), and hence the restitution-pressure is proportional to the distortion. This elasticity may in solids be observed more or less perfectly to obtain, whether the distortion be that of form or of volume; while liquids have elasticity of volume alone, never of form.

To the same order of Molecular Forces must be attributed the effects of Cohesion between masses or particles of the same substance, and of Adhesion between those of different substances; and also the phenomena of Chemical Affinity, the potential energy of chemical separation, and the liberation of energy attendant on chemical combination.

CHAPTER X.

SOLIDS.

THE special properties of solids are due to the relative contiguity of their molecules. Their definite free surface is due to the mutual attraction of their molecules, and is retained in virtue of the same forces which in the aggregate manifest themselves as causes of cohesion, tenacity, etc., and the result of which is that a solid can persist under the action of a stress not evenly applied — that is, of a stress which is not hydrostatic.

Still, the particles appear to have some power of travelling past one another, though not much; carbon soaks into iron in the old "cementation" steel-making process; it can also travel through porcelain; in crystals made up of successive layers of different alums the different layers become more or less blended; in nickel-plating steel plates for printing purposes, the nickel sinks into the steel to some depth; powders of potassium nitrate and sodium acetate, when mixed and compressed, form hygroscopic nitrate of sodium and potassium acetate; and some solids are, to a small extent, affected by an electric current as if they were liquids containing moveable molecules (Electrolysis). The phenomena of Magnetism point towards the molecules of iron being turned round when the iron is magnetized.

Cohesion is the mutual attraction of the particles of a solid for one another, and is measured by the amount of force which must be applied in order to overcome it. The term *cohesion* is generally applied to the mutual attraction of particles of the same substance, *adhesion* to that of different substances. When two pieces of white-hot iron or platinum are brought in contact they weld by cohering. When a piece of silver and a piece of platinum are brought in contact at 500° C. they adhere. If metals in the state of dust be mixed and exposed to a pressure of 7000 atmospheres they will form a firm metallic mass, and will even combine and form an alloy. Even sulphides and arsenides may thus be formed; for pressure promotes contact. Cohesion is manifested by two surfaces of glass, which, if ground exceedingly smooth and placed in contact, will cohere firmly; and the well-known Barton's cubes are little cubes of metal pol-

ished so smoothly that mere apposition causes them to cohere, the force of cohesion being so great that a string of a dozen may be supported in the air by this mutual attraction alone. Common graphite is ground to powder and purified by boiling with nitric acid and chlorate of potash: it is then washed and dried; the powder is placed in a mould and exposed to extreme pressure produced by a hydraulic press; after compression the black powder is found to have been converted into a solid mass of coherent pencil-graphite, which may be sawn into strips and used for pencils. If a leaden bullet be cut into two with a sharp and heavy knife, the two halves will cohere firmly if pressed together by their bright surfaces.

Hardness — Softness. — A body is said to be harder than another when it can be used to scratch the latter but cannot be scratched by it. In this sense the diamond is the hardest of all solids. The scratching body must not have too sharp a point, for this would prove a pin to be harder than glass, which is not the case. Hardness is a property that cannot be measured. All that we can do is to make up a list of substances in their relative order of hardness, and to express the hardness of any particular substance by stating its place in that series. The standard series, due to Mohl, is the following: —

1. Green laminated Talc. 2. Crystallised Gypsum. 3. Transparent Calcspar. 4. Crystalline Fluorspar. 5. Transparent Apatite. 6. Pearly cleavable Felspar (Adularia). 7. Transparent Quartz. 8. Transparent Topaz. 9. Cleavable Sapphire. 10. Diamond. Flint scratches quartz with difficulty, but is easily scratched by topaz: hence its hardness is set down as 7·25 on this arbitrary scale. The rapidity of movement of the attacking substance is a matter of practical importance: thus the sandblast (a stream of sand rapidly blown from a tube) is capable of cutting through rocks and even through steel with relatively great rapidity; and the same result is seen in the mechanical operation of filing.

Mr. Edison finds that platinum wire may be rendered as hard as steel pianoforte wire by heating *in vacuo*, keeping up the vacuum, and gradually increasing the temperature. The particles of platinum have all air removed from their interstices, they cohere very firmly, and the metal welds together.

Hardness — Fragility. — This is a distinct use of the word Hardness. In this sense the diamond possesses little hardness, for if struck a blow with a hammer it flies to pieces.

Malleability, the property of yielding to the hammer without breaking at the edges. — Gold can be hammered out into leaves extremely thin. A half square-inch of gold of the thickness of letter paper is hammered out to 81 square inches; each square inch of this thin sheet is again hammered out into 81 square inches, of which each one is in its turn again hammered out to 81 square inches. Antimony, on the other hand, flies to powder at the first blow of the hammer.

Plasticity. — Some solids can be moulded, as lead in a bullet-mould, coins at the Mint, bars in a rolling-mill. Any force above a certain limit produces a permanent set. Plastic solids, under pressure, follow the laws of the motion of liquids.

Ductility. — Some metals can be drawn through fine apertures in a draw-plate, and wires can thus be formed: other metals are incapable of this, for they snap. The order of ductility is — Gold, Silver, Platinum, Iron, Copper, Palladium, Aluminium, Zinc, Tin, Lead. Platinum wires of exceeding tenuity, such as are adapted to the eye-pieces of microscopes for micrometric work, are made by constructing a thick silver bar with a thin platinum core, drawing this out to an extreme fineness, and dissolving off the silver by steeping the drawn wire in nitric acid.

Resistance to Deformation. — When a solid undergoes deformation under the influence of an applied force, a condition of equilibrium is ultimately reached, and is thereafter maintained, except in so far as excessive forces or protracted duration of the experiment may bring to view the fact that there is always a slow yielding, even of the most rigid solids, under continuously applied forces. This equilibrium is one between the Force acting and an equal and opposite reaction or counter-force or **Resistance** developed by the body during the deformation; and this resistance is so developed, whether the force applied be tensile, compressive, shearing, or torsive.

Cubical Compressibility. — Most solids are only slightly compressible under the action of a pressure equably applied to their whole surface. Their Resistance to Compression is called their Elasticity of Volume.

When a uniform pressure, p dynes per sq. cm., applied to the whole surface of a solid, reduces its volume $\mathfrak{v}$ to $(\mathfrak{v} - \delta\mathfrak{v})$, the Compressibility is measured by the proportionate change of volume, $\delta\mathfrak{v}/\mathfrak{v}$, produced per dyne of applied pressure per sq. cm. It is therefore equal to $\delta\mathfrak{v}/\mathfrak{v} \div p$. The Resistance to Compression, or the Elasticity of Volume, is the reciprocal of this, and is $p \div \delta\mathfrak{v}/\mathfrak{v} = \mathfrak{k}$. The work done on compression is $\frac{1}{2}p \cdot \delta\mathfrak{v} = \frac{1}{2}\mathfrak{k} \cdot (\delta\mathfrak{v})^2/\mathfrak{v}$.

Shearability varies greatly in solids; compare steel and indiarubber; the latter is much more readily pressed out of shape.

Shearability is measured by the Shear ($= \tan\theta$, Fig. 25) produced, per unit of Tangential Force applied per unit of area, when AB and CD (Fig. 25) are one unit distance apart; that is, it is $\tan\theta./\mathfrak{f}$. The reciprocal of this is the **Rigidity**, the Resistance to Transverse Distortion, $\mathfrak{n}, = \mathfrak{f}/\tan\theta$: a shearing force $\mathfrak{f}$ will produce a shear $\tan\theta = \mathfrak{f}/\mathfrak{n}$; and to prôduce a given shear, $\tan\theta$, the shearing force applied per sq. cm. must be $\mathfrak{f} = \mathfrak{n}\cdot\tan\theta$, proportional to the rigidity $\mathfrak{n}$.

The amount of Work done in producing a given shear, $\tan\theta$, in a layer whose thickness is d, is equal to the product of the average Resistance, $\mathfrak{f}/2$, overcome by the shearing force $\mathfrak{f}$ ($= \mathfrak{n}\cdot\tan\theta$), into the space ($= d\cdot\tan\theta$) through which the moving plane AB (Fig. 25), or CD (Fig. 99), is displaced parallel to itself; that is, it is $\frac{1}{2}\mathfrak{n}\cdot d\cdot\tan^2\theta = \frac{1}{2}\mathfrak{f}\cdot d\cdot\tan\theta$, for each sq. cm. of the moving plane AB or CD so displaced; or $\frac{1}{2}\mathfrak{n}\cdot\tan^2\theta = \frac{1}{2}\mathfrak{f}\cdot\tan\theta$ for each cub. cm. of volume so sheared.

Extensibility — Inextensibility. — Some substances can, like indiarubber, be extended greatly by the application of a stretching force or Traction: others, like baked clay, very little. When bodies are so treated, they mostly become thinner at the same time. The ratio of the Elongation produced to the Traction $\mathbf{t}$ is the "extensibility"; this Elongation being measured by the ratio of the increase in length to the original length.

Under a given longitudinal traction, $\mathbf{t}$ dynes per sq. cm. of cross-section, E, the actual lengthening, is equal to $l\cdot\mathbf{t}\cdot(1/3\mathfrak{n} + 1/9\mathfrak{k})$, where l is the length of the rod or wire in centimetres; the Elongation E/l is therefore $\mathbf{t}\cdot(1/3\mathfrak{n} + 1/9\mathfrak{k}) = \Lambda\cdot\mathbf{t}$, where Λ is the Coefficient of Extensibility; or it is $\Lambda\mathbf{T}/o$, where $\mathbf{T}$ is the Total Tension applied, in dynes, and o is the cross-section of the rod or wire employed.

At the same time, the rod thins out, unless it be like cork, exceptionally compressible; its transverse measurements are all diminished by $\mathbf{t}\cdot(1/6\mathfrak{n} - 1/9\mathfrak{k})$ cm. per linear centimetre. The ratio of this proportionate Contraction per cm. to the simultaneous proportionate Elongation is $(3\mathfrak{k} - 2\mathfrak{n}) \div (6\mathfrak{k} + 2\mathfrak{n})$, which is called Poisson's Ratio.

When $\mathbf{t} = 1$, $E/l = \Lambda$; Λ therefore measures the Elongation produced by a Traction of one dyne per sq. cm. cross-section. If, on the other hand, $E = l$, $\mathbf{t} = 1/\Lambda$; and $1/\Lambda$ then measures the Traction per sq. cm. which would be necessary in order to double the length of the rod or wire, if that strain could be effected without rupture.

In Cast Steel, tempered,	$\Lambda =$	$\{1 \div (2520,000000 \times 981)\}$
Wrought Iron	„ „	$\{1 \div (2000,000000 \times 981)\}$
Copper	„ „	$\{1 \div (1050,000000 \times 981)\}$
Wood	„ „	$\{1 \div (10,000000 \times 981)\}$
Leather	„ „	$\{1 \div (175000 \times 981)\}$
Fresh Bone	„ „	$\{1 \div (230,466000 \times 981)\}$
Tendon	„ „	$\{1 \div (16.341000 \times 981)\}$
Nerves	„ „	$\{1 \div (1,890000 \times 981)\}$
Living Muscle at Rest	„ „	$\{1 \div (95000 \times 981)\}$
Arteries	„ „	$\{1 \div (5200 \times 981)\}$

Problem.

How many grammes' weight would be necessary in order to double the length of a piece of steel wire 1 sq. mm. in cross-section, if that were possible? Here $\mathbf{t} = E/l\Lambda = 1/\Lambda$, for $E = l$; therefore $\mathbf{t} = (2520,000000 \times 981)$ dynes: and $\mathbf{t} = \mathbf{T}/o$; whence the Total Tension required is $\mathbf{T} = \mathbf{t} \cdot o = 0{\cdot}01\mathbf{t} = 0{\cdot}01 \times (2520,000000 \times 981)$ dynes = the weight of $\mathbf{T}/g =$ 25,200000 grammes.

French engineers are in the habit of reducing these inconveniently large physical constants by expressing extensibility in terms of the number of kilos.' weight which would be required to double the length of a bar whose sectional area is one square millimetre: the resultant numbers are $\frac{1}{100000}$ of those obtained when the extensibility is measured in terms of the number of grammes' weight which would be required to double the length of a bar whose sectional area is one square centimetre.

Muscles are more extensible when they are in a state of contraction than when they are at rest; and if a muscle when loaded by a certain weight be stimulated to contraction, the mere effort to contract may so diminish the resistance to extension or increase the extensibility that the contracting effort may be more than counterbalanced by the mechanical stretching of the muscle produced by the weight hanging upon it, and the overloaded muscle may actually stretch when stimulated to contract. Muscles also become a little less resistant or more extensible, under a given load, shortly after death.

There is no substance of which wires or rods could be loaded with indefinite weights, or even with such weights as would double the length: there is for each substance a special limit of tenacity or cohesion, when extension can go no farther, and the rod is ruptured. This **breaking weight,** per sq. cm., measures the cohesion.

According to Wertheim, bone ruptures when 800,000 grammes are suspended on it, per sq. cm. of its cross-section; tendon, 625,000; nerve, 135,100; veins, 18,500; arteries, 13,700; muscle, 4500. Thus a nerve whose section is ¼ sq. cm. could bear a stretching force equal to the weight of 33·7 kilogrammes or over 5 stone; but the danger of stretching an artery or a vein by mistake is obvious. There is a great difference in the breaking weight of the same tissue in persons of different age and habit. Wertheim found that the fibula of a young man of thirty had a breaking weight of 1,503,000 grammes per centimetre, while that of the same bone in an old man of seventy-four was reduced to 432,500.

One of the highest breaking-weights is that of steel pianoforte wire. Wire 1 mm. in diameter may sustain a pull equal to 142 tons per sq. inch, or 22,120000 grammes' weight per sq. cm.

The reciprocal of the Coefficient of Extensibility, Λ, is $1/\Lambda$, the **Coefficient of Resistance to Extension,** or **Young's Modulus, $\mathfrak{y}$**; it is the fraction (Longitudinal Traction per sq. cm. of cross-section) ÷ (the Elongation produced).

This is $(1/3\mathfrak{n} + 1/9\mathfrak{k})^{-1} = 9\mathfrak{n}\mathfrak{k}/(3\mathfrak{k} + \mathfrak{n}) = \mathfrak{y}$; and in order to produce a given lengthening or extension E in a rod of a given original length l, we must apply a longitudinal traction $\mathbf{t} = \mathfrak{y} \cdot E/l$ dynes per sq. cm. of cross-section.

In steel, for example, $\mathfrak{y} = 1/\Lambda = (2520{,}000000 \times 981)$, in dynes per sq. cm.; and this is equal to the Weight of $\mathfrak{y}/g = 2520{,}000000$ grammes' mass suspended per sq. cm. cross-section; or to the weight of $\mathfrak{y}/g\rho = (2520{,}000000 \div 7{\cdot}8)$ linear cm. of the wire which is being experimented upon, whatever be its cross-section.

The Work done in producing extension is the average Resistance × the space through which it is overcome; and this is equal to half the product of the Total Tension into the Extension.

If a rod be exposed to a traction $\mathbf{t}$ dynes per sq. cm. of its cross-section, stretching will go on until the ultimate resistance arrived at is in equilibrium with the traction. When this is the case, $\mathfrak{y} \cdot E/l = \mathbf{t}$. The average resistance encountered by the traction is half this, or $\frac{1}{2}\mathfrak{y} \cdot E/l = \frac{1}{2}\mathbf{t}$, per sq. cm. of cross-section of the rod. The space through which the resistance is overcome is $s = E$. The work done = average resistance × space = $\frac{1}{2}\mathfrak{y} \cdot E^2/l = \frac{1}{2}\mathbf{t} \cdot E$, per sq. cm. of cross-section of the rod; or $\frac{1}{2}\mathfrak{y} \cdot o \cdot E^2/l = \frac{1}{2}\mathbf{T} \cdot E$ for the whole rod, of cross-section o.

Linear Compressibility follows the same laws as extensibility. Within narrow limits the coefficients of compressibility and of extensibility have the same value, Λ. Excessive compression leads to crushing, by lateral dilatation; and each substance has its own Crushing Weight, found by experiment on masses of determinate size.

Flexibility.—In every rod undergoing flexion, if this be due to the weight of a mass suspended from a free end, there must be a certain extension of the upper aspect of the rod, a compression of the lower, and a Neutral Line between, which retains its original length. If the flexure be due to weight pressing down the middle of the rod which is supported at its extremities, the extension is in the lower aspect of the rod, the compression in the upper. In the former case a cut in the upper aspect would weaken the rod; in the latter the same effect would only be produced by a cut on the lower aspect. Flexion may bring about compression and extension beyond the range of the breaking or crushing strengths, and the body may thus be broken. If this occur before there has been any perceptible flexion, the body is said to be **brittle**: if it allow a considerable range of flexion it is said to be **tough**—it bends much before breaking. The crystalline or granular or fibrous structure of a substance has much to do with its brittleness or toughness. For example, tin, which is very crystalline, is very brittle; wrought-iron axles,

which are at first fibrous and very tough, are subject to a molecular rearrangement facilitated by vibration, and become crystalline and brittle.

The amount of Flexure of a rod depends upon Young's Modulus. Thus, if a beam, supposed weightless, be fixed at one end, and if its free end be loaded with a mass m; then, if its length be l, its horizontal breadth b, and the vertical depth of its rectangular section be d, the free end will descend through a height $h = 4mgl^3/bd^3\mathfrak{y}$.

Torsibility of a solid may be measured in the simplest case — that of a rod or wire — by specifying the *angle* through which a unit force, applied at a distance of 1 cm. from the axis of the wire, can twist it. This angle is $1/\mathfrak{t}$; and it is inversely proportional to the fourth power of the radius of the wire. Its reciprocal, $\mathfrak{t}$, is the **resistance to torsion.**

This angle, $1/\mathfrak{t} = 2l/\mathfrak{n}\pi a^4$, where l is the length of the twisted wire, a its half diameter, and $\mathfrak{n}$ its coefficient of rigidity to transverse distortion.

If a force F be applied at the end of a lever r, the torque or twisting moment is Fr, and the angle of twist becomes $\theta = \mathrm{F}r \cdot 2l/\mathfrak{n}\pi a^4 = \mathrm{F}r/\mathfrak{t}$. Hence, $\mathrm{F}r = \mathfrak{t}\theta$. Conversely, if such a wire is to be twisted through an angle θ, the torque to be applied must be equal to $\mathfrak{t}\theta$, whatever the distance of the point of application of the twisting force F may be.

The work done in producing torsion θ is $\frac{1}{2}\mathfrak{t}\theta \cdot \theta = \frac{1}{2}\mathfrak{t}\theta^2 = \frac{1}{2}\,\mathrm{F}r \cdot \theta$.

If a bar, loaded so that its Moment of Inertia is N, be suspended horizontally at its midpoint by a wire, and if it be turned round its point of support through a horizontal angle θ, it will oscillate with a period $\mathrm{T} = 2\pi\sqrt{\mathrm{N}/\mathfrak{t}} = 2\pi\sqrt{2\mathrm{N}l/\mathfrak{n}\pi a^4}$, where $\mathfrak{t}$, l, $\mathfrak{n}$, and a are data pertaining to the suspending wire, twisted by the oscillation.

A bar suspended at its midpoint by a wire capable of twist, and acted upon by a torque or twisting moment Fr, will rotate and cause the lower end of the wire to rotate with it; if, however, the upper end of the wire be at the same time twisted in an opposite sense, to so great an extent that the reverse twisting moment due to the torsion θ of the wire itself becomes equal to Fr, there is then no change in the position of the suspended bar at the lower end. The counter-force at the point of application of the force F to the bar is $\mathfrak{t}\theta/r$; this is numerically equal to F, which it holds in check. Any other force F′, similarly applied, would be equal to $\mathfrak{t}\theta'/r$; whence $\mathrm{F}:\mathrm{F}'::\theta:\theta'$; and Forces may be compared by observing the ratio between the angles of opposite twist that must be given to the one end of a wire or fibre in order to prevent those forces, similarly and successively applied, from causing twist at the other end.

By means of fibres of quartz $\frac{1}{1000}$ cm. thick, obtained by suddenly drawing asunder a drop of melted quartz, Professor Vernon Boys has repeated Cavendish's experiment (p. 202), and measured the attraction between masses of 800 grammes and 1 gramme, at such a distance that this amounts to about $\frac{1}{150000}$ dyne, or the weight of $\frac{1}{154,000000}$ gramme. With fibres $\frac{1}{6\cdot 54}$ times this diameter, it would be easy to measure the attraction between two No. 5 shot, or $\frac{1}{2000}$ of the former amount. These fibres are very perfectly elastic, are unaffected by moisture, and have extreme tensile strength.

The Dimensions of $\mathfrak{k}$, $\mathfrak{n}$, and $\mathfrak{y}$ are $[\mathrm{M/LT^2}]$; those of $\mathfrak{t}$ are $[\mathrm{ML^2/T^2}]$.

Elasticity. — "Elasticity is the property in virtue of which a body requires force to change its bulk or shape, and requires a

continued application of the force to maintain the change, and springs back when the force is removed; and if left at rest without the force, does not remain at rest except in its previous bulk and shape "(Lord Kelvin).

Power of Restitution. — There are two properties, **Resistance** and **Restitution,** which must concur in any given body before it can be said to be elastic. Resistance is not the only criterion of Elasticity. A body may resist extension, compression, torsion, shear, and yet not be elastic. In order that it may be perfectly elastic, it must have all the following properties: —

(1.) It must offer a definite resistance to distortion.

(2.) The distortion is not permanent, and if the deforming force be removed, the distorted body springs back to its original form or bulk.

(3.) The distorting force must be continuously maintained in order to keep up the distortion.

(4.) So long as the distorting force is kept up, there is a counter-pressure or **restitution-pressure** (P) developed and sustained in the elastic substance. As this holds the deforming force (F) in check, and is in equilibrium with it, thus setting up a condition of stress in the substance, it must be numerically equal and opposite to it; $P + F = 0$.

(5.) The restitution-pressure does not become diminished by lapse of time.

In both extension, compression, shear, and twist, the Restitution-Force is opposite and equal to the Displacing Force. Thus, in Extension, $P + \mathfrak{T} = 0$, or $P = -\mathfrak{T}$; *i.e.*, $P = -\mathfrak{y} \cdot E \cdot o/l$, or $P \propto -E$. If the distortion $(E \cdot o/l) = 1$, $P = -\mathfrak{y}$, and the restitution-force is represented by a number, the Coefficient of Restitution (or "coefficient of elasticity"), which is equal to the Coefficient of Resistance to Extension. In general, when the Deformation ($\tan\theta$, $\delta\mathfrak{v}/\mathfrak{v}$, E/l, or the angle of torsion θ) is unity, the restitution-force per unit of area (or, in the last case, the restitution-torque) is represented by the coefficient of resistance to deformation, $\mathfrak{n}$, $\mathfrak{k}$, $\mathfrak{y}$, or $\mathfrak{t}$, as the case may be; and when the Deformation has any other value, the restitution-force per unit of area (or, in the case of torsion, the restitution-torque) is equal to the product of that value into the corresponding Coefficient of Resistance. Hence the restitution-pressure on any particle is proportional to the Displacement of that particle, and is oppositely directed.

Under any given distortion within the limits of restitutive power, the restitution-pressure is equal to the product of the Coefficient of Restitution into the distortion; the coefficient of restitution being numerically identical with the reciprocal of the deformability. It is usual to profess to measure the elasticity of a solid by a "*coefficient of elasticity*," which is stated

to be equal to the resistance to distortion. There is an equality, a numerical identity, between the Resistance to distortion and the Coefficient of Restitution (upon which the amount of restitution-pressure depends), provided that any one system of units be strictly adhered to, that the body be perfectly elastic, and that the distortion be unity. It seems, however, strange to set up a method of measuring elasticity based on a tacit fundamental assumption that the bodies dealt with are perfectly elastic.

If there be two bodies, of which one has a low, the other a high coefficient of restitution, and if the same displacement be effected in both, the restitution-pressures in the two substances differ in the same ratio as their respective coefficients: and in these two bodies the relative amounts of work stored up in the form of tensional or potential energy also differ in the same ratio. Elasticity thus presents two aspects, the Statical and the Dynamical. On liberation of a strained body, the whole of the energy stored up in it may be restored in the kinetic form.

This restitution may be due to the solid body being a conservative system of particles, a small displacement amongst which acts as a disturbance of masses in stable equilibrium: by such a displacement an aggregate force is called into action which tends to produce restoration to the original form or bulk. In an elastic body the greater the displacement or distortion the greater the restitution-pressure, and that in direct proportion (**Hooke's Law**).

Perfect and Imperfect Elasticity. — A body is perfectly elastic when any given stress produces no permanent set or deformation, restitution being always complete. It is imperfectly elastic when it does permanently retain such a set. It is said to be strained beyond its **Limits of Elasticity** when it is so far strained that it retains such a set: it is said not to be strained beyond its limits of elasticity when it is not deformed so far that it cannot exactly return to its original form or bulk. When the limits of Elasticity are narrow, as in the case of lead (which, though exceedingly easily bent so as to take a permanent set, can yet be induced to enter into vibration, and must therefore be elastic within narrow limits), the body is again said to be "imperfectly elastic," or to possess little **elastic toughness.** When it can suffer distortion within wide limits without taking up a permanent set, it is said to have great elastic toughness; and a body which has infinitely wide limits of elasticity is said to be perfectly elastic. There is no body perfectly

elastic, but any body may within the limits of its elasticity be considered as a perfectly elastic body.

In popular language a body is said to be very elastic when it has, like indiarubber, great elastic toughness—*i.e.*, when it can be distorted through wide ranges without taking up a permanent set; but this use of the word should be discouraged in favour of that use in which it is made to signify conjoined powers of Resistance to deformation, and of Restitution of shape, of bulk, and of work done upon the elastic object.

The elastic toughness exemplified in a Toledo sword-blade must be distinguished from the ordinary ultimate toughness or breaking toughness; the former may be much less than the latter.

Residual Restitution with **Deferred Restitution-pressure.**—When a body which has been distorted is left to itself without vibration, it may, when it has come to rest, be fixed between supports; it then exerts no elastic pressure; but in the course of a little time it will be found to be exerting an elastic pressure which has been in the meantime slowly developed, and which tends to restore the body more nearly to its normal condition. Mechanical disturbances—such as vibration, shaking, jarring, etc.,—which allow the molecules to glide past one another, facilitate the development of this deferred restitution. Boltzmann found that successive torsions, differing in amount and in sense, caused the subsequent successive emergence of deferred restitution-pressures whose order of succession was the inverse of that of the torsions which had given rise to them.

Vibrations due to Elasticity.—When a body is distorted, not beyond the limits of elasticity, and liberated, the work done upon it is restored. The body exactly regains its original form or bulk, but at the moment of complete restitution the energy possessed by the body (if perfectly elastic) has wholly assumed the kinetic form, and the body passes rapidly, if it be free to do so, through its mean form or bulk, and suffers an equal distortion or alteration of volume in the opposite sense. Again a restitution-pressure is developed, and the body swings back through its mean position. This is repeated, and thus we have vibrations produced as the result of elasticity. The force bringing back every particle towards the mean position is proportional to the displacement from that mean position, and this is the criterion of harmonic motions. Hence in a solid body, which is a system of particles, any displacement sets up an intermolecular restitution-pressure, which results in harmonic motion (Fourier-motion) of the separate particles, and the particles of a disturbed tuning-fork or stretched string may execute harmonic vibrations, particles equidistant from one another generally assuming equal

differences of phase in their respective S.H.M.'s. The whole body executes, like a pendulum, isochronous vibrations; as, for example, the vibrating mainspring of a watch.

Viscosity of Elastic Solids.— When an elastic body has entered into vibration it appears more or less rapidly to lose its energy; its vibrations wane away. This waning away is due to the "*viscosity*" of the solid: the energy of vibration becomes converted into heat. The amplitude of each successive oscillation bears to that of the one immediately preceding a constant ratio. If the amplitudes of the first and second oscillations be $1 : a$, the third will be a^2, the nth will be a^{n-1}. On account of this viscosity a tuning-fork cannot be made of lead or zinc, the vibrations of which too rapidly die away: but even pipe-clay can slightly vibrate in this manner. This "Viscosity" is what is frequently understood by the term **imperfect elasticity**: the restitution of form or bulk may be perfect, but that of energy is not, for some of it is dissipated in the form of Heat.

Fatigue of Elasticity. — When a tuning-fork is kept (as by an electromagnetic arrangement, p. 735) continuously vibrating for a long time, it stops almost instantaneously when the exciting cause is removed. The steel requires periods of rest: if it be kept continuously vibrating it has a tendency to become viscous, and to return comparatively slowly to its mean form after each displacement.

Effect of repeated variations of Stress. — Metal requires intervals of rest in order to enable it to recover from fatigue; and if these be not allowed, it will break down and fracture under the repeated application of forces far less than the breaking weight. The greater the variations of stress, and the more frequent their recurrence, the sooner does the metal collapse.

Velocity of propagation of a compressional wave-motion. — The elastic-restitution-pressure developed in consequence of a Compression varies as $\mathfrak{k}$, the coefficient of restitution; the acceleration produced by the restitution-pressure varies as the restitution-pressure; the velocity in the circle of reference (in S.H.M.) varies as the *square root* of the acceleration; the velocity of propagation varies as the velocity in the circle of reference: therefore the velocity of propagation varies as the square root of $\mathfrak{k}$, the coefficient of restitution, or of resistance to compression.

Given the same elastic pressures and the same work done upon two bodies whose respective densities are ρ and $\rho_{\prime}$, the energy being equal, the respective velocities produced must vary inversely as the square roots of ρ and $\rho_{\prime}$. Hence v varies as $\sqrt{\mathfrak{k}/\rho}$; and it can be shown that no multiplier intervenes, and that v is equal to $\sqrt{\mathfrak{k}/\rho}$.

In this it is assumed, however, that in a compressional wave the rigidity

$\mathfrak{n}$ may be neglected. This is practically the case in gases, to which the formula $\nu = \sqrt{\mathfrak{k}/\rho}$ is applicable, subject to further discussion (p. 324) as to what the true value of $\mathfrak{k}$ may be; but in solids the rigidity does come into play, even in compressional waves; and for such waves, in a tridimensional elastic solid, $\nu = \sqrt{(\mathfrak{k} + \frac{4}{3}\mathfrak{n}) + \rho}$.

The velocity of propagation of a **transversely** distortional vibration is $\sqrt{\mathfrak{n}/\rho}$ in a tridimensional medium, and $\sqrt{\mathfrak{t}/\rho}$ along a uniform and perfectly flexible stretched string; that of a **longitudinal** vibration is $\sqrt{\mathfrak{y}/\rho}$ along a wire or rod, stretched or unstretched; that of a **torsional** vibration is $\sqrt{\mathfrak{n}/\rho}$, along a wire or rod.

Hence along a steel wire ($\rho = 7{\cdot}85$, $\mathfrak{y} = 981 \times 2520{,}000000$, $\mathfrak{k} = 1{\cdot}84 \times 10^{12}$, $\mathfrak{n} = 0{\cdot}95 \times 10^{12}$), a longitudinal compressional wave, such as a sound-wave, will travel with a velocity $\nu = \sqrt{\mathfrak{y}/\rho} = \sqrt{981 \times 2520{,}000000 \div 7{\cdot}85} = \sqrt{314{,}919{,}745{,}223} = 561177$ cm. per second $= 5611{\cdot}77$ metres per second; whereas in an extended mass of steel the rate of propagation of a compressional wave will be $\sqrt{(\mathfrak{k} + \frac{4}{3}\mathfrak{n}) \div \rho} = 655000$ cm. per second, and that of a pure transverse-distortional wave (without change of volume) would be $\sqrt{\mathfrak{n}/\rho} = 348000$ cm. per second.

The property of Elasticity is not inconsistent with brittleness: glass has very narrow limits of pliability, and is accordingly brittle, but within these limits it is eminently elastic.

Physiological Examples of Elasticity. — The whole ligamentous system affords examples, and many of the bones also possess this property. The ligaments of the elastic arch of the foot, the vertebral ligaments, and the intervertebral discs acting against the down-dragging weight of the viscera; those ligaments which by their very molecular constitution (however this may be accounted for) are always on the stretch, such as the elastic ligament of the eye, the filled arteries, the ligaments of the *symphysis pubis*; the combined flexion and twist of the ribs in inspiration and their elastic restitution in expiration; the ligaments of the lamellibranch shell, the tracheæ of insects, — all furnish examples of Elasticity.

The Mechanical Advantages of Elasticity. — These can be studied in a well-hung vehicle with light springs. Any sudden jolt or jar is not communicated to the body of the vehicle with its original abruptness, but gives rise to a wave-motion, which lifts the body of the carriage and allows it to oscillate until it returns to relative rest. If a person jump, landing on his feet, the shock is partly broken by the elastic arches of the feet; but further, before it reaches the brain it has to pass through a succession of elastic discs, the ultimate effect of whose intervention is a gradual and not an abrupt arrest of the downward movement of the head. Were it not for this the brain would be ruptured by exceedingly small leaps.

Work is directed by elastic intermediaries so that it may become useful and not harmful. Jolts and jars — which, as

we have seen under Momentum, involve the disappearance of Energy in doing harmful mechanical work — are converted into smooth wave-motions. Thus energy is saved, mischief prevented, and the mechanism rendered more durable. If a person run over a gravelly road with a heavy vehicle attached to his person by a non-elastic cord, he will feel greatly bruised. If he interpose a steel spring or a thick piece of indiarubber between himself and the vehicle, the pain is infinitely lessened and the actual energy expended is about 25 per cent less (Prof. Marey and M. Hirn); work has not been spent in jolting and jarring himself and the vehicle.

The use of elastic intermediaries suggests itself in all cases where jolts of any kind would be injurious. Compare the effects of an involuntary movement made by a patient, whose limbs are under extension by a weight and non-elastic cord, with the effect of the same movement when a light spring intervenes between the limb affected and the extending weight. The spring persists and keeps up the tension, but it yields to the momentary twitch; the weight rises a little and sinks back to its former position.

If a piece of thin cord, tied round a somewhat heavy mass of iron, be pulled with a jerk, it may snap without lifting the heavy mass; whereas, if an indiarubber band be interposed somewhere between the hand and the iron, the same jerk may first extend the indiarubber band which, in its turn, may then lift the heavy mass.

Strength of structures as depending on their form. — This is the special study of the Engineer. Here we can only state a few principles.

Galileo found that a given weight of material if disposed in solid bars presents less resistance to crushing or bending than the same material arranged in the form of tubes, provided that the walls of these tubes be not excessively thin. The following table is from Weisbach's *Engineering Mechanics*: —

	Resistance to Breaking.	Resistance to Crushing.
Massive cylinder, rad. = 4. Mass = $\pi r^2 l\rho = 16\pi l\rho$.	1000	1000
Hollow; radii 5 and 3. Mass = $25\pi l\rho - 9\pi\lambda\rho = 16\pi l\rho$.	1700	2125
Massive cylinder, rad. = 5.	1000	1000
Hollow, radii 5 and 4.	870·40	870·40
5 and 3.	590·40	590·40

Hence, mass for mass, the hollow tube is stronger: diameter for diameter, the solid is the stronger. The strongest tube for all purposes has the relative radii 11 and 5.

Examples of this kind of structure we find in the hollow stems of plants, in the feathers of birds, in the long bones of the human body.

Economy of material may be carried still farther by the adoption of the lamellar or trabeculated structure. We see in lattice-girders how the judicious arrangement of struts which support each other makes a structure strong enough for all practical purposes, though very light; often much stronger than if it were burdened with the excessive weight of its own substance which, if it were solid, it would have to support.

In the spongy structure of bones we find a similar arrangement. In the upper end of the femur we find a disposition of horizontal, vertical, and oblique trabeculæ, which together form a rigid triangular framework supporting the weight of the body. In the astragalus we have a comparatively light and porous structure, but the trabeculæ are so arranged as to resist and to distribute the downward pressure of the body and the compressing pressure exerted by those bones, the os calcis and the scaphoid, which abut against it in the arch of the foot.

CHAPTER XI.

OF LIQUIDS.

THIS chapter may be divided into three parts, treating of (1) Molecular Actions, (2) the Statics of Liquid Masses, (3) the Kinetics of Liquid Masses.

1. MOLECULAR ACTIONS.

Cohesion. — If a ring of iron wire be dipped into a solution of soap, it will be seen on taking it out that the cohesion of the liquid causes a film to be formed, which remains stretched across the ring. The force of cohesion is also manifested by a drop of water hanging from a glass rod. Such a drop may be gradually increased in size until, at a certain maximum, its weight overcomes its cohesion, the water breaks asunder, and the drop falls. A thin rod of glass or wire of metal may similarly be fused at the end, and the fused drop may be increased in size by continued fusion until the molecular forces can no longer counteract its weight. A little water on the end of a thick glass rod will retain a piece of paper placed in contact with it, even though some grains' weight be suspended from the paper.

The above examples furnish us with indications merely, and do not enable us directly to measure the attractions inside a liquid. These cannot be directly measured, because no apparatus can be applied to the interior of a liquid without creating a new surface at its own boundary. But we can infer their amount. Referring to Fig. 100, we see that one of the particles is at the surface, and that the molecular forces acting upon it are only half those acting upon an interior particle. To move a particle from the interior to the surface would consume a certain amount of work; to remove it from the interior through the surface, as on boiling, would require twice as much. If we suppose some water to be boiling in a tube whose cross-sectional area is 1 sq. cm., and the level in which is maintained constant, then, as it is known that the Energy which must be supplied in the form of Heat in order to boil away one gramme of water is (p. 390) equal to $(536 \times 41{,}593000)$ ergs, half this amount, or $(536 \times 41{,}593000 \div 2)$ ergs, would bring, molecule by molecule, one gramme of water from the interior to the surface. But as

1 gramme of water = 1 cub. cm., and as the area of evaporating surface is 1 sq. cm., the path of the molecules is on the average $\frac{1}{2}$ cm.; and the internal pressure overcome is $\{(536 \times 41{,}593000 \div 2) \div \frac{1}{2}\}$ dynes per sq. cm., or 22000 atmospheres. The internal forces are thus seen to be enormous.

Now let the average diameter of the molecules be taken as $1/x$ cm.: then there will be x^3 molecules in one cub. cm., and a layer one molecule thick, made up out of one gramme or one cub. cm. of water, will have x^2 molecules per sq. cm., and will cover x sq. cm. To make such a layer or film out of 1 cub. cm. of water would be the same thing as to bring 1 cub. cm. of water, molecule by molecule, from interior to surface without evaporation. This, as we have seen, would require $(536 \times 41{,}593000 \div 2)$ or 11146,900000 ergs. To produce in a mass of water 1 sq. cm. of additional free surface would therefore require $(11146{,}900000/x)$ ergs. The numerical value of this would of course depend on the diameter of the molecules of water; and as the diameter $1/x$ is evidently very small, the divisor x is very great, and only a very small part of the internal attraction can make itself obvious at the surface by resisting stretching or causing contraction of the free surface. Still, a measurable proportion of it does so, and gives rise to the phenomena described in the succeeding paragraphs.

Surface-Tension. — It has already been remarked that the molecular forces are most strikingly manifest at the surface of a liquid, and that every liquid may be regarded as bounded by a superficial skin or film, which behaves like a stretched membrane, and which in time reduces the contained liquid to that form which gives to the greatest cubical content the least superficial area. The tension of this superficial film is the Surface-Tension of the liquid. A raindrop, a shot falling from a shot-tower, assumes the globular form because the sphere is the simplest geometrical form which fulfils these conditions. A bubble of air in water assumes a spherical form — not perfectly so on account of the resistance to its ascent. The most convenient way of studying the various forms assumed by masses of liquid under the influence of surface-tension is to relieve them of the effect of gravity by floating them in liquids of their own specific density, with which they will not readily mix.

A mixture of alcohol and water is made, of the same specific density as olive oil. Masses of olive oil placed in this fluid will neither rise nor sink, but will assume the globular form. If they be not free to assume the globular form, but have limiting conditions imposed upon them,* they may assume geometrical forms of great interest, all having the smallest superficial area possible under the given conditions. If, for example, into such a globular mass of oil, an oiled circular disc of iron be suspended, having a diameter greater than that of the mass, the mass of oil will spread over each face of the disc, and will form on each side of it a segment of a larger sphere. If such

* Refer to an exceedingly charming work by M. Plateau, *Statique des Liquides soumis aux seules Forces moléculaires*, a treasure-house of experiments devised by a *savant* afflicted with total blindness.

a disc be brought up to the globular mass by one face only, the oil will not pass round the edge of the disc. The curvature of the segments of spheres produced may be modified by adding or removing oil by means of a pipette. If a short oiled cylinder with open ends be put into the dilute alcohol, and a mass of oil inserted by means of a pipette, the oil will fill the cylinder and form a kind of biconvex lens of oil; by means of a pipette, oil may be taken from the mass until it becomes a biconcave lenticular mass; or, if the operation be stopped at the right instant, a plane-ended cylindrical mass of oil will be obtained. If a circular ring be immersed in a large mass of oil, and some of the oil be then removed, the mass will assume a lenticular form. If a little iron framework be constructed, representing in outline the sides (one inch) of a cube, and hung into a mass of oil which is then gradually removed, the mass of oil will have part of its periphery determined by the iron framework, and will assume the appearance successively of a cube with convex sides, of a cube with plane sides, of a cube with concave sides.

But we can study the effect of surface-tension to even greater advantage when we diminish the mass of a liquid without decreasing the area of the superficial film. This we can do by using soap films or collodion films.

Soap Films. — Plateau's method. 1 part fresh moist Marseilles soap (much better, pure oleate of soda) cut into very small pieces; dissolve with moderate heat in 40 parts by weight of distilled water. Filter through moderately-thin filter paper. Mix thoroughly 15 volumes of this solution with 11 volumes of Price's glycerine; let the mixture stand for seven days. On the eighth day cool down to 3° C. for six hours; a considerable deposit is formed. Filter through porous paper, but take the precaution of placing in each filter along with the solution a little closed stoppered bottle full of ice. This will prevent the re-solution of the precipitate formed by cold. The first filtrate is turbid, but this must be refiltered till it is limpid. Films and bubbles made with this solution last for eighteen hours if kept under a glass shade in very slightly moist air.

Collodion Films. — (*Gernez.*) Ether 89 parts by weight, absolute alcohol 5½, photographic gun-cotton 5½; dissolve. Decant. To 100 parts by volume of the clear solution add 70 to 100 parts of pure castor oil. This mixture is tenacious enough to permit the use of frameworks 8 cm. in diameter.

Prof. S. P. Thompson's Films. — Rosin 46 by weight; Canada balsam 53; melt; add 1 of turpentine. For use, heat to a little above 100° C.

If a roughened iron ring be dipped into any of these mixtures, and taken out, a plane film will be found stretched across it. A pipette (whose point has been dipped into the same mixture) may be employed to blow bubbles and place them on this film, and then to enlarge or diminish these bubbles. Such films and bubbles stretch themselves into the most singularly beautiful forms when iron frameworks forming the complete angular periphery of cubes, pyramids, cylinders, and so forth, are substituted for the roughened ring above described; and these

forms may be infinitely varied by modifying the size of the bubbles placed on the films, or by breaking with a hot needle the connection of the film with one or more of the peripherical bounding lines; in the latter case the most beautiful skew-surfaces are formed, all presenting the least area possible under the limiting conditions.

If on a simple film stretched over a ring, a piece of silk thread (moistened beforehand with the same solution) be laid in such a way that the thread crosses itself at some one point on the film, and if the film be pierced inside the loop of thread, the loop will fly open and form a perfect circle: for the rest of the film tends to occupy as small an area as possible. If a drop of alcohol be laid within the loop, the loop flies open in the same way; although the film is not broken, yet its surface-tension is diminished.

If a shallow dish containing mercury be tilted over, so that the mercury is on the point of pouring out; if then some of the mercury be drawn over so as to start a flow, the stream will drag the mercury out of the dish.

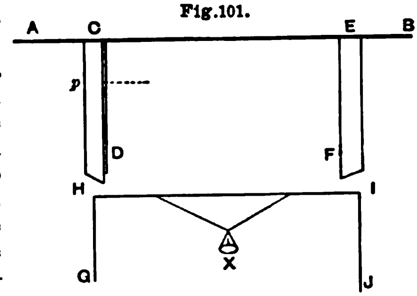
Mercury can even be blown or shaken into bubbles by means of water in place of air, and a film of it can be produced on a small amalgamated copper ring.

If a piece of camphor be placed on clean water it partly dissolves in the water. A strong solution of camphor has less superficial tension than a weak solution, which in its turn has less tension than pure water. At any part of the surface where the solution happens to be more dilute, there the weaker solution, having the greater tension, pulls the camphor towards itself. The camphor accordingly flies about the surface of the water. This is generally prevented by allowing the finger to touch the water, unless the finger be beforehand specially purified; so slight a trace of fatty matter communicated to the water prejudices its surface-tension to so great an extent. If a drop of ether be suspended by a glass rod close to a thin layer of water, the rest of the water is observed to flee from the spot; the surface-tension of the rest of the water is unchanged, but just under the drop of ether the tension is diminished by absorption of the ether-vapour. A thin layer of water, into the centre of which a drop of alcohol is thrown, leaves the alcohol for a similar reason. If a glass of strongly alcoholic wine be tilted so as to moisten the side of the glass, the film of wine left will gradually lose some alcohol, and becoming more aqueous it will

have a greater superficial tension than the wine; it will pull itself up the sides of the glass and gather into drops. A thin layer of water on a metallic plate, the midpoint of which is heated, withdraws to the edges.

Measurement of Surface-Tension. — A soap or collodion film has two surfaces, and if the film be not too thin, these are independent of one another. Consequently the tension of a film is double that of a single surface of the same liquid and of the same area. The tension of a film can be measured directly. A little framework, consisting of a transverse bar AB, and two grooved slips CD and EF, allows the piece of wire GHIJ to slip freely up and down the grooves. The wire is pushed home, and a quantity of the liquid is applied between HI and CE. The little pan X may be loaded with sand until the wire HI is pulled out to a certain distance C*p* from AB. When it is in that position, the film has an area CE · C*p*. The Weight *mg* of the total mass *m* suspended on the film, and the Tension over the area CE · C*p*, are equal to one another. If the total weight applied be increased, the area assumed by the film is increased in direct proportion. The total weight *mg* is distributed over the breadth CE; whence, if *T* represent the superficial tension across *unit of length* of CE, then $mg = T \cdot \mathrm{CE}$. The energy of the film is the work done upon it; weight or force *mg* has pulled the film through a space C*p*: the energy is $mg \times \mathrm{C}p = T \cdot \mathrm{CE} \cdot \mathrm{C}p$. The energy may also be represented as the product of *S* (the energy per unit of area) × CE · C*p* (the area) $= S \cdot \mathrm{CE} \cdot \mathrm{C}p$. Hence

Fig.101.



$$T \cdot \mathrm{CE} \cdot \mathrm{C}p = S \cdot \mathrm{CE} \cdot \mathrm{C}p$$
$$T = S.$$

The energy per unit of area (which in the case of a soap film is 54·936 ergs per sq. cm.) is numerically equal to the surface-tension across each unit of length (which in the same case is 54·936 dynes per cm.). These are independent of the form of the film, and depend only on its actual area, not on its curvature. For each single surface *T* and *S*, as found by experiment on films, must be halved.

T may also be measured by observing the height to which the tension of the curved surface will raise the level of liquid in a clean open capillary tube, wetted by the liquid and dipped into the liquid. The liquid wetting the tube, and the superficial layer of the liquid in the tube, contract towards one another: the result is a curved surface whose outer boundary is $2\pi r$, where r is the radius of the tube. If the superficial tension across unit of length of this boundary be *T*, the total superficial tension will be $2\pi r T$. The Weight of the liquid lifted by this tension is its volume $\times \rho g$; that is, $\pi r^2 h \rho g$, where h is the height of the column supported. Hence in proper units $T = rh\rho g/2$; and the height of the column is inversely proportional to the diameter of the tube. Again, if m be the maximum mass of a large hanging drop depending from a small wetted circular disc of radius r, the

boundary is $2\pi r$; mg, the Weight of the hanging drop, $= 2\pi r \cdot T$; and $T = mg/2\pi r$ dynes per cm.

This Modulus of Superficial Tension, T, is, in all the instances which we have considered, that of a surface between the liquid and the surrounding air. In the case of pure and perfectly clean water and air, this modulus is equal for each surface to 81·96173 dynes per cm., very nearly three times the superficial tension of a single surface of soap solution in contact with air. The tension of the bounding surface separating olive oil and air is 36·8856 dynes; that of the surface between olive oil and water is 20·56176 dynes, per cm. At the meeting-place of oil, water, and air, these three surfaces meet; the tension of the water-air surface decidedly preponderates, and the edge of an oil-drop floating on water is drawn out indefinitely. If a drop of water be placed on chloroform — the respective tensions being water-air 81·96173, chloroform-air 30·6072, and chloroform-water 29·5281 — its surface-tension (water-air) at first preponderates and pulls it into a drop. When water, air, and clean glass are placed in contact there is again a triplet of tensions, the resultant of which pulls the water over the glass, which is thus wetted by the water. The water tends to stand so that its surface makes a certain angle with the glass; this is the **angle of capillarity**, 180° between water and wet glass, 45°·30 between mercury and glass.

In the case of water, this angle is such that the upper surface of water in contact with glass is concave; in the case of mercury, the upper surface is convex. Water will, however, spread over perfectly clean mercury.

On the curvature of the upper surface, thus determined, depends the direction in which the contractile tension of the superficial film acts. The concave surface of water tends to contract and become flat, and it does so in proportion to the curvature imposed on it.

The narrower a capillary tube is, the greater is the curvature of the surface of any liquid standing in it, and therefore the greater is the contractile tendency of that surface. The effect of this tendency is, in the case of water, to exert an upward pull, to neutralise to some extent the downward effect of gravity, and to support a column of water in the tube. Hence water stands at a higher level in a narrow tube whose lower open end is dipped in water than it does in a wider one; and the height of the column supported is inversely proportional to the radius of the tube at the spot where the curved surface of the liquid is situated. The height at which a solution stands depends on its strength and on the salt dissolved. Mercury under similar circumstances stands at a lower level.

This tendency of a curved surface to exert traction or pressure on a fluid may be seen in a conical capillary tube; if a drop of water be introduced, the smaller concave surface will pull the drop towards the apex, if a drop of mercury, the smaller convex surface will push the mercury from the apex.

Capillary phenomena are thus phenomena of surface-tension; and it is futile to explain the rise of sap in plants or the passage of fluids through

minute vessels by "capillary attraction" when there is no free surface. An experiment which may, on the other hand, illustrate these movements, consists in oiling the interior of an open capillary tube, filling it with water, and dipping the end of the tube in oil; the attraction of the sides of the tube for oil will cause the oil to run along the tube and to drive out the water; this, however, is not an exclusively capillary phenomenon.

If two plates of clean glass be set to stand vertically and parallel to one another in a shallow dish of water, the water will rise up the sides of each to a height *half* that which it would attain in a tube whose diameter is equal to the distance between the plates. If the two plates have two vertical edges in contact, the liquid will rise indefinitely where they are in contact; at other parts it rises to a height inversely proportional to the local distance between the plates, and it thus presents the outline of an equilateral hyperbola.

Just as the surface of a liquid is raised against a fixed plane of clean glass, so a floating vessel of clean glass may by surface-tension be pulled down, so as to lie more deeply in the liquid than its specific gravity would lead us to expect. A floating hydrometer mostly gives an abnormally low reading on this account; it is pulled into the liquid, so that the liquid appears to be lighter than it really is. If a little vapour of ether be poured on the surface of the liquid so as to diminish the surface-tension, the hydrometer rises. If the water have any grease on its surface, the same effect follows. If the hydrometer be greasy, it is repelled and stands abnormally high in the liquid. Hence great confusion and inaccuracy may result from films of grease on the glass or on the fingers of the manipulator.

Objects which are wetted by the liquid in which they float are thus apparently attracted by it; those which are not so are apparently repelled. Two wetted objects floating on water seem to attract one another; two objects floating on a liquid which does not wet them seem also to attract one another. This may be seen by throwing upon the surface of water a number of wooden balls, of which some are smoked with lampblack, while others are purified first with soap and water, then with pure water; the smoked balls approach each other, the clean ones approach each other, but the clean balls appear to avoid the smoked ones.

We may mention another consequence of surface-tension. A jet of water issuing from a rectangular orifice is most acted upon by surface-tension at its narrow edges. These are pressed together; they meet, and when they do so, spread out laterally;

the same action is repeated, and the whole jet is a succession of flat segments at right angles to one another. At first sight such a jet seems to have a screw form.

The distances at which molecular forces act are not immeasurably small. Quincke found that while water stands against glass at one angle, against silver at another angle of capillarity, yet against glass coated with silver it stands at such an angle as to show that the influence of the glass is felt through the silver when the layer of silver is less than ·000,005 cm. thick; this thickness being one-tenth of the average length of a wave of light, and being further (Meyer) very much the same thing as the mean free path.

Superficial Viscosity. — This is a property of the superficial film of liquids after exposure to the air for some time: and it is independent of the surface-tension. If a magnetic needle be so adjusted as to have its lower surface in contact with the surface of a solution of saponine, it will remain in any position, in defiance of the directive force of the earth's magnetism. On the surface of most other liquids it will move into the magnetic meridian, but the whole superficial film of the liquid will move with it, as may be shown by strewing lycopodium over the surface. The superficial film is, as a rule, exceedingly viscous as compared with the interior mass; it is consequently hard to move or to break. If a liquid have great superficial viscosity and small surface-tension (as in the case of soap-and-water), a bubble rising through the liquid may raise the surface film, which the tension is not able to break: the bubble may therefore persist. If a wire ring, bearing a soap film, be swept rapidly through the air, the air may fill and stretch the film, and separate part of it in the form of a complete bubble. A bubble rising with very great rapidity through a liquid may tear off some of the viscous superficial film and form a complete bubble: this is seen when a mixture of olive oil and strong sulphuric acid is vigorously stirred.

Pure water has great surface-tension, which is able to overcome the superficial viscosity. Perfectly clean water has no superficial viscosity. Thus pure water will not froth. Some liquids, such as a solution of gum arabic or of albumen, will froth when shaken, but their superficial viscosity is not sufficiently great to enable bubbles to be blown with them. Alcohol, sulphuric ether, bisulphide of carbon, and some other liquids, have a superficial viscosity less than their internal viscosity, and

consequently, when alcohol is mixed with a superficially viscous liquid, it neutralises its relative superficial viscosity, and frothing is rendered impossible. Hence the practice of adding a few drops of spirit in order to check frothing in pharmaceutical operations.

To this toughness of the superficial film, the floating of an oiled needle or the walking of an insect on water must be in part ascribed. The depth of the dimple produced by the needle is not sufficient to account, by displacement, for the support afforded to so heavy a body: the superficial tension is diminished by the oil: the tenacity of the surface film plays its part in supporting the needle. To the same cause we may attribute the smoothing of the surface of a rough sea when oil is poured upon it: the new surface has great superficial tenacity and small superficial tension, and is not readily broken up into surf. The new surface of the sea is relatively rigid; waves press against it from beneath, but their energy is spent in producing, not ripples, but eddies below.

The superficial film of a liquid is thus seen to be a seat of energy and to be physically different from the interior.

A bubble in bursting does so with an audible sound: it scatters particles of its substance and of the contained gas to a height of three or four feet; this happens during the effervescence of sewage which is undergoing fermentation.

Cohesion-Figures. — If the surface of a tumblerful of salt water (½ teaspoonful to the tumbler) be touched with a pen not too full of ink, the ink will, in falling through the liquid, assume very remarkable vortical movements. A shower of rain falling on a troubled sea produces similar vortex-rings, which are carried down into regions of comparative stillness, and moderate the turbulence of the water by equalising its distribution of momentum. The forms assumed by drops of water or of mercury falling on a flat surface, at the instant when they spread out and break, are very remarkable, and may be seen when the spreading drops are momentarily illuminated by the electric spark. The edge of the spreading drop breaks up into thinner and thicker nodes and loops which vibrate: very roughly the result may be seen in a cooled splash of candle-wax.

Solubility of Solids in Liquids.— When a solid is dissolved in a liquid, work is done in overcoming its cohesion. This cohesion is overcome by the adhesion between the solid and the liquid. Ice put into sulphuric acid has its superficial particles

successively torn off, and a mass of dilute sulphuric acid (which on account of liquefaction assumes a low temperature unless heat be supplied) is produced. Such union may or may not be associated with a play of chemical affinities; in the case of ice and sulphuric acid there is a tendency to the production of definite hydrates of sulphuric acid, the formation of which is accompanied by the evolution of a certain amount of heat. If sulphate of magnesia be placed in water it will be dissolved to a certain limited extent; if the salt be added in excess above this limit, no more will be dissolved; when this limit has been reached the solution is a **saturated solution.** This limit is expressed by the **coefficient of solubility,** a number indicating the quantity of solid which can be dissolved and remain in solution in unit-mass of the liquid at the particular temperature for which the coefficient is or ought to be specified. A saturated solution can dissolve no further quantity of the same salt at the same temperature, for the adhesion of such a solution to the salt is no longer greater than the cohesion of the salt itself: or, in other words, just as many particles then leave the liquid for the salt as leave the salt for the liquid. If the cohesion of the salt be lessened by heat, more may be dissolved; and as a general rule, with but few exceptions — hydrate of lime, sulphate of soda, phosphate of magnesia — salts are more soluble in hot than in cold water. The adhesion of a liquid to the solid which it holds in solution may be relatively great or feeble; and its relative amount may be indicated, though not measured quantitatively — (1) by a high or low coefficient of solubility, (2) by the amount of energy which must be imparted to the molecules in order, by boiling, to tear the water away from the salt, or, in other words, by the high or low boiling-point of a saline solution; (3) by the relative effect of charcoal filters in retaining the salts of a saline solution while allowing the water to pass, a property made use of in the analysis of poisons; and sometimes (4) by the detachment of the liquid from the solid by a stronger molecular attraction, as in the case of iodide of starch, a solution of which is precipitated by acetate of potash, the water leaving the iodide of starch and adhering to the salt.

There is a general relation of concurrence between the solubility and the fusibility of a salt; but there are important exceptions, *e.g.*, chloride of silver, which is fusible, but not soluble in water.

Dissociated Molecules in Solutions. — When such a chemically inert substance as sugar is dissolved in water, its mole-

cules seem to remain undecomposed; but in an aqueous solution of a salt, of a chemically strong acid or base, or generally of any substance which presents in solution a marked chemical activity or susceptibility to chemical reaction, there is, somehow, more or less Dissociation of the molecules of the dissolved substance into sub-molecules or free **Ions.** For example, NaCl splits up, on solution in water, into Na and Cl atoms; H_2SO_4 into H, H, and (SO_4), $Al_2(SO_4)_3$ into Al, Al, (SO_4), (SO_4), and (SO_4). This seems quite contrary to experience; but it is clear that the physical properties of the solution can only be explained by assuming that there is within it a Number of molecules or sub-molecules, which cannot be accounted for on any other hypothesis; and then, as the dilution or the temperature of the solution increases, the more nearly is the increase in the number of molecules such as to correspond exactly with their derivation in the above manner. An extremely dilute solution of a salt thus does not contain the salt, as such; it only contains ions. If an aqueous solution of hydrochloric acid (in which there is almost complete dissociation) be mixed with one of potash, in which the condition is the same, the reaction on neutralisation is $(H + Cl) + (K + HO) = K + Cl + H_2O$. Water is formed on neutralisation, but the ions K and Cl remain, for the most part, separate until crystallisation takes place.

The physical properties, the peculiarities in which have led to the foregoing conclusion, are the Osmotic Pressure, the Freezing-Point, the Vapour-Pressure, the Density, the Colour, and the Electric Conductivity of aqueous solutions of those substances which are chemically most active or undergo chemical reactions in the shortest time.

When a saturated solution is cooled, the coefficient of solubility diminishes, and the solid segregates in a separate form: thus hot saturated solutions may be set aside to cool, and on cooling they crystallise, the materials dividing into crystals of the salt and an ordinary cold saturated solution of the same. Sometimes, as in the case of sulphate of soda, such a solution (though cooled down to a temperature at which it cannot permanently retain all the salt which it holds in solution) does not crystallise, but forms a **supersaturated solution.** Such a solution is in a state of unstable molecular equilibrium, and the instant it is touched with a crystal of the same salt or, with less certainty, by a crystal of an isomorphous substance, or by the dust of the air containing the same substance, or by an oil

(especially if somewhat oxidised), or by a bubble of gas soluble in the liquid, or when it is exposed to the least vibration, the whole molecular arrangement topples over, and the excess of salt assumes the solid form. It does so with evolution of heat, if the act of solution had been accompanied by cooling.

A similar delay in solidification occurs in the case of melted phosphorus, which can be kept fluid at 10° C. (its solidification point being 44°·2 C.) for weeks, especially if the water lying above it contain a trace of potash hydrate or of nitric acid. The slightest shake or contact with a piece of phosphorus determines solidification.

Miscibility of Liquids. — If a bottle be filled with oil and water, and shaken, the layers separate as soon as the disturbance ceases, though there is, in such cases, always a certain small amount of evaporation of the one liquid into the other. Alcohol and water treated in the same way mutually dissolve each other, and mix perfectly in any proportions. Ether and water will each take up a certain proportion of the other, which proportion depends upon the temperature, and when shaken together they separate into two layers, the one a solution of ether in water, and the other a solution of water in ether. These two liquids are miscible only in certain proportions, which depend upon the temperature; in some cases a sufficiently high, in others a sufficiently low temperature brings about complete miscibility. Very often, as in the case of alcohol and water, there is a contraction of volume and evolution of heat, there being some potential energy of separation somehow liberated by the approximation of mutually attracting molecules of the different substances, or there may be expansion and cooling, as in the case of alcohol and carbon bisulphide.

Imbibition. — Porous objects, such as a lump of sugar, blotting paper, a heap of sand, a sponge, a lamp-wick, absorb liquids with a rapidity which depends on the nature of the porous substance itself and on that of the liquid absorbed, and which is greater if the materials be warm. This takes place by reason of an attraction between the solid and the liquid (which Chevreul called *affinité capillaire*), and heat is evolved when this attraction is satisfied, as in the case of a wetted rope, which rises in temperature from 2 to 10° C. (part of this effect being due to the concurrent shrinkage of the rope). When a porous body which has thus taken up a quantity of liquid is subjected to pressure, the whole of the liquid can by no means be squeezed out; some water still remains, which can be evaporated away. Imbi-

bition will fill the pores of a solid with a liquid, but will not set up a permanent current in those pores unless, as in the case of a lamp-wick, there be constant removal of the liquid at one extremity of the porous object while imbibition goes on at the other.

Diffusion — Jar-diffusion.* — If a phial, filled to within say half-an-inch of the top with a saline solution, be placed in a jar; if water be poured into the jar so as to surround the phial, and if more water be cautiously added until the phial is covered with a layer of water of about half-an-inch in depth, the whole being set aside in a quiet place, the solution in the phial will diffuse into the surrounding water. The quantity of substance diffused into the water in a given time depends (1) on the length of that time; (2) on the quantity of substance in the phial solution, being (within narrow limits) proportional to its strength; (3) on the temperature, being, for dilute solutions, nearly proportional to the absolute temperature; (4) on a Coefficient of Diffusibility special to each substance. Other things being equal, urea and salt diffuse twice as fast as sugar, sugar twice as fast as gum arabic, gum arabic more than four times as fast as egg-albumen.

Sugar travels as far in a column of water in two days as albumen in fourteen. The following numbers indicate the relative times necessary for the process of diffusion to convey in water through equal distances equal amounts of the several substances: — Hydrochloric acid, 1; chloride of sodium, 2·33; sugar, 7; sulphate of magnesia, 7; albumen, 49; caramel, 98 (Graham).

The rate of diffusion of all substances is increased by moderate heat, but in those substances whose coefficient of diffusibility is small, it is more increased by heat than it is in those substances which are already very diffusible. Hence the greatest proportionate differences in diffusion-rates are found in the coldest solutions.

Some liquids, such as water and sulphuric acid, ether and chloroform, mercury or molten gold or silver and molten lead, diffuse into one another with considerable rapidity.

If a mixture be placed in the diffusion-phial, the approximate rule is that each component of the mixture is diffused out at its own rate, and independently of the others. There is, however, a departure from strict adherence to this rule, in the sense that the ordinary differences of diffusibility are exaggerated in such a mixture. If the phial contain a double salt, such as alum, diffusion may effect chemical decomposition: sulphate of potash and sulphate of alumina are separated, the former being diffused more rapidly.

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* See Graham's *Chemical and Physical Researches.*

A high boiling-point of any solution (which indicates adhesion of water to the salt dissolved) is associated with rapid diffusibility of the same salt; but there is no close relation between the rapidity of diffusion of a salt and its solubility.

Colloids and Crystalloids. — On surveying a number of objects which have a wide range of relative diffusibilities, we see that at one end of the scale we have such things as urea and chloride of sodium, and at the other such things as starch, gum, gelatine, albumen. The former are bodies of rapid diffusibility, have generally a certain chemical stability and a crystalline form, and are called **Crystalloids.** The latter are bodies of slow diffusibility, have large molecules very probably compounded of groups of their simplest molecules, have in general (with rare exceptions, such as the blood-crystals) the non-crystalline amorphous glue-like character which gives them the name of **Colloids,** and are for the most part in a state of unstable equilibrium when in the moist condition. Colloids have great cohesion, and adhere firmly to other colloids: thus isinglass heated with acetic acid forms a cement which adheres firmly to glass; and when they dry they tend to contract firmly, so that a strong solution of gum arabic, drying in a test tube, will sometimes break the tube. They often, when drying up, extrude their contained water, and form clots, on the surface of which the water presents itself in drops. Colloids also in many instances possess the power of taking up alcohol or olein in the room of their water of constitution. This property is possessed even by such a substance as colloid silicic acid.

An animal tissue, which is in great part composed of colloids, may have its water expelled and replaced by alcohol, by dint of repeated washing in that liquid.

Colloids being very slightly diffusible are tasteless; they do not reach the nerve-ends. For the same reason they are very indigestible — *e.g.*, gelatine — unless peptonised; peptones being, by exception, diffusible though otherwise colloidal.

If a layer of pure jelly be laid on a layer of jelly charged with soluble salts and also with caramel, the salts will diffuse into the upper layer of solid jelly nearly as fast as if it were pure water; the caramel will not travel at all. If a film of colloid matter (starched paper) be placed between a mass of pure water and a saline solution containing colloid matters, the colloid septum will offer little obstruction to the passage of the salts into the water, but will prevent the colloid matter from passing

through. Colloid matter is thus impervious to the diffusion of other colloids, but does not hinder the diffusion of crystalloids.

Diffusion through Membranes — Osmosis. — If three layers of liquid, chloroform, water, ether, be placed in a closed bottle and set aside, it will be found that in course of time the ether travels into the chloroform, but that the water does not to any appreciable extent allow the chloroform to pass into the ether. The ether dissolves to some extent in the water and diffuses through it: it is removed from the water by the chloroform: step by step the upper layer of ether may wholly travel into or through the water. A thin caoutchouc membrane lying between alcohol and water allows the alcohol to pass through it into the water; but the reverse passage of water into the alcohol is barred. If an organic septum be used it is wetted, and the water passes into the alcohol. If hydrochloric acid and water be separated by an animal membrane, the hydrochloric acid passes through in greater quantity: both fluids wet the membrane; the hydrochloric acid is most attracted. Hence molecules may travel through a septum devoid of perceptible pores as well as through one in which pores exist.

If the membrane employed be porous, we have the process of Osmosis. The membrane becomes penetrated by that one of the two liquids ("liquid A") for which the walls of its pores have the greater attraction or affinity. When each small capillary column of the liquid A comes at the farther surface of the membrane into contact with the liquid B, the molecules of liquid B diffuse into it. Thus the end of the column of liquid A comes more to resemble that liquid B which is less attracted by the walls of the pore, and it is extruded from the pore and pushed into liquid B. This process is continuous, and a stream of liquid A is drawn through each pore of the membrane into liquid B. Down the centre of the stream there is, however, a backward diffusion-current of molecules passing from the liquid B. This happens if the pores be wide enough to allow the centre of the stream to be comparatively out of reach of the immediate influence of the walls of the channels, an influence which we have seen to extend to a distance of $\frac{1}{20000}$ mm. or $\frac{1}{500000}$ inch. If the liquid stream be not too rapid, these molecules will make their way against it into liquid A. If the channels be very narrow the liquid stream is relatively accelerated; thus the ratio between the amount of water that passes through a porous membrane into a saline solution and the amount of salt

that passes in the opposite direction is increased by diminution of the pores. This ratio is called the **Endosmotic Equivalent.** It is not a constant, but depends on the original concentration of the solution and on the nature of the membrane; and even with the same membrane it differs according to its thickness or state of freshness, and may be increased by tanning with tannin or chromic acid, which diminish the size of the pores.

Thus for a membrane on one side of which is dry common salt, on the other side water, if the membrane be a piece of cow's pericardium, for every grain of salt which passes into the water, 4 grains of water pass into the salt; with a piece of cow's bladder, the endosmotic equivalent is 6. If on one side of an animal membrane there be placed a strong solution of sulphate of magnesia and on the other a quantity of blood serum, the fluid of the blood serum will pass into the saline solution, taking some albumen with it, and some sulphate of magnesia will pass into the blood serum (Milne-Edwards).

The mechanical structure of the membrane has a marked influence on the process; thus water will pass more readily inwards through frogskin, more readily outwards through eelskin.

The matters already moistening the membrane also affect the rate of transmission; thus albumen more readily passes through a membrane previously moistened with alkalies. If between alcohol and water there be arranged a membrane previously soaked in oil, the membrane cannot be wetted, and the alcohol now passes into the water.

If the saline solution be in a state of movement relative to the membrane, the particles are drawn away from the membrane, and the diffusion-stream is hindered; if the water into which the salts are passing be constantly renewed, the molecular diffusion is accelerated.

Heat increases the rapidity of Osmosis. An electric current (the "electrodes" being on opposite sides of the membrane) has the singular effect of, as it were, pushing the liquid bodily through the membrane towards the negative electrode. Even gelatine and the fatty matters of milk can be thus driven through a membrane.

If a mixture of different substances be exposed to osmosis through a porous membrane, the colloids will remain or will pass through in very small quantities, the crystalloids pass through freely. This is the basis of the process of **Dialysis.** Various mechanical arrangements for carrying out dialysis suggest themselves: a phial with the bottom cut off, or a wide glass tube, over the lower end of which a piece of membrane is stretched; the material to be dialysed being placed in this, and

the whole suspended in water. The most convenient arrangement in many respects is a piece of parchment paper (the leaks in which are stopped with albumen coagulated by heat) or, better, gold-beaters' skin, laid upon a wooden ring, into which a smaller ring is thrust so as to form a dish with a membranous bottom; this is floated on a mass of water, and the substance to be dialysed is placed in a thin layer on the dish. The crystalloids (strychnine, etc.) pass into the water, the colloids (mucus, etc.) remain in the dish. This method is peculiarly applicable to the separation of poisons from animal matters.

If the mixed solution exert pressure upon the membrane, colloids as well as crystalloids may be found to pass in considerable quantities through that membrane, along with the fluid forced through by the pressure.

If peroxide of iron be dissolved in a solution of perchloride of iron, and the whole be then dialysed, the chloride of iron will pass through the membrane, leaving the colloid oxide of iron behind in solution. Neutral Prussian blue (as used in microscopical work) is also a colloid, and may be purified in the same way: so is sucrate of copper, a soluble compound of copper oxide with sugar, which is reduced on heating. Albumen may also be obtained in a relatively pure form by separating it by dialysis from the greater part of the salts that it may contain.

If the membrane used be the gastric or intestinal membrane, taken after death, it is found that curare or snake poison will not pass through it, while they are absorbed readily by the dermis or by serous membranes. They seem not to wet the former; hence the selective absorption of poisons has a certain physical basis.

Absorption by the dermis is seen to be a physical process; the walls of the vessels, both lymphatic and venous, are known to be physically permeable to osmose, and the salt, if it be placed on a vascular region, is quickly absorbed, the osmose being accelerated by the flow of liquid in the vessels. Substances brought in contact with the pulmonary epithelium are also very rapidly absorbed. Lymph acts towards blood as water does towards a saline solution, and the tendency of osmotic action is to carry the fluids of the body into the blood-stream. Repletion of the vessels checks this tendency. Adhesion between water and oil is greatly increased if a little alkali be dissolved in the water. When the mucous membrane is covered with bile it has much more affinity for oil globules, which are, besides, each endowed by emulsionising with an aqueous or soapy covering, which makes them act like minute masses of water, and enables them not to experience any relative repulsion when carried with the rest of the aqueous stream.

Osmose through porous membranes is thus related to capillary affinity and to diffusion, but it bears no exact numerical relation to either of these, for it depends on the relation between the pores and the solid parts of the membrane, upon the nature of the material (colloidal or otherwise) of the membrane, upon

the width of the pores, upon the temperature and electrical condition, upon the mutual action of the fluids, and in physiological cases (Milne-Edwards, *Physiologie*, tome V) it seems to depend on the influence of the nervous system.

Solution-Pressure or Osmotic Pressure.—The above phenomena are explained as follows:—The molecules of any chemically inactive substance, such as sugar, when dissolved in water, act precisely as if they were molecules of an independent Gas, which exerts its own Pressure. Accordingly, where there are differences of concentration within a solution, there are differences in the pressure exerted by these molecules; and the molecules of the dissolved substance tend proportionately to travel towards the region of less concentration, thus giving rise to the phenomena of Diffusion; and equilibrium is not attained until their quasi-gaseous pressure, the Solution-Pressure or Osmotic Pressure, and along with it the concentration of the solution, have become equalised throughout the mass. This equalisation is slow, because the liquid obstructs the transference of the molecules. If the liquid be contained in a vessel terminated above by a long tube, and if it be separated from pure water by a membrane or pellicle (such as that formed by precipitation through the contact of a solution of copper with one of a ferrocyanide), which is permeable by water but not by the dissolved substance, water will enter through the membrane until the liquid stands in the tube at a height which measures the osmotic pressure. It is then found, if the substances dissolved be not decomposed or dissociated by the act of solution, that the pressure exerted by the dissolved substance is the same as would have been exercised by its molecules if it had been reduced to a **gas** at the temperature and volume of the solution; and that this pressure is proportional to the absolute temperature (p. 364), and is independent of the nature of the septum. In saline solutions, on the other hand, the phenomena are of precisely the same kind, with this exception, that the pressure is mostly greater than with solutions of indifferent substances: and this tends to show that there is Dissociation of the molecules, which dissociation is more complete the greater the dilution or the higher the temperature. Where the septum is more or less permeable to the substance dissolved, as well as to water, we have the phenomena of ordinary dialytic osmosis, as through parchment paper or animal membrane.

2. The Statics of Liquid Masses.

Liquids are incapable of resisting a change of shape when acted on by force which is not equally applied over the whole surface, and they **flow** when thus acted on, unless supported on all sides.

All soft masses which cannot in the aggregate permanently resist a change of shape are practically liquids, and are subject to hydrostatical laws.

Dilatancy.—Granular masses, such as loose sand, alter in volume when their shape is changed. If their volume cannot alter, neither can their shape: they are then rigid. If they have been well shaken up, they occupy the least

possible volume; and any change of shape involves increase of volume. If water lie between the granules, the water may fail to fill the spaces between the granules if the volume of the whole be thus increased: and the mass becomes rigid whenever any change of shape would thus result in a tendency to a vacuum between the granules. When footprints are impressed upon wet sand, the change of shape under the foot is enabled to go on by drawing water from the neighbouring sand, which becomes dry. (Osborne Reynolds.)

It is often convenient, in discussing the equilibrium of liquids, to imagine little elements of the liquid, floating in and forming part of the liquid, to become solidified or otherwise to become separately recognisable, while not altering their other relations to the surrounding mass. Then, if the liquid as a whole be at rest, each of these little elements of mass must also be at rest.

This being so, the forces acting on each little element of mass must be in equilibrium, and their resultant must be nil. This can only occur (since each fluid element is subject to pressure on all sides, as may be understood by considering the rush of fluid from all sides that would occur if the little element of mass were suddenly annihilated) if the pressure on all sides be equal; and since the element may be reduced to a material point, the proposition follows that at any point in a liquid the pressure in all directions is equal.

The pressure at any point of the surface of a liquid at rest must be at right angles to the surface. If it were not so, it must be oblique; being oblique, it would be resoluble into a component at right angles and one parallel to the surface. The latter could not fail to act, the surface being that of a liquid; hence the liquid would not be at rest; whence there is no such component, and the pressure is at right angles to the surface. Conversely, when a liquid is at rest, the pressure which it exercises on the vessel containing it is at right angles to the walls of the vessel, for the walls of the vessel coincide in aspect with the surface of the liquid.

If in a liquid at rest, expressly supposed to be **not** under the influence of **gravity,** two elements were imagined to be in contact, and yet to be subject to different pressures, there would at the point or surface of contact be a relative difference of pressures which would necessarily cause movement of the liquid; but the liquid is supposed to be at rest; hence there can be no difference between the pressures of any two contiguous elements, and the pressure throughout a **weightless** liquid at rest is every-

where the same (**Pascal's Principle**), and is the same in all directions. It is the same within the liquid as it is at right angles to the surface; and therefore, instead of considering **p** the pressure per sq. cm. between the liquid and the vessel containing it, and at right angles to the surface of the liquid or of the vessel, we may replace this by the pressure p per sq. cm., numerically equal to **p**, but exerted within the liquid in all directions. This is called the **Hydrostatic Pressure.**

If a pressure be applied from without to some of the particles of a liquid, and if that liquid be free to change its shape, it will do so; if it be not free to flow, the particles pressed on will press against contiguous particles, and these against their neighbours; thus the pressure becomes equalised throughout the whole of the liquid. This is the principle of the so-called Transmissibility of Fluid Pressures. The pressure applied to any area of the surface of a liquid not free to flow becomes equally felt over every equal area of the surface.

If a wide cylinder, with a piston whose area is a sq. cm., be placed in communication by a tube with another cylinder, narrower, and provided with a piston whose area is b sq. cm., and if both cylinders and the communicating tube be completely filled with water, a total effective pressure P applied to the smaller piston will produce an equal pressure P on every b sq. cm. of the surface of the fluid, and therefore on every b sq. cm. of the larger piston, and a proportionately greater pressure, $P' = P \cdot a/b$, on the whole surface (a sq. cm.) of the greater piston. This is the principle of the **Hydraulic Press,** by which a smaller force, P, acting on a smaller piston, may produce a greater force, $P' = (P \cdot a/b)$, distributed over the inner surface of a larger piston; and as the area a may bear any proportion to the area b, the force obtained may bear any proportion to the force applied. The principle of the Conservation of Energy holds good, however; the volume of water remains constant, and if the smaller piston move through a space s, the larger piston moves through a shorter space $s' = (s \cdot b/a)$. The work done upon the smaller piston, total force $\times$ displacement $= Ps$; that done by the larger piston, $P's' = (P \cdot a/b) \times (s \cdot b/a)$, gives the same product, Ps.

An analogous action takes place in an aneurism. A small aperture of communication with the artery allows the arterial blood-pressure to be communicated to the whole interior of the aneurismal sac; the total pressure exerted is very great, the rate of distension comparatively slow.

If the action of a hydraulic press be reversed, a great total pressure applied to the larger piston will have the effect of producing a smaller total pressure on the inner surface of the small piston. A small resistance applied to the smaller piston will have the effect of checking the onward motion of the larger piston under the influence of the powerful force. If a bladder full of water be connected with a narrow upright glass tube, heavy weights placed on the bladder will be able to uphold only a very small quantity of liquid in the tube, this arrangement being in fact a hydraulic press worked backwards. If the tube be shortened down so as to form simply the neck of the bladder, the total expulsive pressure exerted by the bladder upon the contents of the neck may seem to be very small when compared with the total pressure exerted over the walls of the bladder upon the whole contents. Here we have apparent destruction of force.

Heavy Liquids.— Let us suppose a cylindrical vessel, filled with liquid, to stand upon a plane base; the area of the base is A sq. cm.; the height of the liquid is h cm.; the density of the liquid is ρ; and the local acceleration of gravity is g. The quantity of matter standing on the base is $Ah\rho$, and the weight of that mass is $Ah\rho g$. The total pressure on the base is therefore $P = Ah\rho g$, and the pressure per unit of area of the base is $p = h\rho g$.

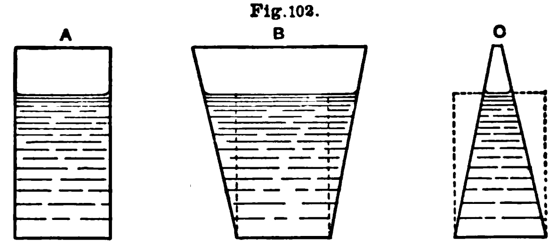
If the unit of area on which the pressure is to be found be plane, but not horizontal, it may be considered to lie at an average depth equal to the depth of its centre of figure. Then the pressure p on a plane of unit-area, chosen anywhere in the fluid and looking in *any* direction, is equal to the product of ρg into the vertical distance h between the surface of the liquid and the centre of figure of that plane; and if the plane have any area A other than unity, the pressure is the product of the area A, $\times\, h$ the vertical depth of the centre of figure, $\times\, \rho$ the density, $\times\, g$.

For all points in the same horizontal layer the depth h is the same, and therefore in a heavy fluid the pressure is the same throughout the same indefinitely-thin horizontal layer. The lateral pressure on the rim of the stratum is equal to the vertical pressure at that level — *i.e.*, $p = h\rho g$ per unit of area.

In Fig. 102 let A, B, C represent three vessels, each having a base whose area is A square centimetres, and each filled with water to a height of h cm. The whole pressure on the base is the same ($P = Ah\rho g$) in all the cases, though the weights of the masses of water differ greatly.

In the first case the lateral pressure against the walls of the cylinder produces a reaction which has no vertical component and does not affect the pressure on the base. In the second we may isolate a cylinder of the fluid in the fluid; the lateral parts of the fluid have a certain weight: the walls of the vessel are exposed to a certain pressure P which is equal to the product of their area × the depth of their centre of figure ($= \frac{1}{2}h$) into ρg. This pressure may be resolved into a horizontal and a vertical component, to each of which the corresponding reactions of the walls of the vessel are equal and opposite: the one reaction resists outward yielding, the other supports the

Fig. 102.



weight of the fluid. It will be found that the upward reaction of the sloping walls of vessel B is exactly equal to the Weight of the fluid overlying them; the walls support the whole weight of the lateral masses. In vessel C the reaction of the walls of the vessel may be found in the same way and resolved into horizontal and vertical components. The latter acts downwards upon the fluid, and will be found to be precisely equal to the weight of that quantity of fluid that would lie vertically above the base if the column of fluid were perfectly cylindrical and of the height h, but which, owing to the form of the vessel, does not so lie.

The total pressure on the base of a vessel containing liquid depends on the height (h) of the liquid and the area (A) of the base, the density ρ of the liquid, and g the local acceleration of gravity, but does not depend on the actual Mass or Weight of the liquid employed. It is $P = Ah\rho g$. If a flask filled with water be fitted with a cork in which a long narrow tube is fixed upright, a very small quantity of water poured into the tube will be competent to burst the flask.

This proposition — that the same amount of water may produce widely-differing amounts of pressure on the vessel in which it is contained, these amounts depending on the form of that vessel — is said to be a Hydrostatic Paradox; the only paradoxical element about it is, however, its discrepancy with a certain uninformed intuitional belief in the Conservation of Force.

A slack bag containing liquid, and set to rest upon a plane surface, exerts a pressure upon that surface which is equal to the product of the area of contact × the height of the centre of gravity of the liquid. So for semi-fluid masses.

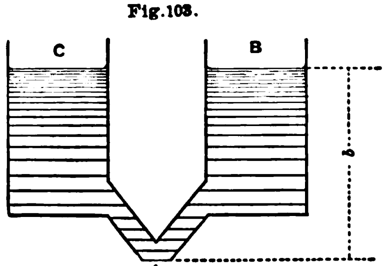
When the surface of a liquid is exposed to the atmospheric

pressure of 760 mm. or 76 cm., it bears on each sq. cm. of surface the weight of 76 cub. cm. of mercury, or 1033·3 grammes; this is equal to (1033·3 × 981) dynes: or if the barometer stand at x cm., the pressure p on each sq. cm. of surface is ($13{\cdot}596x \times 981$) dynes. This number of units of force per sq. cm. may be expressed by the symbol π. Then the Total Atmospheric Pressure on area A sq. cm. is $\Pi = A\pi$. The liquid pressure on the area A at the mean depth h cm. would have been $Ah\rho g$ if there had been no pressure at the surface. When the atmospheric pressure acts at the surface of a liquid, the total pressure on any plane, whose area is A and whose mean depth below the surface is h, amounts to $(A\pi + Ah\rho g)$.

When the human body (as in ordinary circumstances) has the head in the highest position, the blood in the head is exposed to the ordinary atmospheric pressure. If the head be downwards, the pressure on the blood-vessels of the head is increased by the weight of the column of blood in the inverted body, and hence there is congestion. If the body float submerged in a liquid of its own sp. density, head up, the pressure on the blood vessels of the head is the ordinary atmospheric pressure increased by the weight of the column of liquid immediately overlying the head; but if the head be suspended, though the increased depth causes a correspondingly-increased external pressure on the head, yet the equally-increased internal pressure of blood balances this effect, and there is no congestion. This may be illustrated by a loop of thin indiarubber-tubing filled with water: suspended in air, the depending part is distended: suspended in water, it is relieved from distension.

Communicating Vases. — "Water seeks its own level." If there be two communicating vessels containing the same liquid, the lowest part of the communicating channel may be considered as a common base: its area is A. Regarding it as the base of vessel C (Fig. 103), we see that the pressure P on it must be $Ah\rho g$, and the height of the liquid in C is h; regarding it as the base of vessel B, the pressure (which must be the same, for the liquid is at rest) is equal to $Ah_{\prime}\rho g$: whence $h = h_{\prime}$, the height of the liquid in the two vases must be the same, and the level must be the same in two communicating vases, whatever be the shape of the communication, so long as the communication-pipe is continuously filled with liquid. This implies that sufficient time for assuming equilibrium is allowed.

Fig. 103.

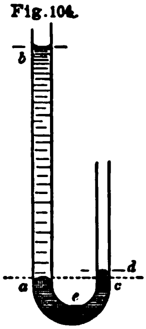


If the liquids in the two communicating columns be not of the same density, the effect is an inequality in the heights of the columns, which vary inversely as the density.

The two pressures are $Ah_{,}\rho_{,}g$ and $Ah\rho g$; these are equal; $\therefore h\rho = h_{,}\rho_{,}$ or $h : h_{,} :: \rho_{,} : \rho$.

In Fig. 104 the column of water *ab* and that of mercury *cd* balance one another because they produce an equal pressure on the base *e*. If a U-tube contain water, of which that in one limb is heated while that in the other remains cool, the liquid in the hotter limb will stand at a higher level than that in the cooler. The relative specific densities of fluids may be estimated by methods based on this principle.

Fig. 104.



The accuracy of the "water-level" may be interfered with by capillarity. If both limbs of a U-tube be narrow, but unequally so, water will stand at a greater height in the narrower limb.

If a U-tube be taken, of which the narrower limb is the shorter, the quantity of water placed in the tube may be regulated so as to afford the following three conditions: — (1) The shorter limb filled with water, the upper surface of which is concave, while the water stands at a lower level in the wider tube; (2) The shorter limb completely filled with water the upper surface of which is plane, and the concave surface of the water in the wider tube at nearly the same level, but a little higher; (3) The shorter limb completely filled with water the upper surface of which is convex, while the water stands at a higher level in the wider tube, its surface being concave.

Every liquid tends to set the whole of its free surface at right angles to the force of gravity.

When a cylindrical vessel containing a liquid is rotated round its longitudinal axis, the surface of the liquid assumes a parabolic form which is maintained constant so long as the rotation is uniform.

Thus the form of the free surface of liquids is affected by gravity, by molecular forces, and by rotation.

Archimedes' Principle. — If an element of mass of a liquid be supposed to be solidified, this will not affect its equilibrium in the midst of the fluid of which it had previously formed part; it will neither rise nor sink. Even though its nature be altered, provided that it do not become either lighter or heavier, it will neither sink nor rise: it has apparently lost its weight. If it become heavier than the liquid it will sink; if it become lighter it will rise. Gravity has no effect in making a body rise or sink in a liquid except in so far as there is a difference between the density of the liquid and that of the body suspended in it. This

leads to Archimedes' principle:—"A body suspended in a fluid apparently loses as much weight as is equal to the weight of the mass of fluid which it displaces." The application of this principle to the study of specific density we have already seen.

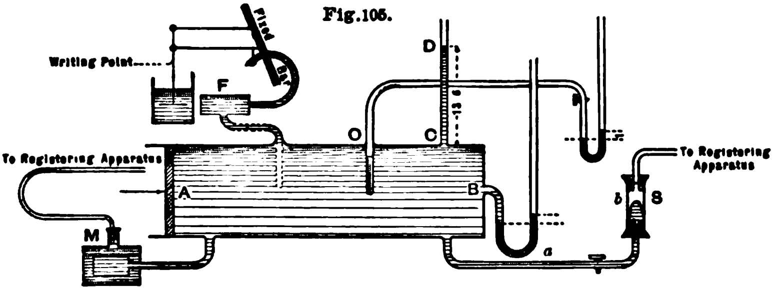
A body lighter than water may be loaded with just so much mass as will sink the light body without that additional mass itself entering the liquid; the whole will then float, the lighter body displacing a bulk of water equal to its own bulk; the weight opposing the buoyancy of the water is the weight of the body *plus* that of the load placed on it; and the ratio

$$\frac{\text{weight of body}}{\text{weight of(body+load)}} = \text{specific density of the floating body.}$$

That a body, even though sufficiently light to float, tends to sink in water until the weight of the water displaced becomes equal to the weight of the whole body, may be shown by a very simple experiment. Take two similar phials, two small elastic bands, and four nails which must not be too heavy. With these may be constructed a couple of rough models representing a person with his arms kept down by his sides, and a person whose arms are elevated above his head. On putting these models into water the difference in floating capacity will be very obvious.

Measurement of Pressure.—The pressure to which the surface of a liquid is exposed can be measured by the height of the liquid column which that pressure can support. If in Fig. 105 the water contained in the cylinder AB be exposed to a certain pressure communicated by a piston at A, and if a side

Fig. 105.



tube (a piezometer tube) placed at C be in communication with the liquid, water will rise in the tube until there is equilibrium. This equilibrium is between the Pressure of the fluid in AB (together with the atmospheric pressure acting through A), tending to push upwards the column of water CD, and the downward pressure upon C, due to the Weight of that column,

which (together with the atmospheric pressure acting on D) tends to make it sink back into the cylinder. The whole outward Pressure P exerted by the liquid on the orifice C must be equal to the downward pressure due to the Weight of the column CD.

The latter is (if the area of the orifice at C be A, and h the height of the column) equal to $g \times$ the mass of the column $= Ah \cdot \rho \cdot g$. As this is distributed over an area A, on every unit of area of the surface of the fluid its amount must be $h \cdot \rho g$. Whence at C the outward pressure exerted by the liquid is, per unit of area, $p = h\rho g$.

Let us suppose that the column CD is one of water, 13·596 cm. high; the pressure per unit of surface is $h\rho g = (13{\cdot}596 \times 1 \times 981)$ dynes per sq. cm.

If at B a U-tube (a manometer tube) be fixed, containing in its bend a quantity of mercury, the mercury will stand at the same level in both branches so long as the internal pressure and the external are equal; but if the internal pressure be increased, the mercury will be depressed in the branch nearer the cylinder, and will rise in the other.

In the case supposed it would (setting aside any difference of pressure due to difference of level between C and B) sink through ½ cm. in the nearer and rise through ½ cm. in the farther limb: a difference of 1 cm. of mercury being thus established. This column of mercury is that whose weight balances the internal pressure: its weight is (1 cub. cm. $\times$ 13·596 $\times$ 981) dynes, acting upon every square centimetre. Hence —

The pressure on the surface of the liquid in the cylinder, AB of Fig. 105, may be equally well represented in brief phraseology as a pressure of say 13·596 cm. of water, or one of 1 cm. of mercury.

Exploration of the pressure in the interior of a stationary liquid mass. — In Fig. 105 let there be an aperture in the walls of the cylinder, at O; through this aperture pass a tube which exactly fits it. The inner end of this tube is furnished with a flexible and elastic cap. The outer end is connected directly or by means of indiarubber — or, better, of leaden — tubing, first with a stopcock (the bore of which is the same as that of the tube), and then with a manometer tube. Before the tube is passed through the orifice O, the level of the mercury in the manometer must be adjusted. This is done while the stopcock is open, by pouring mercury into the manometer tube and bringing it to an exact level by the addition or subtraction of mercury in the outer limb; the stopcock is then closed, and the tube adjusted with its elastic closed end in the body of the cylinder. The stopcock is then opened; the pres-

sure of the fluid in the cylinder on the indiarubber cap (if it differ from the atmospheric pressure) alters the shape of the cap, and the mercury in the manometer assumes a difference of level which indicates the pressure in the interior of the cylinder. If the cap be so small that it is collapsed by a given pressure p, it cannot be used to record pressures of greater amount than p. This defect can be remedied either by using a larger cap or else by using capillary manometers of uniform bore, in which the displacement of a very small quantity of mercury (and therefore a small compression of the indiarubber cap) will serve to indicate high differences of pressure. If the cap be at all inflated before it is inserted within the cylinder, the elastic recoil of the cap adds an unknown quantity to the internal fluid pressure, and the readings of the instrument are untrustworthy, unless special contrivances are made use of for ascertaining the exact effect due to this cause.

Fig. 105 F shows the essential parts of another instrument by which the pressure in the cylinder AB may be measured; it is substantially identical with Bourdon's Steam Gauge. A hollow tube of elastic metal having an elliptical cross-section, bent into the shape of a ɔ, and filled with liquid (alcohol, glycerine, water, or oil), suffers changes of shape under the influence of changes of pressure in the contained fluid. When the internal pressure increases, the ɔ straightens out; when it decreases, it becomes more curved.

The pressure increasing, the cross-section tends to become more circular (the circle being a figure of greatest area for least circumference): the surface and the mean curvature are constant; the curvature across the tube increasing, that along the tube diminishes, and the tube straightens out. The same principle is applied in some Aneroid Barometers, in which a coil of elliptical tubing tends to straighten out when the external pressure diminishes; and *vice versâ*, tends to flatten and curl up when it increases.

Such a tube is continuous with a box or cavity containing liquid, which may in its turn be continuous with the liquid of the cylinder when the surface-pressure has to be found, or may be connected merely with an indiarubber cap like that inserted as an explorateur in orifice O of the same figure.

For physiological work this principle is applied in Fick's Federmanometer, in which the ɔ-tube is filled with alcohol, and the tubes which intervene between it and those blood-vessels in which the blood-pressure has to be determined are filled with a solution of bicarbonate of soda of a sp. gr. of 1·083; this being (Cyon) the strength of solution which most markedly checks any tendency to coagulation.

A given amount of bend of the ɔ-tube may be interpreted as signifying exposure to a certain amount of pressure, if the instrument be previously graduated by finding the relation between certain known pressures and the distortions produced by them.

The instrument S in Fig. 105 is the sphygmoscope of Marey. The tube *a* is closed by an elastic cap which projects into the lumen of the wider tube *b; a* and its cap are filled with liquid, which is continuous with that of the cylinder; the pressure within the cylinder forces the fluid into the cap until the elasticity of the cap and the pressure of the liquid are in equilibrium: the air in the tube *b* is compressed, and the pressure is communicated to a manometric capsule or other registering apparatus, the displacement of the lever of which may be made by preliminary graduation to indicate, in terms of mercury-column, the value of the pressure to be measured.

The instrument M is the manomètre métallique inscripteur of Marey. An elastic metallic capsule filled with liquid, which is continuous with that of the cylinder AB, plays in this instrument a part which, in principle, is exactly the same as that of the elastic cap in the sphygmoscope S.

Measurement of Variable Pressure. — If the pressure in the cylinder AB of Fig. 105 be variable — as, for example, if the piston A oscillate — the various manometers represented in the figure will give oscillating readings. The manometers at B or O and the piezometer at C are subject in action to the defect that, when a single momentary increase of pressure produces a rise of the liquid or of the mercury in the column, the column does not return promptly to its mean position when the additional pressure is taken off, but oscillates like a pendulum for a period of time more or less protracted, until at length friction and viscosity bring it to rest. If the piston A oscillate, its movements are not faithfully reproduced by the oscillations of the mercury manometer, for the latter depend on (1) the weight of the column of liquid lifted at each displacement from the mean position; (2) the variations of internal pressure tending to make the column assume new mean positions; and (3) on friction; and they are the result of the composition of two sets of oscillations, the one due to the variations of pressure in AB (and agreeing with these variations in period, but not in form or amplitude or phase), while the other set, the pendulum-oscillations of the manometer-column (which may even overpower the former if the mass of mercury

be great or if the tubes be wide and offer little resistance), are due to the inertia of the mercury, but vanish if the frictional resistance be very great. The oscillations of the mercury may be checked by making one part of the manometer-tube capillary (Marey's manomètre compensateur), or by interposing a stopcock (Setschenow) the orifice of which can be narrowed till all oscillations are cut off, the instrument then recording merely the slow variations of mean pressure.

Fick's instrument F is damped (prevented from oscillating in virtue of its own elasticity) by connecting with the writing levers a disc immersed in glycerine, as shown in the figure: the viscosity of the glycerine causes all secondary oscillations rapidly to die away. The result is that the Federmanometer is very trustworthy as a recorder of the general form of the variations of pressure in AB. The sphygmoscope S and the metallic inscriptor M, not having much inertia to combat, render accurately the general form of the variations of pressure, especially if in the liquid surrounding the elastic capsules in the latter instrument there be lightly packed a number of bits of sponge to check elastic vibrations of the capsules; but all the different forms of pressure-indicators, with the exception of those shown at O, C, and B, require preliminary graduation before their indications can be held to denote the absolute value of the pressures; and further, this preliminary graduation must be frequently repeated.

3. The Kinetics of Liquid Masses.

Streams. — When a liquid flows in a stream, its particles do not become separated from one another to any perceptible extent, and the liquid usually preserves its mean density. The liquid moves as a whole and has inertia, as may be seen in a rapid and full stream leaping over a chink into which a slow or meagre stream would be pulled by gravity.

This principle is sometimes made use of in order to prevent an excess of rain-water entering drain-pipes; a sloping gutter has chinks in it, opening into the drainage system: when the gutters become flooded, the water rushes over these chinks, and the comparatively pure water is directed elsewhere than into the sewage, the excessive dilution of which may be considered as a commercial evil.

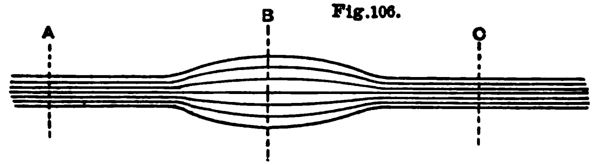
When once a steady stream-flow has been set up, it can in general be maintained by the maintenance of the supply of liquid and of the propelling force.

Steadiness of flow is favoured in actual cases by viscosity, by a free bounding surface, by converging solid boundaries, by a stream passing round a curve with its greatest velocity externally. (Osborne Reynolds.)

In any steady stream there may be drawn a series of imaginary lines, which represent the direction of movement of the elements of liquid through which they pass. These lines are called Stream-lines, or Lines of Flow. So long as a stream retains the same breadth and form, these lines may be considered parallel to one another; if the stream widen out they diverge; if it contract they converge. Such lines are shown in Fig. 106, which represents a steady stream of frictionless liquid flowing either from A towards C, or from C towards A.

Law of Continuity.—If we consider successive sections, equal or unequal, taken across a liquid stream, it is plain that the amount of liquid which crosses each section, during any given interval of time, is equal in each case: otherwise there would be congestion at some part of the stream. In Fig. 106 the amount of liquid which crosses A or C in a second must be

Fig. 106.



equal to that which crosses B: in other words, the Amount of Flow across all sections of a liquid stream is the same. This may be otherwise expressed by saying that at any part of a stream the velocity varies inversely as the area of section at that part: if the stream be broadened out so as to have a tenfold cross-section, its velocity is decreased to one-tenth. The statement of this law is due to Lionardo da Vinci.

Forces producing Flow.—If a perfect liquid, exercising no intermolecular friction and no friction on the walls of the canal or tube conveying it, were once set in motion (say in a closed circuit or circular tube), it would go on moving without the continued application of force. The energy actually expended on the liquid in producing its movement would remain in the fluid-mass, the velocity of which would consequently remain unaltered. Such a liquid might be exposed to a severe hydrostatic stress—as, for example, if such a perfect closed stream were contained in a continuous flexible tube exposed to the weight of a mass of liquid in which it was deeply immersed at

the one level — and yet the flow would not be affected. A hanging loop of tubing containing a circulating liquid, of which the lower part is exposed to a greater pressure than the upper, will present a turgidity of the lower part of the loop if the tubing be distensible, while if it be rigid there will be no expansion of the stream; in the latter case the flow will not be affected; in the former the expansion of the stream affects the local velocities, and therefore the distribution of the energy of the system, but the mean velocity may remain constant. In the case of a suspended loop of distensible tubing the indirect effect of gravity is thus to diminish the velocity of the lower part and to increase that of the upper part, both of the descending and of the ascending parts of the stream; but on the amount of flow it may produce no effect.

Flow, on the one hand, and Hydrostatic Pressure uniformly applied, on the other hand, are thus seen to be perfectly distinct conceptions, and in a perfect fluid they might be independent of one another; but in every physical fluid viscosity and friction come into play, and flow can only be kept up by maintaining a difference of pressure within the fluid, considered as a whole from end to end. As the flow is kept up, so is it started: liquid in equilibrium may be made to flow by locally increasing the pressure or by locally diminishing it.

If the pressure at a point A be p', and that at a point B be p'', the difference of pressure between these two points is $p' - p''$. The difference of pressure per unit of distance is $(p' - p'')/\mathrm{AB}$. The force producing the flow depends on this ratio; and the greater this ratio, the greater (but not in direct proportion, see p. 311) is the velocity produced. Liquid tends to flow in the direction in which the pressure falls off most rapidly; and the Force, acting on a cubic cm. of its volume, is numerically equal to the rate of decrease of pressure, per linear cm. in that direction — that is, to the **Pressure-Gradient** or **Pressure-Slope** in that direction.

Small velocities are associated with small gradients of pressure; or, in other words, with relatively great distances between points whose difference of pressure is equal to any predetermined quantity, say a unit of force. When the velocity is great, the points between which the difference of pressure is unity are relatively near to one another. The theory of Flow from this point of view resembles that of Potential: surfaces of equal pressure correspond to equipotential surfaces; Lines of Flow or Stream-Lines correspond to Lines of Force.

When the liquid is driven through a long uniform tube there is, at the orifice of inflow, a certain initial pressure; at

the other, the orifice of outflow, there is no pressure at all. If the liquid be driven by an equal force through a shorter tube, the pressure vanishes in the same way, but does so more rapidly, and — since a greater difference of pressure per unit of length is associated with greater velocity — the velocity is greater than in the longer tube. The shorter the tube the greater the velocity, other things being equal. The shortest tube possible would be a plain aperture in the side of the vessel from which the liquid issues. In this case the liquid at once assumes the greatest velocity which it can acquire under the action of a given pressure.

"Head" of Liquid. — In the case of a vessel containing liquid which passes out through an aperture, the pressure driving the particles through the orifice is the hydrostatic pressure on that orifice; it is therefore equal (if the area of the orifice be A and the height of the surface of the liquid above the centre of figure of the orifice be H) to $A \cdot H \cdot \rho g = P$ over the whole area, or $p = H\rho g$ per unit area. The height H is known as the **Head** of the liquid producing the pressure; and the "Head of Water" is a term familiar to hydraulic engineers. Head of liquid may be real, as where the flow is fed from a cistern; or it may be virtual, as where an equivalent pressure is produced by mechanical means. In the latter case, $H = P/A\rho g = p/\rho g$, where P is the total pressure on the orifice, and p the pressure per unit area.

Any pressure exerted on a liquid may be stated in terms of Head H of the same liquid; for $p = H\rho g$. Then, g being known and ρ also known, it is sufficient to specify the pressure by stating the value of H; or *vice versâ*.

The pressure produced by compression, as in pressing home a syringe, the negative pressure produced by rarefaction, as in pulling up the handle of a syringe, may all be measured in the same way.

Torricelli's Law. — If v be the constant velocity of outflow of a stream passing out of a vessel under the pressure of a constant head H, $v = \sqrt{2gH}$. If the aperture be in the sides of the vessel, the liquid issues with velocity v at right angles to the walls of the vessel; this velocity becomes combined with a new downward fall due to gravity, and the liquid travels in a parabolic path, forming a continuous parabolic Jet. The form of the parabola indicates the proportion between v and g; and thus v is found to differ very little (one per cent) from Torri-

celli's value, $v = \sqrt{2gH}$. It is somewhat greater the more convex the wall of the vessel. The amount of outflow per unit of time is not, however, the product of the area of the aperture into the velocity; it is only about $\frac{62}{100}$ of that amount.

Since $H = p/\rho g$, $v = \sqrt{2gH} = \sqrt{2p/\rho}$.

In general, whenever any fluid is acted upon by a number of pressures corresponding to the respective heads of the same fluid H, $H_{,}$, $H_{,,}$, etc., the velocity of outflow of the fluid through an orifice is $v = \sqrt{2g(H + H_{,} + H_{,,} + \text{etc.})}$.

Example. — The pressure at the bottom of a column of water 1033·3 cm. deep is equal, per sq. cm., to the weight of 1033·3 grammes. The atmospheric pressure on the surface of a liquid is equal to the same. Hence the atmospheric pressure is equal to that of a head of water of 1033·3 cm. Water at a depth of 1033·3 cm. under a water-surface exposed to the atmosphere is exposed to as much pressure as if it lay at a depth of 2066·6 cm. of water under a free surface not exposed to the atmosphere. In a vessel filled with water, 1033·3 cm. deep, provided with an aperture in its lower surface, this aperture communicating with a vacuum, and the upper surface of the liquid communicating with the atmosphere, the velocity of outflow will, according to Torricelli's law, be $\sqrt{2 \times 981 \times 2066{\cdot}6} = 2013{\cdot}6$ cm. per sec., while, if the lower aperture were also in communication with the atmosphere, the effective head of water would be 1033·3 cm., and $v = \sqrt{2 \times 981 \times 1033{\cdot}3} = 1423{\cdot}8$ cm. per sec.

Torricelli's law shows that the velocity with which a liquid issues through an aperture varies as the square root of the Head of that liquid or of the pressure; or that the Head of liquid, or the pressure p, necessary to produce a certain velocity in a free stream of liquid, subject to no resistances, is proportional to the square of the velocity.

For a given head H, the velocity of outflow, $\sqrt{2gH}$, does not depend upon ρ, the density; all liquids — ether and mercury — issue with equal velocities under the action of equal heads of their own substance: but for a given pressure p, the velocity of outflow, $\sqrt{2p/\rho}$, is inversely proportional to the square root of the density of the liquid.

Energy of Jet. — If Torricelli's law held perfectly good, that $v = \sqrt{2gH}$, the velocity would be the same as if every particle had fallen from the surface of the liquid to the orifice, and had passed out of the orifice with a velocity due to its fall through the height H.

The outflowing jet would thus convey with it kinetic energy equal to $v^2/2$, or to $gH = p/\rho$, ergs per gramme; or to p ergs per cub. cm., where p is the pressure, in dynes per sq. cm., on the orifice.

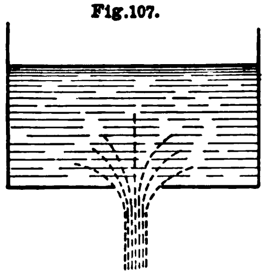
This would be absolutely the case were it not for friction

and viscosity. If the level of liquid be maintained constant by a continued supply, the velocity is constant. At the instant when the whole of the original liquid has passed out through the orifice, the experiment may be stopped. The liquid which has passed out has conveyed with it energy $= \frac{1}{2}mv^2 = mg\text{H}$ if Torricelli's law be true. At the commencement of the experiment it had potential energy (mass m at an average height of $\frac{1}{2}\text{H}$) of $\frac{1}{2}mg\text{H}$ only. It has therefore gained energy $= \frac{1}{2}mg\text{H}$. This energy has been lost by the liquid which has replaced it, and sunk from the surface to an average depth of $\frac{1}{2}\text{H}$ below the surface, thus losing potential energy $\frac{1}{2}mg\text{H}$ without any compensating gain of energy in any other form.

The same principle is illustrated in the following experiment. One cork of a Woulff's bottle completely filled with water is fitted with a piece of glass tube drawn out so as to form a jet; the other cork admits a tube leading from a vessel containing mercury; the mercury is caused to fall into the bottle. Some of the water which already fills the bottle is driven out with great velocity in a thin stream. The mercury sinking through the water loses energy proportional to its density ($mg\text{H} = \mathfrak{v}\rho g\text{H}$); the water forced out acquires this energy, and hence has a great velocity imparted to it.

The Vena Contracta. — The issuing jet may be observed (especially when it is directed upwards) not to be perfectly cylindrical, but to diminish in diameter from the aperture to a spot called the *vena contracta*, whose position is sometimes somewhat difficult to define. This conical form is due to the fact that the onward flow of liquid is not confined to that part of the fluid which exactly faces the aperture, because the lateral parts of the liquid converge on the orifice; thus the most external stream-lines of the jet, which are at first tangential to the wall of the vessel, assume a direction at right angles to this wall, changing their direction gradually, and therefore presenting a curved form as shown in Fig. 107. From the vena contracta onwards the jet is approximately cylindrical, and presently breaks up into drops, which (especially if any vibration affect the vessel from which the jet issues) are found to be oscillating in form, each becoming alternately a prolate and an oblate spheroid. The unaided eye cannot perceive these separate drops, but recognises the vein as continuous though troubled. When, however, the jet is instantaneously illuminated by the electric spark, and its momentary

Fig. 107.



shadow upon a screen observed, the existence not only of these separate drops, but also of others of a smaller size occupying intermediate positions, may be demonstrated with ease; for the instantaneous impression on the retina persists for the sixth part of a second, and the shadow of the jet appears stationary on the screen. The jet may also be looked at through a Stroboscopic Disc, a rotating disc provided with equidistant narrow apertures. Through each aperture, as it passes the eye, a glimpse is caught of the jet in a certain position. If the rate of rotation of the disc be properly adjusted, each successive glimpse is caught just when each falling drop has had its place taken by its successor; and thus, on the whole, under such a succession of glimpses the jet appears to be stationary.

This phenomenon is one of free fall in the air, for the break-up into drops depends greatly on surface-tension; a liquid cylinder of excessive length and with a free surface first assumes an undulating contour, and then breaks up into separate vibrating drops, as Plateau has shown. The vibrations of liquids in tubes are therefore not to be explained as phenomena of this kind.

Ajutages.—The amount of outflow from an aperture in the wall of a vessel is greatly influenced by the form of the ajutage or mouthpiece through which the liquid passes. This may be made so as to present the same form as the jet itself, and if it be prolonged just as far as the vena contracta, the amount of outflow becomes equal to the product, area $\times$ v $\times$ time; not because the outflow is itself altered, but because the area of the orifice of outflow is reduced so as to become equal to {amount of outflow/vt}, the terms of this ratio being unaltered. If the ajutage project inwards, the outflow and velocity are materially diminished. If the ajutage project outwards, being cylindrical, the cylinder, if its walls be wetted by the liquid, is completely filled by it, the jet is cylindrical, and the outflow is greater than when there is no such ajutage. The liquid is drawn towards the sides of the cylinder, and conversely, the sides of the cylinder are drawn towards the liquid. Hence there is no pressure exerted on the walls of the tubular ajutage; on the contrary, there is suction, and if any part of the walls of the tube be mobile, it will be drawn into the stream.

The peculiarly beautiful forms presented by jets under various circumstances are described and figured by Savart in the *Annales de Chimie et de Physique*, vols. 54 and 55.

If two vessels having an aperture in each of the same size and shape, and at the same level, be so arranged that these apertures are exactly opposite one another and close together: if liquid be poured into the one vessel, it will

run into the other. The vessels may then be removed to a certain distance from one another, and the liquid will continue to pass from the one vessel into the other, through a tube formed of its own superficial film, until the same level is nearly attained; then the liquid begins to flow out of both vessels, and the two jets, meeting, spread out into a sheet which is driven back and fore between the two orifices as the liquid in the one or the other vessel stands for the moment at the higher level.

Recoil. — The law of action and reaction perfectly applies to liquid jets and to the vessels from which they issue. The Hydraulic Tourniquet is an example: a cistern containing water and capable of rotating on an axis: pipes ending obliquely issue from its sides: water runs out of these pipes: and by reaction they are driven backwards. Since they are not fitted to an immovable cistern, but to one free to rotate, the whole rotates, and thus the contrivance may be used to convey water-power, the water constantly running into the rotating cistern, and running out of the obliquely-set exit pipes.

Resistances. — When a fluid stream passes through a tube or a channel it experiences different retarding resistances, which convert energy of motion into heat, and of which the following are the chief: — Surface Adhesion, Surface Friction, Inequalities of the Surface of the bounding solid, Eddies, and Fluid Viscosity.

Surface Adhesion. — If a liquid wet the walls of the tube or channel through which it passes, the layer of liquid which is in contact with the walls does not change except by molecular diffusion and exchange. It remains *in situ* while the liquid flows past; in other words, there is infinite friction between this layer and the walls wetted by it. This surface adhesion, when once the flow has been set up, does not directly cause any waste of energy. While the walls are being wetted there is a slight liberation of heat, due to the satisfaction of the mutual molecular attractions between the liquid and the walls.

Surface Friction. — If the liquid do not wet the tube through which it passes, the surface of the moving liquid and the walls of the vessel rub against one another, and energy is lost in overcoming this friction. Loss of kinetic energy is also caused by roughnesses on the walls of the tubes or channels, which give rise to little eddies or whirlpools.

Eddies are produced when a moving fluid is subjected to unsymmetrical retardations. The cases in which eddies, whirlpools, vortex-rings, rolling and tumbling water, and the like, are produced are extremely numerous. Water flowing in a tube

which suddenly widens or suddenly narrows generally presents such eddies at the point of sudden enlargement or contraction.

The production of eddies is favoured by mobility of the liquid, by variations of velocity at different parts of the cross-section of the stream, by rigid bounding walls, by diverging boundaries, by curvature with the greatest velocity internally. (Osborne Reynolds.)

Viscosity. — When a disc or cylinder suspended in a liquid is caused, by twisting the supporting wire or wires, to enter into oscillation, it is found that the oscillations soon die away; though they continue isochronous, their amplitude diminishes; and the amplitudes of any two successive oscillations stand to one another in a constant proportion. If the disc or cylinder be wetted by the liquid, the layer immediately in contact with the solid remains in contact with it; this film, moving with the solid, sets in motion the film next in contact with it, and that in its turn sets the next in motion. Each film goes through a displacement somewhat less extensive and more retarded than the one gone through by the film which sets it in motion. Continuous rotation of the disc or cylinder would in time cause the whole liquid to rotate; but the influence of an oscillating disc travels a very short distance, for half an inch away from the disc the liquid remains undisturbed. Within this small distance the liquid performs oscillations which in period resemble those of the oscillating disc, but which in amplitude are less, and in phase more retarded, the greater the distance from the disc. This lagging behind on the part of the liquid has the effect of dragging on the disc and of gradually bringing it to rest.

If the disc be wetted the retardation is independent of the nature of the material of the disc, for there is no velocity lost by friction between the solid and the liquid. If the disc be not wetted, there is distinct friction (external friction) in addition to the viscosity (internal friction).

The Coefficient of Viscosity serves as the means of measuring the viscosity of a substance. We have already seen (p. 227) that it is equal numerically to the force which is necessary to maintain a flow of one layer of one unit-area past another of the same area with a relative velocity of one unit, the distance between the layers being unity, and the space between them continuously filled with the viscous substance.

If F be the total force required to keep up the flow of two layers past each other, their area being each A, their respective distances from a plane of reference being $d_{,}$ and $d_{,,}$, and their distance from each other therefore

$d_, - d_{,,}$; if their respective velocities be $v_,$ and $v_{,,}$, and their relative velocity $v_, - v_{,,}$; and if the coefficient of viscosity be η, $F = \eta \cdot A\,(v_, - v_{,,})/\overline{d_, - d_{,,}}$ $= \eta \cdot A \cdot \tan \theta./t$, where $\tan \theta$ is the total shear effected in time t.

The Activity required to keep up this flow is the product of the force F acting, into the mean relative velocity, $\frac{1}{2}(v_, - v_{,,})$, of the moving liquid; that is, it is equal to $\frac{1}{2}\eta \cdot A(d_, - d_{,,}) \cdot (\tan \theta./t)^2$, or, per cub. cm., to $\frac{1}{2}\eta \cdot (\tan \theta./t)^2$.

The dimensions of η are $[M/LT]$.

In the case of water at 0°·6 C. this coefficient is 0·0173, at 45° C., 0·005833, at 90° C., 0·00339 (Meyer), while that of air,* which obeys the same laws, is ·00017 $(1 + 0{\cdot}00733t)$, where t is the C. temperature; all expressed in C.G.S. units.

Though the density of air is $\frac{1}{770}$th that of water, its viscosity is as much as $\frac{1}{100}$th that of water. For brass, η is about 300,000,000000. Moist air is more viscous than dry air: hot air is more viscous than cold air.

Hot water is less viscous than cold. Most saline solutions are more viscous than water, saltpetre solution being an exception. Most saline solutions are more viscous the more concentrated they are, saltpetre solution being again an exception (Meyer).

The experimental determination of the coefficient η by means of observations made with the aid of an oscillating disc involves much mathematical computation, and it is often quite sufficient to record the so-called **Logarithmic Decrement** or *log. dec.* special to each liquid. Let us suppose that the oscillating disc or cylinder first turns through the angle δ; that at the next oscillation its deviation from its mean position is $\frac{99}{100}\delta$; that at the third it is $\frac{99}{100} \times \frac{99}{100} \times \delta$; and so forth. Then each successive angle is equal to the one immediately preceding multiplied by $\frac{99}{100}$; its log. is equal to the log. of the preceding angle of oscillation *plus* that of $\frac{99}{100}$, or *minus* the log. of $\frac{100}{99}$; that is, *minus* ·0043648. Such a constant difference in the logarithms of the successive angles of oscillation is the *log. dec.* for the particular substance whose viscosity it measures. Under Poiseuille's Law (p. 315) we shall find a simple method of measuring the value of η.

Effect of Viscosity on a stream of liquid. — The external layer is at rest. The axial parts of the stream are less influenced by viscosity. The velocity of the axial part of the stream is greater than that of the peripheral; the fall of pressure is therefore greatest in the centre of the current. The pressure being least in the centre, the external parts of the stream tend to move into the centre, and to have their velocity accelerated. In capillary tubes the axial stream travels with a greater speed than the average as determined by Poiseuille's Law, to be presently stated.

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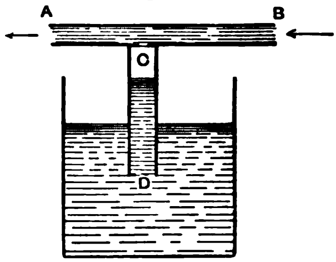
* O. E. Meyer, *Pogg. Ann.*, vol. cxlviii., 1873.

Lateral diminution of pressure. — If through a tube of the form shown in Fig. 108 there pass a current of liquid in the branch AB under a pressure which is barely sufficient to keep up a stream filling the tube, the mutual attraction of the walls of AB and the liquid will put the liquid in AB in a state of tension and diminish the pressure in AB. In the side tube CD a certain column of liquid can be supported in consequence of the diminished pressure in AB. If this rise to the point C, the upper layers of the column DC will be constantly carried off by the stream BA, and thus a stream is set up in the direction DC. If the pressure in the main pipe AB be too great, liquid will be driven down CD.

Fig. 108.

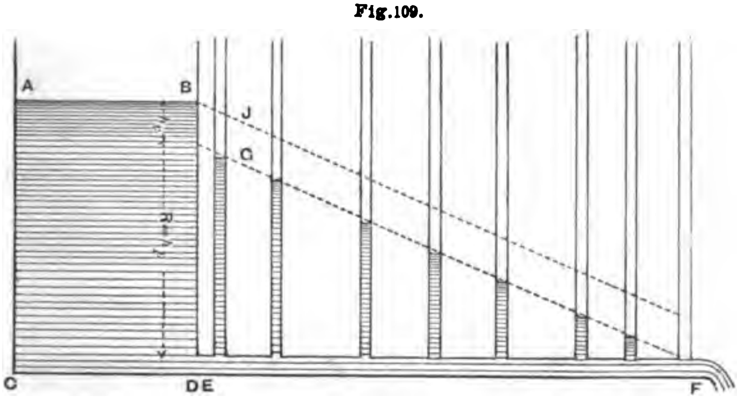


The former action is by some considered as explaining the flow of lymph up the thoracic duct.

The same kind of suction-effect may be perceived in the older forms of washhand-basins connected with a house drain-pipe by a simple bent tube or trap; a downrush of liquid along the main pipe produces a deficiency of pressure, which allows the atmospheric pressure communicated through the basin to drive the liquid which seals the trap into the drain-pipe, and thus to leave a channel patent to the entry of sewer gas.

Constant flow through uniform rigid pipes. — The pressure which is necessary to keep up a continuous flow of water in a uniform pipe, EF in Fig. 109, may be produced by a total

Fig. 109.



head H of water in a vessel (a p r e s s u r e - v e s s e l, ABCD in the figure), this height H being maintained constant. The water is observed to issue from F with a constant velocity v'; this veloc-

ity would (if there had been no resistances) have corresponded to a head $H_v = v'^2/2g$; this may be considered (so far as the velocity and the kinetic energy of the outflowing stream at F are concerned) to be the effective head of water at F, the orifice of exit: it may be called the "**velocity-head.**" This velocity-head, GJ, is equal in all parts of the tube.

The hydrostatical pressure in the immediate neighbourhood of F is necessarily null; that at E, just within the pipe EF, is less than the pressure ($=H\rho g$ per unit of area) corresponding to H, the original head of water; it corresponds to a head H_p (the **Pressure-head**), which differs from H in the first place in consequence of a certain slight waste of head caused by the formation of eddies between D and E, and in the second place differs from H by the amount of the velocity-head itself. If we neglect the effect of these eddies we may say that the velocity-head and the pressure-head are together equal to the total head: $H_v + H_p = H$.

The hydrostatic pressure in the tube (if the tube be uniform) dies away uniformly, as is shown by the level assumed by the water in the successive piezometer-tubes of Fig. 109.

If the tube were lengthened there would be a similar — but necessarily a slower — dying away of the pressure; the velocity would be less throughout the tube; the velocity-head being less, the pressure-head would be greater: there would therefore be a greater pressure at E.

The hydrostatic pressure at any part of a stream measures the resistance which has yet to be overcome. If there were no resistance (as in the imaginary case of a perfect liquid) there would be no lateral pressure, no pressure-head; and the whole of the original total head H would be taken up in producing a velocity $v = \sqrt{2gH}$.

The greater the velocity of a stream, the greater the resistance encountered by it within a tube of given dimensions. The resistance at any point thus depends not only on the dimensions of the tube between that point and the orifice of outflow, but also on the velocity of the stream.

The relation is $U = l(av'^2/r + bv'/r^2)$ (Haagen); U being the measure of the resistance, l and r the length and radius of the tube yet to be traversed by the stream, a and b constants to be found by experiment.

U is not a number of units of force, but it is the height (in cm.) of a lateral column of water whose weight can be supported by the stream-resistance. Its weight is Ug dynes per sq. cm., and the local Resistance at any point of the stream is therefore $R = Ug$ dynes of force per sq. cm. of transverse section of the uniform stream passing that point.

Given that the tube has a certain length l, and internal radius r, and a certain constant driving head of water H, the velocity v' must so adjust itself that the three equations $H = H_p + H_v$; $H_v = v'^2/2g$; and $R/g = U = H_p = l(av'^2/r + bv'/r^2)$, shall all hold good. If the tube be exceedingly long, the resistance becomes proportionally very great and the velocity very small: yet in a tube of any assignable length there would be a constant velocity, and the pressure would uniformly (though slowly) diminish from one end of the tube to the other. The **pressure-line**, GF of Fig. 109, would in such a case—a case of low velocity—have a gentle slope. When the tube is very short, the resistance is initially small and rapidly falls; thus great velocity is associated with steep slope of the pressure-line.

If the driving pressure increase or diminish (the dimensions of the tube remaining unchanged), the velocity produced by it and the resistance brought into play both increase or both diminish. If the dimensions of the tube be altered while the driving pressure remains unchanged, the resistance and the velocity will vary in contrary senses: increased resistance, diminished velocity; diminished resistance, increased velocity. If the resistance be increased by increasing the length or lessening the diameter of the tube, the velocity and the amount of flow cannot remain constant unless the driving pressure be also increased. (Hypertrophy of the heart when the placental is added to the ordinary circulation.)

If the hydrostatic pressure be found to have increased (higher columns being supported in the piezometers), the plain inference is either that the driving pressure has been increased, or else that the peripheral resistance has been increased by narrowing or lengthening or perhaps by roughening the tube. If the pressure be found to have been diminished, either the driving power or the resistance, or both these, must have been also diminished.

If more than one of these elements vary, the result may be either accumulation or compensation of effects. Higher head or narrowed tubes both increase the pressure; with lowered driving pressure on the one hand and narrowed or lengthened tubes on the other, the pressure may remain the same, though, in this case, the velocity is diminished. Hence it is necessary to observe both the pressure and the velocity in order to investigate the local condition of any stream.

Flow due to variable pressure in uniform rigid tubes.— If the driving pressure be reduced, the pressure-head becomes a greater, and the velocity-head a less, fraction of the reduced total head; the velocity-head is thus lessened not only in proportion to the diminution of driving pressure, but in a still greater ratio. Conversely, if the driving pressure be increased, the velocity-head is increased in a greater ratio. Since the velocity is proportional to the square root of the velocity-head, it is not the velocity but the square of the velocity which is a little more than doubled by doubling the driving force. Hence a curve indicating the variations of velocity agrees in general form, but not in its amplitudes, with a curve indicating the variations of driving pressure.

Interrupted flow through uniform rigid pipes.— *a.* The driving force may be applied intermittently, and may cease during the intervals. A perfect incompressible fluid, treated in this way, would move like a solid rod struck endwise by a hammer: all its particles would move simultaneously, and liquid would pass through the orifice of exit without any interval of time. A physical liquid is hurled upon itself, compresses itself, and resiles. Thus the suddenness of outflow at the orifice of exit is somewhat modified; but even with physical liquids the more rigid the tube the more abrupt is the onflow. (Atheromatous arteries.)

b. The pressure being continuous, the flow may be suddenly stopped by an obstruction, say by a stopcock suddenly closed. Beyond the stopcock the liquid runs on somewhat and rarefies itself, or even produces a vacuum near the stopcock; it returns and oscillates until it comes to rest. Between the driving pressure and the stopcock there is a sudden increase of pressure. If a house water-tap be suddenly turned off when water is running from it, a jar or jolt may be given to the water in the pipes, which may be audibly perceived throughout a large building. This jolt is due to the sudden stoppage of the water, which has already acquired momentum. The water compresses itself, rebounds and oscillates, producing waves of condensation and rarefaction which travel back into the mains. The pressure in the pipes is greatly increased by this mode of treatment; an original pressure of 30 lbs. per sq. in. may be raised to one of 120 or 130 lbs.

This principle is economised in the Hydraulic Ram. A stream of water is alternately cut off and allowed to flow: every cut-off enables the

stream, whose pressure is thereby greatly increased, to force a valve which it could not otherwise force, and water is thus driven into a small chamber containing a limited volume of air. This air is compressed, and its elasticity enables it to force the water out through a narrow jet, at a pressure nearly equal to the greatest pressure experienced by the liquid during the cut-off.

Flow through bent tubes. — Bends increase the resistance and diminish the proportionate velocity. The forward momentum of the liquid is destroyed, the reaction of the walls is called into play, and by the elasticity of the liquid and of the walls a new path is given to the liquid. Energy is consumed in this process, particularly in producing eddies in the stream, and the piezometer-tubes show that the pressure in the water, which is about to meet the obstacle, is much greater than in that which has just left it (Fig. 110). If the driving pressure be applied

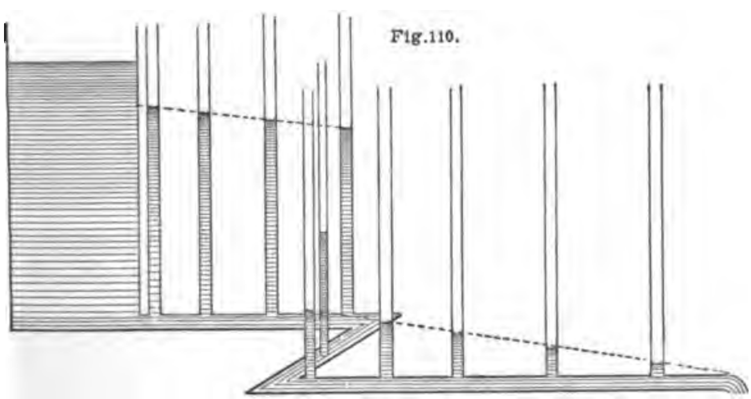


Fig. 110.

intermittently, the liquid between the driving apparatus and the rigid bend may be sharply compressed before it can pass round the bend; it is driven against the bend like a solid, and if the bend be at all extensible it is driven forward. (Locomotive pulse.)

Flow in tubes not of uniform diameter. — We are apt to think that when a fluid passes from a wide into a narrow tube the pressure is increased; and conversely, when a fluid runs out of a narrow into a wide tube, that it is relieved of pressure. The reverse is the case. To understand this we must consider the flow as already set up and constant. The law of continuity shows that when a rapid stream passes into a wide channel, it travels more slowly. The velocity-head suffers a diminution, and the pressure-head increases: the kinetic energy possessed by the rapidly entering narrow stream is partly spent in dashing

that stream against the comparatively stationary layers in the wider channel, and is thus partly converted into potential energy. A certain degree of compression is thus produced, and a corresponding pressure, which is additional to the hydrostatic pressure already existing. Conversely, when a stream narrows it runs more rapidly: its kinetic energy becomes greater (mass for mass), and there is a tendency to stretch or rarefy the narrowed and accelerated stream. This tendency to rarefaction, or even to tearing asunder the stream, corresponds to a defect of pressure in the narrower tube; and it has been utilised in Venturi water-meters, which consist of manometers connected with a wider and a narrower part of the same water-pipe. The readings are different at different velocities.

In the case of a liquid passing from a narrower channel into a wider, we have a flow from a place of lower pressure into one of higher. This apparently anomalous flow is explained by the fact that the pressure, even in the wider channel, can never (on account of the gradual disappearance of pressure-head in the production of heat) exceed, but must always be less than, that corresponding to the difference between the original head H and the local velocity-head.

Flow in branched rigid tubes. — If the total cross-sectional area of the branches do not exceed that of the main tube from which they spring, the parietal surface-area of the stream is increased; this increases the resistance, and the velocity falls. If the total cross-sectional area do exceed that of the main tube, the channel is widened and the resistances are relatively diminished: they may even be diminished by this cause more than they are increased by the increase of the total surface. The resistances are, on the whole, absolutely diminished in this case, and the velocity of the whole system may be greater than that in an unbranched tube of corresponding length.

If we compare two branched systems: the one large, containing many branches, each of which would, if the stream were driven through it alone, offer much resistance, but all together affording the stream a wide bed for its flow; the other system small, containing few branches, of which each is capable of offering only a small resistance, but which, by their small number, cause the stream to flow in a narrow bed; it is possible that the driving pressure necessary to produce a given velocity may, in these two cases, be the same. The advantages of the aggregate wide channel in the first system are neutralised by the great

resistances; the advantages of the small resistances in the second system are counteracted by the narrowness of the channel.

Thus both small and large animals have approximately the same blood pressure in the aorta.

Where branches are given off, the pressure either increases or begins to fall off less rapidly, because the velocity diminishes; where the branches reunite, the pressure rapidly falls off. If the whole system be quite symmetrical, the pressure in the middle of the system is greater than half the initial pressure.

The pressure in the capillaries is more than half the initial pressure in the aorta, though their joint sectional area is very great and their resistance accordingly very small; and the pressure very rapidly falls as the veins unite. At the same time the velocity increases as the sectional area diminishes, and the amount of flow into the auricles is equal to that from the ventricles.

When in a system of branched tubes, some of the branches are relatively shorter or wider, the amount of flow through these is to some extent relatively more rapid.

When in such a system the flow is once fairly established, if the branches as a whole become narrowed, the resistance is increased and the velocity falls. If some only be narrowed, while the driving pressure remains the same, the velocity in the remaining branches may be increased, for the channel is narrowed, being wholly or partly restricted to the latter. The pressure in those which are narrowed is increased; but the pressure in the unnarrowed branches may also be increased, for the peripheral resistance of the system as a whole is rendered greater, and the velocity-head is diminished. The effect on the unnarrowed vessels may thus be the same as if the driving power had been increased.

Flow through capillary tubes. — Poiseuille found that the law regulating the velocity of the flow of liquids through tubes is materially altered when the diameter is very small. Through capillary tubes he found that the volume of liquid flowing in unit of time is $\mathfrak{b}/t = k \cdot r^4 \mathrm{H}/l$, where $\mathfrak{b}$ is the whole volume observed to flow in the course of time t, r and l the radius and length of the tube, H the head of liquid, and k a constant to be determined by experiment. This determination is effected by actually observing the number of seconds taken to drive a given volume of liquid through a capillary tube of given length and diameter. The constant k does not depend on the nature of the walls of the tube, if the walls be wetted by the liquid;

it depends only on the nature of the liquid and on the temperature. Water near the boiling point passes five times as rapidly through capillary tubes as water near its freezing point. We can do no more here than assert with a reference* that theory indicates that (on the assumption that the layer of liquid in contact with the solid wall is at rest, an assumption verified by the fact that it is immaterial what is the nature of the substance of the tube, provided that it be wetted by the liquid) the coefficient $k = \pi\rho g/8\eta$: whence it is easy to find the value of η, the coefficient of viscosity, for any liquid at any given temperature.

Since $k = \pi\rho g/8\eta$, the Volume of liquid passing through a capillary tube in time t is $\mathfrak{v} = \pi\rho g \cdot r^4 \mathrm{H} \cdot t/8\eta l$ or $\pi r^4 \cdot t \cdot p/8\eta l$, where p is the pressure, in dynes per sq. cm., upon the surface of the liquid as it is delivered into the capillary tube; the pressure at the other end of the tube being *nil*.

The Mean Velocity v' = Volume of fluid flowing across any section in unit of time ÷ Area of that section. Hence, in a capillary tube, $v' = \mathfrak{v}/\pi r^2 t = k \cdot r^4 \cdot \mathrm{H}/l \div \pi r^2 = k \cdot r^2 \cdot \mathrm{H}/\pi l = pr^2/8\eta l$.

The flow in capillary tubes is proportional not to the square, but to the fourth power of the radius; the velocity is proportional not to the square root of the pressure, but to the pressure itself.

The resistance, $R = p$, in capillary tubes varies directly as the velocity; in wide tubes approximately as the square of the velocity. This seems discrepant; it is due to the formation of eddies in the wider tubes; in a capillary tube the flow is steady.

But what is a "capillary" tube? For water it is a tube under 1/50 inch in diameter. Would it be the same for treacle? No; a long inch-wide tube behaves with treacle as a 1/50 inch tube does with water; the flow of treacle through it obeys Poiseuille's Law. Professor Osborne Reynolds has made the very important discovery that steadiness of flow and obedience to Poiseuille's Law cease only when the expression (Velocity of stream × Width of stream ÷ Viscosity of fluid) has reached a certain critical value. Too great a velocity, too wide a stream — in either case the stream breaks up into eddies and the movement is like that of water in a wide tube; but even in a wide tube — not too wide — the effect of great viscosity may keep the above expression down below its critical value and the flow may be steady like that of water in a capillary tube. If the stream be wide enough the above expression will be above its

* Helmholtz and Piotrowski, *Sitzungsberichte d. Wien. Acad.*, *Math.-naturw. Cl.* XL. 1860; and O. E. Meyer, *Wiedemann's Ann. d. Physik u. d. Chemie*, 1877, vol. ii., and literature there cited.

critical value; a Mississippi of treacle would flow turbulently round any obstruction; lava-streams flow like water. One and the same tube may be made, by increasing or diminishing the velocity of flow through it, to play the part of a wide tube with eddies or of a narrow tube with steady flow.

Measurement of the pressure at any point of a stream. — This cannot be effected by cutting the tube and fixing a manometer in it. The result in that case would be a determination of the original driving pressure or head of liquid, for the condition becomes statical and the liquid seeks its level. The manometer must be fixed laterally and at right angles, and the flow must be allowed to proceed without any hindrance. This being seen to, any one of the forms of manometer already described may be used.

Measurement of the velocity of a stream.—It is needless to point out that the velocity cannot be calculated from a single observation of the pressure at any point.

The velocity may be observed by direct or by indirect methods with more or less accuracy.

A. Direct Methods.—*a. Optical.*—The velocity of bodies floating in the stream may be measured by observing the distance traversed by one of them in a given time.

Objection to this method. — The velocity of floating bodies does not necessarily represent the velocity of the stream.*

A body of the same sp. density as the liquid moves in the axial stream: the larger it is, the more it is delayed by the peripheral layers and the more slowly it moves. It moves without rolling, unless it gets into the peripheral layers and is twisted out of them into the centre of the stream.

A body heavier or lighter than the liquid is pressed against the upper or lower wall. It rolls in the peripheral layers, for it is urged forward by unsymmetrical forces; and it is retarded. Within certain limits, the larger it is, the less it is retarded; but it always travels more slowly than a body of the same density as the liquid.

Of two heavy bodies the heavier moves more slowly; of two light bodies the lighter moves more slowly; an effect due to rolling friction.

A disc which rolls travels at the same rate as a sphere of the same density and radius; if it travel in the axial stream the velocity is the same as that of a sphere or cylinder of the same radius. If a heavy or light disc lie flat against the wall, the friction is increased and the speed diminished.

Bodies nearly filling a tube approximate more nearly in speed than when they are small in relation to the tube; if heavy they tend to check the stream and to permit the pressure to accumulate behind them: if light they tend to roll rapidly, and thus to diminish the pressure behind them.

If the density of the liquid be diminished, the bodies (*e.g.*, red corpuscles),

* For the facts mentioned in these paragraphs I am indebted to the kindness of Prof. Hamilton of Aberdeen, who rejects Schklarewsky's assertion that the same granular substance may float in the axial or in the peripheral stream, according to the *nature* of the other granules with which it is associated in the stream.

which had been just light enough to float in the unaltered liquid, sink and roll in the lower part of the stream. If it be increased, all float. In either case, or if the sp. gr. of the floating bodies be altered, these bodies block the bends of the tube and check or slow the stream (albuminuria, cholera, pernicious anæmia, fatty embolism, injection of milk into the veins). When the stream is slowed, if the particles be viscid they adhere to the sides of the tube and the stream flows past them: after a while they may be torn off and proceed.

b. Chemical. — This is a method principally employed in physiological work and suggested by Hering. A soluble chemical substance easily recognised is introduced into the stream: at a certain distance the stream is tapped, and samples of the liquid are there collected at regular intervals of time. The interval of time which elapses before the chemical substance can be detected in the liquid at a given distance affords a datum from which the mean velocity can be roughly calculated.

c. Volumetric. — 1. The vessel may be cut and the amount of outflow measured. This is objectionable (1) because the resistance is diminished and the velocity increased, and (2) if the stream be a closed circuit, opening into it causes loss of liquid and a modification of the driving pressure.

2. A tube filled with liquid may be placed in the course of the stream. The liquid in the tube is driven into the stream, liquid from the stream taking its place. The time taken to empty the tube is observed. The objections are (1) resistance interposed, and (2) difficulty of recognising the exact instant at which the liquid is wholly replaced.

3. In Ludwig and Dogiel's Stromuhr, used by physiologists, there are two large cavities; the one nearer the heart is filled with oil; the cavity nearer the periphery is filled with defibrinated blood (the introduction of which into the animal's circulation does little harm). When the stream flows, the oil passes from the proximal chamber into the peripheral one: the defibrinated blood of the peripheral chamber passes into the animal: the proximal cavity of the stromuhr becomes filled with the fresh blood of the animal. Then by a play of stopcocks (effected just when the oil is on the point of escaping into the vessels of the animal) the stream in the instrument is reversed, and the animal's blood flows into the chamber now occupied by the oil, the oil passing back into the chamber which it had originally occupied, and the blood which had freshly entered that chamber being returned into the circulation. The oil-chamber is always functionally in the rear. This may be repeated several times, and thus the amount of time taken to pass a certain large volume of blood through the instrument may be ascertained. The animal suffers on the whole no loss of blood, and there is no material increase (2 to 5 mm.) of resistance in the circuit of fluid; while if the stream be periodically reversed with attention and promptitude, relatively great accuracy is attainable in the determination of the mean velocity.

B. Indirect Methods. — The mechanical effects of a stream of liquid are derived from its forward momentum.

a. "Hydrostatic Pendulum." — If some object be attached by its upper end to the walls of the tube, and swing freely in the stream, the quicker the flow the more will the lower free end be displaced. A box containing such a contrivance (the so-called hydrostatic pendulum of engineers) may be inserted (as in Vierordt's Haemotachymètre) in the course of a stream; or the pendulum itself may consist simply of a needle thrust

through the elastic walls of the tube or (as in Chauveau's Haemodromomètre) through elastic parts of the wall of the tube. In the latter case, as the immersed end of the needle is driven by the stream in one direction, the external end moves in the opposite direction; and the elastic walls of the tube exercise a constant pressure upon it, tending to adjust it in its normal position at right angles to the wall of the tube or to the axis of the stream. The whole arrangement is very sensitive to variations of velocity.

Chauveau's instrument has been so modified that at one and the same part of the circulation the pressure may be found by a sphygmoscope, and the velocity ascertained by a hæmodromometer which, being coupled with self-registering apparatus, has acquired the name of hæmodromograph. The condition of a stream cannot be thoroughly investigated unless the pressure and the velocity are both ascertained.

b. Pitot's Tubes. — If a piezometer-tube be prolonged into the axis of a stream, and if it be bent at the submerged end so that its lower orifice faces the stream directly, the liquid will be forced up in it to a certain height, varying with the velocity, and greater than that which corresponds to the lateral pressure at the same part of the tube. If its lower orifice be turned away from the stream, the column of liquid is lower than it would have been in a plain piezometer. If two such lateral tubes be fixed near to one another in the walls of a main tube, the mouth of one facing, that of the other turned away from the stream, there will be set up a difference in the heights of the columns in these tubes; this difference depends entirely on the velocity, and varies with it. This principle has been applied by Marey (*Trav. du Lab.*, 1875, p. 347) in the formation of a registering instrument of great excellence, but in physiological work unfortunately not suitable, because coagulation is promoted by the projection of the lateral tubes into the lumen of the main tubes, the vessels through which the blood passes.

c. Venturi-meters. — See p. 314.

All instruments in which indirect methods are applied must be graduated by exposing them to the action of streams of various known velocities, and marking the corresponding positions at which the index stands.

Work done in keeping up a stream. — The initial head H would (if no energy were wasted in overcoming resistances, etc.) produce a mean velocity $v = \sqrt{2gH}$, and the kinetic energy imparted to a mass m of fluid would be $\frac{1}{2}mv^2 = mgH$. This is the whole energy lost by the water falling out of the cistern, and this is independent of the amount of the resistances. The work done in keeping up a stream is therefore independent of the length of the pipes; the pressure at any point is not so. If we know m and H it is easy to calculate the total work done by the driving apparatus: if we do not know H, but do know H_p (the height of the maximum piezometer column), the equation

$$\text{Work done } (= mgH) = mgH_p + \tfrac{1}{2}mv'^2$$

enables us to find it when we know v', the velocity of outflow.

This equation is arrived at by combining the two equations, $H = H_p + H_v$ and $H_v = v'^2/2g$.

If the left ventricle of the human heart propel at each systole 180 grammes of blood at a mean pressure equal to that of 12·8 cm. of mercury (that is, the sp. gr. of blood being 1·055, at a head H of 165 cm. of the same liquid, blood), during each systole the left ventricle does work equal to $mgH = 180 \times 981 \times 165 = 29{,}128000$ ergs = weight of 29,692 grammes raised 1 cm. or ·2969 kilos. lifted through 1 metre.* Otherwise, if p be the mean pressure in dynes per sq. cm., the energy expended $= mgH = mp \;\; \rho = p\mathfrak{v}$; and p = weight of 12·8 cub. cm. of mercury $= (12{\cdot}8 \times 13{\cdot}596 \times g)$; whence Work $= (12{\cdot}8 \times 13{\cdot}596 \times 981) \times (180 \div 1{\cdot}055) = 29{,}128000$ ergs during each systole.

If the liquid leave the tubes with actual velocity v', its kinetic energy is then only $\frac{1}{2}mv'^2$. The difference $(mgH - \frac{1}{2}mv'^2)$, having been spent in overcoming resistances, has been transformed into heat.

Streams in elastic tubes. — If the inflow be continuous the internal pressure expands the tube, and continues to do so until the tube exerts a contractile restitution-pressure equal to the expansile pressure of the liquid. Then the stream flows on as in a rigid tube.

If the inflow be intermittent the case is different. We may first suppose the liquid to be injected instantaneously, and the tube to expand as a whole. In such a case, a sudden inflow creates a pressure which expands the walls of the tube, in addition to forcing onwards a certain quantity of the liquid. Work is thus done upon the walls of the vessel. These, being elastic, tend to restore the work done upon them. When the pressure due to the inflow is relieved, the primitive form of the tube is gradually resumed: the potential energy of the stretched walls is transferred to the liquid in the form of kinetic energy. The stream is thus kept up until the original form of the tube is restored.

If there be a quick succession of influxes, each successive inflowing quantity may enter the elastic tube before its predecessor has left it. If the frequency of inflow be sufficiently great, the outflow may be uninterrupted though variable in velocity and amount. The rate of succession necessary for continuous outflow depends on the width of the tube — being greater as this is greater — and also, in the reverse sense, on the extensibility of the walls of the tube and on the mechanical resistance offered to the onflow. The greater the resistances, or the greater

* Nearly 150 foot-pounds per minute, or $\frac{1}{220}$ horse-power on the average; about [illegible] h.-p. during contraction: its own weight raised about 22,000 feet per hour.

the extensibility of the tube, the greater will be the proportionate size of the dilatation or pouch of the elastic tube, and the more continuous will be the outflow: the more deliberate, therefore, may be the succession of influxes necessary to keep up a continuous outflow.

Primary waves in elastic tubes. — The tube does not in fact dilate and contract as a whole, nor does the liquid at each inflow enter instantaneously. The pouching is local, and the inflow more or less gradual. The more gradual the inflow, the less the width and the greater the length of the pouch produced: the more abrupt the inflow, the wider and shorter will the pouch be. The pouch contracts and drives the liquid onwards: this action dilates the walls of the tube beyond the pouch: the dilatation travels onwards, and liquid travels with it. The contraction of a pouch can never produce another quite equal to itself in width: and so, as the dilatation travels along, it becomes narrower and longer. In this case the direction of movement of the liquid as a whole, and that of the dilatation, are the same.

If instead of a sudden inflow of liquid due to pressure there were a sudden outflow due to suction, there would be a local collapse of the walls of the tube. The walls, returning to their original form, will cause a stream to be set up, which travels towards the orifice of suction. The contracted *form* of the tube will travel in a direction opposed to that of this stream.

The dilatation or the contraction of the tube, as it travels, forms a wave, the so-called Pulse-Wave — positive, and travelling in the same direction as the liquid, in the case of an inflow and dilatation; negative, and travelling in the opposite direction, in the case of a suction of the liquid and a contraction of the elastic tube.

The farther down the tube, the later the arrival of this pulse-wave. The velocity of propagation of a wave of this kind depends on $\mathfrak{E}$, the elasticity-coefficient, and on a, the thickness, of the wall: the greater these are, the greater is that velocity. It also depends on d, the diameter of the tube, and on ρ, the density of the liquid: the greater these are, the less is ν, the velocity of propagation, the distance traversed by the wave in one second.

The law is (Moens: *Die Pulscurve*, Leiden, 1878) $\nu = 0{\cdot}9\sqrt{g\mathfrak{E}a/\rho d}$.

The elasticity varies in the case of arteries; the more expanded an artery is, the more elastic it is; and therefore a full pulse travels more rapidly than one of small expansion. The length of such a wave = time of inflow ×

rate of propagation. In the case of the pulse the former is ⅙ sec., the latter is from 10 to 18 metres per sec. The length of the dilatation in the arteries would be 3·33 to 6 metres if the arterial system were long enough to contain a whole wave. The arterial system is never during life wholly relieved from pressure, and is in a state of permanent though variable distention. The elasticity and self-contraction ("arterial tension") of the arteries are opposed to the expansile internal blood-pressure, and at each instant these are either equal to it or are in the act of coming into equilibrium with it.

The form of a simple pulse-wave. — The more abrupt the disturbance, the steeper will be the front of the resulting pulse-wave, and the more abrupt will be the expansion of any part of the tube at which the pulse-wave arrives. The greater the elasticity, the less will be the height of the wave; the greater the length of the wave, the gentler will be the pulse-rise. The greater the resistance, the more abrupt will be the dilatation, and the more slowly will it disappear.

Secondary waves in elastic tubes. — When by a sudden access and sudden removal of pressure a primary wave is produced in an elastic tube, the distal end of which offers no resistance to outflow, the liquid is not found, on removal of the pressure, to stop just when it has regained its position of equilibrium. In virtue of its inertia, it overshoots that mark and passes beyond that position, leaving the tube somewhat collapsed behind it. The tube being elastic regains its form, and thereby exercises suction: a back-rush occurs which in its turn, and for the same reason, is again excessive; and a system of secondary waves is thus set up, of which usually only the first is very important.

The form of the physiological pulse-wave. — The pulse-wave presents first a sudden rise, a steep-fronted primary wave, due to a rapid contraction of the ventricle; then a series of equidistant secondary waves, of which there are commonly two, seldom three, sometimes only one, and sometimes that one (Moens) so strongly marked as to resemble the primary wave in height ("dicrotic pulse"), a result specially observed when the tension of the vessels is small and their "coefficient of elasticity" great. Between the primary wave and the first secondary wave there is a sudden sinking and recovery of pressure which give the appearance of an interpolated undulation. This is (Moens) due to the cessation of the ventricular pressure and to the folding back of the valves under the influence of the pressure in the elastic blood-vessel (aorta or pulmonary artery).

In arteries, the higher a wave the faster it travels: the primary wave travels faster than the secondary; and the distance between the primary and the secondary waves increases as they travel. In caoutchouc tubes, the coefficient of elasticity does not vary with the distention as it does in arteries, and there is no such relative retardation observed.

Reflected waves in elastic tubes. — If the tube be wide

down to the orifice of outflow, which is itself narrow, pouching takes place at the distal end of the tube. There will be greater or less recoil, which is the greater the greater the elasticity of the tube; and this produces reflected waves in the stream; but the outflow through the narrow distal orifice may under such circumstances be singularly uniform.

The reflected wave returning may meet the hinder part of the same on-coming wave, and may complicate its form with secondary undulations (see Marey, *La Circulation du Sang*, 1881).

This does not wholly explain secondary oscillations of the pulse, because the capillary system (which presents a wide channel for the onflowing stream) does not present this kind of resistance.

Amount of outflow from distensible elastic tubes.— Marey showed that when an intermittent inflow was distributed by a λ-tube and divided between a rigid and a distensible-elastic tube (the proximal ends of which were provided with valves to check regurgitation, and the distal orifices of which were narrowed to increase the resistance), the flow from the distensible tube was (if the intermittent inflow were sufficiently frequent) not only continuous, but also absolutely greater in amount than that from the rigid tube. This has been explained as depending on the diminished mean resistance offered by the widened tube; consequently a given initial total-head, H, may have a larger proportion of its own amount devoted (as velocity-head) to imparting movement to the liquid, though, as a result of the widened area, the actual mean rate of flow across each unit of sectional area may be less than that in the rigid tube.

If the distensible elastic vessel became rigid it would be necessary, in order to keep up the same onflow in it, to increase the driving power. [Hypertrophy of left ventricle in atheroma.]

CHAPTER XII.

OF GASES.

Density. — The standard of density of gases is, for chemical purposes, Hydrogen = 1: sometimes air is taken as the standard, in which case the density of hydrogen is $\frac{8957}{129320}$. It is for most physical purposes better to adhere to the C.G.S. system, according to which air has a density $\rho = \cdot 0012932$, and hydrogen a density = .0000895682.

As a rule the density of a gas is determined by first weighing a vessel — a glass or copper vessel or a collapsed indiarubber bag — devoid of contents, and by again weighing it when it contains a known volume of the gas in question.

Elasticity. — Gases, as we have seen, have elasticity of volume alone; and in this they are perfect. In gases, subjected to a compressing force, but not allowed to vary in temperature, the Pressure outwards (acting across each unit of area) is equal to the Resistance to the compressing Force acting inwards (across the same area); the Coefficient of Elasticity of Volume, $\mathfrak{k}$, is, for every temperature, so long as that temperature is kept constant (see p. 368), numerically equal to the Pressure per sq. cm.; $\mathfrak{k} = p$.

The elasticity-coefficient, $\mathfrak{k}$, is any small increment of pressure, $\dot{p}$, divided by the compression, $\dot{\mathfrak{v}}/\mathfrak{v}$, produced; it is therefore equal to $\mathfrak{v} \cdot \dot{p}/\dot{\mathfrak{v}}$. When there is no change of temperature, $p\mathfrak{v} = p'\mathfrak{v}'$ = a constant for any given mass of gas; hence $(p + \dot{p})(\mathfrak{v} - \dot{\mathfrak{v}}) = p\mathfrak{v}$: or, omitting the product $\dot{p}\dot{\mathfrak{v}}$, which vanishes when $\dot{p}$ and $\dot{\mathfrak{v}}$ are infinitesimal, $\mathfrak{v}\dot{p} - p\dot{\mathfrak{v}} = 0$, or $\dot{p}/\dot{\mathfrak{v}} = p/\mathfrak{v}$; whence the Elasticity-Coefficient at constant temperature, $\mathfrak{k}$, $= \mathfrak{v} \cdot \dot{p}/\dot{\mathfrak{v}} = p$; it is numerically equal to the Pressure per sq. cm. When the gas is compressed, but at the same time no heat is allowed to escape from it, the compression is said to be "adiabatic"; and the law of the relation between the volume and the pressure of a given mass of gas, under these circumstances, is stated by the adiabatic equation (p. 395) $p\mathfrak{v}^{k/c} = const.$, where k and c are the two "thermal capacities" (p. 367). On treating that equation in the same way as the former, we find $\dot{p}/\dot{\mathfrak{v}} = k/c \cdot p/\mathfrak{v}$; whence the Elasticity of Volume in Adiabatic Compression is not $\mathfrak{k} = p$, but $\mathfrak{k}' = p \cdot k/c$: and the ratio between the two Elasticity-Coefficients, $\mathfrak{k}$ and $\mathfrak{k}'$, of a gas is the same as that between k and c, the two Thermal Capacities. (See note, p. 370.)

The Pressure within a gas is **hydrostatic.**

Compressibility. — The extent to which a gas can be compressed is indefinite, provided that its temperature be above Andrews's critical point (p. 233); if the temperature be below this point, compression may liquefy the gas.

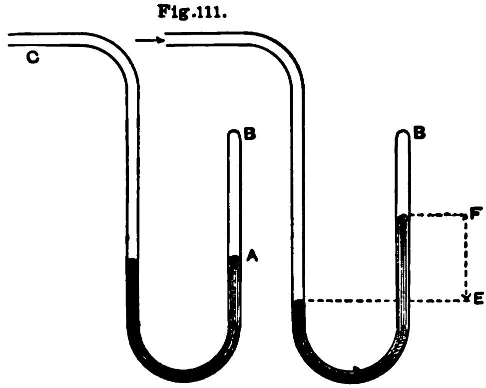
Boyle's Law, already stated, is that the volume of a gas and the pressure acting on it vary inversely.

Float on water a little glass bulb, which contains an adjusted quantity of air, and the interior of which communicates by an aperture with the liquid on which it floats; if the pressure on the surface of the water be increased, water passes into the bulb and compresses the contained air; the bulb as a whole becomes heavier and sinks (Descartes' Diver).

In a fish the air bladder acts during muscular relaxation as a float; during contraction of the muscular walls of the bladder the contained air is compressed, the mean density of the whole body is increased, and the fish sinks.*

Manometers can be constructed so as both to illustrate and to apply Boyle's Law. If a tube, bent and containing mercury as shown in Fig. 111, and enclosing a certain volume of air within the space AB, be exposed to an additional pressure acting through C, that additional pressure will be partly spent in sustaining the weight of the column EF of mercury raised in the tube, partly in maintaining a compression of the air AB within the space FB. By preliminary graduation such an instrument may be made to act as a manometer, and may be added to those of Fig. 105.

Fig. 111.



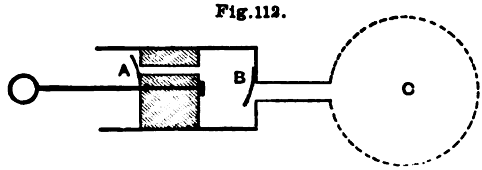
Boyle's law is somewhat departed from by oxygen, carbonic oxide, nitrogen, air, hydrogen, whose bulk at increasing pressures is greater than that law would indicate; while sulphurous acid, carbonic acid, and other easily condensible gases shrink in volume more rapidly when exposed to moderately-increasing pressures than the amount of pressure alone would lead us to expect. The latter gases present very curious aberrations when

* The local contraction of one end of the air bladder causes the other end to act alone as a float, the head or tail being thus tilted up or down. The air bladder is in many cases too near the ventral aspect to sustain the fish in its normal position in the water; the action of the tail sustains the fish in its position of unstable equilibrium throughout the whole period of life.

extremely high pressures — bringing the gas to the verge of liquefaction — are applied.

The Tendency of Gases to Indefinite Expansion is utilized in the **Air-pump**. The primitive type of an air-pump is a cylinder, provided with a piston in which there is a valve A (Fig. 112), opening outwards. The cylinder itself is connected with C, the vessel to be exhausted, by a tube closed by the valve B, opening into the cylinder. Suppose the piston drawn out: the air, within C and the connecting tube BC, expands, thrusts aside the valve at B, and fills the whole space open to it — that is, the cylinder, the tube, and the vessel C. The piston returns: the valve B closes and the valve A opens, because the air between A and B is compressed; the air in AB is driven out through A. By repeating this process often enough the air in C is greatly reduced in quantity and becomes of correspondingly small density. This simple form of air-pump is liable to two objections: — (1) It is tedious to pull the piston against an external pressure which each time becomes more and nearly equal to the entire atmospheric pressure of 15 lbs. per square inch, as the space between A and C becomes more nearly a vacuum; and (2) After a certain number of strokes the expansion of the air in C fails to lift the valve B. The latter objection may be obviated by causing the piston itself to lift the valve; the former is rendered less serious by connecting with the cavity C two such cylinders, and so arranging matters that when the one piston is being driven home the other is being drawn out; the whole being driven by a large and heavy wheel. Not only is a smaller force enabled by the leverage thus gained to resist a great pressure, but the inertia of the flywheel renders the action less painful, because more equable.

Fig. 112.



Sprengel's Air-pump in its simplest form consists of a long tube AB, Fig. 113, provided with a side-branch EF, which communicates with a vessel C, the vessel to be exhausted. At the upper end of the tube AB is D, a supply cistern of boiled mercury, which is allowed to fall down AB. As it passes EF the mercury entangles molecules of the expansive gas in EFC, and these are continuously removed by the falling stream and escape in bubbles at B.

If the lower end of the tube AB be bent upwards, a vessel filled with mercury may be inverted over the upturned end, and as the gas issues at B it can take the place of the mercury in that vessel; the Sprengel-pump may thus be used as a means of transferring small quantities of gas from one vessel to another.

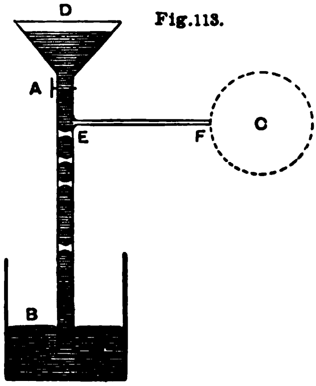


Fig. 113.

When the vacuum in C is tolerably complete, the mercury falls as a continuous mass, containing no bubbles.

If a long rubber tube be connected with a flask, and laid across a table; if it be squeezed against the table by a roller pressed along it, away from the flask; the air in the tube, in front of the roller, is squeezed out, and part of the air in the flask follows the roller, as the tube regains its form behind that roller; the air in the flask thus becomes somewhat rarefied. If this operation be repeated often enough, without ever allowing air to return from the external atmosphere into the rubber tube, a considerable degree of rarefaction may be attained within the flask. The rubber tube should be flexible enough for the roller to squeeze it flat, but at the same time rigid enough to withstand the atmospheric pressure. By arranging such a tube in the form of a coil inside a drum, and squeezing it by a continuously-rotating roller, very considerable rarefaction can be easily and promptly set up; and air-pumps are now constructed, acting on this principle.

The **Absorption** of Gases by solids is sometimes a true **solution,** as in the case of the alloy of metallic hydrogen * with palladium. This is produced by evolving hydrogen from a palladium electrode in the electrolysis of water (p. 658); or by heating a piece of palladium *in vacuo*, and allowing it to cool in an atmosphere of hydrogen, or even by heating it in a tube through which a current of hydrogen passes, and allowing it to cool in that gas. The same kind of colloid solution is exemplified by the alloys of iron and hydrogen, or of platinum and hydrogen, or, again, by the carbonic oxide, which (to the extent of 4·15 vols.) is retained on cooling by cast-iron, or the carbonic dioxide, of which a half-per-cent volume may be retained by indiarubber.

Such absorption may, on the other hand, be due to **surface attraction** and condensation within the pores of the solid, —as in the case of animal charcoal, which can absorb so much oxygen or so much ammonia, that these gases must even be

* This seems (Graham, *Physical and Chemical Researches*) to be a white paramagnetic metal of density 1·95; diamagnetic (Blondlot).

liquefied within its pores, or which can absorb both oxygen and oxidisable gases, and bring them into such close molecular relations that they combine, as in charcoal respirators; or in the case of platinum-black, which, if surrounded by a mixture of oxygen and hydrogen, absorbs both gases and brings their molecules into contact so close that they combine, and do so with evolution of heat so great as to cause the platinum to become incandescent and thus to ignite the remainder of the gas, as in Döbereiner's Hydrogen Lamp.

Chemical affinity may also promote the absorption of a gas by a solid. If a dish containing spirit of wine be suspended over quicklime within a confined space, the mixed vapours of alcohol and of water, which pass by evaporation into the space above the quicklime, are discriminated by it: the water is absorbed, the alcohol not; more water, but not more alcohol, is evaporated from the spirit of wine, and is again absorbed by the quicklime. The result is dehydration of the spirit, which may proceed to an extreme degree.

Where a gas is dissolved freely by a solid, that gas may freely *traverse* that solid. Thus hydrogen leaks freely through a white-hot palladium or platinum tube; so does carbonic oxide through glowing iron;* so do carbonic acid, marsh-gas, coal-gas, and oxygen in small quantities through indiarubber, and coal-gas under high pressures through cool steel. The solids in which this effect is observable are as a rule colloid, or (like non-crystalline metals) resemble colloids, and they behave towards gas just as liquid films do.

A **solution** of a **gas** in a *Liquid* is itself a liquid, of which the gas-molecules form a part. Liquids differ from one another, and saline solutions from pure liquids, in their power of dissolving the same gas; and different gases are differently soluble in the same liquids. The more readily a gas can be liquefied, the more freely will it, in general, dissolve in water or alcohol.

The *Coefficient of Absorption* of a gas in a liquid is the volume of the gas dissolved in 1 vol. of the liquid, the volume of the gas being reduced to 0° C. and 76 cm. barom. pressure. The actual volume of gas dissolved in 1 vol. of liquid at any specified temperature and pressure is called the *Solubility* of the gas in the liquid at the given temperature and pressure.

When a gas is dissolved in a liquid, the liquid increases in

* Cast-iron stoves when red-hot allow carbonic oxide to escape into the air of a room; carbonic oxide in small quantities destroys the red blood corpuscles and produces anæmia. Carbonic oxide forms a volatile compound with iron.

volume, generally in proportion to the volume of gas dissolved. Thus 1 vol. of water, at 0° C. and at atmospheric pressure, dissolves 1049 vols. of ammonia gas and becomes 1·487 vols. of ammonia-solution.

According to Prof. Henry, carbonic dioxide, oxygen, nitrogen, and some other gases, are dissolved in water in the exact ratios of the pressures under which they are exposed to the surface of the liquid; at five atmospheres' pressure five times as much carbonic acid by weight can be dissolved in water as can be dissolved at one atmosphere. The *volume* of each gas dissolved by a given quantity of water at a given temperature is thus always the same. Henry's law cannot be stated as a universal one with perfect numerical accuracy, though it is approximately adhered to by all gases in relation to all liquids: the divergences are greatest when the gas is very soluble in, or chemically unites with the liquid, or acts on a salt dissolved in it.

When a Mixture of gases is exposed to a liquid, each gas is dissolved independently of the rest: each is dissolved (if Henry's law be obeyed) in proportion to the partial pressure exerted by it.

Thus, if water be exposed to air at the pressure of 76 cm. of mercury, the total pressure is made up of $\frac{20{\cdot}9}{100} \times 76 = 15{\cdot}884$ cm. pressure due to oxygen, and $\frac{79{\cdot}1}{100} \times 76 = 60{\cdot}116$ cm. pressure due to nitrogen. The Solubility of oxygen in water at 10° C. is ·03250; that of nitrogen is ·01607; both at the pressure of 76 cm. Hg. It happens that Henry's law applies to both these gases. Thus the volume of oxygen dissolved by 1 vol. of water is $\frac{15{\cdot}884}{76} \times {\cdot}03250 = {\cdot}0067925$ vol.; that of nitrogen is $\frac{60{\cdot}116}{76} \times {\cdot}01607 = {\cdot}0127114$ vol. Hence the air dissolved in water — that which subserves the respiration of fishes — contains oxygen and nitrogen in the ratio of ·0067925 to ·0127114, or, in percentages, 34·82 oxygen to 65·18 nitrogen.

The solubility of gases in liquids not only diminishes with diminished pressure, but also with increased temperature. The gases dissolved in a liquid may, if they form with it a simple solution, — as in an aqueous solution of ammonia, — be entirely removed either by diminution of pressure or by increase of temperature. In some cases there is a chemical union between the gas and the liquid, or some constituent of it. Thus a solution of hydrochloric-acid gas, when heated, first loses some gaseous HCl, and then boils over as a whole: a solution of bicarbonate of soda, when the pressure is greatly reduced, somewhat suddenly loses half its carbonic acid: blood, when the pressure is gradually diminished, first loses the carbonic acid and the oxygen which it holds in simple solution, and then, at a very low pres-

sure, those quantities of these gases which it holds in feeble chemical combination are suddenly given off.

A gas will traverse a liquid diaphragm with great rapidity if it be soluble in it: it is dissolved, diffuses, and emerges on the other side. A soap bubble or a wet bladder, containing hydrogen and surrounded by carbonic acid, absorbs the latter gas and enlarges in size, although, as we shall see, hydrogen runs with the greater speed through dry or indifferent membranes, and a dry bladder under similar circumstances would collapse.

It seems that there may be something of the nature of a solution of a gas in a gas. Oxygen evolved from chlorate of potash may contain no chlorine or any oxides of chlorine recognisable by any chemical test; yet if it be passed through a red-hot tube it will be found that chlorine can now be detected in it (Schützenberger). And further, a gas or vapour may dissolve a solid; boracic acid in steam; naphthalene in coal-gas rich in hydrocarbon-vapours; indeed there apparently never occurs any evaporation from a solution entirely without this result.

Diffusion of Gases. — Two gases in vessels between which a free communication is established are found to mix freely, and if sufficient time be allowed the mixture will become uniform throughout. The rate of diffusion is somewhat rapid. Pure carbonic acid and air placed in communication will diffuse at such a rate that the air at a distance of half a metre will be found in seven minutes to contain one per cent of carbonic acid: hydrogen will similarly travel a third of a metre in one minute (Graham). The lighter the gas the more rapidly does it travel.

This process is molecular, and solid particles floating in either gas remain practically at rest; and thus mere diffusion is not sufficient for purposes of ventilation.

In the lungs, diffusion carries air out of the air-cells and oxygen into them; the oxygen tends to travel more rapidly inwards, and hence there is a small force tending to dilate the air-cells (Graham).

Effusion. — When gas is caused to flow through apertures, such as pinholes in membranes, the law of Torricelli is obeyed, and the velocity of outflow $v = \sqrt{2g\mathrm{H}}$. Air rushing through such an aperture into a vacuum will do so as if the atmosphere consisted of a uniform layer of fluid, of uniform density $= \cdot 0012932$, and throughout which g is constant, all at the freezing temperature and at the pressure of 76 cm. of mercury; the layer having a depth H of 799022 cm. (nearly 5 miles). Then $v = \sqrt{2g\mathrm{H}} = \sqrt{2 \times 981 \times 799022} = 39595 \cdot 2$ cm.-per-sec.

In different gases at the same pressure the height H will vary inversely as their densities; the gases will therefore pass through apertures with velocities inversely proportional to the square roots of their densities. Thus oxygen and hydrogen, whose densities are 16 : 1, will have effusion-velocities $\frac{1}{\sqrt{16}} : \frac{1}{\sqrt{1}}$, *i.e.*, 1 : 4, at the same temperature and pressure.

In any gas the velocity of outflow is not affected by changes of pressure. H is proportional to the pressure p; it also varies inversely as the density; $H \propto p/\rho$. If the pressure be increased, Boyle's law shows that the density is increased in the same ratio; hence p/ρ is constant. Wherefore H is constant, and $v (= \sqrt{2gH})$, the effusion-velocity of each gas (*i.e.*, the volume flowing per second ÷ the area of the aperture), is constant under all circumstances of pressure; and the normal rate of outflow of different gases at constant temperatures depends only on the nature of the gases. Under changes of temperature at constant volume, $p \propto \tau^{\circ}$ *Abs.* (p. 370); $\therefore v \propto \sqrt{\tau^{\circ}}$ *Abs.* Under changes of temperature at constant pressure, $H \propto 1/\rho$; $\therefore v \propto 1/\sqrt{\rho}$. Perturbations are, however, produced by variations in the viscosity of different gases at different temperatures; these cause slight departures from this law.

This phenomenon of outflow or effusion is one of masses, and in it gases act as fluids, practically continuous.

If a gas be driven under pressure through a substance which is porous, but whose pores are too small to allow the mass to traverse it without great resistance, the result is the **transpiration** of the gas, a slow flow under resistance. Transpiration may be studied by driving gases through long capillary tubes, or even through tubes which are not capillary, provided that their length so far (4000 : 1) exceed their diameter that considerable resistance is offered to the onflow of the gas. It is found that in each case the mass of gas passing per second is proportional to the motive pressure, but also varies inversely as the length, directly as the density of the gas (a singular result), and further, depends on a constant, the Coefficient of Transpiration; $m/t = k \cdot p\rho/l$. A film of gas adheres to the sides of the tube, and the gas flows in an axial stream in each channel.

The coefficient of transpiration peculiar to each gas is a very isolated factor, and does not seem to have any intelligible relation to the other properties of gases. The transpiration-coefficients of nitrogen, of nitric oxide, of carbonic oxide, are double that of hydrogen: those of ether and of hydrogen are the same: those of oxygen and nitrogen are related to one another in the ratio 14 : 16, so that equal times are taken by equal masses of these gases to pass through long or capillary tubes.

If a gas be heated it will become lighter, and its transpiration-rate will be lessened: if the barometric pressure rise, it will be compressed and its transpiration-rate will be increased.

Membrane-Diffusion.—Gases placed on opposite sides of an indifferent porous membrane, and exposed neither to the influence of a difference of pressures nor to that of a difference of solubilities in the material of which the membrane is composed, will pass through it in virtue of their own molecular motion.

The velocity with which hydrogen and oxygen, separated by a partition of plaster-of-Paris, or graphite, or biscuitware, will traverse that partition is exceedingly small in comparison with the rate of effusion through a relatively large aperture into a vacuum; but it is found to be proportional to the mean velocity of the molecules in the gas.

We have already seen (p. 250) that the mean velocity of the particles of any gas is inversely proportional to the square root of the density of that gas; and hence the rate of diffusion of any gas through an indifferent membrane is inversely proportional to the square root of the density of that gas.

A dry bladder filled with hydrogen and surrounded by oxygen will partially collapse, for hydrogen leaves it four times as fast as oxygen enters it.

This difference of diffusion-rates may be made to effect a partial separation of gases. If a long porous tube be fitted so as to pass through a vacuum or a neutral gas, and if a mixture of gases be passed through the porous tube, the components of that mixture will escape through the walls of the porous tube in unequal proportions. If the vapour of chloride of ammonium be passed through such a tube, the hydrochloric acid (density $=18{\cdot}25$ when $H=1$) and ammonia (density $=8{\cdot}5$), into which the chloride is dissociated by heat, pass through in the ratio of $\frac{1}{\sqrt{18{\cdot}25}}$ to $\frac{1}{\sqrt{8{\cdot}5}}$: the ammonia thus passes through in excess, and litmus paper placed in the neighbourhood of the porous tube will indicate an alkaline reaction.

A gas may pass through the pores of a solid by liquefaction in those pores: sulphurous acid may pass through charcoal and evaporate on the farther side.

Diffusion of Gases from Liquids.—If a layer of liquid charged with gas be placed upon one free from gas, the gas rapidly permeates the whole liquid. If the two layers be separated by a membrane wetted by both, the diffusion is rapid. If the two layers, thus separated by a thin membrane, be in a state of relative motion, the diffusion-rate may be accelerated if the

velocities be not too great. If two streams so separated move in opposite directions, they may completely exchange gases; for suppose two such streams to be charged, as they arrive at the opposite ends of a certain tract of vessel, with gases A and B: then throughout the whole of that part of their course during which they are contiguous, the A-charged stream passes and diffuses A into a stream which is at every point poorer in A than the A-charged stream itself is at the same point; and *vice versâ:* so that, if the course be long enough, the A-charged stream may lose all its A, and the B-charged stream all its B.

The Statics of Gases.—A gas always fills the whole space within which it is contained. There is no difference in respect of statical theorem between a gas, and a liquid which also fills the whole space within which it is contained. Pascal's principle, that of the so-called Transmissibility of Pressure, that of the perpendicularity of the pressure exerted by a fluid upon its bounding surface—all these apply equally to all fluids: so do the principle of the Hydraulic Press and that known as Archimedes' Principle.

The last must be kept in mind when accuracy is required in weighing. A piece of brass of density 8 and weighing 1 kilo. *in vacuo* occupies 125 cub. cm. ($\frac{1}{8}$ the bulk of an equal mass of water). It apparently loses, when weighed in air, the weight of 125 cub. cm. of air; that is, $125 \times \cdot 0012932$ grammes $= \cdot 16165$ gramme. The substance to be weighed also loses weight, but if it displace more air than the counterpoising mass of brass does, it loses more than the brass does, and an inaccurately large quantity of it has to be used in order to counterpoise the metallic kilogramme.

Balloons and soap bubbles containing coal-gas or hydrogen rise in the air; bulk for bulk they are lighter than air. The lighter they are, the more rapidly they ascend; and they can be loaded until they weigh, bulk for bulk, the same as the air in which they float. If a balloon with its contained gas weigh 100 lbs., and the bulk of air displaced by it weigh 120 lbs., the balloon will rise under an ascensional force equal to the weight of 20 lbs. applied to a mass of 100 lbs.; its upward acceleration will be equal to $g \times \frac{20}{100} = \frac{1}{5}g$.

The pressure on the walls of a closed vessel containing gas is greater the lower the level at which it is measured: the law is exactly the same for gases as for liquids. The effect is seldom perceptible, because within vessels of ordinary size the mere weight of the gas adds little to the atmospheric or other pressure acting.

With vessels of ordinary dimensions a manometer applied laterally at any part will indicate the internal pressure; strictly speaking, in gases, as in liquids, it indicates only the pressure at the horizontal level of the orifice of communication between the manometer and the vessel.

Streams of Gas. — The statements made in the discussion of streams of liquid in Chap. XI. apply also to streams of gas. The Law of Continuity, Torricelli's Law, the distinction between Velocity-head and Pressure-head, the gradual disappearance of the latter, coupled with the simultaneous heating of the flowing fluid, the Lateral Pressure in a main pipe, and the propulsion of the fluid up piezometer tubes or through lateral orifices, — all these apply to gaseous as well as to liquid streams.

In calculations based on Torricelli's Law it is necessary to find H. H is the height of that column of the outflowing fluid which would, if acting alone, produce a pressure equal to that actually undergone by the fluid set in motion. If the gas issue from a vessel in which the pressure is such as to support a manometric column of, say, 24 cm. Hg in addition to the atmospheric pressure; and if the atmospheric pressure at the time, as shown by the barometer, be 76 cm. Hg: the whole pressure on the gas is, for each sq. cm. of its bounding surface, equal to the weight of a column of mercury whose content is 100 ($= 76 + 24$) cub. cm. This is equivalent to the weight of 1359·6 cub. cm. of water. If the density of the gas be $\frac{1}{800}$ that of water, the pressure p per sq. cm. is equal to the weight of $800 \times 1359{\cdot}6 = 1{,}087680$ cub. cm. of that gas, the same gas as is driven out in a jet. This column, standing on a sq. cm. base, is 1,087680 cm. high. Hence, for the gas in question, $H = 1{,}087680$; and the velocity of that gas, rushing into a vacuum under a total pressure of 100 cm. Hg (so long as the pressure is maintained at that value), is $v = \sqrt{2gH} = \sqrt{2 \times 981 \times 1{,}087680}$ in cms. per second; while into the atmosphere it would run with a velocity $\sqrt{2 \times 981 \times (800 \times 13{\cdot}596 \times 24)}$ due to the difference (24 cm. Hg) between the internal and external pressures.

Recoil. — When a stream of gas issues from a jet or burner, the reaction is equal to the action, and there is a tendency for the burner itself to move backwards. This tendency we see turned to account in certain revolving shop-window gas-illuminations.

Viscosity. — The viscosity of gases, which is due to diffusion, is on the whole similar in its results to that of liquids.

A stream of air, driven through air, soon comes to rest. If it have a great velocity, it can cut its way through air to a greater distance than a slower stream can.

If a stream of air be introduced into a room through a funnel-shaped aperture, the broad mouth of the funnel being open to the external air, it will enter the room through the narrow orifice with great velocity, and will pass a considerable distance (being acutely felt as a draught), until at length the process of diffusion between it and the surrounding air relieves it of its relative momentum. If, on the other hand, the narrow end of the funnel be presented to the exterior air, the stream as it enters will (obeying the Law of Continuity) widen out in accordance with the shape of the funnel, and its velocity will be proportionately diminished; the result being that

a considerable amount of air may, through ventilators of such a form, be introduced into a room without producing a perceptible draught.

All objects surrounded by air bear on their surface an adherent film of air which is almost dustless. When a body moves in air, this film rubs against contiguous layers of air, and the movement of the body is retarded by internal friction in the air. Haughton and Emerson Reynolds found that a granite ball suspended in the air, and swung pendulum-fashion, suffered, on each successive swing, a diminution of amplitude of $\frac{1}{6052}$ due to this cause.

The friction within a gas is independent of its density, but increases with its temperature (Clerk Maxwell, *Phil. Trans.* 1866).

A body falling *in vacuo* is not retarded, and falls with a downward acceleration fully equal to g; falling through air, it is retarded, because the viscosity of the air causes friction. A thick piece of gold and a piece of paper fall *in vacuo* at the same rate: through air the gold falls more rapidly, because it presents less surface in proportion to its weight: but even through air a piece of smooth paper and a piece of gold leaf, presenting the same total surfaces and the same weights, or each bearing the same proportion between its surface and its weight, will fall side by side.

Falling water is retarded by the air; and conversely, air is dragged down by falling water. If a stream of water be made to fall through a closed cavity, the water will drag down with it a considerable volume of air; and if a lateral communication be established between this cavity and a vessel containing air, much of the air in that vessel may be extracted. If a stream of air or steam be maintained through a cavity, it is not only itself retarded, but the surrounding air is dragged with it, and the pressure in the cavity is diminished.

This action, due to viscosity, is independent of the general diminution of pressure experienced by fluids in motion. A vibrating tuning-fork brought near a suspended pith-ball seems to attract it; the air between the objects vibrates, the pressure is lessened, and the exterior atmospheric pressure urges the ball against the tuning-fork.

When the density of the vibrating fluid is ρ, at a point in the fluid where the greatest velocity of vibration is v, the diminution of pressure p is $\frac{1}{4}\rho v^2$ per sq. cm.; provided that the cause of the vibration be the to-and-fro movement of solids moving within a finite space of the fluid (Lord Kelvin).

Measurement of Flow.—The amount of flow of gas through pipes may be measured on the same principles as the amount of flow of liquids.

(*a*) The amount of gas actually passed may be collected and measured. It may be collected in a balanced bell-jar, inverted over water like a small gasometer (Hutchison's Spirometer), or in a very large and thin flexible caoutchouc bag (Boudin).

(*b*) It may be made to drive a registering train of wheel-work, like a gas-meter, as in Bonnet's pneumatometer.

(*c*) The principle of the hydrostatic pendulum or

(*d*) that of Pitot's tubes may be employed.

Barlow's Formula for the flow of gases in pipes is $Q = 1350\, d^2\sqrt{hd/sl}$, where Q is the flow in cub. ft. per hour, d the diameter in inches, h the pressure in inches of water, s the sp. gr. of the gas (air = 1), and l the pipe-length in yards. In C.G.S. measures, this becomes $\mathfrak{v} = 222{\cdot}83\, t\, \sqrt{pd^5/\rho l g}$ $= 7{\cdot}115\, t\cdot\sqrt{pd^5/\rho l}$, where p = dynes per sq. cm., d and l are measured in cm., ρ is the density (water = 1) and $\mathfrak{v}$ is the number of cub. cm. passing in time t.*

The Pressure of the Atmosphere.

Most of our experiments and observations are complicated or affected by the fact that we live at the bottom of an atmospheric ocean which exerts pressure upon every surface exposed to it, and which penetrates even into the recesses of everything porous, and there also exerts pressure, unless special appliances be made use of in order to remove it wholly or in part. We live at the bottom of such an atmosphere without inconvenience, just as deep-sea fishes live at the bottom of the ocean: so long as they are in their *habitat*, the internal pressure of the gases contained and dissolved in their organisms is equal to and is in equilibrium with the immense external pressure exerted by the

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* Reductions of this kind are frequently found very troublesome. Here, if d=1 inch, h=1 inch, s=1, and l=1 yard, Q=1350 cub. ft. per hour. Similarly, if d=1 cm.$=\frac{1}{2\cdot 54}$ inch, h = 1 cm. $=\frac{1}{2\cdot 54}$ inch, and l = 1 cm. $=\frac{1}{91\cdot 44}$ yard, $Q = 1350 \times (\frac{1}{2\cdot 54})^2 \times \sqrt{(\frac{1}{2\cdot 54} \times \frac{1}{2\cdot 54}/\frac{1}{91\cdot 44}s)} = (787{\cdot}77 \div \sqrt{s})$ cub. ft. per hour $= (787{\cdot}77 \div \sqrt{s} \times (30{\cdot}48)^3 \div 3600$ sec.) $= (6196{\cdot}4 \div \sqrt{s})$ cub. cm. per second. But if we measure the density of the gas in terms of water as the standard, we use, instead of $\sqrt{s}$, a smaller divisor $\sqrt{\rho}$ $=\sqrt{0{\cdot}0012932s}$, and must compensate for this by multiplying the numerator 6196·4 by $\sqrt{0{\cdot}0012932}$. The number of cub. cm. per second is then $(222{\cdot}83 \div \sqrt{\rho})$ or, if ρ = unity, 222·83 simply. This is the numerical factor which takes the place of the original 1350; and now the number of cub. cm. per second is $222{\cdot}83\sqrt{d^5h/\rho l}$, where all the terms are in C.G.S. units. The pressure is still stated in terms of water-column; to transform it to dynes per sq. cm., we observe that when the manometer-column has a height h, $p = h\rho' g$ where ρ' is the density of the manometer-liquid; but in a water-column, $\rho' = 1$, and $h = p/g$; whence the number of cub. cm. flowing per second is $222{\cdot}83\sqrt{(pd^5/\rho l g)}$.

surrounding water; but when they are brought towards the surface, the external pressure becoming greatly less, the gases contained in the swim-bladder and throughout the tissues undergo expansion, and the fish explode.

The pressure within our organisms cannot be less than the atmospheric pressure, that exerted by the atmosphere on the surface, 1,013663 dynes, per sq. cm., or a pressure equal to the weight of about 15 lbs. per sq. inch. If the internal pressure in any part become less than this, the fluids or the semi-fluid tissues or masses of the body must flow towards the region of diminished pressure. Hence a permanent vacuum within the body, total or partial, is impossible.

The abdominal walls are closely appressed against the viscera: the walls of these are pressed against one another as far as their contents will allow.

The lungs are pressed against the ribs by the atmospheric pressure acting down the trachea and bronchi, and they are thereby expanded when, but for this action, the expansion of the ribs would tend to form a vacuum between the pulmonary pleura and the parietes of the thorax. This expansion does not take place when the thorax is so opened by a wound that, on expansion of the ribs, air can pass through the wound into the pleural cavity, and can thereby equalise the internal and external pressures without the aid of pulmonary inflation.*

The atmospheric pressure acts freely upon and through a mass of gas, if that mass be free to expand or contract, whatever be its temperature. The air in a room may be hot, and yet the atmospheric pressure, acting down the chimney and through all the chinks and orifices of the room, will be undiminished in amount and in effect.

A trap in a wash-hand basin in a room will not be unable to prevent gases from being forced into the room from the drains, simply because the air in the room is warm. It may be unable to do so if the pressure within the drains become excessive, or if the air in the room be rarefied by a strong draught up the chimney, especially where the fittings of the room are so air-tight that the external pressure cannot force air into the room except through the trap.

If any object containing gas or air be placed in a region of space from which the air has been wholly or in part extracted —such as the bell of an air-pump—the internal pressure may overpower the external, and the body will then tend to become inflated and may even burst.

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* In such a case some of the air in that cavity can be expelled by an expiratory effort with closed glottis, and can be prevented from returning by a valve opening outwards.

A little indiarubber balloon, a bladder half filled with air, a shrivelled apple, a dish of soapsuds, present under the air-pump a singular appearance of expansion. If a loaded piece of wood be put in a dish of water, and the whole placed under the air-pump, the wood will appear to effervesce; the air contained in its pores expands and forms bubbles. If soda-water already flat be subjected to similar treatment it will renew its effervescence.

This inflation is not due to any suction on the part of the air-pump, but is due to the expansion of the contained gas, which always tends to expand, but which can only do so when the resistance offered to its expansion on the part of the external pressure is diminished or removed. The gas expands until the internal pressure of the expanded gas is equal to the pressure of the rarefied air or gas; the latter, as we have already seen (Boyle's Law), suffers diminution in the same ratio as the density.

If the pressure within an object or a cavity exceed or be made to exceed the external atmospheric pressure, there is, as in all such cases, a tendency to establish equilibrium by setting up a flow from the place of greater pressure to one of less. Thus, if a bladder containing gas and provided with a stopcock be loaded with a weight, and its stopcock opened, the atmospheric pressure tends to drive air into the bladder, but it is overpowered by the greater pressure within the bladder, and there is an outward flow set up, due to the difference between the internal and the external pressure.

A gasholder, consisting of an inverted bell floating on water, may be loaded so as to exercise any given expulsive pressure. Thus coal-gas driven out "at a pressure of 1 inch of water" is subject in the pipe, when the stopcock is closed, to an internal pressure = atm. pr. + "1 inch of water," and to an exterior pressure at the burners = atm. pr. only.

If air be blown into a flask partly filled with water, partly with air, and provided with a narrow open glass tube passed through the cork, and if the flask be suddenly inverted, water will rush out through the nozzle: the air has been compressed, and its pressure has become greater than the atm. pr.; this difference of pressures causes an outward flow, a jet of liquid.

In the dome of the fire-engine air is compressed in the same way: the inflow is intermittent, the outflow continuous; for the air never ceases to be compressed, and it exercises a continuous pressure.

If a gas-evolution flask containing, say, zinc and sulphuric acid, be fitted with an ordinary safety-funnel dipping into the liquid, the hydrogen evolved will pass out by the intended channel: the liquid of the flask will be observed to oscillate a little in the safety tube, which acts as a manometer indicating the internal pressure. If any obstruction offer, the gas accumulates in the flask, a difference is set up between the internal and the external pressure, and the liquid is forced up the safety tube. The safety tube should dip into the liquid only just so deeply that before the liquid forced up into the funnel can overflow, the level of the liquid in the flask

shall have been so far depressed that nothing but gas can pass out through the safety tube.

If a cistern at a height be connected by a tube with a large flask containing air, in such a way that liquid may pass from the cistern into the flask, air is driven out of the flask: it may be driven out through a tube; this tube may be connected with any cavity through which it may be necessary to drive air. This is one form of Aspirator.

The same principle is applied in the plenum method of ventilation: a local excess of pressure is set up by forcing air into a building, and the air is left to find its own way out.

When the thoracic walls contract, air is driven out of the lungs, and blood out of the thoracic organs in general.

When the abdominal walls contract, a general-contents-pressure is set up, always at right angles to the general surface of the practically-fluid visceral mass, and opposed partially or completely by a uniform atmospheric pressure.

When the external atmospheric pressure exceeds that within an object or cavity, air may be forced into it or it may be compressed, or if these effects be not possible, the existence of the atmospheric pressure generally becomes in some way strikingly manifest.

The Magdeburg Hemispheres, a couple of hemispheres fitting together so as to form a sphere, and ordinarily separable with ease, but when apposed, and the air extracted from between them, not to be separated without great force; the boy's leather Sucker, a piece of moistened leather closely applied to any object and pulled — any residual air still remaining being rarefied — the pressure of air between the sucker and the object becoming very small, and the sucker being thus firmly pressed by the weight of the atmosphere* against the object on which it is placed; the difficulty experienced in pulling the handle of a good Syringe when the nozzle is stopped up, or in the continued working of a reciprocating Air-pump, — all these clearly point out the part played by atmospheric pressure.

In the experiment previously described, in which gas escaped from the pores in a piece of wood kept under water and exposed to the action of the air-pump, it is only necessary to allow the atmospheric pressure again to act to see the water driven by that pressure into the pores of the wood, which thus becomes too heavy to float.

The atmospheric pressure is a prime agent in most of what we usually call the phenomena of **Suction.** A syringe has its nozzle inserted in water; the handle is drawn up: in the body of the syringe there would arise a partial vacuum were it not that the external atmospheric pressure overcomes the feeble internal pressure, and pushes the liquid through the nozzle into the body of the instrument.

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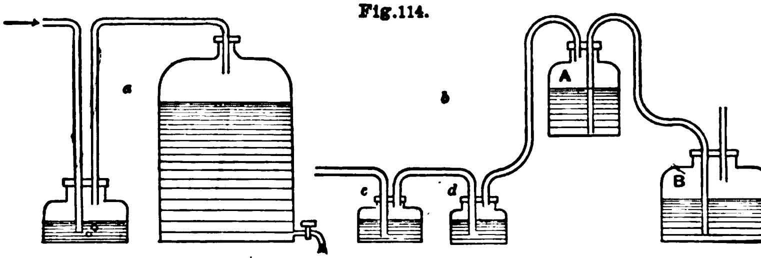
* The air does not force its way between the sucker and the object pulled upon, for the intervening film of moisture is held in place by adhesion.

If the syringe have a thin closed wooden nozzle, and if the vacuum in the syringe be relatively good, the atmospheric pressure can force water or mercury through the pores of the wood.

If there be no safety tube attached to a gas-evolution apparatus, and if the evolution of gas suddenly cease while the gas still continues to be absorbed by the liquid into which it is passed, we find the gas diminishing in amount, and the external atmospheric pressure forcing the absorbing liquid back into the gas-generating flask. If there be a safety tube, the very short column of liquid at its lower end is forced down, and air enters the flask until the internal pressure becomes equal to the external.

Aspirators are generally constructed on this principle. Water flows from a large flask or can, Fig. 114 *a*: air must take its place: this air "is drawn," or rather is pushed by the atmospheric pressure, through a series of flasks which it must traverse on its way from the outer air to its place in the aspirating flask. With the arrangement *b* of Fig. 114, the flexible tube between A and B, being filled with water, acts as a siphon, and water flows

Fig. 114.



out of A: when A is nearly emptied, disconnect it from *d*, and place it at a lower level than B; it then becomes refilled.

The vacuum method of ventilation is an exhaust-method: air is removed at a certain point, by the mechanical action of a fan or by the ascent of heated air in a tall chimney or shaft; air then finds its way from different parts of the building or mine towards this point.

Filtration may be assisted by connecting the filter with a partial vacuum: the funnel is for this purpose fixed into a flask by a cork through which there also passes a tube leading to an aspirator of any kind, a Sprengel pump worked by water, and called a Bunsen pump, being frequently employed. The atmospheric pressure on the liquid in the funnel forces it through the filter into the partial vacuum.

Suction nipples and bleeding cups illustrate not suction but atmospheric pressure: the pressure within them is less than the external pressure; the part of the surface of the body exposed to their action suffers less pressure than the contiguous parts of the skin, which are acted upon by the full atmospheric pressure. The result is as if all parts of the surface except the area operated on were subjected to a powerful squeeze: the fluids are squeezed by the atmosphere towards the area subjected to least pressure.

When the thoracic walls expand, their soft parts are driven inwards, air is driven into the lungs, and blood is driven into the thorax from the parts of the body acted upon by the full atmospheric pressure; all this being the consequence of the so-called negative pressure (*i.e.*, pressure less than that of the atmosphere) in the thorax. The lungs act like a sphygmoscope (Fig. 105, S): they are dilated by internal pressure until their resistance to

further dilatation is equal to the dilating force. The less extensible they are or become, the sooner will this limit be reached: if their extensibility become so small that the limit of expansion would, if the ribs expanded to their full extent, be reached before the pleural cavity is filled, then the blood and the thoracic walls themselves are pressed inwards and the chest-walls lose the habit and the power of expansion. If while the chest is expanding, there be an orifice open in a large vein, the diminution of thoracic pressure allows the atmospheric pressure not only to drive venous blood towards the heart, but also to force air into the open vein, and thus into the circulation.

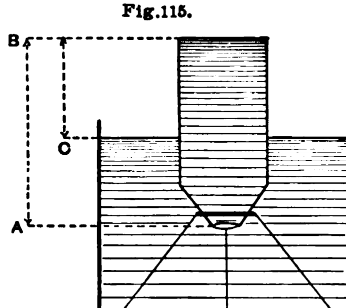
If a test tube be inserted in a larger test tube containing water, it will float. If the whole be inverted, surface-tension may for some time prevent the escape of water; but if any water do escape, the atmospheric pressure pushes the smaller tube up into the larger one, and thus causes it to appear to be sucked up.

After an extreme contraction of the abdominal muscles, there is elastic restitution of position of the abdominal walls, and the intra-abdominal pressure sinks. Apparent suction is thus exercised on the pelvic diaphragm.

When in a joint the bones are separated by extreme flexion or extreme extension, the tendency to form a vacuum between them permits the atmospheric pressure to press skin and tissue between the bones, and thus to form an external dimple.

Columns of liquid supported by the atmospheric pressure. — If a vessel filled with liquid be inverted with its mouth beneath the surface of liquid standing in a larger vessel, we see — provided that the inverted vessel do not exceed a certain height, about 33 feet in the case of water, about 30 inches in that of mercury — that the liquid does not fall out of the inverted vessel, but remains in position, supported by the atmospheric pressure. If in Fig. 115 the inverted vessel have a mouth whose area is A, and if the height of the column of fluid supported be CB, while that of the whole column of liquid above the orifice is AB; and if the density of the liquid be ρ, — the whole pressure tending to drive fluid out through the orifice A is $A\cdot\rho g \times AB$. Opposed to this we have two pressures: — (1) the atmospheric pressure acting through the fluid, equal to Π dynes **per unit of surface**, and therefore equal to $A\cdot\Pi$ over the mouth of the vessel; and (2) the water pressure on that orifice at the depth AC — that is, $A\cdot\rho g \times AC$. The whole pressure tending to drive water up into the vessel is thus $A\cdot\Pi + (A\cdot\rho g \times AC)$. Since there is equilibrium when CB has the greatest possible height — equilibrium brought about without

Fig. 115.



bringing into play the elasticity or rigidity of the upper part of the vessel — we can find the greatest free height CB by the equation —

$$A \cdot \rho g \times AB = A \cdot \Pi + A \cdot \rho g \cdot CA.$$
$$A \cdot \rho g \cdot BC = A \cdot \Pi.$$
$$BC = \Pi/\rho g = H.$$

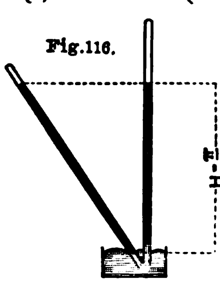
If the vessel be of exactly such a height, or be immersed just so deeply, that its own free height BC is such as to enable it to contain a column of the "barometric" height $H = \Pi/\rho g$, it will be exactly filled.

If $BC = H$, the free height of the vessel, exceed $\Pi/\rho g$, it is not possible that the column of liquid supported should extend to the upper limit of the vessel; for if it did, the weight of that column would exceed the atmospheric pressure which supports it against gravity — an evident impossibility. Hence the column actually supported cannot have a height greater than $\Pi/\rho g$, and the space between the top of the column of liquid and the upper limit of the vessel is a vacuum, the **Torricellian vacuum.**

Thus, if the free internal height of a vessel be equal to $\Pi/\rho g$ or greater than it, the height of the liquid column supported against gravity by the atmospheric pressure can never exceed $\Pi/\rho g$, but will be equal to it, whether there be above it a vacuum or not, and whatever be the size of that vacuum.

The Barometer is in its simplest form a tube filled with liquid and inverted into a cistern. If the tube have a free length CB greater than $\Pi/\rho g$ the liquid will stand in it at a free height H equal to $\Pi/\rho g$. Thus, if the atmospheric pressure be 1,013663·376 dynes per sq. cm., and if the liquid employed be water ($\rho = 1$), the free height of the column will be $H = \Pi/\rho g$ $= (1{,}013663{\cdot}376 \div 981) = 1033{\cdot}296$ cm.; while, if the liquid employed be mercury ($\rho = 13{\cdot}596$), the free height of the barometric column will be $\Pi/\rho g$ $= \{1{,}013663{\cdot}376 \div (13{\cdot}596 \times 981)\} = 76$ cm. Hence a mercury barometer is much more convenient an instrument than a water barometer, for the height of the column in the latter is over 33 feet.

Fig. 116.



If the tube be tilted obliquely, its lower end being kept immersed, the liquid will move upwards in the tube: the vertical height remains unaltered (Fig. 116).

A common water-pump cannot act if it be so deep that during its action the atmospheric pressure would have to support a greater column than one of about 33 feet: a vacuum might be produced at the top of the cylinder of the pump, and yet no column whose height exceeded $H = \Pi/\rho g$ could possibly ascend in it. The Torricellian vacuum

is utilised in the so-called mercury air-pump. A flask is filled with mercury: this flask is connected with a flexible tube also filled with mercury: this mercury is continuous with that in a cistern into which the flexible tube dips. The flask may be raised to a certain height without the mercury leaving it, but if it be raised so high that the upper limit of its cavity comes to an elevation greater than $\Pi/\rho g$ above the surface of the mercury in the cistern, a Torricellian vacuum is formed by some of the mercury leaving the flask. The vacuum may be laterally connected with flasks filled with liquids, the gases contained in which are to be extracted for analysis. When the flask is raised and a vacuum formed in it, the liquids in the lateral flasks effervesce and the gases previously dissolved in them ascend into the mercury flask, which may be disconnected and removed for further research.

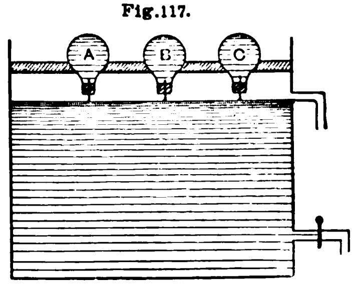
When the free height of the vessel is less than the barometric height $\Pi/\rho g = H$, the column of liquid fills the vessel.

If a card be laid across the mouth of a tumbler completely filled with water, the whole can be inverted; the card will not drop off, and the water will not drop out of the tumbler: atmospheric pressure keeps the whole in place. It is important to observe that there is no tendency for the card to become bulged in any sense.

A pipette completely filled with liquid and closed by the thumb will not allow the contained liquid to escape, unless the lower orifice be so oblique or irregular as to permit successive portions of liquid to trickle away. If it be partly filled and closed by the thumb, the pressure of air in the upper part would neutralise the effect of the external atmospheric pressure, and the liquid would be free to fall were it not in the first place for the surface-tension at the lower orifice, which, if the orifice be very small, may be able to support a considerable column of liquid, and in the second for the rarefaction which is set up by the escape of some drops of liquid.

A gas-holder may contain a certain quantity of gas above and of water below, and even though an orifice be made in the walls of the vessel below the level of the water—provided that it be not too large—none of the gas will escape, for the atmospheric pressure keeps the whole in place.

Fig. 117.



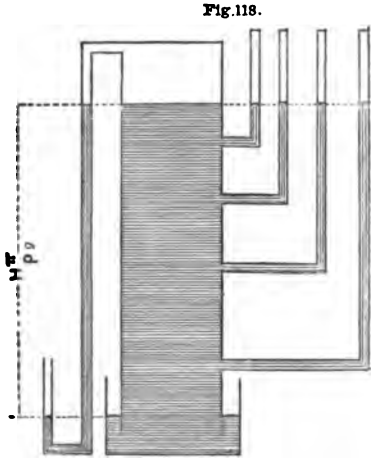
It is often of importance to keep water in a cistern at a constant level. The arrangement shown in Fig. 117 enables this to be done. The instant that the level of the liquid passes below that of the orifices of the nozzles of the flasks A, B, C, air enters these flasks, and water passes into the cistern. The aggregate delivering power of the flasks must not be less than that of the cistern itself.

When a column is supported by the atmospheric pressure, its own lateral pressure differs at different altitudes. This is illustrated by the indications of the lateral manometers of Fig. 118.

If the walls of the tube in which such a column is supported be rigid, these walls will, on account of differences between the internal pressures

and the external atmospheric pressure, be subjected to stress: this stress varies from point to point according to the altitude.

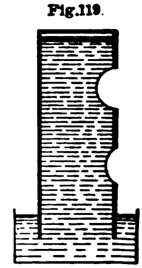
Fig. 118.



If some parts of the walls be flexible, mercury will leave the column, and the tube will yield laterally as in Fig. 119; this it will do until the resistance to further distortion offered by the walls is equal to that difference of pressure which tends to produce it.

If the column be not barometric but closed, and if in the same way the containing vessel have local flexibilities, the upper flexible parts of it will yield inwards, the lower will bulge outwards; in each case equilibrium is established between the internal pressure, the atmospheric pressure, and the elasticity of the walls. If the whole walls be flexible, the whole mass becomes pyriform; here the atmospheric pressure produces no special effect in the determination of form, for it is equably yielded to.

Fig. 119.

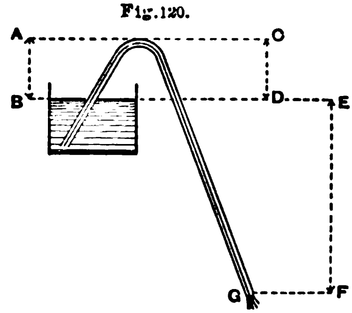


If the upper part of the walls be rigid while the lower are flexible, the lower part will bulge, but the upper will be completely filled, provided that the whole column have a height not greater than $\Pi/\rho g$; if the height be greater, there will be a Torricellian vacuum produced. If the upper rigid part of such a vessel become flexible in whole or in part, it will collapse to some extent, and fluid will pass into the lower part of the column. The amount of collapse of the upper part depends on its extensibility: equilibrium will be established when its restitution-pressure is locally equal to the difference between the internal and external pressures.

Suspended Loops. — A suspended loop is a double closed column, and it presents variations in pressure and in distension similar to those of a single column. The pressure at any altitude is determined by the relative height and the values of ρ and g: the amount of distension at any altitude accommodates itself to the pressure. A loop more than $\Pi/\rho g$ cm. deep must either present a vacuum or else collapse at its upper part. If the ascending part of the loop be more distensible than the descending, or *vice versâ*, the amount of distension will be different in the two parts of the loop, but statically the pressures at equal altitudes in the two parts of the tube will be equal. If an additional quantity of fluid be forced into the loop, it will settle down in greater quantity in the more extensible parts of it. If a constant flow of liquid be maintained in the loop, the more distensible part will contain more liquid, but (when once the relative quantity of fluid in the two parts of the loop has been adjusted) the rate of passage will not be affected by gravity. If an intermittent circulation be kept up in such a suspended loop, each successive increment of fluid is delayed in the more extensible part according to the relative degrees of distensibility; but gravity has no direct effect on the mean velocity of the stream.

A **Siphon**, such as is shown in Fig. 120, is an inverted loop. If it be more than $\Pi/\rho g$ in free height, a Torricellian vacuum is formed in its upper part. The maintenance of columns the heights of which are less than $\Pi/\rho g$ depends on atmospheric pressure; and thus a siphon will not act at all under the air-pump. In Fig. 120 the tendency of the column AB to fall out of the siphon is equal to that of the column CD to fall towards G; but the tendency of the column EF to fall towards G is uncompensated. The whole mass of liquid filling the siphon at any moment is set in motion by the weight of the liquid column whose vertical height is EF, and its cohesion makes it move as a whole.

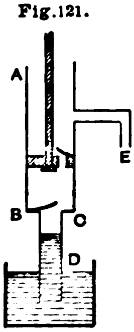
Fig. 120.



Woven tissue or a skein of thread may act as a siphon, as in the draining of a water basin by a towel, one end of which is left in the water, the other hanging over: the fibres may become wetted by imbibition, and once wetted they allow the liquid to pass over in tubes whose walls in part consist of the fibres, and in part of the superficial film of the liquid itself. This siphon-action is impossible under the air-pump.

The Common Pump (Fig. 121).—By an upward stroke of the piston the air in the cylinder AB is expanded and rarefied. The atmospheric pressure drives up a column of liquid along DC. The piston is driven downwards, or else descends by its own weight; the valves now permit a certain quantity of air to escape to the upper side of the piston, but permit none to return to the column CD. At the next stroke the air in AB and CD is again rarefied, and more water rises in DC. This is repeated until the water rises into the cylinder AB, which it will do provided that the column CD be somewhat less than $\Pi/\rho g$ in height. The piston then scoops up the water in the lower part of the cylinder, always allowing it to pass to its upper surface, but never to return, and thus at each upward stroke of the pump water is lifted up and falls out at E.

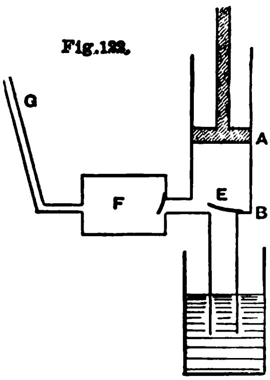
Fig. 121.



By the **force-pump** water may be raised to very great heights. Fig. 122 shows the arrangement of the valves. The piston is solid, and when it is pressed down the valve E is closed,

while air or water is forced through the valve F against the pressure of air or water in the tube G, which tends to close that valve. In the Fire-engine there may be one or two such force-pumps, which drive water into the dome.

Fig. 122.



Digression on Valves. — There are three principal types of valves in use. Of these the first is the ordinary and very familiar clapper valve.

The second is the conical valve shown in Fig. 123. The pressure of the fluid in A may displace the valve: a spring returns it to its place when the relative pressure in A has become sufficiently diminished to permit it to do so.

The third kind is that shown in section on Fig. 124. The piston AB is furnished with a cap of indiarubber, which is slightly smaller than the tube in which the piston moves. In the direction A to B the piston can be freely moved through the liquid; but if the piston be moved in the contrary direction, the indiarubber cap flies open, and it exactly and equably fits

Fig. 123.

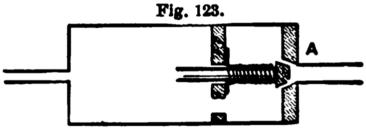
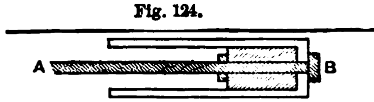


Fig. 124.



the tube so that no water can pass it. A pouch is formed: the greater the pressure within the pouch, the closer the apposition between the indiarubber and the walls of the tube, and the better the action of the valve.

A somewhat similar form of valve is found in the heart. The semilunar valves (pulmonary and aortic) consist of pouches attached to the walls of the vessel; they lie loosely against the walls and allow the liquid to flow past them as it issues from the heart; but when a backward impulse is given to the blood, or the valves are pushed forward against the blood, they are caught by the liquid, the pouches are distended, they touch one another and completely block up the lumen of the tube.

The other valves of the heart are clapper valves, attached to the walls of the cavity of the heart, two or three in each situation, together attached to a complete circumference, acting together, slightly overlapping one another, and completely closing the lumen of the tube, and provided with tendinous and muscular arrangements which prevent their being driven too far towards the auricle when they are impelled backwards by a predominant ventricular pressure.

Measurement of Atmospheric Pressure. — The atmospheric pressure Π per unit of surface may be easily calculated if H, the height of the barometric column, be known, for $\Pi = H\rho g$. The habit of stating the atmospheric pressure in terms of the barometric height, — as thus, "a pressure of 30 inches of mercury," — is general, and if clearly understood is unobjectionable.

The height of the barometric column of mercury is subject to corrections for capillarity and for temperature; the latter involve the consideration of the less density of warm mercury, and of the expansion of the glass of the tube, which expansion involves an alteration in the correction for capillarity.

The aneroid barometer is essentially a hollow box of elastic metal in which there is rarefied air. Any given amount of external pressure produces a corresponding amount of compression of this box; a multiplying arrangement causes a lever to indicate, by its position in reference to the face of the dial, the amount of this compression. Careful preliminary graduation enables the absolute amount of external pressure, corresponding to each indication of the instrument to be recorded.

The pressure $\Pi = H\rho g$, = (say, when H is found to be 76 cm. Hg) 1,013663 dynes per sq. cm., is the same pressure as would be exerted by a uniform atmosphere throughout which g was uniform, whose uniform density was 0·0012932, and its uniform height $H = \Pi/\rho g = 1{,}013663 \div (0{\cdot}0012932 \times 981) = 799{,}022$ cm., or 7990·2 metres. If a barometer on the floor stand at 76 cm., the same barometer raised to the height of 1 metre should stand at a height of 76 cm., *less* ·095 mm., a perceptible diminution.

The pressure does not diminish regularly with the height, as it would in an ocean of incompressible fluid. The lower strata of the air are compressed, and therefore, to set up a given difference of pressure, a shorter vertical ascent among them is sufficient than is necessary among the higher strata.

Each stratum differs from the one below it in two respects: — (1) it has fewer strata above it; (2) it is therefore less compressed, and for equal mass has greater volume. If we imagine the whole atmosphere to be divided into 7990·2 strata, the lowest of them all, which bears the weight of 7989·2 strata, will be 1 metre thick; the next, which bears the superincumbent weight of 7988·82 strata, will have a thickness of 1 metre $\times \dfrac{7989{\cdot}2}{7988{\cdot}2}$; the next stratum will have a thickness greater *than this* in the ratio

$\frac{7988 \cdot 2}{7987 \cdot 2}$; *i.e.*, it will be $\left(1 \times \frac{7989 \cdot 2}{7988 \cdot 2}\right) \times \frac{7988 \cdot 2}{7987 \cdot 2} = \frac{7989 \cdot 2}{7987 \cdot 2}$; and the nth layer will be $\frac{7989 \cdot 2}{7990 \cdot 2 - n}$ metres thick.

Altitudes as indicated by the Barometer.—If h be the vertical height between two stations, H the height of the mercury-barometer at the lower station observed at temperature t° C., and H' the height of the barometer at the higher station at the temperature t', λ being the latitude; then

$$x = 18393 \cdot (1 + \cdot 002837 \cos \lambda) \cdot \log \frac{H}{H'} \cdot \left(1 + 2\frac{t + t'}{1000}\right).$$

(Laplace's Formula.)

Variations in the barometric pressure occur from moment to moment as the atmospheric ocean is disturbed by currents, driven in whirlpools, varied in thickness by superficial waves, or locally varied in its superincumbent mass by expansion (due to heat) and lateral overflow. When any spot has a low pressure, there is a tendency for the surrounding air to rush in from all sides towards that spot, the centre of depression; the greater the difference of pressure between two places—*i.e.*, the steeper the barometric gradient—the greater will be the tendency to an inflow of air towards the centre of depression. This tendency is so modified by the rotation of the earth from west to east (in a direction opposed to the apparent movement of the sun) that the flow does not take place directly towards the centre, but round it in a circular storm or cyclone, whose direction is in the northern hemisphere opposed to, in the southern the same as, that of the hands of a watch (Dove's Law of Storms). The wind whirls round the centre and also towards it; air ascends in the centre: it expands and becomes cooled; moisture condenses; rain falls. "Put your back to the wind, and the barometer is lower towards your left hand (in the northern hemisphere)."—(Buys-Ballot.)

Correction for pressure.—Variations in the barometric pressure render it necessary in measuring quantities of gas by volume to make a correction for pressure, and to reduce the gas to standard pressure—*i.e.*, to state what the volume would have been had the atmospheric pressure at the time of measurement been 76 cm. of mercury. Boyle's law teaches us that the volume varies inversely as the pressure. If therefore the pressure on gas measured as x cub. cm. at 76·1 cm. had been, not 76·1 but 76·0 cm., the volume of that gas would have been greater under the less pressure in the ratio of 76·1 to 76·0.

The general rule is, that a volume of gas measured at a pressure H cm. of mercury must be multiplied by $H/76$ in order to reduce it to the standard atmospheric pressure.

Standard Atmospheric Pressure. — In many modern books, instead of a pressure of 76 cm. mercury, or 1033·296 cm. water, or 1,013663·376 dynes per sq. cm., the standard atmospheric pressure is taken as 1,000000 dynes, or one megadyne, per sq. cm. This is 75 cm. Hg at 1°·8 C., or 1000 cm. of 3 % KCl solution at 8° C. (density = 1·0937).

Gases passed into the Torricellian Vacuum. — If a bubble of gas be passed into the Torricellian vacuum, it will expand so as to fill it; further, it will exert pressure on the top of the column of mercury — it will therefore depress that column; the extent to which it depresses the column measures the pressure which it exerts upon the mercury: conversely, that depression measures the pressure of the mercury upon it, and therefore indicates the pressure under which it itself assumes its actual volume.

Let a barometer tube whose cross area is ¼ sq. in., and whose free internal height is 34 inches, have standing in it a column of 30 inches of mercury. Pass a cubic inch of air (measured under a pressure of 30 inches) through the mercury into the four-inch-long Torricellian vacuum. It would exactly fill that vacuum, exerting a pressure of 30 inches on the top of the mercury. This is impossible. The gas expands; it depresses the mercury through x inches; it is then subjected to a pressure of x inches of mercury as compared with the atmospheric pressure of 30 inches under which it was measured. Its volume is now accordingly increased to 1 cub. in. $\times\ 30/x$. The length of tube occupied by this volume is $(30/x) \times 4 = 120/x$ inches; of these, 4 inches were already taken up by the vacuum. The actual depression is therefore $(120/x) - 4$; but this depression is x itself. Hence $x = (120/x) - 4$; or $x = 9{\cdot}1455$; and the mercury will stand in the tube at a height of 20·8545 inches.

CHAPTER XIII.

HEAT.

Heat is a form of Energy. It would, perhaps, indeed be more correct to say that we designate under the one name Heat two totally distinct forms of Energy. The one of these is the energy of a wave-motion in the Ether, passing from a hot body to surrounding objects across the intervening space, as from the sun to our earth, or from a hot fire to the colder objects upon which it shines: this we call Radiant Heat. The other form is that of a confused oscillatory disturbance of the particles of a body: in virtue of this molecular movement a body may appear to our cutaneous sense of heat (a sense quite distinct from that of touch) to be more or less hot or warm; or in the converse case it may, on account of the small amount of this movement, appear to be relatively cool or cold. The latter form of heat may be called Sensible Heat, or Heat simply, and of it we shall proceed to treat in this chapter. It is the only form of heat for the perception of which we have special sense-organs. We do not directly perceive the undulations of radiant heat by our senses: when the sun shines on us heat-waves strike the skin, throwing it into vibrations, and the sensible heat of the skin, not the radiant heat of space, affects the appropriate nerve-ends. When we touch a hot body it communicates its oscillations to the nervous system: when we approach a hot body we become indirectly sensible of the radiant undulations into which it is throwing the surrounding ether. Thus we may state that our sense of heat is our power of perception of the confusedly-vibrating condition of a body; and that the more pronounced this condition of agitation, the hotter will a body appear. A hotter body may be readily supposed — and rightly so if we confine our attention to bodies formed of the same substance — to have in it a greater amount of Heat than a colder one. And a hotter body can become cold, a colder body can become warm: heat can be supplied to bodies, or they can be deprived of it;

heat can be gained or lost by material bodies. The primitive interpretation of this was that Heat was a substance, a fluid, the so-called Caloric, invisible, imponderable; that a piece of hot iron was a kind of temporary union of cold iron with this subtle imponderable fluid. When a piece of metal was rubbed it became warm: the reason assigned was that Caloric was squeezed out of it, like water out of a sponge. But this material theory of heat became untenable when it was shown that there was absolutely no limit to the amount of sensible heat which might be so produced by the friction of a trifling amount of metal; the amount of water that might be boiled, for example, by heat produced in this way depended only on the mechanical power available (Rumford). The heat evolved by friction — as, for instance, in metal boring or turning — is practically limitless. Even two masses of ice, caused to rub against one another, melt (Davy) — a fact which leads the material theory of heat into helpless confusion. Water was admitted to be ice *plus* caloric; if, then, ice with its caloric rubbed or squeezed out of it and lost — that is to say, ice *minus* caloric — become water, how can the theory stand? Plainly Heat is not material: and the only alternative appears to be that it *is* the Energy imparted to the system. It is equal to the work done upon the system; and we find that Heat and the other forms of Energy are reciprocally convertible.

When a body is sensibly hot its particles are in an active state of motion. The particles strike one another and rebound; the more rapidly they do so, the greater is the mean velocity of the particles, and the greater is the kinetic energy of the whole mass; but it is impossible that the energy of the molecules should be entirely due to such a movement of Translation. They are not material points, and they have — if not in solids or in liquids, yet certainly in gases — six degrees of freedom; when they strike each other they not only rebound but they also spin; to the energy of translation must be added one of Rotation. Further, the molecules are made up of atoms: atoms are not stationary in the molecule, but may be so violently agitated as to leave it altogether, and thus to give rise to the phenomena of chemical decomposition by heat; part of the energy of a heated body is due to intra-molecular or Atomic Oscillations. Lastly, the ether entangled in a molecule is also set in vibration, and absorbs some energy, which appears as kinetic energy of Ether-Vibrations. The sum of these is found, by the agreement of experimental results with calculations based on the hypothesis that such is the law, to be proportional on the average — an average not perceptibly departed from for any appreciable interval of time — to the kinetic energy of translation alone.

Heat is not Motion, for it is neither Change of Position, nor yet Momentum; it is the Energy of Motion. Double the quantity of molecular Motion, and you quadruple the molecular kinetic Energy, that is, the Heat.

Heat is not liberated by Pressure alone: there must be yielding to

the pressure: then the work done, Fs, or the equivalent Energy, has a determinate value, measurable in ergs.

The convertibility or identity of Heat with Energy is independent of the inner mechanism of the moving molecules which possess it; and it is confirmed by instances from all sides.

The Energy of work which is apparently wasted in friction becomes Heat: the heating of a locomotive brake, the ignition of a lucifer match, the heat evolved during the mechanical operations of metal boring or turning, the heat found in a body which has received a sudden blow or a sudden distortion, or suddenly yielded to pressure, — all these prove the proposition.

If work be done in driving a paddle in water, no work being done other than that of churning the water, when the operation is over the work appears to have been wasted and to have disappeared; but the energy is not destroyed; it exists in the water in the form of heat. If 772·55 * foot-pounds of work (measured at sea-level and latitude of Greenwich, Joule) be expended in churning a pound of water, the temperature of that water will be raised by 1° F., from 60° F. to 61° F.; a similar rise of 1° C. in a kilogramme of water will be effected by the expenditure of 423·985 kilogrammetres or 41,593,000000 ergs of work; that is, of 41,593000 ergs per gramme. Hence the water at the base of Niagara Falls ought (setting aside the effect of evaporation and of cooling or heating by the air) to be about $\frac{1}{5}$° F. higher in temperature than at the top, for the vertical fall is 161 feet. Hence also the sailor's maxim that the sea is warmed by a storm.

When in a steam-engine at work the steam at its entrance to the cylinder from the boiler is compared with that which goes to the condenser, it is found that the latter is colder. The difference of heat is found to be equivalent to the work which the engine has done; and if the engine do no work, then the energy which has not been converted into work remains as heat in the out-going steam, and the engine may become heated (Hirn).

When a quantity of gas or of liquid is forced through a tube, as in Fig. 109, the potential energy of the system before the flow is started is greater than the kinetic energy of the outflowing stream. If the resistance be so great that the velocity

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*** The investigations of Rowland and Griffiths have shown that this number is too low, and that it should be above 778, or possibly as high as 779. If we took it as 778.5, we would have to replace the number 41,593000, used in this volume, by 41,914000.**

of outflow is practically null, the whole of the work done on the fluid is spent in heating it.* The work done is equivalent to the heat produced.

We are now able to state the First Law of Thermodynamics. **Heat, being a form of Energy, can be measured in ergs,** in foot-poundals, or in foot-pounds.

This law is usually stated in a somewhat different form. An arbitrary unit of heat is chosen, and designated a *calorie :* this is the amount of heat which is required to raise the temperature of one gramme of water from 0° C. to 1° C. This quantity of heat is found to be 41,593000 ergs. This last number, 41,593000 ergs, is the "Mechanical Equivalent of Heat," or "Joule's Equivalent": † it should perhaps be called the Dynamical Value of the Conventional Unit of heat, the calorie. The first law is, then, that one calorie (*ca*) is equal to 41,593000 ergs.

Another unit of heat has been proposed, the Electromagnetic Unit, the Joule, or 10,000000 ergs; this is the amount of heat developed in one second in an electrical circuit or wire whose resistance is one Ohm when a current passes whose intensity is one Ampère. (See p. 647.)

Heat is energy, and it is the lowest form of energy. It may be said to have no organisation, but to depend on undirected and blind activity of molecules, which dash hither and thither. When in any action energy is liberated which is not guided by the environment into any specialised form, it manifests itself as heat; and when energy is spent in doing work, the equivalent of which appears in no other form, it then appears as heat. This statement is widely applicable and important.

Work done upon a dynamoelectric machine whose circuit is complete appears in the first place as the energy of an electric current: if no exterior work be done, the system as a whole becomes heated.

A voltaic cell can do exterior work: if it do none, the current being allowed to circulate uselessly, the whole of the energy liberated during the chemical combination appears as heat in the circuit.

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* This must be done at a pressure corresponding to a certain definite head H of the same fluid. The fluid is found to rise in temperature by x° C.; a head of H/x cm. would cause it to rise by 1° C.; a vertical free fall of H/x cm. would cause it, if abruptly stopped, to rise in temperature by 1° C.; the amount of energy corresponding to such a fall would be $(H/x) \cdot mg$ ergs; this energy in the form of heat $(H/x) \cdot mg$ ergs, would heat a mass m of the fluid through 1° C.; $(H/x) \cdot g$ ergs would heat one gramme of the fluid through 1° C.; $(H/x) \cdot (g/\sigma)$ ergs (σ being the *specific heat* of the fluid, p. 365) would heat one gramme of *water* through 1° C.

† Joule's Equivalent in its original form was a number (772) which denoted the number of foot-pounds of work found to be equivalent to the heat necessary to raise 1 lb. of water through 1° F.

Heat being a form of energy, many propositions relating to it are merely special cases of propositions relating to energy.

If a certain number of bodies be arranged in a system A whose potential energy — depending on the arrangement of the bodies in the system — is P_A; if the same bodies can be arranged in other systems B, C, D, whose respective potential energies (less than that of the former) are P_B, P_C, P_D, etc.; then the transformation of the more highly-stressed system A into a less-stressed system B, if this be brought about by a rearrangement of its constituent bodies, involves a liberation of energy equal to $P_A - P_B$. If in this case the system A be converted into the system B without doing any exterior work, the whole of the energy liberated appears in the form of heat; and, numerically expressed, the heat thus liberated is equal to the work W which would have had to be done upon the system B in order to convert it into the system A, if that converse operation had been effected; that is, the heat so liberated is equal to $W = P_A - P_B$.

A gramme of hydrogen and eight grammes of oxygen form a system (system A) which after explosion may be converted into nine grammes of water-vapour of the same volume (system B) at a temperature of 136°·5 C. The former is converted into the latter without doing any exterior work. Much energy is liberated in the form of heat, and though the absolute values of P_A and P_B are unknown, their difference is found (see Calorimetry) to be an amount of energy equivalent to 28,580 *ca*.

If the products be cooled down to steam at 100° C., the total amount of heat liberated is equal to 28,738 *ca*, or 1,195300,000000 ergs; if to water at 0° C., it is equal to 34,462 *ca*, or 1,433380,000000 ergs, or 1,433380 megergs. The potential energy which such a mixture loses when its particles clash together and combine is the energy of chemical separation. A mixture of explosive gases may be made to yield up some of this energy in the form of work, as in the modern gas-engine; if no work be done, and if there be no other transformation, the whole of it must appear in the form of heat.

Chemical combination is thus often attended with the evolution of heat. One gramme of carbon burned in oxygen yields 8,080 *ca* or 336071,520000 ergs; 1 gramme of carbonic oxide yields 2,403 *ca* (2,431 *ca*, Andrews); 1 gramme of marsh-gas, 13,063 *ca*; 1 gramme of dry albumen, 4,998 *ca*; urea, 2,206 *ca*; fat, 9,096 *ca*; starch, 3,901·2 *ca*, or 162263,000000 ergs per gramme (Frankland).

When copper or antimony is dropped into chlorine it takes fire, and a chloride is formed: heat is evolved.

In some instances the converse is true; work has to be done upon separate elements in order to force them directly or indirectly to combine: and when their compound decomposes, heat is evolved. Carbon and sulphur will only combine when they are kept hot by an external source of heat: they must be forced to combine: and when CS_2 is violently shaken, as by the explosion of a percussion cap, the carbon and the sulphur fall apart, evolving heat. Nitrous oxide (N_2O) evolves heat when it is decomposed

into nitrogen and oxygen; and hydrogen dioxide (H_2O_2) evolves heat when it is decomposed by contact with platinum.

The heat liberated or absorbed measures the work done by chemical action. Where there is none, there is no chemical change. Thus hydrochloric acid gas and ammonia gas do not change in temperature when mixed hot: there is no combination: it is only on cooling that they combine and liberate heat. Similarly in many other cases, where dissociation takes place on heating or on solution in a liquid.

The Chemical Forces themselves are not measured by the total Heat liberated or absorbed. The reactions may be rapid or slow, and they do not necessarily take a course which will liberate the maximum amount of heat.

A change from the condition B to the condition A (which possesses more potential energy) cannot be effected unless there be energy added *ab externo*, or else unless some of the kinetic energy of the body, if it have any, assume the potential form; in the latter case the body may lose sensible heat, and may become cold.

When a chemical decomposition is effected by heat, if heat had been evolved during the formation of the compound, heat must be continuously supplied to do the work of decomposition. The heat supplied has the effect of throwing the molecule into such agitation that the mutual affinity of the atoms cannot retain them in union. This is the process of Dissociation or Thermolysis. At moderately-high temperatures the atoms reunite with others which they encounter; at very high temperatures (from 2300° to 3000° C. in the case of oxygen and hydrogen) no such reunion is possible, and the decomposition is complete. Thus the proportion of decomposed to apparently undecomposed material varies with the temperature. The process is favoured by one or more of the resultants of dissociation being gaseous. After dissociation, the separated elements contain potential energy equal to the heat expended upon them; and upon cooling they may recombine with the evolution of this energy in the form of heat, which is gradually lost.

Dissociation is easy at low pressures. Hence at low pressures the combustion of a candle is incomplete and its flame is smoky.

Dissociation of the products of combustion is also exceedingly facilitated by the presence of solid surfaces (Sir C. W. Siemens).

In general, *every Change* in the State or Condition of a body or a system of bodies is associated* with a Change in the Intrinsic Potential Energy of the body or the system; and this change is accompanied and manifested either by the liberation of Energy in some form, useful or useless — *e.g.*, work or heat — or else by the disappearance of Energy which is spent in producing the change of state, and is either taken in *ab externo*

* This general conclusion is subject to the qualification that the change of state or condition must be a real one, not one which consists in a mere replacement of the particles occupying a given position by others physically similar, or by a mere change of the direction in which similar parts of the substance lie.

or is transferred from the kinetic energy already possessed by the body, as is shown in the ordinary case by that body becoming cold.

Thus, if a quantity of air in a cylinder be suddenly compressed by means of exterior work done upon it, it becomes hot: if the piston be allowed to return, the air cools down to its former temperature; but if it be kept compressed until it has assumed the temperature of surrounding objects, and if it be then allowed to drive the piston out against atmospheric pressure, it becomes very cold, for it obtains the energy required to do the work of driving out the piston at the expense of its own heat.

If the system A disengage x units of energy (as heat, or in any other form) on being let down to the condition B, and if the same system A disengage y units when it acquires the condition C, then the system B, on being let down to the condition C, will disengage energy $= y - x$. Conversely, if B and C respectively require energy x and y to enable them to become converted into the system A, the system C requires energy $= y - x$ in order to enable it to become the system B.

The relative amounts of chemical energy in organic compounds may be estimated by finding the amount of heat which they evolve when they are burned so as to form carbonic anhydride, water (and nitrogen).

Oil of lemons, turpentine, and terbene, which have the same chemical constitution, seem to have a different intramolecular arrangement, for on combustion they evolve different amounts of heat. This shows that the potential energy of the molecules is different in each case.

What is the intrinsic energy of Acetic Acid? 60 grammes of acetic acid are found (Berthelot) to disengage on combustion 210,000 *ca* of heat: 3500 *ca* or 145,576 megergs per gramme. The total intrinsic potential energy of acetic acid we do not know; the number given indicates the total amount available on combustion with oxygen. Its elements, — 24 grammes of carbon, 4 of hydrogen (and 32 of oxygen), — yield on combustion 332,000 *ca*. The difference between the "Combustion-equivalent" of the 60 grammes of acetic acid and that of the same weight of its component elements — that is, 122,000 *ca* — is the total amount of energy lost by these substances when they pass through the changes (whatever be the number, the nature, or the order of these) in which they pass from the state of free elements to that of acetic acid.

Amorphous sulphur kept in a solution of sulphuretted hydrogen becomes octohedral sulphur with absorption of heat.

When zinc is dissolved in sulphuric acid, a certain amount of energy is liberated and heat is evolved: when zinc is amalgamated with mercury, it becomes cold unless heat be supplied: when amalgamated zinc is dissolved in sulphuric acid it evolves more heat than unamalgamated zinc does, and that by an amount exactly equal to the heat absorbed during amalgamation.

The absolute amount of energy liberated or absorbed during any change of state is independent of the rate at which the change is effected.

A slow change of state (as in the processes of decay or of the oxidation of the tissues of an animal) evolves the same amount of heat as a rapid change; the temperature in the former case is lower than in the latter, because the lapse of time allows a more equable distribution of the heat. Thus, 1 gramme of hydrogen and 8 grammes of oxygen will evolve enough heat to raise 34,462 grammes of water in temperature by 1° C., or 344,620 grammes by $\frac{1}{10}$° C.: this it will do whether the combination be explosive or gradual, as when the gases are induced slowly to combine by the presence of rolled platinum. The final condition of the products must be the same in both cases: if this be not borne in mind, the amounts of energy evolved during combination will appear to differ in the two cases by an amount equal to the work which would have to be done in order to convert the one final state into the other.

When a system of bodies passes from one state to another it is a matter of indifference what the intermediate changes have been, so far as concerns the absolute amount of energy liberated or absorbed; the system A may have assumed the conditions C, D, etc., and that in any order; but the amount of energy liberated depends only on the initial state A as compared with the final state B.

If it had been otherwise, the perpetual motion might be realised; for it might be possible to effect a change from A to B by one series of transformations, and to effect the reverse operation by another series, such that the one series of changes would evolve more energy than the converse one consumed, and the result would be a repeated restoration of the *status quo*, associated with a perpetual supply of energy, available for useful work, and created out of nothing.

The energy absorbed by a system during a given change of state is exactly equal to that which is liberated when the change is reversed.

It is assumed in this statement that no exterior work is done through the instrumentality of the change of state.

The potential energy of every system of bodies always tends to diminish as far as possible. Every system which possesses potential energy thus tends to lose it; its potential energy tends to become kinetic, and, if it assume no other form, to take the unspecialised form of heat. In any system which undergoes spontaneous transformation, the transformation generally tends, unless prevented, to take such a course that the heat evolved by it shall be a maximum. This is, however, only a tendency: and in many chemical reactions the heat evolved is not the maximum possible.

In many cases a single change of state may be analysed into several others. The heat-value of the total change is equal to the sum of the heat-values of the separate component changes.

Thus when a piece of sodium is put into water the following changes occur simultaneously:—(1) decomposition of water into free atoms of hydrogen and oxygen; (2) coalescence of atoms of hydrogen to form molecules; (3) reduction of hydrogen to the gaseous state; (4) exterior work done by the hydrogen escaping against atmospheric pressure; (5) combination of sodium with oxygen and hydrogen atoms to form sodium hydrate; (6) solution of sodium hydrate in water. Each of these changes has its own heat-value, positive or negative, according as it involves the evolution or the absorption of a certain amount of energy. On the whole, potential energy is lost and heat is liberated.

The combustion of 1 grm. H with 8 grms. oxygen yields 34,462 *ca* heat. The same quantity of the same elements combining in the nascent state yields 54,623 *ca*. Hence the heat evolved during the combustion of one gramme of hydrogen is the resultant of an absorption of energy (20,161 *ca*) due to the break-up of the gaseous molecules into atoms, and an evolution (54,623 *ca*) due to the combination of these atoms in the formation of water-molecules and condensation into liquid water at 0° C. The balance of the account shows energy to be liberated as heat.

When a gas is dissolved in water there are two effects:—(*a*) liquefaction of gas with evolution of heat; (*b*) satisfaction of chemical affinity between the water and the gas, with the evolution of still more heat. When NH_3-gas is dissolved in water, there is no evolution of heat corresponding to any union of NH_3 and H_2O to form NH_4HO.

When a solid is dissolved in water the liquefaction of the solid causes the absorption of heat (as in freezing mixtures), while the satisfaction of mutual chemical affinity causes its evolution. When glacial acetic acid is dissolved in water, the absorption of heat caused by imparting greater fluidity to the acetic acid overpowers the evolution of heat due to chemical union.

When two or more changes of state occur concurrently, it may be that some of these changes are accompanied by the liberation, some by the absorption of heat, and that these changes exactly compensate each other; the result being that on the whole there is neither absorption nor liberation of heat.

For example, a gas, while expanding (from whatever cause), against the atmospheric pressure, does work in lifting the atmosphere; if it increase in volume alone, without undergoing any change in its temperature, energy must be supplied to it in order to enable it to do this work; if it diminish in temperature without suffering any change in its volume, it must necessarily lose heat; if, on the other hand, it undergo both these changes—increase in volume and diminution of temperature—concurrently, it is possible that these two changes may be so adjusted that the body, while it undergoes the double change, neither loses heat nor acquires energy from without. Such expansion is called (Rankine) **adiabatic** expansion—expansion during which the substance neither gains nor loses heat by conduction or radiation to or from surrounding objects, and in the course of which,

as it expands, it cools down by reason of its expenditure of energy. This is a kind of operation which could only be perfectly realised in practice if the expansion were infinitely rapid; but any gas suddenly expanded is thus chilled. Conversely, adiabatic contraction of volume is associated with increase of temperature.

We have hitherto regarded any change of state, simple or complex, as a possible antecedent cause of the liberation or of the disappearance of heat. We shall now change our standpoint, and consider the effects (including change of state or of condition) produced *by* the increase of heat in a body or by its withdrawal.

Effects of Heat.

The principal effects of an increase of heat in a body may be the following: —

A. **Internal Work.**

a. Increase of the kinetic energy of the molecules of the body — an increase of the sensible heat of the body; *i.e.* an increase of *temperature.*

b. *Intermolecular* work — work done by or against molecular forces — change of volume, change of cohesion, change of elasticity, etc.

c. *Intramolecular* work — work done within each several molecule — production of intramolecular vibrations, rotations, deformations.

d. *Chemical* work, intermolecular and intramolecular.

B. **External Work.** — Work done by or on a body as it expands or diminishes in bulk.

These effects are not necessarily all produced by the action of heat upon any substance.

There may, as in the following example, be no external work done when a body is heated; the whole energy imparted to the body being spent upon the internal accumulation of energy in the form of heat. Water at 3°·4 C. if heated to 4°·4 C. first contracts and then returns to its original dimensions. On the whole there is in this case no external work done. Neither is there any work done in giving the particles a new position in opposition to the intermolecular forces,* nor is there any chem-

* This statement is only approximately true, for there are physical differences — of viscosity and the like — between water at 3°·4 C. and water at 4°·4 C. The temperature of the maximum density is lowered 0°·0177 C. per atmo. pressure (Tait).

ical effect. The whole heat imparted may thus be held to be spent in raising the temperature by 1° C.

When a bar of iron is heated in a vacuum there are two effects: (1) increase of temperature; (2) expansion of the iron, which represents work done against the molecular forces. When the same bar is heated in air, there is added a third effect, viz., the thrusting aside of the surrounding air by the expanding bar, in consequence of which exterior work is done during expansion. In a bar of iron the exterior work done in this way is very small, and the interior work done predominates so largely that the exterior work may for many purposes be neglected.

When a mass of gas is heated, the work which is done in expanding the gas itself is appreciably null, for this is one of the characteristics of gases; if there be any work done during the expansion, it is all exterior. The effects in this case are two: (1) the increase of temperature; (2) exterior work done in overcoming the exterior (atmospheric or other) pressure.

When water above 3°·9 C. is heated it expands. The effects are — (1) increase of temperature; (2) work done in separating the molecules; (3) a small amount of work done *against* the external pressure.

When water between 0° and 3°·9 C. is heated it contracts. The effects are — (1) increase of temperature; (2) intermolecular work; (3) a small amount of work done *by* the external pressure.

When a piece of caoutchouc is heated it contracts; when pulled, it expands and assumes the dimensions proper to a lower temperature, intermolecular energy is set free, and the caoutchouc becomes warm. A piece of metal suddenly extended becomes cool.

When ice at 0° C. is heated, the whole energy imparted to it is expended in producing the following results: — (1) Fusion, with contraction of volume (intermolecular work — work spent in producing a new arrangement of the molecules); (2) A slight amount of work done *by* the exterior pressure on the body. The latter may be for most purposes neglected; if we do so we may say that all the heat supplied to the ice is spent in doing the interior work of liquefaction, and that none of it is spent in producing any increase of temperature. When, therefore, a piece of ice is heated it melts, but it does not rise in temperature until it has been wholly melted. The water produced has a temperature of 0° C., and it does not begin to rise in temperature until the ice has entirely disappeared: when

this has occurred, the continued action of heat causes the water to rise in temperature.

A gramme of ice at 0° C. absorbs (Bunsen) 80·025 *ca* of heat, and becomes a gramme of water at 0° C. Conversely, a gramme of water at 0° C. must continue to lose heat until it has parted with 80·025 *ca* before it can become a gramme of ice at 0° C.; whence we observe that water does not freeze throughout at the instant of the thermometer's touching the freezing point.

It was obvious that a gramme of water differed from one of ice in somehow possessing 80·025 *ca* of heat; but this was not sensible to the thermometer; hence the heat so possessed by the water was said to be hidden or **Latent Heat.** We now know that it is not Heat of any kind; it is latent or potential Energy; work must be done against molecular forces in order to convert ice into water: water somehow differs from ice at the same temperature in possessing more potential energy.

Direct increase of the kinetic energy of the particles of a heated gas is demonstrated by the Radiometer.

If a surface be heated, a molecule of gas striking against it is heated; it leaves the hot surface with a velocity greater than that with which it had approached it. If the surface be fixed, the gas in front of it is driven away from it by the bombardment of the molecules which have touched the hot surface, and on their return strike their fellow-molecules; in front of the hot surface the gas is therefore under a greater pressure than it would have been had the surface been cold. If the hot surface be not fixed, this increase of pressure has — reaction being equal and contrary to action — a tendency to drive that surface backwards. This tends to knock dust away from a hot surface.

If the hot surface be the front aspect of a disc, the back of which is by some means kept colder than the front, and if this disc be suspended in a gas, the heat of the front surface increases the pressure towards the front, and the gas flows round to the back of the disc. Thereafter the disc is struck on the hotter surface by fewer molecules with greater velocities, on the colder surface by a greater number of molecules with lesser velocities; there is thus compensation; the result is that the disc is equally pressed upon in front and on the back; it does not move.

Let us now suppose that the particles recoiling from the heated surface do not meet other molecules, but impinge on the walls of the vessel. A layer of particles in such a condition is called a Crookes's layer.

This will occur in two cases — (1) when the gas is so rarefied that the mean free path of the molecules exceeds the distance between the hot surface and the walls of the vessel; (2) when, whatever the density of the gas,

the opposite wall is so near the hot surface that the distance between them is less than the actual mean free path of the molecules. These conditions, which are essentially identical, may concur; there may be both rarefaction of the gas and approximation of the opposed surfaces.

In such a case there is no flow of gas from the hotter surface towards the colder one: each molecule which strikes the hotter surface and rebounds with a greater speed adds independently to the recoil which the hotter surface suffers, and if the hotter surface be movable, it is driven backwards. If it be not movable, the particles which rebound from it strike the opposite wall of the containing vessel, and that wall has a tendency to move forward.

There is yet another case: if the rarefaction of the gas be extreme, the particles which strike the heated surface are few in number or none at all, there is little or no recoil, and there is no movement set up when the rarefaction is carried too far.

The disc of which we speak — a disc of which one face is kept hotter than the other — may be a disc covered on the one side with some heat-absorbent material such as lampblack, the other face being whitened. When radiant heat or light falls upon the disc, even in an equally-lighted field, the blackened side becomes hotter. If such a disc be suspended vertically by two threads, it will diverge slightly from the perpendicular. Such a disc may be attached to the end of a counterpoised rod, the whole being suspended by two threads: the effect of heat or light is to twist the suspending threads to a certain extent. If the suspensory arrangement be replaced by a pivoting one, we have the **Radiometer.** A globe of glass, in which a vacuum is made, carries a vertical needle axially fixed, on the summit of which is poised a rotating vane consisting of light rods, to the extremities of which discs are affixed, each similarly blackened on one side. Such an instrument placed in light, even in a uniformly-lighted field, has the black sides of its discs more heated than the unblackened sides, and if the radiance be of sufficient energy the vane rotates. Moonlight is too weak to produce this effect: a candle will make a sensitive radiometer rotate; a paraffin lamp without a globe will at close quarters make the vane fly round so fast as to be invisible.

If a radiometer be floated in water, and if the vane be so constructed — one of its spokes being a magnet — that a powerful magnet in the neighbourhood can hold it motionless, when

the radiometer is exposed to light the bulb itself will rotate in the water in which it floats.

The radiometer is a machine in which heat (generally derived from the transformation of light into heat) is directly converted into the energy of work.

The less the distance between the discs and the walls of the bulb, the greater will be the effect, and the faster will the vane rotate, provided that the rarefaction be less complete than that which gives the greatest effect. Too complete a rarefaction is not an advantage, for it leaves an insufficient supply of working molecules.

When the distance between the disc and the opposite wall is excessively small, the vacuum need not be very good; indeed the effect of repulsion may be made manifest even in the open air.

When a drop of water is placed upon a very hot iron it assumes the so-called **Spheroidal State**; it does not wet the hot iron, but gathers itself into a drop, which rapidly evaporates and alters the local conditions of its surface-tension so as to present an appearance of varying scroll-work on its surface, while the drop oscillates so as to present the form of rosette and other patterns, these being due to the formation of nodes and vibrating loops. The drop may be of very considerable dimensions—several ounces in weight. It does not touch the iron; there is an intervening layer of aqueous vapour on which it floats; through the space intervening between the drop and the hot solid the light of a candle may be seen. This layer of aqueous vapour is a "Crookes's layer;" particles strike the heated surface, rebound, and strike the liquid, thus maintaining a clear space between the metal and the drop. Ether and small drops of bromine float in the same way on the surface of hot water. A lump of carbonate of ammonia, thrown into a red-hot platinum crucible, assumes the spheroidal state superficially, but does not melt. The hand can be safely immersed in melted metal if it be not too dry, and if the immersion be effected with a certain degree of prompt deliberation; a Crookes's layer of water-vapour intervenes between the hand and the metal.

When liquid sulphurous acid is dropped into a white-hot platinum crucible it sinks greatly in temperature, on account of its rapid evaporation and its slow reception of heat across the Crookes's layer; if a little water be added to it, the water freezes. Ice can thus be produced in a white-hot platinum

crucible. A similar Crookes's layer is formed if a quantity of solid carbonic dioxide be *lightly* placed on the tongue; the extreme cold (−80° C.) is not felt.

When the hot solid-body cools down, the Crookes's layer disappears, the liquid suddenly comes in contact with the solid still relatively hot, and the liquid explodes in vapour. This occurs in the case of water and iron at about 180° C.

Melted copper can be cast under water in a canvas mould; and, singularly, it often remains fluid so long and cools down so far in that condition that there is no explosion.

Increase of Temperature. — We have freely made use of the term Temperature because it is a term in common use, and not likely, so far as we have used it, to lead to ambiguity. We have still to defer the consideration of thermometry; but we must now consider increase of temperature as directly due to increase of the molecular kinetic energy of a body. When we double the Molecular Kinetic Energy of a body we double its Temperature.

Observe that it is not asserted that we double the temperature when we double the total energy of a body: some may disappear in doing work, and take the form of the so-called latent heat.

This implies that there must be some point of Absolute Zero of Temperature, independent of the conventions of Fahrenheit, Celsius, and others, afterwards to be explained — a point of Absolute Cold, beyond which no cooling is conceivable.

We have already seen that in a perfect gas — one in which there is no complication due to intermolecular forces — the pressure is proportional to the molecular kinetic energy of a given mass, occupying a given volume; the temperature is, or may by definition be held to be, also proportional to this kinetic energy; it follows that the Temperature is Proportional to the Pressure when the volume occupied by a given mass remains unchanged. It is found that in all gases the pressure diminishes by about $\frac{1}{273}$ for each Centigrade degree of cooling, the temperature of 0° C. being the starting-point, and the volume being maintained constant. If a gas could be cooled down in this way to − 273° C. (a feat unachieved, − 225° C. being the lowest yet reached), it would have no pressure and therefore no temperature, for it would have no kinetic energy, no heat. The Absolute Zero of temperature is therefore −273° C. (or more accurately −273°·72 C.), and the Absolute Temperature of a body whose temperature, as measured by

the Centigrade thermometer (see p. 402), is t° C., is τ° Abs. $=(273+t)^\circ$ Abs.; thus the boiling point of water, 100° C., is 373° Abs.

The true C.G.S. unit of temperature would be the rise of temperature produced by adding one erg to the molecular kinetic energy of one gramme of a perfect gas (p. 369) of unit specific heat, as defined below; but this is an impracticable unit; and the conventional "unit of temperature," the Centigrade-thermometer degree, or ° C., in terms of which temperatures are measured, is equal to 41,593000 such true C.G.S. units.

Specific Heat. — The heat-energy of a molecule of hydrogen is equal to that of a molecule of oxygen at the same temperature; but the latter weighs sixteen times as much as the former, and a mass of hydrogen contains sixteen times as many molecules as an *equal mass* of oxygen under similar physical conditions. Hence a given mass of hydrogen at a given temperature possesses sixteen times as much heat-energy as an equal mass of oxygen at the same temperature.

To produce a given rise in the temperature of a mass of hydrogen we must supply sixteen times as much heat as we would find necessary to produce an equal rise in temperature in an *equal mass* of oxygen; the capacity for heat — the Thermal Capacity — of hydrogen is sixteen times that of oxygen.

The **Thermal Capacity** of a Substance is the number of units, that is, the number of **ergs** of Heat-energy with which a **gramme** of that substance must be supplied in order to raise its temperature by one "unit of temperature," that is, by 1° C.

The **Specific Heat** of a substance is the **ratio** between its thermal capacity and that of **water**: and, since one gramme of water requires 1 *ca* to raise its temperature 1° C., the Specific Heat of a substance is also, numerically, the Number of *calories* of Heat required to raise the temperature of one gramme of that substance by 1° C.

If the specific heat of a substance be σ, its thermal capacity will be 41,593000 σ ergs per gramme (or 41,593000 $\rho\sigma$ ergs per cub. cm.).

The ratio between the Specific Heats of two substances is the same as that between their Thermal Capacities.

The 'thermal capacity of a body,' such as the earth or a lump of copper, is $m\sigma$ *ca*, or 41,593000 $m\sigma$ ergs, per ° C.

In general, the lighter the molecules of which a substance is made up, the more numerous must they be in a given mass, and the higher the thermal capacity of the substance, *i.e.* the

more heat must be expended upon it in producing a given rise of temperature.

The law here indicated—that the specific heat of an element varies inversely as its atomic weight—is based on the assumption that a mass of a heated substance behaves like a group of isolated molecules which have no action on one another. It is not surprising, accordingly, to find, when heat supplied to a body is spent not only in raising the temperature of a body, but also in doing internal and external work, that the law is only approximately obeyed. Still, the approximate obedience is sufficiently striking to have caused Dulong and Petit to enounce it as a law, and as such it bears their name. It has been utilised as one means among others of ascertaining the atomic weight of different elements.

For the formula "sp. heat $\propto \frac{1}{\text{at. wt.}}$" we may substitute sp. heat $= \frac{\textit{const.}}{\text{at. wt.}}$; or sp. heat $\times$ at. wt. $=$ *const.* This constant product, which bears the name of Atomic Heat, is about 6·4; the metals, phosphorus, sulphur, may be said to form a group in which it varies from 5·86 to 6·93. Divergences from the average value are most marked in the case of elements whose atomic weight is below 30. In the case of carbon, silicon, and boron at ordinary temperatures the product is small, being 1·8 to 2·8, 3·9 to 4·2, and 2·5 to 2·7 respectively; but at higher temperatures the specific heat of these substances increases so that the product rises to about 5·5.

Hydrogen possesses a higher thermal capacity than water; its sp. heat at constant volume (p. 367) is 2·411, and that at constant pressure is 3·409 calories per gramme.

The Molecular Heat of a compound—*i.e.* the product of its sp. heat into its molecular weight—is approximately equal to the sum of the atomic heats of its component elements, reckoned with reference to each component atom separately. This rule applies with tolerable accuracy to gaseous compounds formed without condensation; solid and liquid compounds, and even gaseous compounds whose formation from their elements is accompanied by condensation,—compounds which in some sense approximate to the liquid state,—depart from it in a marked degree.

This product—the **atomic heat** of elements, the **molecular heat** of compounds—has the following physical meaning. Of

any substance whose atomic or molecular weight we know, we may take a number of grammes numerically equal to the atomic or molecular weight; for example, 35·5 grammes of chlorine, 16 grammes of marsh gas; we may call such a quantity the gramme-atom or the gramme-molecule of the substance. The Atomic Heat or the Molecular Heat of a substance is the number of calories of heat necessary to raise the temperature of a gramme-atom or of a gramme-molecule of the substance through 1° C. The atomic heats of elementary substances are approximately the same — another form of Dulong and Petit's law; so are the molecular heats of substances of similar composition.

The specific heat of a substance determines the temperature which it will assume when a definite quantity of heat is supplied to it or liberated in it.

Thus when 1 gramme of hydrogen and 8 of oxygen are exploded together, but are not allowed to expand in volume, 28,580 *ca* of heat are liberated. If we could assume the action to be instantaneous, we might assume that none of the heat is lost. The 28,580 *ca* would then be divided among 9 grammes of water-vapour whose sp. heat at constant volume is 0·37; the temperature attained would be $\frac{28580}{9\times0.37} = 8883^\circ$ C. above the temperature (136°·5 C.) proper to a volume equal to the original volume of the mixture. This case is instructive as showing the influence of dissociation; for when a temperature of 3000° C. is actually attained, further combination becomes impossible and the action is arrested, but not wholly, for it is gradually completed *pari passu* with the loss of heat by conduction or by radiation. If, however, the exploding mixture be allowed to expand, doing external work, the temperature of 3000° may never be attained. The possible temperature is also lowered by a progressive increase in the specific heat of the products as the temperature rises.

Where a substance while being heated is not allowed to expand, there is probably no internal work done; neither is there any external work done; all the heat supplied is applied in raising the temperature. The thermal capacity in this case is specially known as c the **thermal capacity at constant volume.** If, however, the substance be allowed to expand while it is being heated, an external pressure being maintained, both external and internal work are done, and in order to effect a given increase of temperature more heat-energy is required than in the former case. The **thermal capacity,** k, of any particular gas under a **constant pressure** is therefore greater than the thermal capacity, c, at constant volume; and it is found, in the case of **air,** to exceed it in the ratio of 1·4058 : 1, whatever be the external pressure, so long as that is maintained constant.

That k (in ergs per gramme) does not depend upon p, follows from the equations $km\dot{\tau} = cm\dot{\tau} + p\dot{\mathfrak{v}}$ (p. 370, note), and $p\mathfrak{v} = \mathfrak{R}m\tau$ (p. 370).

The ratio, $= k/c$, of the thermal capacity of a gas under any constant external pressure, to that at constant volume, may be found in two ways.

I. Let a gas suddenly exchange its pressure p, its density ρ, and its temperature τ°, on the Absolute (centigrade-degree) Scale, p. 364, for others $p_{,}$, $\rho_{,}$, $\tau_{,}^\circ$, the ratio of its thermal capacities being $= k/c$; then from the adiabatic equation $p/p_{,} = (\rho/\rho_{,})^{k/c}$ (see p. 395, footnote), and the equations $p = \mathfrak{R}\cdot\rho\tau$ and $p_{,} = \mathfrak{R}\cdot\rho_{,}\tau_{,}$ (see page 370), we find that, for any given kind of gas, $k/c = \{\log(p/p_{,}) \div \log(\rho/\rho_{,})\}$ (i); or, since $p/p_{,} = \rho\tau/\rho_{,}\tau_{,}$, $k/c = \log(\rho\tau/\rho_{,}\tau_{,}) \div \log(\rho/\rho_{,}) = \{\log(\tau/\tau_{,}) \div \log(\rho/\rho_{,})\} + 1$ (ii); or again, since $(p/p_{,}) = (\rho/\rho_{,})^{k/c} = (\mathfrak{R}\rho\tau/\mathfrak{R}\rho_{,}\tau_{,}\cdot\tau_{,}/\tau)^{k/c} = (p\tau_{,}/p_{,}\tau)^{k/c}$, $k/c = \{\log(p\tau_{,}/p_{,}\tau) \div \log(p/p_{,})\}$ (iii). Hence if two of the changes p to $p_{,}$, ρ to $\rho_{,}$, τ to $\tau_{,}$, can be found, the value of k/c may be calculated, on the assumption that the gas is a perfect gas. The experimental adiabatism necessary is very difficult to ensure; yet Röntgen has performed the following series of operations upon known quantities of air and determined the value $k/c = 1{\cdot}4053$.

1. Air in a reservoir at a pressure p dynes per sq. cm., exceeding the atmospheric, at density ρ, and temperature τ° Abs.

2. Open a stopcock: air rushes out of the reservoir until the pressure p falls to Π, the atmospheric pressure, per sq. cm.

3. *Immediately* close the stopcock. The air within the reservoir is at pressure Π, but has been cooled by doing external work during expansion; it soon comes to the same temperature as surrounding objects — that is, again τ° Abs.: it now has the pressure $p_{,}$ and the density $\rho_{,}$, which can be found at leisure, and the above formulæ applied.

II. From the velocity of sound. This is, in air, 33,200 cm. per sec. Newton's law of the velocity of propagation of waves is that $v = \sqrt{\mathfrak{k}/\rho}$. The coefficient of elasticity $\mathfrak{k}$ is equal numerically to the pressure p in a gas *if the temperature be constant*; $\therefore \mathfrak{k} = \sqrt{p/\rho} = \sqrt{\Pi/\rho}$ if the pressure be the atmospheric. For air $\rho = 0{\cdot}0012932$ grms. per cub. cm. at 0° C. and 76 cm. bar. pr. at Paris; $\Pi = 1{,}013663$ dynes per sq. cm. Hence, according to Newton's law, $v = \sqrt{\frac{1{,}013663}{0{\cdot}0012932}} = 27{,}997$ cm. per sec.; but in fact it is found to be 33,200 cm. There is here to all appearance a material divergence from Newton's law; but it is explained when we observe that the assumption that the temperature is constant is unfounded; that a travelling wave of sound subjects the air to adiabatic compression — adiabatic* because the heat has not time to become diffused; that the elasticity of air so compressed is greater than that of air maintained at a constant temperature; that the ratio of these two elasticities of a gas is otherwise known (p. 324) to be the same as the ratio of their specific heats at constant pressure and at constant volume; and therefore that the coefficient of elasticity in the formula should have been, not $\mathfrak{k}$ the elasticity at const. temp., but $\mathfrak{k}'$ the elasticity under adiabatic compression, $= k/c \times \mathfrak{k}$. Whence $v = \sqrt{\mathfrak{k}'/\rho} = \sqrt{k/c\cdot\mathfrak{k}/\rho}$; $33{,}200 = 27{,}997\ \sqrt{k/c}$; $k/c = 1{\cdot}40622$.

The mean value of k/c is thus $1{\cdot}4058$, for air.

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* If the heat produced had time to become diffused, or if, as might be the case in excessively slow vibrations or rare gases, the gas had time to flow round the vibrating object, so that it could not become compressed and evolve heat, the speed of propagation would tend to approximate to the value $\sqrt{\mathfrak{k}/\rho}$.

Differences in the ratio between the specific heats. — There is a wide range of difference between the ratios observed in different gases; $k/c = 1{\cdot}66$ in mercury-vapour (Kundt); $k/c = 1{\cdot}03$ in oil-of-turpentine vapour. The reason of this is the following: — Heat communicated to a gas at a constant external pressure does at least three things; (1) it is expended in doing external work; (2) it increases the molecular-translational kinetic energy; (3) it increases the rotational and other intramolecular kinetic energy. The first of these is limited; it does not depend on the nature of the gas undergoing a given expansion; it is equal to the constant external pressure p resisted, into the increase of volume, $\dot{\mathfrak{v}}$; *i.e.* it is $p\dot{\mathfrak{v}}$. If we take a gramme of hydrogen at 0° C. and 76 cm. bar. pr., we find that it occupies 11,164·5 cub. cm. On being heated through 1° C., it expands by $\frac{1}{273}$ of its volume; hence $\dot{\mathfrak{v}} = (11{,}164{\cdot}5 \div 273)$ cub. cm. The atmospheric pressure, at 76 cm. of mercury, is 1,013663 dynes per sq. cm. Accordingly, $p\dot{\mathfrak{v}} = (11{,}164.5 \div 273 \times 1{,}013663) = 41{,}454000$ ergs = very nearly 1 *ca* per ° C., per gramme of hydrogen.

The second term, the increment of molecular-translational kinetic energy, cannot exceed this more than 50 per cent; for (p. 249) $p\mathfrak{v} = \frac{2}{3}$ of the whole of this kinetic energy; and similarly, $p\dot{\mathfrak{v}} = \frac{2}{3}$ the increment of this translational energy, which will therefore be, very nearly, 1½ *ca* per ° C., per gramme of hydrogen.

If hydrogen were a substance whose molecules did not rotate or vibrate, 1½ *ca* per degree C. would be an amount of energy sufficient to impart the necessary increment of translational energy to 1 gramme of hydrogen imprisoned within any given space; and the specific heat of hydrogen at constant volume would be 1½. If the one gramme of hydrogen were allowed to expand under constant atmospheric pressure, another *ca* per degree C. would be required in order to thrust away the air, and the specific heat under any constant external pressure would be 2½. The ratio k/c would then be 2½ ÷ 1½, or 1·66. But the specific heat under a constant external pressure is, in hydrogen, more than 2·5 *ca* per gramme; it is 3·409: that at constant volume is more than 1·5, being 2·411. The excess is due to intra- and inter-molecular work; and this work, which differs very much from one substance to another, but which is approximately proportional in any one substance to the translational kinetic energy, so long at least as the specific heat remains constant, causes the ratio k/c to be not 1·66, but 3·409 ÷ 2·411 = 1·414.

An equal bulk of oxygen, 11,164·5 cub. cm., which ought to take up the same amount of heat as the same volume of hydrogen, takes up 3·480 and 2·4816 *ca* respectively; $k/c = 1{\cdot}402$. An equal bulk of benzol-vapour takes up 14·64 and 13·65 *ca* respectively, and $k/c = 1{\cdot}073$: ether-vapour, 17·75 and 16·76 *ca*, and $k/c = 1{\cdot}058$: turpentine, 34·4 and 33·4 *ca*, and $k/c = 1{\cdot}03$. In the last example, any heat which is communicated to the vapour is used to the extent of more than nineteen-twentieths in doing work upon the complex molecules themselves, or in altering their mutual relations. In sharp contrast to this we have the vapour of mercury, whose low specific heat and ratio $k/c = 1{\cdot}66$ point towards extreme simplicity of the molecule, which is, on chemical grounds, otherwise believed to be monatomic.

In a **perfect gas** — one whose molecules did not act upon one another — the thermal capacity at constant volume would be quite independent of the temperature or of the pressure.

In air the specific heat is sensibly, though not perfectly, constant at all temperatures between $-30°$ C. and $+225°$ C., and at pressures from 1 to 10 atmospheres. We shall see that this justifies us in relying upon the indications of the air thermometer. In carbonic acid it increases with the temperature, becoming about doubled at 2000° C.

In a perfect gas the pressure at constant volume and the volume under constant pressure would both vary directly as the Absolute temperature.

The general law is, that $p\mathfrak{v} \propto m\tau$, or $p\mathfrak{v} = \mathfrak{R}m\tau$, or, whatever may be the volume, $p = \mathfrak{R} \cdot \rho\tau$, where $\mathfrak{R}$ is a Constant; this constant is numerically equal* to the Difference between the two Thermal Capacities of the particular gas, at constant external pressure and at constant volume respectively, and measured in ergs per gramme.

In the case of air, $p = \mathfrak{R} \cdot \rho\tau = (k-c) \cdot \rho\tau = 0{\cdot}4058\,c\rho\tau$, where c is the Thermal Capacity at constant volume, measured in ergs per gramme. Hence $c = (p \div 0{\cdot}4058\rho\tau)$ ergs per gramme; or the Specific Heat at constant volume $= \{(p \div 0{\cdot}4058\rho\tau) \div 41{,}593000\}$ *ca* per gramme.

When $p = \Pi = 1{,}013663$ dynes per sq. cm., the density ρ of atmospheric air is $(1 \div 773{\cdot}2833)$ at 0° C. or 273°·72 Abs. Then the thermal capacity at const. vol., $c = \{1{,}013663 \div (0{\cdot}4058 \div 773{\cdot}2833 \times 273{\cdot}72)\}$ ergs, and the specific heat of air at const. vol. $= 0{\cdot}1696$ *ca*, per °C. per gramme. The observed value of the specific heat at const. vol. is 0·1684 *ca* per °C. per gramme, which corresponds to 41,731000 ergs per *ca*.

When a gas is compressed it becomes heated — that is, provided that external pressure have produced the compression, and added energy to the gas by doing work upon it.

When a gas is allowed to expand it becomes cool — that is, provided it expand against external pressure and sacrifice energy by doing external work.

The Work done upon or by the gas appears or is lost as Heat. The rise of temperature may be calculated, on the express assumption that there is no internal work done — an assumption approximately but not perfectly true (Joule) — by

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* Assume a given mass of gas (m grammes) to be maintained under a constant external pressure p; then, Heat imparted in raising its temperature τ through the small increment $\dot\tau$ is $k \cdot m\dot\tau$ ergs; and this is equal to $c \cdot m \cdot \dot\tau$ (mere heating, as if at constant volume) $+ p\dot{\mathfrak{v}}$ (external work done): $k \cdot m \cdot \dot\tau = c \cdot m\dot\tau + p\dot{\mathfrak{v}}$. But $p = \mathfrak{R} \cdot m\tau/\mathfrak{v}$; hence $k \cdot m\dot\tau = c \cdot m\dot\tau + \mathfrak{R} \cdot m\tau \cdot \dot{\mathfrak{v}}/\mathfrak{v}$; or $(k-c) \cdot \dot\tau/\tau = \mathfrak{R} \cdot \dot{\mathfrak{v}}/\mathfrak{v}$. But, at constant pressure, changes of volume are, in perfect gases, proportional to changes of absolute temperature; hence $\dot\tau/\tau = \dot{\mathfrak{v}}/\mathfrak{v}$. Therefore $(k-c) = \mathfrak{R}$, the Thermodynamic Constant.

It is assumed in the above that no internal work is done during the expansion of a gas, or that the Latent Heat of Expansion, L (p. 377), $= 0$. If the gas be not perfect, or if L otherwise not $= 0$, the heat supplied to mass m is $km\dot\tau = cm\dot\tau + \mathrm{L}m\dot{\mathfrak{v}}/\mathfrak{v} + p\dot{\mathfrak{v}}$; and $k = c + \mathrm{L}/\tau + \mathfrak{R}$.

It may be explained that as on p. 324, so in this chapter, $\dot\tau$, $\dot p$, $\dot{\mathfrak{v}}$, etc., are small actual increments, not increments per unit of time. For these symbols the student may substitute $\delta\tau$, δp, $\delta\mathfrak{v}$, etc.; and he may read them as 'the change of temperature,' etc., positive or negative as the case may be.

dividing the whole work done on the gas (measured in terms of calories of heat) by the mass and by the specific heat of the gas: or the work measured in ergs, by the mass and by the thermal capacity.

Saturated vapour behaves in this regard in a peculiar manner. If work be done upon saturated steam at any temperature below 789°·8 Abs. (516°·8 C.), the heat evolved causes the vapour to become a superheated vapour, and heat must be parted with in order to allow the steam to remain saturated. Conversely, if saturated steam below 516°·8 C. be allowed to expand, doing external work while no heat is supplied to it, it loses energy, loses latent heat, and is partly condensed; and it does not fall in temperature during expansion as much as it would do if it were a perfect gas expanding to the same extent, for the liquefaction of the vapour liberates heat. Thus an expanding saturated vapour, such as steam, liberates more energy and can do more work than an expanding gas. Above 516°·8 C. a sudden adiabatic expansion of saturated steam would, on the other hand, produce evaporation of water in contact with it; and compression would produce condensation.

The above facts are comprised in the equation — k being the thermal capacity and σ the specific heat of saturated steam (caused to rise in temperature as saturated steam, without superheating) at constant external pressure, τ the Abs. temperature —

$$k = \left(42{,}136000 - \frac{33{,}280{,}000000}{\tau}\right) \text{ ergs per gramme;}$$

$$\sigma = \left\{ \left(42{,}136000 - \frac{33{,}280{,}000000}{\tau}\right) \frac{1}{41{,}593000} \right\} \textit{ ca} \text{ per gramme.}$$

Below 789°·8 Abs. σ is negative; above that temperature it is positive.

The vapour of bisulphide of carbon acts at ordinary temperatures like that of water below 516°·8 C.; that of ether, on the other hand, is rendered cloudy by compression even at ordinary temperatures.

The specific heat of substances is not perfectly constant at all temperatures: whence the necessity of the qualification "from 0° to 1° C." This want of constancy is, among gases, most remarkable in those which are most condensible; but among solids and liquids the variations of specific heat are still more remarkable, and indicate differences in the amount of internal work associated with changes of temperature at different temperatures, this internal work being done in effecting changes in the density, the intermolecular stresses, the allotropic form, and so on.

The thermal capacity of a body may be expressed by the fraction —

$$\frac{\text{Increment of Heat supplied to unit-mass}}{\text{Increment of Temperature produced}},$$

where both the increments are very small: if an amount of heat, δH ergs, produce a change of temperature δt, the thermal capacity is $\delta H/\delta t$; and this

is one of the **Specific thermal capacities** of a body, of which six may be distinguished.

1. Specific thermal capacity per unit-increase of temperature at constant pressure; the amount of heat required to raise the temperature of unit-mass by 1° C., the pressure being constant, whatever may be its amount. This is called the Thermal Capacity at Constant Pressure, k ergs per gramme.
2. Specific thermal capacity per unit-increase of pressure effected, the temperature being constant. This has no special name.
3. Specific thermal capacity per unit-increase of pressure effected by heat, at constant volume. This has no other name.
4. Specific thermal capacity per unit-increase of volume, the pressure being constant.
5. Specific thermal capacity per unit-increase of temperature at constant volume (the Thermal Capacity at Constant Volume, c).
6. Specific thermal capacity per unit expansion per gramme, the temperature being constant. This is called the Latent Heat of Expansion, L; and it is usually specified as a certain number of calories, not of ergs, per gramme. We adhere, however, to ergs.

Under No. 5, heat $= m \cdot c$ ergs, supplied to a mass m, produces a unit-increase of temperature; a rise of temperature $\dot{\tau}$ is produced by heat $= m \cdot c\dot{\tau}$ ergs. Under No. 6, in the same way, heat $= m \cdot \mathrm{L}\dot{\mathfrak{v}}/\mathfrak{v}$ ergs produces a proportionate expansion $\dot{\mathfrak{v}}/\mathfrak{v}$. We commit no sensible error if we suppose that when the temperature and volume both vary, the amount of heat which must be supplied to a mass m of any substance is found by simple addition, and is equal, if there be any external work, $p\dot{\mathfrak{v}}$ ergs, done during expansion, to $\{m(c\dot{\tau} + \mathrm{L}\dot{\mathfrak{v}}/\mathfrak{v}) + p\dot{\mathfrak{v}}\}$ ergs.

Internal Work. — If any substance were a perfect gas, heat imparted to it would to no extent be spent in doing internal work against intermolecular or intramolecular forces.

In that case the latent heat of expansion, L, = 0.

There is, however, no such perfect gas, as we shall now show.

If our physical gases were perfect gases we would find —

1. That the amount of heat evolved on compressing a gas would be exactly equal (when measured in ergs) to the work done in compressing the gas.

If the original pressure and volume be p_0, $\mathfrak{v}_0$, and the new volume $\mathfrak{v}_{,}$, the work done is $p_0\mathfrak{v}_0 \log(\mathfrak{v}_{,}/\mathfrak{v}_{,}) = p_{,}\mathfrak{v}_0 \log(p_0/p_{,})$: a conclusion deduced, on the assumption that the heat evolved is withdrawn at once, so that there is no rise in temperature, from the two equations —

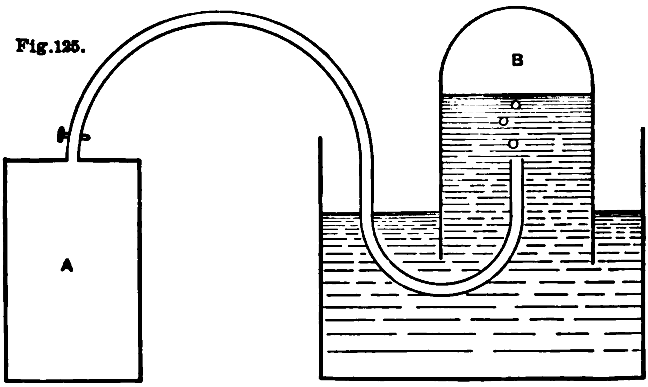
$$(1)\ \text{work done} = \int_{\mathfrak{v}_0}^{\mathfrak{v}_{,}} p\,d\mathfrak{v}\,;\ \text{and}\ (2)\ p_0\mathfrak{v}_0 = p_{,}\mathfrak{v}_{,}\ \text{(Boyle's Law)}.$$

2. That when a gas expands, doing external work, the gas loses energy; and that a perfect gas would in this way lose heat

exactly equal in amount to the external work done, and would accordingly sink in temperature.

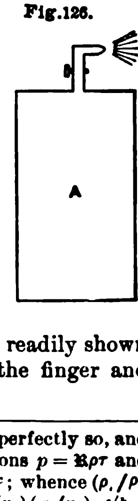
A vessel, A, of compressed air (Fig. 125) is provided with an exit tube

Fig. 125.



furnished with a stopcock: the extremity of the exit tube dips under water in a bell jar B. The stopcock is opened; air flows out; it replaces the water in the bell jar: in so doing it forces water down against the atmospheric pressure: it thus does work; the air remaining in A becomes cold (Joule).

Fig. 126.



A similar vessel of compressed air (Fig. 126); the extremity of the exit tube communicates with the open air. The stopcock is opened; air flows out; it thrusts aside the air immediately surrounding the orifice; the air within A thus does work against the atmospheric pressure: the air remaining in A becomes cold.*

3. That if a stream of a perfect gas were checked, the whole kinetic energy lost by the gas would appear as heat in it.

The heating effect of checking a stream of gas may be readily shown (Verdet) by pinching a rapidly-issuing jet of air between the finger and thumb, or by partly blocking it with the finger-tip.

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* The expansion here is nearly adiabatic; let us assume it to be perfectly so, and the air to be a perfect gas, so that $p\rho_,/p_,\rho = \tau/\tau_,$, from the equations $p = \mathfrak{R}\rho\tau$ and $p_, = \mathfrak{R}\rho_,\tau_,$. The law of adiabatic expansion is (p. 395) $p/p_, = (\rho/\rho_,)^{k/c}$; whence $(\rho_,/\rho) = (p/p_,)^{-c/k}$. Then $\tau/\tau_, = \{(p/\mathfrak{R}\rho) \div (p_,/\mathfrak{R}\rho_,)\} = p\rho_,/p_,\rho = (p/p_,)(p/p_,)^{-c/k} = (p/p_,)^{(k-c)/k}$. Let the initial and final pressures, p and $p_,$, be 6 atmos. and 1 atmo., so that $p/p_, = 6$; let the initial and final temperatures be 0° C. ($\tau = 273°$ Abs.) and the unknown $\tau_,$° Abs.; and for air, $k/c = 1{\cdot}4058$. From these data we find $273/\tau_, = 6^{\frac{0{\cdot}4058}{1{\cdot}4058}} = 6^{0.2886}$, or $\log(273/\tau_,) = 0{\cdot}2886 \log 6 = 0{\cdot}16252 = \log 1{\cdot}4533$; whence $273/\tau_, = 1{\cdot}4533$, or $\tau_, = 273 \div 1{\cdot}4533 = 187°{\cdot}8$ Abs.; this is $-86°$ C., the temperature of the residual air in the vessel A: not that of the escaping air, which reconverts some of its kinetic energy into heat by friction.

It may be noted here that if instead of expansion we have compression, the calculation is of precisely the same kind as the preceding; if, for example, the initial and final pressures be 1 atmo. and 6 atmos., so that $p/p_, = {}^{1}$, instead of 6 as above, the result is that the final temperature is 617° C.

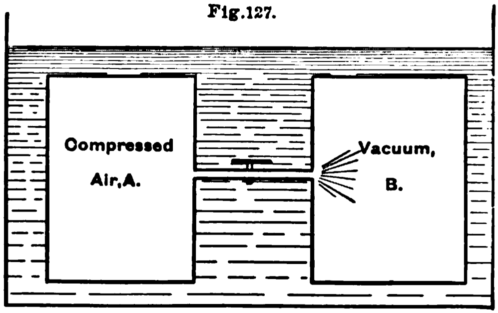
In the same way a jet of high-pressure steam, when liberated into the free air, suddenly expands and partly condenses into scalding droplets; then a little way farther on, by reason of friction against the air and of intermolecular friction, it is deprived of its momentum, and is heated so far as to become superheated or gaseous steam, in which condition it will rapidly dry (and even cool) any moist surface on which it plays; still farther on, it again becomes opaque, and is then scalding steam.

If the vessel A of Figs. 125 and 126 be connected with another in which a vacuum has been produced, the air in A loses energy and is cooled. The part of the gas which first arrives in B is heated by compression exercised by the part which arrives afterwards; the latter is also heated by having its motion checked: the temperature in B thus becomes higher than the original temperature. The result is as if, of the particles in A, those possessed of the higher translatory velocities had escaped into B (Natanson).

4. That expansion of a perfect gas would not, if no external work were done, affect its mean temperature: for, no internal and no external work being done, the amount of kinetic energy possessed by the gas would remain unaltered, and the mean temperature would be unchanged. There is no gas whose mean temperature remains unaffected under such circumstances; therefore there is no perfect gas.

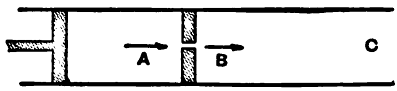
The apparatus of Fig. 127 being immersed in a large vessel of water, the stopcock is opened; the air in A is cooled, that in B is warmed; the amount of heat-energy gained by B is equal to that lost by A: the water surrounding A and B (which must be stirred) is not on the whole perceptibly cooled or warmed. This experiment, made by Joule, was believed to show that air did behave approximately as a perfect gas; for the temperature of the water, and therefore the average temperature of the whole gas in A and B, remained unchanged after opening the stopcock.

Fig.127.



The objection to this experiment is, that a rise or fall of temperature in the gas, even though by no means insignificant, would under such circumstances be imperceptible. The mass of water surrounding the vessels A and B cannot be made much less than about 7 kilogrammes: the specific heat of water is high, that of air is low; and, besides, it is desirable that the experiment be continuous, and that the effects, if there be any, be accumulated.

Fig.128.



Hence a new form of the experiment was devised by Joule and Thomson (Lord Kelvin). A tube obstructed by a diaphragm with a narrow orifice takes the place of the

vessels A and B. Air is forced from A, Fig. 128, towards B. The pressure within A is greater than that within B; the gas which passes into B ultimately becomes simply the same gas with a larger volume: it cannot become cooler by reason merely of its thrusting the exterior air at C out of the tube, for it simply acts as a buffer between the air in A and the exterior air at C, and the exterior work which it does is equal to that done upon it. If the air were a perfect gas, the temperature at B would be the same as that at A. It is found in such apparatus to vary from spot to spot on account of eddies; these must be got rid of. This is done by substituting for the diaphragm with the single opening a porous plug of graphite or of cotton wool. It is then found that air is not a perfect gas; the temperature in B is a little lower than that in A. Energy has been consumed in doing internal work—probably in separating the particles of the gas—to the extent, when the pressures in A and B both differ little from the atmospheric pressure, of about $\frac{1}{477}$ of the whole work spent upon the gas in forcing it through the plug. The proportion of the total energy spent in doing internal work varies from substance to substance, and from condition to condition. In carbonic dioxide, at a pressure varying little either in A or B from the atmospheric pressure, it amounts to about $\frac{1}{77}$; in air at a pressure in A of 19 atmospheres, it amounts to as much as $\frac{1}{63}$ of the whole.

In the case of hydrogen, curiously, there is a slight *increase* of temperature: the expanded gas has *more* kinetic energy than the unexpanded gas: energy is liberated when hydrogen expands; its particles seem to repel one another.

At equal temperatures, therefore, compressed air contains less intrinsic intermolecular *potential* energy than an equal mass of rarer air; compressed hydrogen the reverse.

High-pressure steam, treated after the fashion of Fig. 127, becomes superheated or gaseous steam.

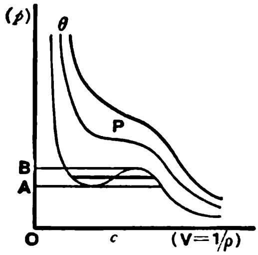
Van der Waals's Law.—Van der Waals has been able to get the departures from Boyle's law, presented by various gases, approximately dealt with by using, in the formula $p\mathfrak{v} = \mathfrak{R}\tau$ (where the quantity of gas referred to is one gramme), instead of the observed pressure p a quantity $(p + a/\mathfrak{v}^2)$, and instead of the observed volume $\mathfrak{v}$ a quantity $(\mathfrak{v} - b)$. Then the equation $p\mathfrak{v} = \mathfrak{R}\tau$ becomes $(p + a/\mathfrak{v}^2)\cdot(\mathfrak{v} - b) = \mathfrak{R}\tau$. The former of these terms is the theoretical value of the pressure, that is, the sum of the observed pressure p and a mutual attraction which varies directly as the square of the density, and is most observable when the gas is approaching liquefaction; in the latter, b is 4 times the volume (per gramme-mass) occupied by the molecules themselves.

We shall now write these quantities in the more general form $(p + a\rho^2)$ and $\{(1/\rho) - b\}$ or $(1 - \rho b)/\rho$. Then, corresponding to the equation $p = \mathfrak{R}\cdot\rho\cdot\tau$ (p. 370), we have $(p + a\rho^2) = \mathfrak{R}\cdot\rho\,(1 - b\rho)^{-1}\cdot\tau$. When multiplied out, this becomes $ab\cdot\rho^3 - a\cdot\rho^2 + (bp + \mathfrak{R}\tau)\rho - p = 0$, or $\rho^3 - \rho^2/b + \rho\,(p/a + \mathfrak{R}\tau/ab) - p/ab = 0$. This is a "cubic equation" in ρ. Now, every cubic equation has three roots, of which the whole may be real, or one may be real and two imaginary, not corresponding to any physical reality, or there may be three equal roots. Which of these results corresponds to any particular case depends on the actual coefficients in the equation. When there are three equal roots in the equation $x^3 - \alpha x^2 + \beta x - \gamma = 0$, we have $x = \alpha/3 = \sqrt{\beta/3} = \sqrt[3]{\gamma}$. Hence there are three coincident and equal solutions of the cubic equation above, when $\rho = 1/3b = \sqrt{p/3a + \mathfrak{R}\tau/3ab}$

$= \sqrt[3]{p, ab}$. By elimination among these equations we find that $p = a/27b^2$, and $\tau = 8a/27\mathfrak{R}b$: and when these are the values of p and τ, ρ has only one value, $1/3b$.

The other relations between p, τ, and ρ may be represented by a diagram, Fig. 128*a*, in which ordinates represent pressures, and abscissæ the reciprocals of the density, or the relative volumes of a given mass of gas, while the different curves correspond to different fixed values of τ. It will be seen that the curve marked θ is the first curve, going upwards from a low to a high temperature, in which it ceases to be possible to find three values of $\mathfrak{v} = 1/\rho$ for some given pressure p, with a minimum and a maximum lying between these. The point P in that curve is the point at which the three values above-mentioned ($\tau = 8a/27\mathfrak{R}b$, $p = a/27b^2$, and $\rho = 1/3b$) concur: and at that point, τ is θ, the **Critical Temperature**, p is ϖ the **Critical Pressure**, and ρ is the **Critical Density**, the reciprocal of ϕ the **Critical Volume** per unit-mass of the gas. If the temperature be above θ there is only one real value of ρ for any value of p; if it be below θ, there will, within certain limits of value of p, be three real values of ρ for each value of p.

Fig. 128a.



In the curve $\tau_{,}$, at pressures below OA, the substance is a gas: at pressures above OB, it is a liquid. At pressures between OA and OB, however, if the curve corresponding to the cubic equation were completely verified, the condition of the material would be unstable. At the volume O*c*, it would be increasing in volume with increasing pressure. What does happen is, that between A and B the curve of unstable condition is replaced by a line representing nearly uniform pressures; the substance is partly gas, partly liquid. Now increase the temperature; the range AB diminishes, and the volume is also somewhat higher. Continue raising the temperature; the range AB disappears, and at the Critical Temperature θ the volume of the compressed gas, the inaccessible volume in the unstable condition, and the volume of the liquid at that temperature and pressure, all come to coincide.

At temperatures above the Critical, the swerve in the curve diminishes, and the higher the temperature, the more nearly does the curve coincide with the rectangular hyperbola of a perfect gas. At temperatures above the Critical Temperature, there is thus no condensation, no separation of liquid from a compressed gas as the pressure rises, and the higher the temperature the more nearly does the condition of the substance approximate at all pressures to that of a perfect gas. At temperatures below θ, on the other hand, the lower the temperature, the smaller is the pressure required to condense the gas into a liquid.

Wroblewski calculated from the behaviour of hydrogen, which gave data for the constant numerical terms a, b, and $\mathfrak{R}$ in the equation, that the critical temperature and pressure of hydrogen are respectively 32°·6 Abs., or − 240°·4 C., and 13·3 atmospheres. For nitrogen the critical temperature is −146° C.; for oxygen −118°·8 C. (Olszewski).

It will be understood that what we have called the swerve in the curve will cause the appearance of anomalies. As the pressure increases, the volume first diminishes somewhat more rapidly, then less rapidly than it would

have done in a perfect gas: so that there is always some point at which the observed pressure p and the observed density ρ are such that p/ρ, $= p\mathfrak{v}$ per gramme, is a minimum. If now we compare different gases at the same temperature and pressure, these anomalies seem to be inexplicably different; but if we compare the different gases at temperatures and pressures which are equal multiples of their respective critical temperatures and pressures, it is found that they all behave similarly. The curves are then approximately identical for them all. For example, if any gas be heated to a temperature τ° Abs. $= 1{\cdot}4$ times its critical temperature, p/ρ is always a minimum when the pressure $p = 3\tilde{\omega}$; but if the temperature rise to $\tau_{,}{}^{\circ}$ Abs. $= 3\theta$, p/ρ is a minimum when the pressure $= \tilde{\omega}$: and if it rise to, say, 5θ or 6θ, p/ρ is a minimum only when the pressure is exceedingly small. In hydrogen, therefore, at 0° C. $= 273^{\circ}$ Abs., $=$ about $8{\cdot}4 \times \theta$, the gas is more compressible at all high pressures than Boyle's Law would indicate; but at -183° C., the temperature of boiling liquid oxygen, which is about 3θ on the Absolute scale, it has been observed by Wroblewski that the compressibility of hydrogen begins to diminish at 14 atmospheres' pressure, and the gas behaves as ordinary gases do at ordinary temperatures. In carbonic acid, on the other hand, whose critical temperature θ is $30^{\circ}{\cdot}92$ C. $= 304^{\circ}$ Abs., and whose critical pressure $\tilde{\omega} = 73$ atmospheres, the minimum compressibility would be found at about 219 atmospheres and 153° C. ($= 426^{\circ}$ Abs. $= 1{\cdot}4 \times 304$), or again, at 73 atmospheres and 639° C. ($= 912^{\circ}$ Abs. $= 3 \times 304^{\circ}$ Abs.); beyond which limits carbonic acid would behave like hydrogen at ordinary temperatures and pressures.

In gases the amount of heat which disappears during expansion in doing internal work is generally small in proportion to the external work done against the atmospheric pressure: in solids and liquids the internal work done is relatively much greater.

When a substance is heated and rises in temperature without being allowed to expand, so much heat is absorbed during a given rise of temperature; when expansion is permitted, an additional supply of heat is required. The **Latent Heat of Expansion**, L, of a substance may thus be found by difference.

When temperature varies, the volume being constant, the heat, H, supplied to a mass m is equal to $m \cdot c\dot{\tau}$ ergs; when the pressure is kept constant and expansion is allowed, the heat supplied, $H_{,}$, is equal to $m \cdot (c\dot{\tau} + L\dot{\mathfrak{v}}/\mathfrak{v}) + p\dot{\mathfrak{v}}$; whence $L = (H_{,} - H - p\dot{\mathfrak{v}})\mathfrak{v}/m\dot{\mathfrak{v}}$. $H_{,}$ is equal to $m \cdot k\dot{\tau}$, where k is the thermal capacity at constant pressure, whence L, in ergs, $= \{(k - c)\,\dot{\tau}\mathfrak{v}/\dot{\mathfrak{v}} - p/\rho\}$, per gramme. In a perfect gas $L = 0$.

Latent heat of expansion is difficult to measure. To ascertain that of water between 0° C. and 100° C., for instance, it would be necessary to compare the amounts of heat required to heat a certain mass of water from 0° to 100° C. when it is free to expand and when it is prevented from expanding: but the latter investigation would require the application of a pressure of 8772 atmospheres.

In the same way wrought-iron heated through 15½° Fahr. exercises a pressure of 1 ton per square inch.

The Latent Heat of Expansion of a substance is, as a numerical coefficient, the amount of heat (usually reckoned in calories) required to effect unit-expansion in a gramme of that substance — that is, to double its volume — and which disappears in doing that work, without affecting the temperature.

The work of expansion is, however, associated with that of raising the temperature; only in idea can we form an abstract conception of the amount of heat required to effect a certain expansion while the temperature is supposed to remain unchanged. Temperature and volume vary simultaneously, and the physical constant known as the **coefficient of expansion** states numerically the relation between these associated effects of heat.

A substance whose volume is $\mathfrak{v}_0$ at a temperature τ_0 assumes a volume $\mathfrak{v}_,$ at the temperature $\tau_,$; the change of temperature is $\tau_, - \tau_0$; the proportionate change of volume is $(\mathfrak{v}_, - \mathfrak{v}_0)/\mathfrak{v}_0$; the quotient $(\mathfrak{v}_, - \mathfrak{v}_0)/\mathfrak{v}_0(\tau_, - \tau_0)$ is the coefficient of expansion. If $\tau_, - \tau_0 = 1°$, the coefficient of expansion is $(\mathfrak{v}_, - \mathfrak{v}_0)/\mathfrak{v}_0$.

The coefficient of expansion of any substance is the ratio between the *increase of volume* which it undergoes when its temperature is raised by 1° C., and its *original* volume.

If a cube of volume $\mathfrak{v}_0$ assume volume $\mathfrak{v}_,$, its side $\sqrt[3]{\mathfrak{v}_0}$ becomes $\sqrt[3]{\mathfrak{v}_,}$; its coefficient of linear expansion is therefore $\dfrac{\sqrt[3]{\mathfrak{v}_,} - \sqrt[3]{\mathfrak{v}_0}}{\sqrt[3]{\mathfrak{v}_0}}$, or approximately $\frac{1}{3}(\mathfrak{v}_, - \mathfrak{v}_0)/\mathfrak{v}_0$. Thus, if a body measuring a cubic foot, on being heated 1°, assume a volume of 1·0003 cub. ft., the side of the cube (1 foot) has become nearly 1·0001 linear foot.

Since we have in general to deal with expansions proportionately very small, we may say that the coefficient of linear expansion — the proportionate increase in length, breadth, or thickness per degree centigrade — is equal to one-third the coefficient of cubical expansion.

If L be the coefficient of linear expansion of a body whose length at τ_0 is l_0, the length of the body at the temperature $\tau_,$ is $l_, = l_0 + (L \cdot l_0 \cdot (\tau_, - \tau_0))$ or $l_0(1 + L(\tau_, - \tau_0))$. In this equation there are five terms, any four of which being known, the fifth can be found.

In some cases — many crystalline bodies — the coefficients of linear dilatation are not equal in all directions. Crystals have three axes, in the directions of which the coefficients of expansion ($L_,$, $L_{,,}$, $L_{,,,}$) are not always equal to one another; thus the

angles of crystals are often modified by changes of temperature. Substances belonging to the regular system have the coefficients equal in the three axial directions, and they preserve similarity of figure when heated; dimetric crystals have two axial coefficients equal, the third different; trimetric crystals have all three coefficients unequal. In general the cubical coefficient $= (L_{,} + L_{,,} + L_{,,,})$.

Take plates of gypsum, cut parallel to the prismatic axis: cement them together so that the direction of the axis of one plate forms a right angle with that of the other. Heat until the cement is melted; allow to cool. The unequal contraction in cooling will warp the whole (Fresnel). In the case of this substance a contraction in one direction is associated with expansion in two others.

Indiarubber and iodide of lead, iodide of lead and silver (PbI_2, AgI), iodide of silver up to 156°·5 C., and garnets, as well as water between 0° and 3°·9 C., contract when heated: their coefficient is negative. In some substances (zinc and iron) the coefficient of expansion slowly alters with lapse of time.

When a hollow body such as a flask or thermometer-bulb is heated, it expands almost exactly as if it were solid: a glass tube expands as if it were a glass rod. It follows that when a hollow body is heated, its internal cavity increases in volume in the same proportion as it would have done if it had been occupied by a solid the same as that which surrounds it.

Examples of Expansion by Heat.—Bodies which, when cold, exactly fill certain apertures, when they are warmed will not enter these. Railway rails are not laid in exact contact; allowance must be made for their summer expansion and winter contraction. In designing lattice-girders for bridges, the same necessity must be taken into account. Railway-distance signals are controlled by rods, which differ considerably in length at night and by day; provision must be made for tightening them up or the reverse. If the neck of a stoppered bottle clasp the stopper too tightly, it may be loosened by causing the neck to expand while the stopper does not do so; this may be effected by winding a string round the neck and pulling it backwards and forwards so as to produce heat by friction; the neck is heated before the stopper itself is affected. Glass suddenly heated expands superficially while the inside is still cool: under the stress set up the glass may break; hence the thinner a flask, the less risk there is of its cracking when it is heated. A cart-wheel tire is fitted on when it is hot; when it cools down it

contracts and holds the rim, spokes, and hub firmly together: if it be originally too small it may break itself by its own contractile tension. The lead on a roof expands by day and contracts at night; gravity aids the one and checks the other tendency; the lead creeps down. The same theory has been applied to glacier movement.

Applications of Expansion.—The Compensation-pendulum is a pendulum constant in length, whatever be the temperature. A simple bar of metal would, by its variations in length, produce oscillations irregularly unequal, the clock going slow in summer, fast in winter. In order to correct this, the bob of the pendulum is suspended from a framework of bars of iron and brass, so arranged that expansion of the bars of iron tends to depress the bob: that of the bars of brass tends to raise it; by proper adjustment of the lengths of the bars these effects compensate one another.

The bob itself is sometimes made of a tube containing quicksilver: the expansion of the suspending bar tends to lower the centre of gravity of the pendulum: that of the mercury tends to raise it; a proper adjustment of the quantity of mercury in the bob produces sensibly accurate compensation.

Sometimes the rod of a pendulum bears a transverse bar, which is loaded at each end with heavy masses. This transverse bar consists of strips of different metals; in weather warmer than the average the lower strips expand most, distort the bar, raise the heavy masses, and thus raise the centre of gravity of the whole pendulum: in colder weather the reverse effect is obtained, for the lower strips contract most. These effects may be adjusted so as to neutralise the effect of the lengthening or shortening of the pendulum itself.

Measurement of Coefficients of Expansion.—In solids the coefficient of linear expansion is found by direct observation. A bar is heated to a known temperature; its original length and temperature are known. The elongation of the bar may be measured by a traversing bar with micrometer, or by the method of Fig. 5, or by the expansion of the bar in a tube pushing out a piece of porcelain, which can move outwards but cannot return. The first-mentioned method is by far the least liable to error, especially when the distance between two distinctive points on the bar is observed at two given temperatures.

$$\frac{l_{,} - l_0}{l_0} \cdot \frac{1}{\tau_{,} - \tau_0} \text{ is the coefficient of linear dilatation.}$$

The coefficient of cubical expansion may be found by multiplying the coefficient of linear expansion by 3; or, better, by finding the different specific densities of the solid at different temperatures.

The mass ($=$ weight$/g$) remaining the same, ρ and $\rho_{,}$ being the densities, the volumes $\mathfrak{v}_0$ ($= m/\rho$) and $\mathfrak{v}_{,}$ ($= m/\rho_{,}$) are easily found; and $(\tau_{,} - \tau_0)$ being the difference of temperatures, $\left(\frac{\mathfrak{v}_{,} - \mathfrak{v}_0}{\mathfrak{v}_0} \cdot \frac{1}{\tau_{,} - \tau_0}\right)$, the coefficient of cubical expansion, can be computed.

If a solid be heated in a flask with a narrow orifice and completely filled with mercury, the mercury expands, the flask expands, and so does the solid immersed in it. The absolute expansion of the mercury is previously known, that of the glass vessel must be known, and the amount of mercury which would fall out of the flask if the flask were completely filled with mercury and heated to the same degree is already known; when the solid is immersed in the mercury, a different quantity of mercury escapes from the flask when heated; the difference is due to the difference of dilatation between mercury and the immersed solid: the coefficient of expansion of the immersed solid can thus be calculated.

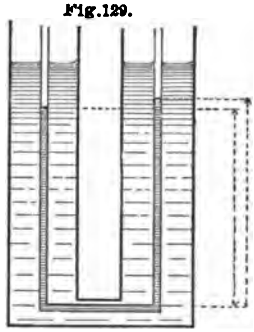
In liquids the expansion may be found by observation of the apparent increase of bulk undergone by a liquid contained in a flask. The width of the neck may be ascertained by the addition of known quantities of mercury: an apparent rise of the liquid in the neck may be interpreted as corresponding to so many cubic cm. apparent increase of bulk. But it is important to bear in mind that the cavity of the flask also expands, and that the real expansion of the liquid is the sum of the expansion of the cavity of the flask, and the apparent expansion of the liquid in the neck. If the liquid have the same cubical coefficient as the glass, there will be neither a rise nor a fall in the neck; if it have a less rate of expansion than glass it will sink in the neck, and will then apparently contract; only when it has a greater coefficient of expansion than the glass will it rise in the neck, and thus under such circumstances manifestly appear to expand.

When a thin glass flask filled with water is suddenly heated it expands before the water contained in it has had time to become heated, and the liquid in the first place appears to shrink into the flask. Then the liquid becomes heated and rises in the neck of the flask.

The expansion of a liquid may also be found by observing

its *density* at different temperatures. This may be done by means of separate observations. It may also be done by the observation of the simultaneous heights of a hotter and a colder column of the same liquid, which balance one another in a U-tube. The heights are reciprocally proportional to the densities, and thus it is easy to find the coefficient of expansion per degree centigrade. Fig. 129 shows that each limb of the U-tube is maintained at a constant temperature by surrounding baths (of water, mercury, oil, etc.) whose temperatures are known. The heights of the columns may be measured by means of a cathetometer. The absolute expansion of mercury is by this method found to be per degree centigrade (between $-36°$ C. and $100°$ C.) almost exactly $\frac{1}{5555}$ of its total bulk at $0°$ C.; above $100°$ C. it increases rapidly with the temperature. The total amount of expansion is thus not exactly proportional to the rise of temperature.

Fig. 129.



In *gases* the coefficient of expansion is nearly uniform, about $\frac{1}{273}$ for every degree centigrade. Not quite uniform; for all gases are not necessarily in the same physical condition merely because they are at the same temperature, for some may be near, others far from, their point of condensation; and the volume of gases is not exactly proportional to their absolute temperature.

The coefficient of expansion in gases may be determined by *direct* observation, the volume being allowed to vary, while the pressure is maintained constant during a given change of temperature; or *inferentially*, by observation of the increase in pressure exercised by a gas when its volume is kept constant during a given change of temperature, coupled with the assumption that *Boyle's law* is perfectly obeyed, and that the volume and the pressure bear an exact inverse ratio to one another. The latter method, as we shall see, is more valuable in thermometry than in the determination of the actual coefficient of expansion of a gas.

If we assume Boyle's law and Charles's law to be both true, we have the equation $p\mathfrak{v}/\tau = const.$ If the same quantity of gas change in pressure, volume, or temperature, again $p_{,}\mathfrak{v}_{,}/\tau_{,} =$ the same *const.* Hence $p\mathfrak{v}/\tau = p_{,}\mathfrak{v}_{,}/\tau_{,}$. This enables us to solve, to a first approximation, such problems as the following:—

Fifteen litres of air at 0° C. and 761 mm. bar. pr. are heated to 10° C. while the barometer sinks to 759 mm.; what volume does the air assume?

$$\frac{p\mathfrak{v}}{\tau} = \frac{p_{\prime}\mathfrak{v}_{\prime}}{\tau_{\prime}};\ \frac{761 \times 15}{273} = \frac{759 \times \mathfrak{v}_{\prime}}{283}.$$

$$\text{Whence } \mathfrak{v}_{\prime} = \left(\frac{761}{759} \times \frac{283}{273} \times 15\right) \text{ litres.}$$

Again, 15 litres of air at 0° C. (273° Abs.) and 762 mm. Hg pressure are, when they are heated to an unknown temperature and exposed to a pressure of 1000 mm. Hg, doubled in volume: what is the unknown temperature?

$$\frac{p\mathfrak{v}}{\tau} = \frac{p_{\prime}\mathfrak{v}_{\prime}}{\tau_{\prime}};\ \frac{762 \times 15}{273} = \frac{1000 \times 30}{\tau_{\prime}};$$

$$\text{Whence } \tau_{\prime} = \frac{1000}{762} \cdot \frac{30}{15} \cdot 273 = 716^{\circ}{\cdot}5 \text{ Abs.} = 443^{\circ}{\cdot}5 \text{ C.}$$

We may combine with these equations the two following propositions:—

1. The specific density of a gas is numerically equal to half its molecular weight.

2. One gramme of hydrogen measures 11·1645 litres at 0° C. and 760 mm. bar. pr.

Problem.

Fourteen litres of carbonic acid are measured at 10° C. and 759 mm. pressure: what is their mass?

First reduce the 14 litres to the volume which they would occupy at 0° C. and 760 mm. bar. pr.—*i.e.*,

$$\left(14 \times \frac{273}{283} \times \frac{759}{760}\right) \text{ litres.}$$

Each litre of carbonic acid at 0° C. and 760 mm. weighs $\frac{1}{11{\cdot}1645} \times \frac{44}{2}$ grammes. The whole weighs

$$\left(14 \times \frac{273}{283} \times \frac{759}{760} \times \frac{1}{11{\cdot}1645} \times \frac{44}{2}\right) \text{ grammes.}$$

Problem.

What bulk is occupied by 20 grammes of ammonia gas at 15° C. and 740 mm. bar. pr.?

One gramme of hydrogen occupies at 0° C. and 760 mm. a bulk of 11·1645 litres; at 15° C. and 740 mm. it would have a volume of $(11{\cdot}1645 \times \frac{288}{273} \times \frac{760}{740})$ litres; but ammonia gas has a sp. density $= \frac{17}{2}$; hence 20 grammes of ammonia occupy a bulk

$$\left(20 \times \frac{2}{17} \times 11{\cdot}1645 \times \frac{288}{273} \times \frac{760}{740}\right) \text{ litres.}$$

It may be left to the student as an exercise, to find what corrections should be applied, to reduce the apparent weight of a substance weighed in air at a given temperature to the real weight at a standard temperature, say 0° C., the coefficients of expansion of air, of the counterpoising weights, and of the substance weighed, being supposed to be known.

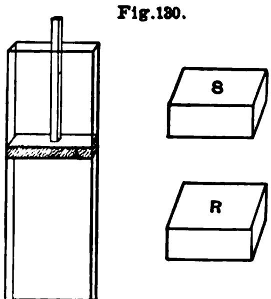
Fusion. — Heat sometimes operates liquefaction of solid bodies. The temperatures at which fusion is effected differ widely: the fusing point of solid alcohol (–130°·5 C.), that of mercury (–40° C.), and that of platinum (about 1775° C.) which can only be fused by the oxyhydrogen blowpipe or the electric arc, may be taken as examples.

Fusion upon heating, and solidification upon cooling, occur normally at the same temperature; Melting points and Freezing points are the same, except in cases of overcooling, in which the temperature may fall below the freezing point, and in which solidification may be made to start by dropping in a piece of the solid. During fusion or during freezing or solidification, when this has once begun, the temperature remains the same until the process is complete. Energy is being absorbed or liberated in the form of Heat. See Latent Heat, p. 361.

In general there is expansion during fusion; in such event there may be a small amount of work done against external pressure. If the external pressure be increased, the amount of heat-energy that must be supplied, in order to effect this external work in addition to the internal work of fusion, is proportionately increased. The temperature of fusion is thus in most cases raised by increase of pressure. In the cases of water, antimony, cast-iron, and many rocks, the freezing point is lowered by pressure, because these substances expand when they freeze. Tables of melting points therefore denote the melting points of substances at the atmospheric pressure.

We may here state the reasoning by which it was predicted* that an increase of pressure would be found to lower the melting point of ice; though some of the steps will not be understood until after we have considered Carnot's cycle of operations and his "perfect engine."

Fig. 130.



A cylinder with a square base, 1 cm. square, contains one gramme of water — *i.e.* 1 cub. cm. S is a *source* of heat at 0° C. (which must be situated within a sufficient space entirely devoid of air). R is a refrigerator situated within a region where the atmospheric or other pressure is equal to p dynes per sq. cm.; it is maintained at a constant temperature $-t°$ C., very slightly below 0° C.

1. The cylinder is kept upon the source S until the water assumes the temperature 0° C. We now have 1 cub. cm. water at 0° C.

* By Prof. James Thomson; experimentally confirmed by his brother, Prof. Sir William Thomson, now Lord Kelvin.

2. Put the cylinder on the refrigerator R; keep it there until the water is wholly frozen to ice at 0° C. We now have 1·0908 cub. cm. ice at 0° C. (the sp. density of ice being ·91674, Bunsen).

Work has been done during expansion; the piston has been thrust upwards through ·0908 cm. against an external pressure p dynes per sq. cm.; the work done by the expanding substance is 0·0908 p ergs.

Put the cylinder again on the source: the temperature of the source is supposed to be by an infinitely small amount higher than that of the ice. In course of time the ice melts; now we again have 1 cub. cm. of water at 0° C. (while no work has been done upon the melting ice by any exterior pressure). The melting ice has had heat imparted to it equal to the latent heat of fusion of 1 cub. cm. of water—that is, 80·025 ca = 3,328,480000 ergs. This amount of heat has been absorbed from the source at 0° C.; heat has been lost to the refrigerator at $-t$° C. The piston returns to its normal position, as we have seen, and the whole contrivance, perfectly imaginary, will act as a "perfect engine," with ice or water as its working substance, provided that t has a certain value to be deduced from the equation—

$$\frac{\text{Work done}}{\text{Heat absorbed}} = \frac{\text{difference of temp. between source and refrigerator}}{\text{absol. temperature of source}} = \frac{t}{273}.$$

$$\frac{0{\cdot}0908\,p}{3{,}328{,}480000} = \frac{t}{273}.$$

$$t = \frac{273 \times 0{\cdot}0908}{3{,}328{,}480000}\,p.$$

When the external pressure p changes by Π = 1,013663 dynes per sq. cm. —that is, when it changes by an amount equal to one atmosphere—t changes by ·0074° C. This means that such an engine is reversible, and its operation is theoretically perfect, when the freezing operation is conducted at a temperature lower than 0° C. by an amount equal to ·0074° C. for every additional atmosphere-pressure suffered by the freezing water. If the freezing could occur at a higher temperature than this, there would be production of work by the expanding ice, accompanied by a withdrawal of heat from the source insufficient to account for the work, and the perpetual motion would become possible.

When a piece of ice is placed in contact with another, both being at 0° C., a very slight pressure will, by lowering the melting point, cause a certain quantity of ice at the point of contact to melt. When the pressure is relieved, the mass solidifies and becomes continuous ice.

Ice is not without plasticity at temperatures not far from 0° C., and can slowly flow down a slope of 1 in 4 under a pressure equal to the weight of 300 feet of ice-cliff (Moseley and Browne); but at temperatures between 0° C. and about $-\frac{1}{2}$° C. it can be driven through narrow passages by the above process of Regelation, for when crushed the fragments are relieved of pressure and reunite, again to be crushed and forced onwards. To the small plasticity of ice and to the process of crushing or regelation, as well as to creeping (p. 380), is to be mainly ascribed the flow of glaciers.

Sometimes the fusion-point of a mixture is below that of its ingredients. A mixture of common salt with about $2\frac{1}{2}$ parts of crushed ice melts at about -18° C. or 0° Fahr.: above this tem-

perature it is liquid; and when ice and salt are mixed, the result is very cold liquid brine.

When the pavements in snowy weather are cleared by means of salt, the brine thus formed being at a temperature of 0° Fahr., or at "thirty-two degrees of frost," penetrates the shoe-leather and chills the feet of pedestrians, while it refuses to dry, the salt being hygroscopic — that is, having a great affinity for water.

This example is a particular case of a general proposition, that a solution of a solid in a liquid has a lower freezing point than the pure liquid itself.

The extent to which the freezing point of a liquid is lowered by dissolving a substance in it varies directly as the number of molecules dissolved, inversely as the number of molecules in the solvent liquid, and directly as a constant which depends upon the nature of the solvent. For example, if n molecules be dissolved in 10000 molecules of water, the freezing point will fall by $0{\cdot}0063n$°C. This simple relation is most nearly adhered to in the most dilute solutions; certain abnormalities are observed however, which are interpreted as showing either (1) that there is coalescence of molecules of the substance dissolved, which fall asunder on increasing the dilution, particularly with water, which is somehow unfavourable to molecular coalescence or polymerisation; or (2) that there is a break-up or dissociation of the molecules. This latter particularly occurs in aqueous solutions of electrolytes (p. 590) or acids, bases, and salts, in which the ions (p. 281) become separated in the solution; and each of the ions produces its own independent effect upon the freezing point of the solution.

Sublimation. — When a solid on being heated becomes a vapour without passing through the liquid state it is said to be sublimed. Examples of this are furnished by arsenic trioxide and pentasulphide, metallic arsenic, and some metallic chlorides, as well as by many organic substances.

Sometimes the word sublimation simply means the distillation of a solid, as in the case of sulphur, bichloride of mercury, or benzoic acid, all of which melt before vaporising.

Sulphide of zinc and sulphide of cadmium are not volatile when pure; but when mixed with traces of metallic zinc or cadmium respectively, they are very volatile.

Boiling or ebullition is a rapid process of reduction of a liquid to vapour. Evaporation is thus distinguished from ebullition; in evaporation particles possessing more than the average kinetic energy fly from the surface and mingle with the particles of gas or vapour already existing in the neighbourhood of the surface of the liquid, and drive or repel only a certain proportion of them away from the surface: in boiling, the particles which fly from the surface bombard the surrounding particles so hotly as to drive them all from the neighbourhood

of the surface of the boiling liquid, and to take their place. Thus the vapour of a boiling liquid has to exert a pressure which is just a little greater than the atmospheric, or, in general, the exterior pressure, whatever that may happen to be; the vapour of an evaporating liquid exerts a pressure which is only a fractional part of the atmospheric or exterior pressure.

This pressure, just a little greater than that of the atmosphere, may be made up in different ways. Volatile-oil vapour, led into water, or, equally, water-vapour led into oil, will cause boiling of the water or the oil at temperatures below the boiling point of the oil; each liquid contributes its own quota of pressure independently. Two liquids partially miscible, in two layers, distil at a constant temperature until one of the layers has disappeared, the temperature being generally higher the greater the mutual solubility and therefore the greater the diminution of the joint vapour-pressure by reason of the mutual attraction of the two liquids: and the vapour-pressure, at any given temperature, of a saturated solution of either liquid in the other is the same. A mixture of miscible liquids presents two cases: ethylic alcohol and water on repeated distillation ultimately give a distillate of nearly pure alcohol; a mixture of propylic alcohol and water gives a distillate containing 75 per cent of propylic alcohol which cannot be separated by farther distillation; while if the liquid distilled be richer than this in propyl-alcohol, water and propyl-alcohol pass over, and propyl-alcohol remains in the retort. In such cases, there are differences in the action of the components of the mixture on one another, and of the attraction of the boiling liquid for the components of the mixed vapour; and the boiling point depends upon the resultant vapour-pressure.

The boiling point of a solution may differ considerably from that of the solvent: thus a saturated solution of caustic soda in water boils at 215°·5 C., and one of calcium chloride at 179°·5 C.

Vapour-Pressure of a Solution.—The pressure of the vapour of a solution is less than that of the solvent alone at the same temperature, and the boiling point is correspondingly higher. Whatever be the temperature and the concentration, and whatever be the nature of the solvent and the substance dissolved, the Fall of Vapour-Pressure is proportional to the ratio of the number of molecules of the substance dissolved to the total number of molecules in the solution (Raoult). Apparent departures from this law, in the direction of an excess in the fall of vapour-pressure, sometimes manifest themselves, particularly in solutions of acids, bases, and salts: but these departures are interpreted as showing that the number of molecules in the substance dissolved is altered by Dissociation taking place when solution occurs.

Besides, the process of evaporation is restricted to the exterior free surface: that of boiling occurs both at this surface and at the internal surface of bubbles in the interior of the liquid.

A liquid may be heated to a temperature above its boiling point, and if there be no bubbles formed, no point at which the action may preferably start, the whole liquid may become over-

stressed, like a Rupert's drop, and when it does give way and form vapour, it may do so explosively. This kind of explosive boiling may be observed when water void of air is heated, or when drops of water are suspended in a mixture of light and heavy oils of the same specific density as water and then heated, or when water is heated in a glass vessel, especially if it have been carefully cleaned with sulphuric acid. In the last case the surface of the vessel is very uniform, and there is no sharp point or roughness at which a bubble may commence: thus the temperature rises above the boiling point until it is brought down by a sudden outburst of vapour, and bumping ensues. There is less of this in a smooth-metal vessel than in a glass one; still less in a rough-metal vessel; still less where jagged pieces of platinum or stone have been immersed in the liquid to be boiled. The process of boiling depends to a great degree for its regularity on the presence of air-bubbles: we may sometimes see that water, when long boiled, ceases to evolve bubbles and evaporates only at the surface, with an occasional outburst of steam.

A bubble of air or vapour, produced in the interior of a hot liquid, is increased in size by molecules escaping into it from the surrounding liquid; if the temperature of these molecules, their energy, their velocity, their pressure, be such that they can expand the bubble against the surrounding pressure and against the surface-tension within the bubble itself, the bubble enlarges and rises. If we artificially produce bubbles in the interior of a heated liquid, as when we electrolyse hot water, the liquid boils very rapidly at the electrodes, where gaseous oxygen and hydrogen are being given off.

The uniformity of the boiling point is interfered with by variations in the size of the bubbles, and therefore in the inward tension of their liquid boundaries, which resist expansion.

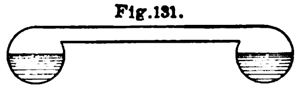
The boiling point at different pressures.—The greater the external pressure to be overcome, the greater must be the energy, and therefore the greater the temperature, of the rising vapour. The temperature of ebullition and the external pressure are not directly proportional to one another, but are found experimentally, and recorded in tables such as those great tables of Regnault's, to be found in his *Relation des Expériences*.

At mountain heights the atmospheric pressure is less and the boiling point is lower; thus at Quito, at a height of 9540 feet, water boils at $90^{\circ}\cdot1$ C.

If a flask containing water, boiling at 100° C., be corked and set aside until it has cooled, say, to 90° C., and if the upper part of the flask, the part containing the vapour of water, be suddenly cooled by cold water dashed upon it, the vapour in it will be partly condensed, and a partial vacuum will be formed: the water will find itself at a temperature of 90° C. under a pressure of less than 525·45 mm. of mercury, and it will again begin to boil: the water is thus seemingly induced to boil by the application of cold to the flask containing it.

If a cryophorus tube, Fig. 131, of which both bulbs are half filled with water, have one bulb immersed in a freezing mixture, the vapour in the cold bulb is condensed; the vapour in the tube is pushed into the cold bulb by the uncompensated pressure of particles rising from the liquid in the warmer bulb; this process is continuous; work is continuously done in maintaining the flow of vapour, which is as continuously condensed; the liquid in the warmer bulb continuously evolves vapour, and does so so rapidly, the pressure being small, as to boil; it continuously does work, but receives no energy; it cools and ultimately freezes, even while evaporating.

Fig. 131.



Boiling and evaporation may thus involve not only the giving of momentum to particles of the liquid, but also external work done against resistances; and during **evaporation** there may be **cooling** due not only (1) to the latent heat of evaporation absorbed in producing change of state, but also (2) to the external work which is done by the evaporating body — work which generally takes the form of thrusting aside the external air.

Examples of cooling due to evaporation are:—The cooling of the skin by perspiration or by a draught of air, even though the air be warmer than the skin; a dog cooling himself by panting with his tongue exposed; a porous water-cooler or *alcarraza*, the evaporation at the surface of which cools the contained water; the practice in some hot countries of cooling a room by throwing water over the floor; the cooling of air supplied for the ventilation of coalpits by injecting water-spray into it; the cooling of the compressed air of refrigerators by the same means; the cooling undergone by a liquid which is being rapidly evaporated, as, for example, the rapid cooling of sulphurous anhydride or of ammonia, which is effected in the course of the process of artificial ice-making by the rapid evaporation of the liquefied gases under a powerful air-pump; the cooling of a jet of liquefied carbonic acid when allowed to escape into the air, so that the substance is in part solidified.

Ethylene (olefiant gas) may be liquefied by cold and pressure; on being rapidly evaporated under the air-pump it becomes so cold that air, greatly compressed, can be liquefied by it. This liquefied air, when allowed to evaporate freely, produces temperatures apparently below $-210°$ C. (Olszewski).

The latent heat of evaporation of steam is $\lambda = (33011{,}504000 - 33{,}200000\tau)$ ergs per gramme, where the temperature of ebullition is τ° Abs. At $994^{\circ}\cdot32$ Abs. or $720^{\circ}\cdot6$ C., $\lambda = 0$, and this temperature is for steam the Critical Temperature, beyond which there is no change of state when liquid water becomes water-vapour.

Saturation-pressure. — In the case of every vapour we find that for each particular temperature there is a maximum density; if we compress the vapour beyond this density, a portion of it will be liquefied. If we allow it to expand, then — provided that the temperature be kept constant, and that the vapour be kept in contact with its own liquid — a portion of the liquid will be evaporated; thus the density is maintained constant and the vapour is kept saturated. Each volatile liquid has its own saturation-pressure for each temperature, this being the pressure necessary to bring the vapour to its maximum density.

A vapour which is not saturated may by compression, exerted until the pressure of the vapour is equal to the saturation-pressure, be made saturated, and by further pressure will be caused partly to condense.

The saturation-pressure of any vapour at any temperature is the same as the pressure at which the corresponding liquid boils at that temperature.

Even in contact with ice, water-vapour has a saturation-pressure, and evaporation will go on until this pressure is attained. A strong wind blowing over a snowfield may remove much of the snow by true evaporation without liquefaction.

Saturated steam in contact with ice at t° C. has a pressure $p = \{107\cdot2 + (6255 \times 1\cdot080^{t})\}$ dynes per sq. cm. (Regnault).

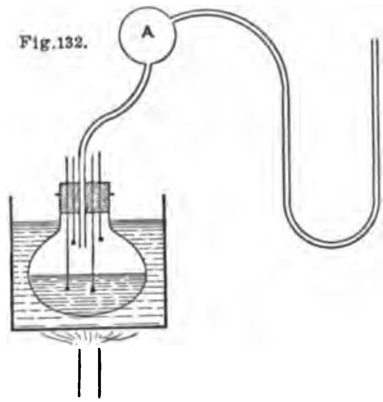
As a general rule each component of a mixture of gases exercises its own pressure, and is not affected by the others which accompany it. Yet this rule is not absolute; for if we heat in a flask a certain quantity of air alone, we find that it exerts a certain pressure; a certain quantity of water-vapour introduced alone into a vacuum would exert a certain pressure; but when both the water-vapour and the air are introduced into the same vessel, the joint pressure falls somewhat short of the sum of the several pressures, and thus it is shown that there is an attractive action between water and air.

Vapours at variable pressures and temperatures generally obey Boyle's law with tolerable regularity until the pressure comes up to about $\frac{8}{10}$ of the saturation-pressure, and that whether they be alone or mingled with air or with other vapours.

Measurement of Vapour Density at different temperatures.

a. By measurement of the pressure exercised by the vapour of liquid at a series of known temperatures.

This is effected by the arrangement sketched in Fig. 132. The mean temperature of boiling is indicated by four thermometers, two in the liquid, two in the vapour: the vapour is condensed in A and returned to the flask: the pressure is measured by a manometer.



The use of this method depends on a tacit assumption that Boyle's law is obeyed throughout all ranges of temperature; but this method is not applicable except at low temperatures and low pressures; for at high pressures the vapour assumes abnormally small volumes as it approaches its saturation-pressure.

β. By measurement of the volume occupied by a known weight of fluid, or by measurement of the weight of vapour which can occupy a known volume.

The first of these methods is that of Gay Lussac. A tube filled with mercury is inverted like a Torricellian barometer in a vessel of mercury, and has a Torricellian vacuum at its upper part; the whole is immersed in a bath of liquid kept at a definite temperature. A little bulb containing a known quantity of the liquid to be vaporised is passed up into the tube; being heated it bursts; the vapour occupies a certain volume of the tube; the mercury stands at a certain height in the tube. The mercury stands at a different height in an ordinary barometer; the difference of readings indicates the pressure exercised by the vapour. Its weight is known, its volume, and its temperature. A series of observations is made at different bath-temperatures. It is difficult to ensure that all the substance is liquefied. V. and C. Meyer use a large long-necked closed flask: this is heated until no more air escapes; when this is the case, a little glass globe containing a known quantity, m grammes, of the liquid to be tested is dropped from the cool top of the neck: it breaks and the liquid evaporates: the vapour drives out, say, $\mathfrak{v}$ cub. cm. of air from the flask: then, in C.G.S. measures, the vapour density at the temperature and (corrected) pressure of observation is $m/\mathfrak{v}$.

The second method is that of Dumas. A bulb with a long-drawn neck is filled with liquid and immersed in a heated bath. The liquid in the bulb violently rushes out in the vapourous state through the narrow neck; this ceases and equilibrium is set up; the bulb is filled with vapour at the temperature of the bath. The end of the neck is then sealed by a blowpipe-flame; the whole is removed, cooled, weighed. This gives the weight of bulb + vapour; already the weight of the bulb, its volume, the bath-temperature, are known; the density of the vapour occupying the bulb at the temperature of the bath can be thus found. At high temperatures bulbs of porcelain or iron, and baths of mercury-vapour, sulphur-vapour, or zinc-vapour, may be used (Deville and Troost).

The density of saturated vapour.—Fairbairn and Tate found the density of saturated steam by introducing into a recipient of known capacity

and devoid of air a known quantity of water, and by measuring the temperature at which the whole of the water was evaporated.

The measurement of the pressure of unsaturated vapour, if it present itself alone, is simply the measurement of gaseous pressure, and calls for no further remark.

The measurement of the pressure exercised by an unsaturated vapour which forms one of the components of a mixture is in one case — that of Aqueous Vapour in the Air — a matter of importance. A numerical example will illustrate this. If water be exposed to a pressure of 9·16 mm. of mercury (= 0·01205 atmos.), it will boil at 10° C.; if water-vapour, of such density (supposed constant) that it exerts a pressure of 0·01205 atmos., be exposed to a temperature *above* 10° C., it will be unsaturated; *at* 10° C., it will be saturated; at any temperature *below* 10° C., it will be in part condensed. 10° C. is, then, the **Condensation-Temperature** for aqueous vapour of this pressure of ·01205 atmos., just as the latter is the saturation-pressure for aqueous vapour at a temperature of 10° C. If, now, we take moist air containing aqueous vapour and air in the proportion of 0·01205 to 0·98795, at the ordinary atmospheric pressure: at any temperature above 10° C. it will not deposit moisture; at 10° C. it will begin to do so. 10° C. is the condensing temperature or **Dewpoint** for air containing this proportion of moisture. To other proportions of moisture other dewpoints correspond; these can be found in any table of the boiling points of water at different pressures. Hence, if we can find the temperature at which air containing aqueous vapour begins to deposit moisture, we can by reference to such tables find the proportion of aqueous vapour in the air. This temperature is ascertained by a Hygrometer.

The essential part of a hygrometer is a glass — or, better, a smooth silver — surface, which can be cooled down until the moisture of the air begins to deposit as a film upon it, and whose temperature at the instant of the dimming of its brightness can be accurately ascertained. The surface may be fashioned into a bulb: this bulb may contain ether; the bulb may be cooled by blowing through and thus rapidly evaporating the ether; the temperature at the instant of dimming of the surface can be read off on a thermometer whose lower end is dipped in the evaporating ether. The whole may be allowed spontaneously to become warmer; as it does so, the film disappears: the temperature at which this occurs is noted. The film is again caused to appear and disappear; by dint of repetition a mean point between the highest temperature of appearance of the film and the lowest temperature of its disappearance is obtained, which is the Dewpoint required.

Another method for ascertaining the dewpoint — one for doing so by a single observation — is the following: — If a thermometer bulb be by any means kept cool by evaporation — being covered with a wet piece of linen which dips in water, or the like — the bulb is cooled; the extent of cooling depends on the rapidity of evaporation: the rapidity of evaporation depends on the Humidity of the air — that is, on the ratio between the amount of aqueous vapour actually present in the air, and that which, at the temperature of the air, would be present if the air were saturated with moisture. The less the humidity of the air, the greater will be the evaporation, and the greater will be the difference between the readings of a thermometer kept cool in this way and those of a thermometer subjected to normal circumstances. Tables have been constructed in which, for each reading of the "dry bulb" and of the "wet bulb," the corresponding percentage of aqueous vapour in the air is recorded.

Dew. — When, on a clear night, the earth, stones, plants, etc., become cool by free radiation, their temperature may sink below the condensation-temperature proper to the particular percentage of aqueous vapour in the air. When the temperature thus sinks below the dewpoint, the moisture of the air is partly deposited in the form of dew; and the more highly charged with moisture the air had become during the day, the earlier and the heavier is the deposit of dew at night.

The soil immediately underneath the surface is at the same time warmer than the air or the surface of the soil; moisture is condensed on the under surface of cold stones, etc. Much of what is called Dew is, however, liquid transuded from plants themselves (J. Aitken).

Transformations of Heat.

Transformation of Work into Heat may be effected directly by the agency of friction, or indirectly by the transformation of kinetic energy into the energies of noise, light, electrical condition, which are in their turn converted into heat. Even the conversion, apparently direct, by the agency of friction may be due in the first place to the generation of local electrical currents or conditions, the energy of which is afterwards converted into heat.

Transformation of Heat into Work. — From our previous discussion of the Indicator-Diagram we understand that the work done by any

Fig. 133.

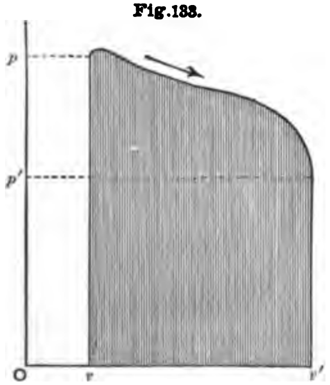
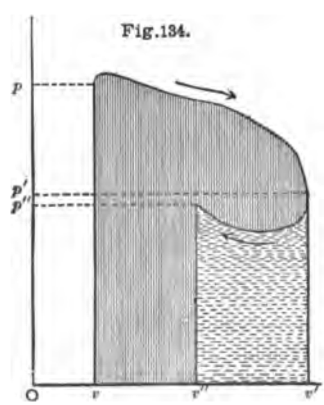


Fig. 134.



substance during expansion can be represented by the area PP′V′VP (Fig. 133), where OP, OP′ represent the original and final pressures, OV and OV′ the original and final volumes. The work is positive, done *by* the expanding substance (steam, air, etc.) if the expansion be positive, from OV to OV′;

negative, done *upon* it if the expansion be negative, as from OV′ to a less value OV.

Where work is done both by and upon the working substance, as in Fig. 134, the negative work P′P″V″V′P′ being subtracted from the positive work PP′V′VP, there is left an area PP′P″V″V, which represents the work done.

If the curve PP′P″ be complicated, the total work done may be found by dissecting the figure; any complex operation may be resolved into a number of simple ones, of which each produces its own effect; the work done is found by a process of summation of positive and negative areas.

When the working substance returns to its original volume and pressure, as in Fig. 135, the shaded area again indicates the amount of work done by the working substance, just as if in Fig. 134 the line P″V″ had been made to coincide with PV. The work is positive if the change of pressure and of volume have been effected in the direction of the arrows; negative if effected in the contrary sense. Such an operation is a **Cycle**.

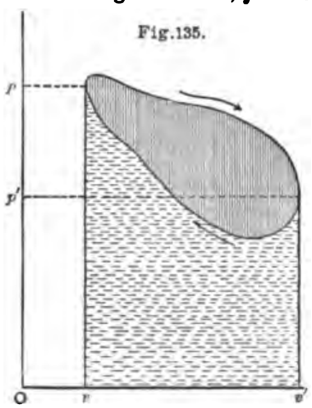


Fig. 135.

The advantage of studying the amount of work done by a working substance operating in a cycle is that we are not called upon to take any internal work into account. The body returns at the end of the operation to its primitive condition, and there is no balance of work done either by or against internal forces.

Into the consideration of a cycle we introduce an assumption that it is possible for a working substance to return to the same condition as regards pressure and volume at the original temperature; this might not have been true as regards any actual substance, though it is theoretically true as regards perfect gases; it is, however, actually true as regards physical gases, for the elasticity of gases is perfect.

We must choose some particular kind of cycle for our ideal operations; that to be explained is the one best adapted for the study of the relations between work and heat, and was devised in its primitive form by Sadi Carnot; it is hence known as **Carnot's cycle**.

If a gas expand at constant temperature, we know by Boyle's law that the pressure and the volume vary inversely; this law can be expressed graphically by an equilateral hyperbola, for in that curve $xy =$ const. The pressures and volumes at different temperatures are represented by points on different hyperbolas. Imagine the curves of Fig. 136 to represent portions of the hyperbolas corresponding to temperatures τ_1° Abs. and τ_2° Abs. for a given mass of substance. This substance, at the temperature τ_2 and pressure p_1, will have the volume $\mathfrak{v}$: to the pressure p_2 at the same temperature corresponds volume $\mathfrak{v}_2$; if the temperature be τ_1 and pressure p_1, the volume will be not $\mathfrak{v}$, but $\mathfrak{v}_1$, a point on the higher hyperbola, on the line — the so-called **Isothermal** line — corresponding to the higher temperture τ_1° Abs.

Expansion of a gas involves a more rapid fall of pressure when it is effected adiabatically than when effected at constant temperature, for the gas cools down: the **Adiabatic** lines, which express the relations between pressure and volume when heat is neither supplied nor allowed to escape,

slope more steeply than the isothermal lines for the same substance. The equation by which any one of these lines may be traced out is called the **Adiabatic Equation**, and it is $p/\rho^{k/c} = const.$, or, for a given mass of gas, $p\mathfrak{v}^{k/c} = const.$,* where k/c is the ratio of the two thermal capacities of the gas in question. Fig. 137 represents these lines, and shows the relations between the pressures and volumes of a substance starting from conditions p_1, $\mathfrak{v}_1$, τ_1, and p_1, $\mathfrak{v}$, τ_2, which correspond to those of the previous figure.

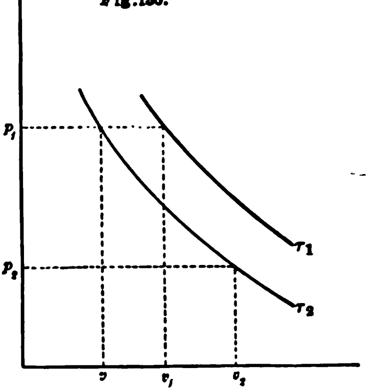


Fig. 136.

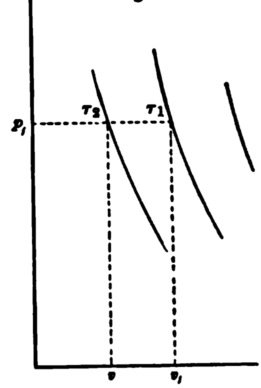


Fig. 137.

Let us now superpose the two figures 136 and 137, and we obtain Fig. 138, and are now prepared to understand Carnot's cycle in its modern form.

The steps of Carnot's cycle:—

1. Starting with our working substance at the condition p_1, $\mathfrak{v}_1$, τ_1° Abs.

---

* From $\mathfrak{R}\cdot m\tau = p\mathfrak{v}$ [i] we get, by differentiation, $\mathfrak{R}\cdot m\dot{\tau} = p\dot{\mathfrak{v}} + \mathfrak{v}\dot{p}$ [ii]; and also, p. 370, $\mathfrak{R} = k - c$ [iii]. Of any small element, $= k\cdot m\dot{\tau}$ ergs, of Heat supplied, $c\cdot m\dot{\tau}$ ergs would be consumed in raising the temperature by $\dot{\tau}$, where c is the thermal capacity at const. vol.; and the remainder would do external work equal to $p\dot{\mathfrak{v}}$ ergs; whence the heat supplied $= k\cdot m\dot{\tau} = c\cdot m\dot{\tau} + p\dot{\mathfrak{v}}$ [iv]; but this $= 0$ under adiabatic conditions; whence $c\cdot m\dot{\tau} + p\dot{\mathfrak{v}} = 0$ [v]. From equations [v], [ii], and [iii], we get $k\cdot p\dot{\mathfrak{v}} + c\cdot\dot{p}\mathfrak{v} = 0$, by eliminating $\dot{\tau}$ and multiplying by $k - c$; and this, on being transformed into $k\dot{\mathfrak{v}}/\mathfrak{v} + c\dot{p}/p = 0$, may be integrated, and we then get $p^c\mathfrak{v}^k = const.$, or $p/p_1 = (\mathfrak{v}_1/\mathfrak{v})^{k/c}$. Then, whatever the mass m may be, $p/p_1 = (\rho/\rho_1)^{k/c}$; and $p^c/\rho^k = const.$ It is assumed in this that the gas is perfect, and $L = 0$.

Otherwise.—Suppose a body of mass m grammes to possess on the whole H ergs of Heat, at a temperature τ° Abs.: then the quotient $H/m\tau$ is the **Entropy** or Thermodynamic Function φ. This function φ represents a certain condition of the body; it increases if the total Heat H in the body increase, and *vice versâ*; and it cannot change unless Heat enter or leave the body, so long as the body is single, and is not a system of unequally-heated parts, of which some gain while others lose heat. If the body be thus single, and not such a system, the isentropic curves must be the same as the adiabatic curves. Any small amount of Heat, δH ergs, supplied to a given mass m at an average temperature τ, would produce a change $\delta H/m\tau = \dot{\varphi}$ in the Entropy; but $\delta H = (c\cdot m\dot{\tau} + p\dot{\mathfrak{v}})$: and therefore $\dot{\varphi} = \delta H/m\tau = (c\dot{\tau}/\tau + p\dot{\mathfrak{v}}/m\tau)$ $= (c\dot{\tau}/\tau + \mathfrak{R}\dot{\mathfrak{v}}/\mathfrak{v}) = \{(c/\mathfrak{R}m\tau)(p\dot{\mathfrak{v}} + \mathfrak{v}\dot{p}) + \mathfrak{R}\cdot\dot{\mathfrak{v}}/\mathfrak{v}\} = \{(c/p\mathfrak{v})(p\dot{\mathfrak{v}} + \mathfrak{v}\dot{p}) + (k-c)\cdot\dot{\mathfrak{v}}/\mathfrak{v}\}$ $= (c\dot{p}/p + k\dot{\mathfrak{v}}/\mathfrak{v})$; which on being integrated gives $\varphi = c\log p + k\log\mathfrak{v}$, whence $p^c\mathfrak{v}^k = \epsilon^{\varphi}$, and finally, whatever be the value of m, $p/p_1 = (\rho/\rho_1)^{k/c}$. Here ϵ is the base of the Naperian logarithms, and ϵ^{φ} is, for any determinate value of φ or $H/m\tau$, a constant quantity.

(point A), we allow it to expand at the temperature $\tau_1{}^\circ$, this temperature being maintained constant. Running through successive pressures and volumes represented by successive points on the isothermal line τ_1, it assumes a pressure, say, p_1' and volume $\mathfrak{v}_1'$ (point B).

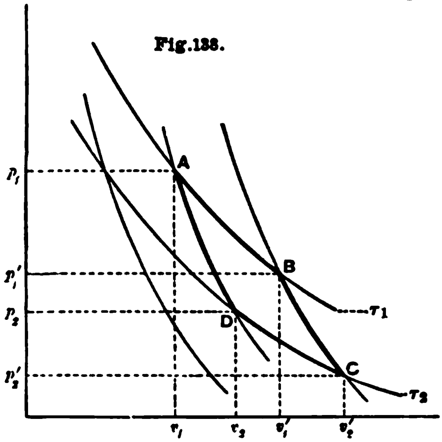


Fig. 138.

Work is done equal to $p_1\mathfrak{v}_1 \log(\mathfrak{v}_1'/\mathfrak{v}_1) =$ the area $ABV_1'V_1$. This work is done at the expense of heat-energy supplied to the working substance from an external source.

2. Starting from the condition p_1', $\mathfrak{v}_1'$, τ_1 (point B) we allow the working substance to expand adiabatically, until it assumes the temperature $\tau_2{}^\circ$ and the corresponding condition p_2', $\mathfrak{v}_2'$, $\tau_2{}^\circ$ (point C). BC is a part of the adiabatic line passing through B and cutting the τ_2 isothermal in C.

Work equal to the area $BCV_2'V_1'$ is done by the expanding substance, but at the expense of its own heat-energy, for no heat is supplied to it.

3. The substance is now compressed until it assumes the condition p_2, $\mathfrak{v}_2$, τ_2—that is, until it runs from C so far up the isothermal line τ_2 as to meet at D an adiabatic line, which passes through the original point A. Work is done equal to the area $CDV_2V_2' = p_2\mathfrak{v}_2 \log(\mathfrak{v}_2'/\mathfrak{v}_2)$: but it is done *upon* the working substance, for that substance is compressed: and heat to a corresponding amount is lost by the working substance, for it passes to surrounding objects, and may be wasted by conduction and radiation into all the universe.

4. The body, from which no more heat is allowed to escape, is now supposed to be still further compressed until it has regained its original condition p_1, $\mathfrak{v}_1$, $\tau_1{}^\circ$. Work is done on the working substance thus compressed, but appears as heat in the substance, not as external work either positive or negative, and the temperature rises, for no heat is allowed to escape.

During the adiabatic expansion in Step **2**, the change of temperature is from τ_1 to τ_2; and (see footnote, p. 373) in that case $\tau_1/\tau_2 = (p_1'/p_2')^{(k-c)/k} = (\mathfrak{v}_2'/\mathfrak{v}_1')^{(k-c)/k}$. Similarly, in Step **4**, the temperatures are again τ_2 and τ_1; and $\tau_1/\tau_2 = (\mathfrak{v}_2/\mathfrak{v}_1)^{(k-c)/k}$. Therefore $\mathfrak{v}_2'/\mathfrak{v}_1' = \mathfrak{v}_2/\mathfrak{v}_1$; or $\mathfrak{v}_2'/\mathfrak{v}_2 = \mathfrak{v}_1'/\mathfrak{v}_1$.

The whole energy supplied to the working substance from the source is $p_1\mathfrak{v}_1 \log(\mathfrak{v}_1'/\mathfrak{v}_1)$; that wasted is $p_2\mathfrak{v}_2 \log(\mathfrak{v}_2'/\mathfrak{v}_2) = p_2\mathfrak{v}_2 \log(\mathfrak{v}_1'/\mathfrak{v}_1)$; that utilised is $[(\log(\mathfrak{v}_1'/\mathfrak{v}_1)\cdot(p_1\mathfrak{v}_1 - p_2\mathfrak{v}_2)) \div (p_1\mathfrak{v}_1 \log(\mathfrak{v}_1'/\mathfrak{v}_1)] = \dfrac{p_1\mathfrak{v}_1 - p_2\mathfrak{v}_2}{p_1\mathfrak{v}_1}$ of the whole. But $p_1\mathfrak{v}_1 = m\cdot\mathfrak{R}\tau_1$; $p_2\mathfrak{v}_2 = m\cdot\mathfrak{R}\tau_2$, where τ_1 and τ_2 are the respective temperatures. Hence the proportion of energy utilised is $\{(m\mathfrak{R}\tau_1 - m\mathfrak{R}\tau_2) \div m\mathfrak{R}\tau_1\}$ or $(\tau_1 - \tau_2)/\tau_1$ of the whole.

The working substance operating in such a cycle acts as a distributor of energy; it divides δH, the heat-energy supplied to it from the Source of heat, into two parts: one part, $\delta' H$, passing to the Condenser, is lost by conduction and radiation; the remainder, W, is usefully converted into external Work.

The heat δH is supplied at the higher temperature τ_1; the quantity of

heat $\delta' \mathrm{H}$ is lost to surrounding objects at the lower temperature τ_2; the **Efficiency** of such an ideal arrangement is the ratio

$$\frac{\text{Heat utilised}}{\text{Heat supplied}} = \frac{\delta \mathrm{H} - \delta' \mathrm{H}}{\delta \mathrm{H}} = \frac{\mathrm{W}}{\delta \mathrm{H}} = \frac{\tau_1 - \tau_2}{\tau_1}.$$

Thus, so far as Carnot's cycle is concerned, even though we could find a working substance and construct a machine which could carry the cycle out in practice, yet there would be a great waste of heat-energy, unavoidable unless we had a condenser at a temperature of absolute zero. If the temperature of the boiler of an ideal engine competent to work out Carnot's cycle were 120° C. (393° Abs.), and that of the condenser 0° C. (273° Abs.), the work done by such an engine could not exceed $\frac{393-273}{393}$, or about 30·6 per cent of the whole energy supplied as heat.

The cycle above considered is **reversible**; each step in it can be retraced —or could be retraced if we could construct an engine capable of working without waste of energy in noise, friction, excessive conduction and radiation of heat, and the like—work being done not by but upon the engine as it is driven backwards.

The effect of reversing such a cycle would be that work W being done upon the engine, the quantity $\delta' \mathrm{H}$ of heat would be taken from the condenser, and the quantity $\delta \mathrm{H}$ of heat would be communicated to the source.

Any engine which operates through periodic cycles must be a reciprocating engine: and in every reciprocating engine there is an absolutely necessary waste of energy arising from the necessity of restoring the engine to its primitive position in order that its piston may repeat its effective thrusts.

Carnot's ideal "perfect" engine is one which, with a working substance capable of returning to its primitive condition, will work out the *reversible* cycle above described, and thus attain the efficiency above indicated: an engine which wastes no energy otherwise than by restoring the primitive condition of its working substance.

The perfection of a perfect engine depends not on the nature of the working substance, but on the reversibility of the cycle which it operates, and the efficiency of such a reversible engine depends only on the temperatures between which it works. **Carnot's Principle**, as enounced by himself, is—the motive power of heat is independent of the material agents employed to realise it; its quantity is determined solely by the temperatures between which the "transport of Caloric" * is effected.

The efficiency $\frac{\mathrm{W}}{\delta \mathrm{H}} = \phi(\tau, \tau - \dot{\tau})$, where $\dot{\tau} = \tau_1 - \tau_2$.

$$= f(\tau, \tau) - f'(\tau, \dot{\tau}) = 0 - \psi(\tau)\dot{\tau}.$$

The efficiency depends upon $\dot{\tau}$, the difference of temperatures between the source and condenser, and upon $\psi(\tau)$, a function of τ which is called **Carnot's function**, C.

We have also seen that efficiency $= \frac{\text{difference of temperatures}}{\text{temperature of source}} = \frac{\dot{\tau}}{\tau}$.
Hence $\mathrm{C} = 1/\tau$; and Carnot's function is the reciprocal of the Absolute Temperature of the Source.

* An expression implying, as in his day, the material theory of heat.

The **Efficiency of a Carnot's Reversible Reciprocating Engine is greater** than that of **any other** reciprocating engine. If it were possible to devise a more efficient reciprocating engine it might be employed with the expenditure of a certain amount of heat to drive a reversible reciprocating engine backwards; the source and the condenser of the Carnot's engine might be the same as those of the more efficient engine: the Carnot's engine would be occupied in restoring to the source the heat taken from it by the better engine; on the whole, a surplus of work would during each cycle be done by the conjoined mechanism — a surplus not accounted for by heat lost by any body — a creation of energy.

If the better engine were employed in driving a larger Carnot's engine backwards, there might be no surplus, no external work done; but a greater amount of heat would be conveyed to the source by the reversed Carnot than would be taken from it by the more efficient but smaller engine, and the whole heat of the universe might be, step by step, induced to travel through the condenser into the source of the conjoined mechanism — a conclusion evidently absurd.

That this conclusion is absurd, or at any rate contrary to experience, so long as we cannot deal like Clerk Maxwell's Demon (p. 52) with single molecules, it is the aim of the **Second Law of Thermodynamics** to state: — Heat cannot of itself pass from a colder body to a hotter one, nor can it be made so to pass by any inanimate material mechanism: and no mechanism can be driven by a *simple* cooling of any material object below the temperature of surrounding objects.

The word simple, or some equivalent word, is necessary in the above statement of the second law for the following reason: — A quantity of compressed gas *can* do external work, and in so doing cool itself below the temperature of surrounding objects; but its cooling is not a simple loss of heat-energy; there is a concurrent change of condition of the gas, a change which cannot be reversed without the expenditure of heat exceeding in amount the heat converted into work by the expanding gas.

This being admitted, we may reason backwards and arrive at the ratio of efficiency $\frac{\tau_1 - \tau_2}{\tau_1}$ in a reversible engine as a direct corollary of the proposition; and the statement of that ratio of efficiency in a reversible reciprocating engine is also known as the Second Law of Thermodynamics.

This Protean law assumes another form, apparently different from but essentially identical with both the preceding. Temperature being assumed proportional to the total heat-energy, the amount of heat-energy, δH ergs, supplied at the higher temperature τ_1, is proportional to τ_1; $\delta H = \dot{\varphi} m \tau_1$; $\delta H / m\tau_1 = \dot{\varphi}$. Similarly $\delta' H$, the heat lost to the condenser at the lower temperature τ_2, is $\delta' H = \dot{\varphi} m\tau_2$; $\delta' H / m\tau_2 = \dot{\varphi}$.* Hence $\delta H / m\tau_1 = \delta' H / m\tau_2$; and from this we may not only derive the former equation $\frac{\delta H - \delta' H}{\delta H} = \frac{\tau_1 - \tau_2}{\tau_1}$, but also the equation $\delta H / m\tau_1 - \delta' H / m\tau_2 = 0$; an equation which, in the most general case, takes a form applicable to the most complex reversible cycle, namely, $\Sigma(\delta H / m\tau) = 0$, or, when the mass of gas referred to is a unit-mass, $\int dH/\tau = 0$ (Lord Kelvin); an expression very convenient for mathematical purposes, but difficult to translate into words. — In a perfect, a reversible

* The value of $\dot{\varphi}$ is the same in both these cases, because in both cases the change of entropy is the difference between the entropies of the isentropic or adiabatic lines AD and BC, Fig. 138.

cycle, the Entropy,* the sum of the equivalences of all the transformations, is zero (Clausius). In a non-reversible process the sum of the transformations is positive, and since all processes are non-reversible, the sum of the entropies in the universe tends to a maximum. According to Rankine's mode of expression, substantially identical with the preceding, the second law is: If the absolute temperature of a uniformly-hot substance be divided into any number of equal parts, the effect of each of those parts in causing work to be performed is equal. This implies that the absolute temperature is proportional to the total heat-energy, and so merges into the preceding form of the second law.

Lastly, Carnot's Principle itself is often called the Second Law of Thermodynamics.

We have already studied the direct transformation of heat into work in the radiometer. In the steam-engine the heat of the steam may be in part converted into work; the piston is bombarded by the particles of the steam, and if the resistance to its onward movement be not excessive, it is thrust forward by the joint impact of the particles which impinge on it, their several components of motion parallel to the piston-rod being effective in this respect.

Even under the most favourable circumstances which can be conceived, heat cannot be wholly converted into work by any form of continuously-acting mechanism. The efficiency of the ideal perfect engine — small though that efficiency be — is never approached in practice; and the efficiency of the human body considered as a machine — one-fifth of the total energy supplied to it being capable of utilisation — is remarkable when we consider the narrow limits within which it operates.

Work can thus be wholly converted into heat, but heat can never be wholly converted into work; whence a universal tendency to the Degradation of Energy into Heat, the lowest of its forms.

Measurement of Heat.

Temperature we have now seen to be, when measured from an absolute zero — a zero of absolute cold — (1) proportional to the absolute amount of molecular kinetic energy, and (2) the reciprocal of Carnot's function.

What is meant by equal degrees of heat? Why is the difference between 0° C. and 1° C. supposed to be equal to that

* Clausius, dealing always with unit-masses, has applied the term Entropy to the expression $\Sigma(\delta H/\tau) = \Sigma(\dot{\varphi})$; and it will not be difficult to see that where the sum is positive, more heat is given to the engine by the source than is given when that sum $= 0$, the work done, W, remaining unchanged; but this excess is wasted by passing through the condenser to the external universe.

between 100° C. and 101° C.? — In a perfect gas equal differences of temperature correspond to equal increments of energy.

In a diagram containing a system of adiabatic and isothermal lines, the isothermal lines must be so drawn as to cut off equal areas between the adiabatic lines.

Absolute zero would correspond to total absence of molecular kinetic energy.

If we had a perfect gas at command we might measure temperature by its means in either of two ways: —

(1) We might observe its **pressure** at constant volume: equal increments of pressure correspond to equal increments of temperature.

(2) We might observe its varying **volume** at constant pressure: the volume is proportional to the absolute temperature, and equal small increments of volume approximately correspond to equal small increments of temperature.

The former is the more accurate method.

We have no perfect gases to experiment upon: air, etc., are not perfect gases. Yet we may perform either of the above operations on a quantity of air confined in a flask, and thus construct an air thermometer. The former method — that of observation of pressure — is here doubly preferable to the latter — that of observation of expansion — because in the former there is no waste of energy in doing either internal or external work, and the increase of pressure is appreciably the same as that of a perfect gas. The indications of an air thermometer used in this way may hence be assumed as an approximate standard of comparison.

For the corrections necessary, see the table in Tait's *Heat*, p. 340.

By the air thermometer we find that for a fall of 1° C. (from 1° to 0° C.) on the mercurial thermometer, the pressure sinks in the ratio of 274 to 273; hence the temperature sinks in the same ratio, absolute zero is $-273°$ C., and Carnot's function has the numerical value of $\frac{1}{273}$ for a temperature of 0° C., and of $\frac{1}{273+t}$ for a temperature of $t°$ C.

Two bodies are said to be at different temperatures when the one has a tendency to lose heat to the other; to have the same temperature when there is no such tendency: and bodies are at the same temperature when they have the same kinetic energy *per molecule*, not per unit of weight.

Differences of temperature may be roughly perceived by the hand; the sense of temperature can even be cultivated like

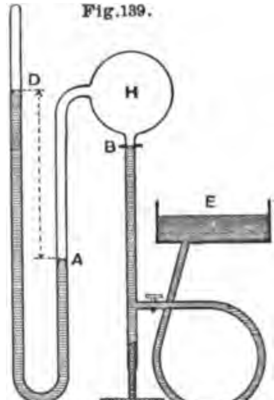
that of musical pitch so as to arrive at approximate accuracy without actual recurrence to a standard of known temperature.

Any of the effects of heat may be used for detecting the presence of heat and for constructing a thermoscope. Arbitrary graduation of any thermoscope will enable it to be used as a thermometer.

Bréguet's metallic thermometer is a spiral strip composed of three metal strips soldered together by their broad surfaces: the different rates of expansion cause the spiral to roll or unroll according to the variations of temperature, and thereby to move a pointer.

The air thermometer, one of whose forms is shown in Fig. 139, is principally used as a standard of reference. AD is a manometer, in which above D there is a Torricellian vacuum: H is an air chamber, E an auxiliary cistern of mercury. As far as the mercury A is depressed below a certain mark, so far is the level of mercury at B raised by raising the mercury cistern E, closing the stopcock, and effecting a fine adjustment by means of the screw C. The volume of the gas between B and A is thus made constant, and the column of mercury AD measures the pressure of the gas in H.

Fig. 139.



The same thermometer may, by an adjustment of the height of the column AD, be used as a constant-pressure-and-variable-volume air thermometer.

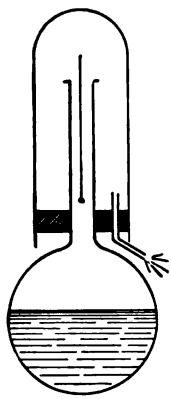
The air thermometer presents the disadvantage of being extremely unwieldy; but it has the advantage that the comparatively small expansion of the glass produces little effect in causing any difference between the apparent and the real expansion of the air, or in vitiating the adjustment to constant volume.

The ordinary mercurial thermometer is a familiar object. Its simplest form would be that of a flask with a long neck. If the neck were open, the mercury would be in danger of accidental loss and of evaporation; the neck must therefore be closed. If it were simply closed, the air contained in the neck would at high temperatures be compressed; the bulb would burst; hence a vacuum must be produced in the upper part of the neck. This vacuum is produced by closing the tube while

mercury is boiling within it; on cooling, the mercury contracts and retracts, leaving a space containing only a certain quantity of the vapour of mercury.

As to the graduation of the mercurial thermometer, this might be effected by comparison with an air thermometer, a troublesome process, resulting in degrees true but unequal in size; or by taking advantage (Renaldini) of the fact that the "freezing point" of water — or, better, the melting point of ice — and the "boiling point" of water — or, better, the temperature of steam at the pressure of 760 mm. Hg — are constant temperatures, and may be taken as fixed points; that the height assumed by the mercurial column at these two temperatures may be marked on the tube; and that the tube between these two marks may (Newton) be mechanically graduated by e q u a l division into degrees — a method certainly convenient, but only approximately correct.

Fig. 140.



The boiling point of water is estimated by inserting the thermometer in an atmosphere of steam surrounded by a steam-jacket (Fig. 140), intended (Berthelot) to check irregular condensation. The pressure must be the standard, 760 mm. Hg. The "freezing point" must then be at once determined by the position assumed by the mercury when the water which trickles off melting ice flows in a stream over the mercury bulb, the whole being surrounded by a jacket of melting ice.

On the Centigrade thermometer (Linnaeus) the "freezing point" and the boiling point are respectively 0° and 100° C.; on the Fahrenheit scale they are 32° F. and 212° F.; 0° F. being the lowest temperature attained by Fahrenheit (*Phil. Trans.* 1724) by means of a mixture of ice, water, and salt or sal-ammoniac.

A temperature of t° C. is, accordingly, equal to $(\frac{180}{100}t + 32)^\circ$ F.; and one of x° F. to $\frac{100}{180}(x - 32)^\circ$ C.

Fahrenheit did not use the boiling point of water as a standard, but imagined his zero to be an absolute zero, and then made or intended to make the freezing point of water to stand at one-third between this absolute cold and the temperature of the human body, which for convenience he called 96°.

Water has been used as the expanding substance in thermometers; it is objectionable on account of its point of maximum density. Alcohol is used at very low temperatures, because it is not readily frozen. Mercury, which is very advantageous on account of its low specific heat and its ready response, was brought prominently into notice by the astronomer Halley.

The s e n s i t i v e n e s s of thermometers — the power of

revealing minute variations of temperature—is increased by narrowing the tube or by enlarging the bulb. A large bulb is, however, inconvenient; because it is difficult of insertion in apertures—a fault which may be remedied by giving the bulb a cylindrical form; because it may alter materially the temperature of the object whose temperature is to be ascertained; because it slowly equalises its temperature with that of the object. A narrow tube is inconvenient because a narrow thread of mercury is difficult to see; this may be remedied by using a tube of flat elliptical section, and by enamelling the back of it.

The main causes of error in the use of a thermometer are that the graduation alters, the "zero rises," or a thermometer inserted in melting ice comes in course of time apparently to indicate a temperature somewhat above 0° C. or 32° F., this effect being probably due to a slow yielding of the bulb to atmospheric pressure; and further, that it is not always possible to ensure that the whole of the mercury is at the same temperature.

In testing a thermometer it is important to see that the "freezing point" and the "boiling point" are accurately indicated by it, or that it agrees with a thermometer in this respect correct; and that the bore of the tube is uniform, so that a little detached portion of the thread of mercury may occupy an equal length in all parts of it.

For accurate comparison of thermometers they should be immersed together in a cooling fluid rather than in one which is being heated (Fourier); the temperature indicated by a thermometer in a cooling fluid is always a little higher than that of the fluid.

For practical details connected with testing thermometers see Gscheidlen, *Physiologische Methodik*, p. 76.

For observations of the temperature of the skin it is well (Colin) not to cover the bulb with flannels, or to leave the thermometer in such circumstances for too long a time, for the skin assumes the temperature of the interior; rather should quickly-acting thermometers be used. Apply a thermometer quickly, fresh from the pocket or the hand: keep it closely in contact with the skin; avoid blowing on the bulb; put a little cupola of paper or cotton over the bulb, but not in contact with it.

Special Forms of Mercury Thermometers.—In the Maximum thermometer, above the column of mercury, a small bubble of air is introduced; above this a little thread of mercury. When the temperature rises, the air is compressed, the thread is pushed upwards; when the temperature falls back, the thread of mercury does not return.

The Minimum thermometer is usually a spirit thermometer with a little broad-headed piece of wire loosely fitting in the spirit. It is ad-

justed with its head touching the surface of the thermometric liquid. When the liquid contracts, surface-tension drags the wire with it; when the temperature rises, the liquid passes the wire without forcing it upwards: the position of the end of the wire nearest the free surface indicates the lowest level to which the surface had sunk, and therefore the lowest level which had been attained since the last observation.

In Metastatic thermometers any part of the mercury may be removed from the column and shaken aside into an apical cavity, or restored in whole or in part to the main thread; the thermometer, a very delicate one, being thus competent to read to very small fractions of a degree at any part of the scale chosen at will. See Gscheidlen, p. 84. The principle of overflow — liquid being caused to expand and overflow, or vapour (iodine, mercury) being boiled out of a heated flask, what remains being weighed when cooled — is utilised in the construction of some pyrometers.

High temperatures, beyond the reach of the mercury thermometer, may be measured by '**pyrometric**' means, of which the chief are the following: — (1) readings of air-, or rather of hydrogen-thermometers; (2) the dilatation of solids, such as porcelain ($\frac{1}{4,000000}$ its length per °C.), made manifest by the method of Fig. 5; (3) the fusion of masses of known melting-point (a series of fusible porcelains or of fusible alloys); (4) variations of viscosity of air and consequent variations in the flow of air through apertures, at different temperatures; (5) the temperature attained by a uniform stream of water directed, at a known speed, through the hot region; (6) exposure of a known quantity of a substance (such as iron or platinum), of known specific heat, to the temperature in question, then dropping it into a known quantity of water, and finding what temperature is attained by the water; (7) the colour of the glow, white-hot or red-hot or otherwise, of the heated object, observed with the naked eye or, more accurately, by means of a piece of cobalt glass or of a rotatory-polarisation apparatus (p. 566); and (8) by electric methods (p. 628).

Calorimetry or the quantitative measurement of Heat. — The Calorie (*Ca*) is the amount of heat required to raise the temperature of 1 kilo. of water (or $1/\sigma$ kilos. of any substance whose specific heat is σ) from 0° C. to 1° C. The calorie or small calorie (*ca*) is the amount of heat similarly required to heat one gramme to the same extent. The latter is the C.G.S. unit; the former is much used by French and German writers.

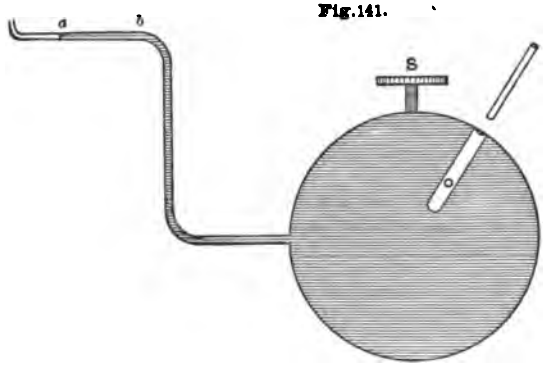
1. The Method of Mixtures. — This may be illustrated by a numerical example. How many calories of heat does a gramme of mercury absorb when it is heated from 0° C. to 1° C.? — *i.e.*, what is the specific heat of mercury?

One gramme of mercury at 100° C. and one of water at 0° C. are mixed: the result is a uniform temperature of 3°·194 C. The water has gained 3·194 calories; the mercury has lost the same. Mercury on losing 3·194 *ca* per gramme is cooled through 96°·8 C.; cooling through 1° C. involves a loss of ·033 *ca*. The specific heat of mercury is thus $\sigma = \cdot 033$; and the amount of Heat contained in the mass of mercury mixed with the water was (373 × ·033) *ca*, 373 being its absolute temperature and its mass being unity.

One gramme of water and one of mercury are, together, thus equivalent in calorimetric calculations to 1·033 grammes of water; and their joint '**water-equivalent**' is said to be 1·033 grammes, *i.e.*, (the mass of the water × its sp. heat) + (the mass of the mercury × its sp. heat).

A modification of this method is that of Fig. 141. A globe filled with mercury: the free surface of the mercury at

Fig. 141.



a; the screw S, which alters the position of the surface *a* so as to bring it to the zero of a scale marked on the horizontal tube *ab;* the hot substance, introduced into a depression at *o*, heats the mercury, expands it, and causes the capillary surface *a* to assume a new position.

Otherwise, instead of observing the direct expansion of the fluid heated, its temperature may be taken by a thermometer.

Dulong's water calorimeter is of this kind: a copper chamber containing a living animal supplied with air by afferent and efferent pipes: round this a water-jacket, the water in which assumes a certain observed temperature. The number of calories taken up by the water and by the copper or other vessels (considered as equivalent to σ times their mass in water, σ

being their specific heat) is found, and that amount thus measured is the amount of heat given out by the enclosed animal.

2. Latent heat methods. — The amount of heat of a hot body may be measured by the amount of ice melted by it — this being ascertained roughly (Lavoisier and Laplace) by the amount of water which trickles from ice amid which the hot body is thrust; or, better (Sir J. Herschel and Bunsen), by observing the actual decrease in volume of a mixed mass of ice and water when some of the ice is melted; or by the amount of liquid — water, ether, acetic aldehyde — which the heat of a hot body (or living animal — Rosenthal, *Arch. f. Anat. und Physiol.*, 1878) can evaporate.

Transference of Heat.

When two masses or parts of the same mass are in contact, the molecular agitation of each is in part communicated to the other: if they be equally hot, each receives as much heat as it gives up: if they be not equally hot, that which has the more molecular energy loses more than it receives, while the other, the colder, gains more heat than it loses. The flow in one direction thus overpowers that in the other, and, on the whole, heat is transferred from the hotter mass to the colder.

We regard in general only the difference, and not the common part; the surplus which flows from the hotter, and not the compensated and non-apparent flow from the colder body.

This tendency is universal. Heat always tends to pass on the whole from hotter to colder bodies, and if these be in contact, the transference is effected by **conduction**; whence all bodies possess some degree of **conductivity** or power of transferring heat through their substance.

When two points in a substance are at temperatures constantly differing by $\delta\tau$, and are at a distance d, a flow of heat is set up between them. The amount of heat which passes from the one point to the other in time t is proportional (1) to the length of time during which the flow proceeds; (2) to $\delta\tau$, the difference of temperatures; and (3) it is inversely proportional to d, the distance between the points.

Otherwise, the Quantity of Heat which flows is H, which $\propto t \cdot \delta\tau / d$; or $H = \Theta \cdot t \cdot \delta\tau / d$. Here Θ is a coefficient, the Coefficient of Conductivity, and $\delta\tau / d$ represents the Fall of Temperature per unit of distance, the Temperature-gradient, *G*.

The **coefficient of conductivity** varies from substance to substance, being greatest in the metals; some substances permit a rapid, some only a slow transfer of heat; compare a horn spoon and a silver one inserted in a hot liquid.

If the one point be maintained at the temperature τ and the other at the temperature $\tau_{,}$, intermediate points have temperatures which from point to point sink uniformly with the distance from the hotter point, if there be no loss of heat on the way between these points, as by radiation or convection. There is thus set up a condition of **Steady Flow** of Heat.

Across a plate of thickness d whose sides are maintained at an *actual* * and constant difference $(\tau - \tau_{,})$, the flow of heat per unit of area will in time t be $\Theta \cdot t \cdot (\tau - \tau_{,})/d$; across area A the flow will be $\Theta A \cdot t \cdot (\tau - \tau_{,})/d$.

This is $\Theta A \cdot t \times$ the temperature-gradient G.

Let the flow of heat be measured in ergs: then Θ is called the Dynamical Coefficient of Thermal Conductivity. In copper, for example, $\Theta = 36{,}675000$; and the Heat-flow, in ergs, $= 36{,}675000 t \cdot \delta\tau/d$. If the heat flowing be measured in the larger unit, the calorie ($= 41{,}593000$ ergs), the proper coefficient will be a smaller one; it is ϑ, the Calorimetric Coefficient of Thermal Conductivity, and is the one commonly employed; in copper the Heat-flow in calories is $H_{,} = 0{\cdot}88176 t \cdot \delta\tau/d$. If the units chosen be such that one unit of heat can raise the temperature of 1 cub. cm. of the conducting substance itself through 1° C., these units are each equal to $\rho\sigma\, ca$, where ρ is the density and σ the specific heat of the substance: and the Heat-flow, measured in such units, is $H_{,,} = \mathfrak{p} \cdot t \cdot \delta\tau/d$; where the coefficient $\mathfrak{p}$, called the Coefficient of Thermometric Conductivity or of Thermal Diffusivity, is equal to $\vartheta/\rho\sigma$ or to $\Theta/41{,}593000\rho\sigma$. In copper, when $\rho = 8{\cdot}6$ and $\sigma = 0{\cdot}95$, $\mathfrak{p} = 0{\cdot}8819$. In iron, $\Theta = 6{,}290000$ and $\vartheta = 0{\cdot}15123$; and since $\rho = 7{\cdot}6$ and $\sigma = 0{\cdot}114$, $\mathfrak{p} = 0{\cdot}183$. In air, $\Theta = 2381$ and $\vartheta = 0{\cdot}0000558$; and since $\rho = 0{\cdot}0013$ and $\sigma = 0{\cdot}1684$, $\mathfrak{p} = 0{\cdot}256$.

Example.—The earth is found to be about 1° C. hotter for every 30 metres of vertical descent: the coefficient ϑ for rock is, on the average, 0·0045; what is the approximate loss of heat from the surface of the earth, in calories per sq. metre per annum? $H_{,}$ (in ca) $= \vartheta \cdot A \cdot t \cdot (\tau - \tau_{,})/d =$ 0·0045 × 10,000 sq. cm. × 31,556929 seconds × 1° C. ÷ 3000 cm. = 473,000 ca per sq. metre per annum; enough to melt a layer of ice 0·644 cm. thick.

The surface of the earth is about $(5{\cdot}093 \times 10^{18})$ sq. cm.; each sq. cm. loses 47·3 ca per annum; the total yearly loss is $(240{\cdot}9 \times 10^{18})$ ca. The specific heat of rock is about 0·5 ca per cub. cm.; the volume of the earth is about $(1{\cdot}0866 \times 10^{27})$ cub. cm.; the "thermal capacity of the earth," if it were wholly similar to surface rock, would be $(0{\cdot}5 \times (1{\cdot}0866 \times 10^{27}))$ $= (0{\cdot}5433 \times 10^{27})$ ca per °C. The heat lost by the earth at the present time would, on the same assumption, correspond to an average fall in tem-

* The flameward side of a steam-boiler plate is not at anything like the temperature of the flame beneath it.

perature, throughout the earth, of $[(240 \cdot 9 \times 10^{18}) \div (0 \cdot 5433 \times 10^{27})]$ °C. = 0·00000045° C. per annum.

The thermometric coefficient $\mathfrak{p}$ has also the following physical meaning. It serves as a measure of the rate of rise of temperature under a varied distribution of heat in a mass; the *temperature* tends to become uniform and flows in the mass; the rate of rise of temperature at any point in the mass is proportional to the local mean rate of change of gradient of temperature between two points, unit distance apart in the direction of the flow of heat, and is equal to $\mathfrak{p} \times$ that mean rate. If two faces of a slab, of area A and thickness δx, be supposed to have the respective Temperature-Gradients G and $(G + \delta G)$, the outflow at the one face would, in time δt, be $H_t = \vartheta \cdot A \cdot \delta t \cdot G$ calories; and the inflow at the other would be equal to $\vartheta \cdot A \cdot \delta t \cdot (G + \delta G)$. The surplus remaining in the slab would be $\vartheta \cdot A \cdot \delta t \cdot \delta G$, or $\vartheta \cdot A \cdot \delta x \cdot \delta t \cdot \delta G / \delta x$ calories. But this slab contains $A \cdot \delta x$ cub. cm., of density ρ; its mass is therefore $A\rho \cdot \delta x$ grammes: its specific heat is σ *ca* per gramme: it will therefore require $A \cdot \delta x \cdot \rho\sigma$ calories to raise its mean temperature through 1° C.; its mean rise in temperature will be $\{(\vartheta \cdot A \cdot \delta x \cdot \delta t \cdot \delta G / \delta x) \div (A \cdot \delta x \cdot \rho\sigma)\}$ degrees Centigrade $= (\vartheta / \rho\sigma)(\delta t \cdot \delta G / \delta x)^{\circ}$ C., in time δt. The rate of increase of temperature per unit of time will therefore be $(\vartheta / \rho\sigma)(\delta G / \delta x)^{\circ}$ C. per sec.; and this $= \mathfrak{p} \cdot \delta G / \delta x, = \mathfrak{p} \cdot \delta G$ when $\delta x = 1$ cm.; which is the proposition stated above.

If a bar be heated at one extremity, the amount of heat which will arrive at a sectional area a given distance along the bar will depend upon the thickness of the bar and its proportional surface. A thin iron wire may be melted at one end but not have its temperature raised by 1° C. at a distance of 6 feet; so much heat is lost on the way, being spent in warming the surrounding air and in keeping up radiation from the surface. For the same reason, the most volatile oil may be burned in a lamp with a sufficiently long wick-tube.

In such a bar, maintained at a uniform distribution of temperature, the heat flowing across a given cross-section can be measured by a process of summation or integration. The temperatures at different points beyond the cross-section are observed; the rates of cooling of a similar bar at different known temperatures are also observed; from these data the loss of heat by radiation and convection can be ascertained; and this is kept up by, and is equal to, the flow of heat across the sectional area. The flow H is thus known; so is $\delta\tau / d$, the temperature-gradient at the cross-section; so is A the area: whence the value of ϑ can be calculated.

In bars of different thicknesses, the distances from the heated extremity at which the same temperature can be kept up by heating the extremity of the bars to the same temperature are to one another as the square roots of the thicknesses; and in bars of the same thicknesses but of different lengths the flow of heat into the bar varies as the square root of the cube of the length.

A hot point in space conceived to be maintained permanently hot will be the centre of a flow of heat symmetrical in all directions. The points in the surrounding space which are at the same temperature may be connected and found to lie on concentric spheres, or spherical **Isothermal Surfaces.**

The heat travels by the shortest path from one surface to another, by **Lines of Propagation**, or Lines of Flow, at right angles to both; and there is on the whole no lateral propagation over an isothermal surface. The whole system of surfaces and lines closely resembles a system of equipotential surfaces and lines of force. The difference of temperature per unit of distance along the lines of spherical propagation decreases with the distance, being proportional to $(1/\text{radius}^2)$. The greater the curvature of a hot body, the greater will be its loss of heat by conduction. Hence an ellipsoidal body maintained at a uniform temperature loses most heat where the curvature is greatest — a proposition closely resembling one in the theory of electricity.

We must distinguish a Flow of Heat from a Flow of Temperature. The latter depends, inversely, on the specific heat $\rho\sigma$ per unit of volume; and if we compare the passage of heat through two substances similarly heated, we find that even though the one substance have a greater conductivity than the other, yet, if its specific heat per unit of volume be greater in a still greater proportion, a given temperature may take a longer time, travelling in the better conductor, to reach a point at a given distance from the source of heat, than it does in the worse conductor.

The rate of propagation of a given temperature depends upon the thermometric conductivity $\mathfrak{p} = \vartheta/\rho\sigma$. Thus in copper, a given Temperature travels faster than it does in iron: and so does it in still air, though the actual quantity of Heat carried by conduction in still air is extremely small.

When a body is exposed to a superficial periodic variation of temperature, the variations are propagated as waves of temperature according to the same law as if they were displacements in a vibrating but more or less viscous solid.

The waves diminish in amplitude — that is, in thermometric range — as they penetrate, and that in geometrical progression; and the depth at which the amplitude is reduced in a given ratio varies as $\sqrt{T}$, and also as $\sqrt{\mathfrak{p}}$ or $\sqrt{\vartheta/\rho\sigma}$. Yearly variations of temperature are thus felt at depths beneath the earth's surface 19·11 times as great as the daily variations are; for $\sqrt{365} = 19{\cdot}11$.

Where a substance is not physically similar in all directions, as in the case of crystals, the conductivity may be unequal in three directions. Thus, a plate cut out of any crystal belonging to the binaxial system, and covered with a film of wax, will, if heated by a hot wire passed through its centre, so conduct the heat that the wax melts not in a uniform circle — as in glass or a crystal of the regular system it will do — but in an ellipse.

Some solids are extremely bad conductors of heat. Down is perhaps the worst of all conductors; hare's fur, sand, asbestos,

are examples of substances within which warm objects may be placed and remain without losing their heat to any material extent for some time. Flannel, cork, etc., appear warm when they are touched by the bare skin, because they carry away by conduction less heat than the air had been removing before these materials had been touched. Wood is, in the radial direction, a bad conductor: this has a certain effect in preserving the tree in life.

The actual amount of the loss of heat suffered by a cooling body depends directly on the effective cooling surface: whence the natural tendency in warm weather to lie at full length, in winter to roll the body up into small compass.

The conductivity of the skin as a whole is greatly diminished by a layer of fatty tissue. The muscles are exceedingly bad conductors.

When a hot body is surrounded by one or more concentric jackets with layers of air between them, the loss of heat is remarkably diminished. A single layer of linen diminishes the loss of heat from the human body by about two-thirds; a double layer effects a much greater economy of heat, and so forth. The practice in cold countries of using double windows proceeds on this principle, and hence also the hygienic advice to multiply the number of light garments in cold weather rather than their weight.

The conductivity of liquids is as a rule greater than that of gases, which in the form of true conduction of heat-energy, as distinguished from convection, is very small.

It is impossible to keep the hands in water at 52° C., while it is quite possible, as observed by Banks, to remain for five minutes in air near the boiling point of water.

When a hot body is placed in air it sets up a number of **Convection currents.** Air becomes heated and rises, carrying away the heat of the hot body: colder air takes its place.

Newton's law of cooling in a current of air is, that at each instant the amount of heat lost varies as the difference of temperature between the solid and the air. This law seems to be adhered to within narrow limits.

In an undisturbed atmosphere the law of cooling by convection is, that the velocity of cooling is proportional to $p^a \tau^{1.233}$, where a is a constant (·45 for air), p the pressure, and τ the *excess* of temperature (Dulong and Petit).

In hydrogen the process of cooling is very rapid.

The carbonic acid, etc., of the atmosphere are mixed thoroughly and equably, not by diffusion, which would take several hundred thousand years to accomplish the task, but by convection currents.

Convection currents, as they pass colder or warmer strata of air, exchange molecules with them by diffusion; the temperature of the whole mass thus rapidly becomes uniform.

Convection currents may be demonstrated by throwing some coloured powder into cold water and proceeding to heat the liquid over a lamp; by looking at distant objects through the heated gases which arise from a heated boiler or wall: the rise of smoke itself is an example of solid particles borne upwards by convection currents — particles which, when the ascending air has become cool, again fall, and may aid in producing fogs by the condensation of water around them.

Though two bodies be not in contact with one another, they may yet exchange heat across the intervening space, and the hotter body, giving out more heat than it receives, is said to **radiate heat** to the colder body. This transfer of heat is effected by means of the Ether of space, and we shall, in the meantime, defer the consideration of the transfer of heat by radiation until we can take a general view of waves in the Ether.

Dulong and Petit found that between 0° C. and 200° C., the aggregate amount of radiation is proportional to $(1{\cdot}0077^{\tau} - 1)$, where τ is the *excess* of temperature above the surrounding enclosure.

With diminishing pressures of air or gas, the rate of loss of heat falls, at first more rapidly, then less so; it then remains sensibly constant (Kundt), but with still further exhaustions the rate again falls (Crookes).

Transport of Heat from place to place may be effected by storing up work-energy in springs which, on being released, set a mechanism at work which evolves heat by friction; or by storing up heat as "latent heat," or by raising the temperature of a substance whose specific heat is high. The former method is not effective, because so large a number of units of work correspond to so small an amount of heat; the latter are exemplified in heating by hot water or by steam. A hot-water bottle contains several calories of heat, according to its size and its temperature; these can be liberated by conduction at any desired situation. If filled with crystallised acetate of soda, melted by heat, the cooling is protracted, for the melted salt, as it slowly solidifies, gives out its latent heat. Steam at 100° C., when condensed, liberates at the point of condensation 546 calories of heat for every gramme of water condensed, and can still, in the form of hot water, surrender more heat to surrounding objects.

CHAPTER XIV.

ON SOUND.

THE word Sound is used in four different senses: —

1. The physiological sensation perceived by means of the ear.

2. The complex harmonic motion of sounding bodies — the Fourier-motion, the periodic or vibratory motion of elastic masses whose vibration is the physical cause of sound.

3. The disturbances of the air which affect the ear. "Sounds," says Newton (*Princip.* ii. Prop. L, Prob. xii. Schol.), "since they arise in tremulous bodies, are no other than waves (*pulsus*) propagated in the air."

4. The energy of a sounding body. "Heat converted into Sound," etc. It is better in this sense to say explicitly, "the Energy of Sound."

A sounding body is a vibrating body.

Cause a tuning-fork to sound in the usual way — by striking it on the knee or drawing a violin-bow across it, or by forcing a steel rod between its prongs and drawing it through the point of the fork. Apply the point of the vibrating tuning-fork to the lips, to the surface of water, to a piece of glass. Bring a vibrating tuning-fork under a light splinter of wood lying upon two points of support; on contact, the light body will be hurled upwards. Cautiously bring a vibrating tuning-fork or bell into contact with a pith-ball suspended by a thread.

Pluck one of the strings of a violin: look at it as it vibrates: touch it. Look at a harmonium or concertina reed while it is in action.

Observe the distinct tremor caused by a large organ pipe while sounding, or even by a large drum.

Relatively deep, grave sounds are produced by slower vibrations; higher, shriller sounds by more rapid vibrations.

Take a long strip of iron — say a strip 4 feet long; fix it in a vice; pull it aside and let it go; it will oscillate transversely at a rate such that the oscillations can be counted; remove it, and refix it so that only 2 feet of it are now free to move; — it will now oscillate four times as frequently: 1 foot

free — sixteen times as frequently as at first; 6 inches free — sixty-four times as frequently, and so on. The oscillations now become so rapid, the number of them in a second (*i.e.*, their *frequency*) becomes so great, that they can no longer be counted directly; now we hear a sound; the shorter the vibrating part, the more rapid become the vibrations, the shriller the sound.

The transmission of sound from a vibrating body to the ear involves, as a rule, the formation of sound-waves in the air.

This may be rendered impossible, *e.g.*, where the sounding body—a bell suspended or placed upon wadding within the bell of an air-pump from which the air is exhausted—has no contact with air, and therefore no means of transferring its own vibration to air; in such a case the ear perceives no sound, even though the bell be struck, for there are no air-waves set up.

But it may be impossible for another reason. Air will not oscillate in waves such as can be propagated to a distance, unless there be some well-marked compression or rarefaction produced at the centre of disturbance. Take as extreme instances of sound produced by well-marked compressions or rarefactions the effect of the discharge of a cannon, which abruptly adds a mass of gas to the already-present atmosphere, and thereby produces great and sudden compression; or the rarefaction produced by the sudden collapse of a weak boiler when the steam contained in it has cooled down. Thus a vibrating body, before it can act as a sounding body, must produce alternate compressions and rarefactions in the air, and these must be well marked. If, however, the vibrating body be so small that at each oscillation the surrounding air has time to flow round it, there is at every oscillation a local rearrangement — a local flow and reflow — of the air, but the air at a little distance is almost wholly unaffected by this. The same result follows if the medium surrounding the vibrating body be rare (*e.g.*, hydrogen) or rarefied (*e.g.*, rarefied air); then, on account of the small inertia of the medium, it is easily induced to flow round the vibrating body; in such cases there is but little wave-motion caused at any distance, and thus there is but little sound produced.

A string stretched between two points of a rigid and massive framework produces surprisingly little sound when caused to vibrate: it does not act upon the air otherwise than by setting up local flow and reflow. If the same string be stretched over bridges upon a sounding-board, the string gives, at each oscillation, an impulse to the sounding-board which causes it to yield slightly; and thus the string causes the sounding-board to vibrate. But though the amplitude of its vibration is small, the sounding-board is broad,

and the air cannot, by flowing round its edge, evade compression and rarefaction; the air is, accordingly, alternately compressed and rarefied, and thus a system of waves is effectively set up in it. Thus the loudness of the sound produced by a string may, by the use of a sounding-board, be multiplied many thousandfold. A similar experiment may be performed with a vibrating tuning-fork suspended in the air by a string, and the same fork vibrating while its shank is pressed against the panel of a door.

In these cases the energy of vibration of the string or tuning-fork is very much more rapidly dissipated, while the large-surfaced sounding-board is enabled to produce an intenser or louder sound than is produced when the string or the fork vibrates alone; and the vibration sooner comes to an end.

The speaking-trumpet is in part an application of the same principle. Instead of a comparatively small surface, the oral aperture, being the source of sound, the much broader aperture of the trumpet is practically converted into the source, and the broad sound-waves thence issuing are only slightly weakened at their origin by lateral flow.

As a general rule it is therefore advisable, when sound is to be heard at a distance, to make the sources of sound of the largest size convenient. Smallness of size may, however, be compensated by quickness of vibration.

Thus the chirp of certain insects is produced by such extremely rapid movements — as many as 12,000 to-and-fro vibrations per second — that the air is alternately compressed and rarefied on each side of the wings or in the neighbourhood of the stridulating organs, without having time to flow round them.

Characteristics of Sounds. — The Fourier-motions which may produce sounds differ amongst themselves in their

(*a*) **Frequency** — the number per second of the slowest component-oscillations.

An oscillation is a complete oscillation, once to-and-fro. The frequency of a seconds pendulum is ½; in one second it performs half a complete oscillation. In French works we find that a "*vibration simple*" is half a complete oscillation, a swing over from one side to the other; and a seconds pendulum is held to effect such "*vibrations simples*" at the rate of one per second. The reason for the apparently more artificial mode of defining an oscillation here used will be seen on considering the meaning of *period* in S.H.M. (p. 82); a complete oscillation restores the oscillating body to its starting point.

(*b*) They differ as to their **Energy.** Proportional to the energy are the Intensity and the Square of the Amplitude.

(*c*) They also differ as to the **Relative Amplitudes** of their **Components.**

Of these three particulars, the first, the frequency, depends on the vibrating body itself, its form, its material, etc., and upon its tension, but is very slightly affected by its viscosity; the second depends entirely on external causes; the third depends

partly on the form, the tension, the rigidity, etc., of the vibrating body, partly on the manner in which it is set in motion.

By variations in these particulars an infinite variety of Fourier-motions may be produced in vibrating or sounding bodies; and as a natural consequence we might expect to find, as we do find, an infinite variety of musical sounds actually occurring in nature.

Musical sounds may differ from one another in three corresponding respects, viz. — **Pitch, Loudness,** and **Quality** or *Character*.

Pitch. — The pitch of a clear musical sound depends on the Frequency of the Fundamental Vibration of the sounding body. Suppose a string to vibrate harmonically, and its component vibrations to occur 261, 522, 783, 1044, etc., times per second: then that string would have a *fundamental vibration* whose frequency is 261 per second; and a sound of this fundamental frequency is recognised by our musical sense as the note 

The **loudness** of a sound increases with the amplitude of oscillation of the vibrating body; if two strings, otherwise similar and similarly circumstanced, oscillate through ranges of $\frac{1}{8}$ and $\frac{1}{4}$ inch respectively, the latter has twice the amplitude and tends to produce four times as much sound as the former: the loudness or intensity of sound being, among sounds of the same pitch, proportional to the energy of vibration, and therefore to the square of the amplitude. Mark, however, that the relative loudness of *different* sounds as perceived by the ear is not to be measured by their physical intensity or the square of the amplitude of the vibrations at their source, for the ear is not necessarily, and is not in fact, equally sensitive to sound of every pitch.

Viscosity of a sounding body, while it scarcely affects the pitch, aids in causing the amplitude of the vibration, and therefore the loudness of the sound produced, gradually to dwindle away.

As to their **Quality** or **Character,** we find among sounds an infinite variety. We can distinguish a sound produced by a violin from one of the same pitch and loudness produced by a clarionet, a flute, or a pianoforte; we can distinguish the sound of a viola from that of a violin; one violin from another; one player from another on the same violin; one person's voice from that of another; the voice of the same person in different

moods or states of health. The basis of all this variety lies in the endless differences that may exist between Fourier-motions which, though they agree as to the frequency of their fundamental or slowest component and as to the total energy involved in their movement, do not necessarily coincide in the relative amplitudes of their component harmonic motions.

But if, as this theory indicates, an extended series of component vibrations go to make up the aggregate vibration of a sounding body, ought we not, in the sound produced by a sounding body, to hear a series of tones corresponding to the series of vibrational components? If a string produce the note , corresponding to a fundamental vibration whose frequency is 261 per second, ought we not, at the same time, to hear other sounds corresponding to 522, to 783, to 1044, etc., vibrations per second? The reply is that we do actually hear such tones; but we do not attend to them, and for practical purposes we are therefore deaf to them. We are accustomed to interpret a sound produced by a single sounding-body — the voice of a person, for example — as a single sound; from earliest infancy we unconsciously train ourselves to listen only to the fundamental tone of any single note: and the presence of the other tones of the really-compound sound produced by a single vibrating-body has the apparent result of determining the Character of that tone to which alone we consciously listen. In many cases, when we listen for the higher component sounds, knowing what to listen for, we can hear them, even with the unaided ear: after practice the ear acquires the power of recognising the presence of these **harmonics** with great readiness — a power which may easily become oppressive to its possessor. The special training which confers this power differs only in degree from that which enables one to discriminate the different notes which make up a chord, sounded in harmony; for to the untrained ear even a chord, if it be well in tune, seems to be a single mass of sound.

Noise. — If all the keys of a piano within the compass of one or two octaves be simultaneously struck, the result is a confused jangle, a Noise. Here we have the Superposition of Fourier-motions resulting in an apparently irregular disturbance of the air. This may go still farther; the Fourier-motions, which are superposed on one another, may have no relation of frequency and little or no individual persistence. The more markedly this is the case, the less musical will be the sound

produced, and the more markedly will it bear the character of noise. The general hum of a town is made up of sounds and cries, each of which, taken singly, may perhaps not be unmusical; but because they are not related to one another by any simple numerical ratio of frequency, they together produce the disagreeable effect of a noise. Noises, then, such as the sound of steam escaping from a boiler, wind rushing through trees, the clatter of falling objects, and so forth, may be considered to be produced by the superposition of a number of distinct musical sounds. Some of these may predominate in intensity and in persistence; and thus a noise may have a distinguishable pitch. We may recognise differences in pitch between the noises produced by drawing the thumb-nail at various speeds over the cover of a book bound in cloth, by blowing across the mouth of keys or tubes or flasks of various sizes, by letting boards of various sizes fall on a wooden floor, by blowing through glass tubes on which bulbs of various sizes have been blown, and so forth. Even where the original disturbance is in the highest degree irregular, as where bricks are pitched out of a cart, the elasticity of the bricks, small though it be, affects the pitch of the noise produced, for the thuds produced by soft porous bricks are graver than the clinks produced by hard glazed-bricks of the same size.

If we listen to a continuous noise with the aid of a resonator (p. 430) tuned to some particular tone, we can often recognise the presence of that tone as a component of the noise; the resonator will, if that tone be present as a component, sound it forth—continuously if it be continuously present; intermittently if it occur at intervals only.

Even a single vibrating-body may, when struck, produce a noise. A bell is not, with ease, so cast as to be perfectly uniform; when struck it tends, if not quite uniform, to divide into unequal sectors, each of which pulsates at its own rate; the physical result is a number of simultaneous vibrations bearing no simple relation to one another, and the physiological result is a mixed sensation, a jangle, a kind of noise.

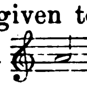
Thus sounds originate in Fourier-motions; a musical note in a single Fourier-motion; a noise in a number of simultaneous Fourier-motions whose fundamental frequencies bear to one another no simple numerical relation; and, as we shall afterwards see, the sensation of Harmony in a number of simultaneous Fourier-motions whose fundamental frequencies have simple numerical relations to one another.

The simplest possible sound would be one produced by a vibration in which the Fourier-motion was represented by one component; such a sound would be a **pure Tone.**

The **pitch** of the sound or note produced by a vibrating body is the pitch of the gravest component, the fundamental *Tone*; and it may be specified in two ways: —

(1.) **Physically,** by stating the number of vibrations per second which correspond to that fundamental tone;

(2.) **Musically,** by referring the tone to its place in an arbitrary scale of pitch in conventional use among musicians.

To find the frequency of vibration corresponding to any given note: — As the note in question let us take, for the sake of example, that produced by an ordinary "A" tuning-fork. A card or strip of metal is placed so as to touch at one end the cogs of a little cog-wheel, while the other end is firmly fixed; the wheel is rotated slowly — each cog makes one click; more rapidly — the clicks blend into a hum; still more rapidly — the hum rises in pitch, and the faster the rotation the shriller becomes the sound; at a certain rate of rotation the sound is neither graver nor shriller than that produced by the tuning-fork; this rate of rotation is such that the card is struck 435 times per second; 435 impulses per second given to the card, and by the card to the air, produce the sound , "$a' = 435$." Higher sounds are due to more rapid, lower sounds to slower, vibrations than this. This arrangement is known as *Savart's Wheel.*

Another contrivance, devised to the same end, is the *Syren.* A rotating disc is pierced by holes arranged equidistantly in a circle, whose centre is in the axis of rotation of the disc. A tube brings a current of air to a spot near the disc, so situated that in some positions of the disc the air can blow clear through one or other of the holes, while in others the current of air is almost cut off by the disc itself. Rotate the disc; the current of air is alternately cut off by the disc and allowed to blow through it. If there be 87 holes in the circle of holes, and if the disc rotate five times per second, there are then produced 435 puffs of air per second, and the note "a'" is heard: its quality is, however, decidedly inferior, for the principal sound heard is the noise made by the current of air when it strikes the disc. If the current be divided by 87 pipes, so as to blow through the 87 holes simultaneously, and to be simultaneously cut off from them all, the sound is very much clearer and louder than when there is only a

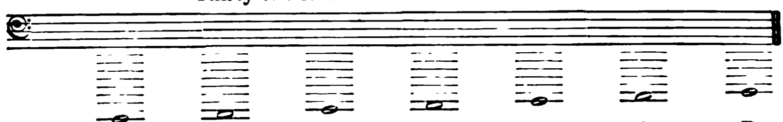
single stream of air blowing through one hole at a time. Instead of 87 pipes issuing from a wind-chest, we may employ a wind-chest capped by a fixed disc containing 87 holes, arranged in a circle like that of the rotating disc: the rotating disc rotates in the immediate vicinity of the fixed one: simultaneously the air rushes through all the apertures of the rotating disc, simultaneously it is cut off from them all. The number 87 is in practice never used; some such number as 24 or 48 is chosen. Connected with the rotating disc is some form of mechanism for recording the number of rotations effected by it in a given time. The rotating disc is caused to rotate at such a speed as causes the desired sound to be produced: the number of apertures in the disc, multiplied by the number of rotations per second, gives the number of impulses per second imparted to the air, and thus determines the frequency of the tone in question. The syren works under water as well as it does in air.

The experiment already described on page 412 also gives roughly the means of finding the frequency of any given tone. The thin strip of metal is, by successive trial, carefully withdrawn into the vice, until its free part gives, when set in vibration, a sound of precisely the same pitch as the tone whose frequency is to be determined. Say that this length is 1 inch; and also that if 30 inches of the strip be free, it executes 29 complete oscillations per minute. The number of oscillations varies inversely as the square of the length; whence (1 inch)^2: (30 inches)^2 : : 29 : x, or $x = 26{,}100$ vibrations per minute, 435 per second.

Still another method of determination of the frequency of vibration of sound, of a given pitch, is graphically to record the actual vibrations of the sounding body. A tuning-fork has a little feather-barb attached by cement to one of its prongs: the extremity of the barb is brought into contact with slightly-smoked paper spread over the surface of a cylinder. The cylinder is caused to rotate; the point of the barb draws a straight line on the smoked paper. The fork is caused to vibrate: the barb now describes, on the rotating cylinder, a sinuous line which records the oscillations of the tuning-fork. An independent mechanism can be made to mark the cylinder once every second, and thus the absolute number of oscillations made by the tuning-fork during each second can be counted on the permanent record. The same principle may be applied to many forms of vibrating body, such as strips of metal, membranes, etc.

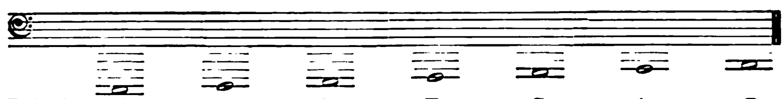
Musical Pitch.—The arbitrary scale of pitch in common use, and typified by the white keys of a pianoforte, is the following:—

Thirty-two foot Octave—Subcontra Octave.



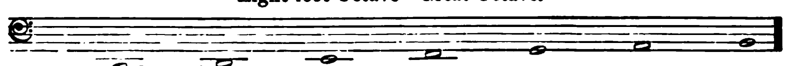
British	C$_{,,}$	D$_{,,}$	E$_{,,}$	F$_{,,}$	G$_{,,}$	A$_{,,}$	B$_{,,}$
German	C$_{,,}$	D$_{,,}$	E$_{,,}$	F$_{,,}$	G$_{,,}$	A$_{,,}$	H$_{,,}$
French	ut$_{-1}$	re$_{-1}$	mi$_{-1}$	fa$_{-1}$	sol$_{-1}$	la$_{-1}$	si$_{-1}$
No. of Vibrations	16·3125	18·3515625	20·390625	21·75	24·46875	27·1875	30·5859375.
Ratios	16 :	18 :	20 :	21·$\dot{3}$:	24 :	26·$\dot{6}$:	30.

Sixteen-foot Octave—Contra Octave.



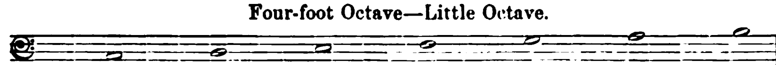
British	C$_{,}$	D$_{,}$	E$_{,}$	F$_{,}$	G$_{,}$	A$_{,}$	B$_{,}$
German	C$_{,}$	D$_{,}$	E$_{,}$	F$_{,}$	G$_{,}$	A$_{,}$	H$_{,}$
French	ut$_0$	re$_0$	mi$_0$	fa$_0$	sol$_0$	la$_0$	si$_0$
No. of Vibrations	32·625	36·703125	40·78125	43·5	48·9375	54·375	61·171875.
Ratios	32 :	36 :	40 :	42·$\dot{6}$:	48 :	53·$\dot{3}$:	60.

Eight-foot Octave—Great Octave.



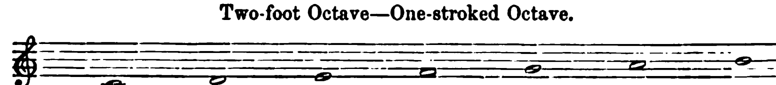
British	C	D	E	F	G	A	B
German	C	D	E	F	G	A	H
French	ut$_1$	re$_1$	mi$_1$	fa$_1$	sol$_1$	la$_1$	si$_1$
No. of Vibrations	65·25	73·40625	81·5625	87	97·875	108·75	122·34375.
Ratios	64 :	72 :	80 :	85·$\dot{3}$:	96 :	106·$\dot{6}$:	120.

Four-foot Octave—Little Octave.



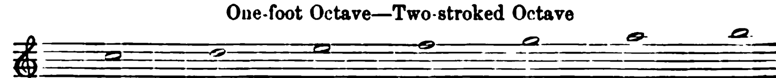
	c	d	e	f	g	a	b
	ut$_2$	re$_2$	mi$_2$	fa$_2$	sol$_2$	la$_2$	si$_2$
No. of Vibrations	130·5	146·8125	163·125	174	195·75	217·5	244·6875.
Ratios	128 :	144 :	160 :	170·$\dot{6}$:	192 :	213·$\dot{3}$:	240.

Two-foot Octave—One-stroked Octave.



	c$'$	d$'$	e$'$	f$'$	g$'$	a$'$	b$'$
	ut$_3$	re$_3$	mi$_3$	fa$_3$	sol$_3$	la$_3$	si$_3$
No. of Vibrations	261	293·625	326·25	348	391·5	435	489·375.
Ratios	256 :	288 :	320 :	341·$\dot{3}$:	384 :	426·$\dot{6}$:	480.

One-foot Octave—Two-stroked Octave



	c$''$	d$''$	e$''$	f$''$	g$''$	a$''$	b$''$
	ut$_4$	re$_4$	mi$_4$	fa$_4$	sol$_4$	la$_4$	si$_4$
No. of Vibrations	522	587·25	652·5	696	783	870	978·75.
Ratios	512 :	576 :	640 :	682·$\dot{6}$:	768 :	853·$\dot{3}$:	960

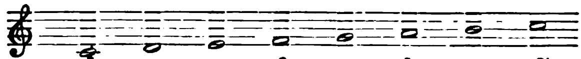
Six-inch or Three-stroked Octave. Three-inch or Four-stroked Octave.

	c'''	d'''	e'''	f'''	g'''	a'''	b'''	c''''	d''''	e''''	f''''	g''''	a''''	b''''	c'''''
	ut_5	re_5	mi_5	fa_5	sol_5	la_5	si_5	ut_6	re_6	mi_6	fa_6	sol_6	la_6	si_6	ut_7
No. of Vibrations	1044	1174·5	1305	1392	1566	1740	1957·5	2088	2349	2610	2784	3132	3480	3915	4176.
Ratios	1024 :	1152 :	1280 :	$1365\cdot\dot{3}$:	1536 :	$1706\cdot\dot{6}$:	1920 :	2048 :	2304 :	2560 :	$2730\cdot\dot{6}$:	3072 :	$3413\cdot\dot{3}$:	3840 :	4096.

The starting-point of this notation is the a' tuning-fork, made to vibrate 435 times per second, or the second string of the violin, made to vibrate in unison with such a fork. Under this system the c'' tuning-fork makes 522 complete oscillations per second. This is entirely a matter of convention. The number 435 was chosen by the Académie des Sciences of Paris; 433 by the Philharmonic Society under Sir George Smart in 1826; 440 by the German Society of Nature-researchers at Stuttgart in 1834; 452 is used in the British Army; while a pitch $a'=426\cdot\dot{6}$ has been highly recommended, on the ground that under such a system the tones $C_{,,}$, $C_{,}$, C, c, c', etc., are produced by 16, 32, 64, 128, 256, etc., vibrations per second — an arrangement which has the advantage of giving very simple numbers to deal with, but which has, on the other hand, the practical disadvantage of giving a pitch which is too low to please instrumentalists, and the didactic disadvantage of tending to conceal the real arbitrariness of the convention which assigns to the a' or the c'' fork the particular number of vibrations chosen in practice. In practice there is, indeed, a great lack of agreement; instrument-makers are constantly raising the pitch for the sake of increasing the brilliancy of orchestral music, while vocalists are made to suffer. Modern concert pitch has thus risen as high as $a'=460$ vibrations per second, about $1\frac{1}{2}$ semitone above what it was in England in the time of Handel ($a'=424$), while the organ-pitch in England was, in the middle of the eighteenth century, as low as $a'=388$. If the standard number of vibrations chosen for a' be any other than 435, the whole series of numbers given in the table must suffer a proportionate increase or reduction. The accuracy of such a scale depends not upon precision of absolute numbers of vibrations so much as upon correctness of the ratios of the several numbers to one another.

The successive tones of the scale of C are related to one

another, with respect to their frequency, in the following manner: —



d	r	m	f	s	l	t	d'
256 :	288 :	320 :	341·3 :	384 :	426·6 :	480 :	512
1 :	$\frac{9}{8}$:	$\frac{5}{4}$:	$\frac{4}{3}$:	$\frac{3}{2}$:	$\frac{5}{3}$:	$\frac{15}{8}$:	2.

Here C ($c' = 256$) is a keynote, and upon it we have raised a diatonic major scale, **d r m f s l t d'**.

Such a scale is found by experience to be satisfying to the ears of the Western nations; and whatever tone be chosen as the keynote, there can always be sung or played on instruments of the violin or of the trombone class a scale of this kind, in which the intervals are felt to be pleasing and in tune, in which the intonation is felt to be just, and in which each tone, when it is carefully listened to while the keynote is borne in mind, is felt to have its own peculiar mental effect, this depending on its relative place in the scale, and not on its absolute vibrational frequency. Singers who have sung much together, string players who have practised together without pianoforte accompaniment, naturally use the tones of such a scale without knowing or even caring what the numerical ratio of the frequencies of the various tones of the scale may be.

Intervals. — We may now identify the various intervals occurring within the diatonic scale —

Minor second, "*semitone*"	**m : f** or **t : d'**	15 : 16
Grave major-second	**r : m** or **s : l**	9 : 10
Major second	**d : r**, **f : s**, **l : t**	8 : 9
Grave (or Pythagorean) minor-third	**r : f**	27 : 32
Minor third	**m : s** or **l : d'**	5 : 6
Major third	**d : m**, **f : l**, **s : t**	4 : 5
Perfect fourth	**d : f**, **r : s**, **m : l**, **s : d'**, **t : m'**	3 : 4
Acute fourth	**l : r'**	20 : 27
Augmented fourth	**f : t**	32 : 45
Grave diminished fifth	**t_1 : f**	45 : 64
Grave fifth	**r : l**	27 : 40
Perfect fifth	**d : s**, **m : t**, **f : d'**, **s : r'**, **l : m'**	2 : 3
Minor sixth	**t_1 : s**, **m : d'**, **l : f'**	5 : 8

Major sixth . . .	d : l, r : t, s : m' . .	3 : 5
Acute major-sixth .	f : r'	16 : 27
Grave minor-seventh	r : d', s : f', t : l' . .	9 : 16
Minor seventh .	m : r', l : s' . . .	5 : 9
Seventh	d : t, f : m' . . .	8 : 15
Octave	d : d', r : r', etc. . .	1 : 2

Musical intervals are equal to one another when the constituent tones in each have the same relative frequency. Thus **d : s : : 1 :** $\frac{3}{2}$, and **m : t : :** $\frac{5}{4}$: $\frac{15}{8}$; the ratio of 1 to $\frac{3}{2}$ is equal to that of $\frac{5}{4}$ to $\frac{15}{8}$ — that is, it is 2 : 3; whence the musical interval between **d** and **s** is equal to that between **m** and **t.**

Transition. — Any tone may be chosen as a keynote. Let us choose $g' = 384$ as our keynote, and then compare the tones of the scale of the key of G with those of the scale of C. Retaining the same ratios, the scale of G is

d : r : m : f : s : l : t : d'.
1 : $\frac{9}{8}$: $\frac{5}{4}$: $\frac{4}{3}$: $\frac{3}{2}$: $\frac{5}{3}$: $\frac{15}{8}$: 2.
384 : 432 : 480 : 512 : 576 : 640 : 720 : 768.

Comparing the two scales we find: —

Scale of C ("Key C").

c' d' e' f' g' a' b' c'' d'' e'' f'' g'', etc.
. . . . 384 : 426·$\dot{6}$: 480 : 512 : 576 : 640 : 682·$\dot{6}$: 768.

Scale of G ("Key G").

384 : 432 : 480 : 512 : 576 : 640 : 720 : 768.

The tones agree with the exception of the *a*'s and the *f*'s. The *a'* of the scale of C and the *a'* of the scale of G differ from one another in the ratio of 426·$\dot{6}$: 432, or 80 : 81. The two tones are perfectly distinct, and an ear that has become accustomed to the pure scale of C is pained, especially in harmony, by the substitution, for the proper *a'* in that scale, of the slightly sharper *a'* which belongs to Key G. The difference between the two tones is called a Comma; and they may be respectively written *a'* and *'a'*. The *f''* of Key C and the corresponding tone in the scale of G differ more widely from one another; their frequency-ratio is 682·$\dot{6}$: 720, and the interval between them, $\frac{135}{128}$, is sometimes called a semitone.

In order to play in correct tune music written in Key G as well as music written in Key C, we would require not only the tones of the Key of C, but also two additional tones in each

octave. Every transition from one Key to another "more remote from" the Key of C multiplies the demand for new tones; and that to an extent twice as great as the current notation, which neglects differences of a comma, would seem to indicate.

In the table, pages 426 and 427, are given the tones of the scale of C, together with a number of tones derived from related keys. The relative, not the absolute, number of vibrations has been shown in each case.

If a singer were called upon to produce a note of 324 vibrations per second, the feat would be impossible. This number is, however, 1·265625 × 256; and hence if c' have 256 vibrations per second, the note required is the *re* of Key D. A tuning-fork $c' = 256$ is set in vibration; call the note of the fork *do*; sing *do*, *re*; fix the attention on *re* (d'); call it *do* without changing its pitch; dwell on it a moment; then sing some such phrase as *do*, *mi*, *sol*, *do*, *mi*, re, *do*; and the desired note, $'e'$, a note differing by a comma from e', the *mi* of Key C, has been produced, the sense of tonality and key-relationship having carried the singer into the correct sound.

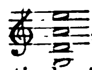
The column headed "logarithmic increments" contains figures which measure the intervals between the successive tones; for it gives the logarithms of the frequency-ratios between each tone and its predecessor; and the most convenient method of comparing ratios is to compare their logarithms. When the logs. are equal the ratios are equal; when the ratios are equal the intervals are equal. Thus the intervals between C and C♯, D♭ and ‘D, ‘D and ‘D♯, D and D♯, E♭ and E, E and E♯, F and F♯, 'F and 'F♯, ‘G♭ and ‘G, G♭ and G, G and G♯, A♭ and A, A and A♯, 'A and 'A♯, ‘B♭ and ‘B, B♭ and B, B and B♯, are all equal, being measured in that column by the logarithm ·0177288, which is the log. of the ratio $\frac{25}{24}$. Again, we find a number of lesser intervals whose log. is ·005395, and whose ratio is $\frac{81}{80}$: these are C and 'C, C♯ and 'C♯, ‘D♭ and D♭, ‘D and D, ‘D♯ and D♯, ‘E♭ and E♭, E and 'E, F and 'F, F♯ and 'F♯, ‘G♭ and G♭, ‘G and G, G and 'G, ‘G♯ and G♯, ‘A♭ and A♭, A and 'A, A♯ and 'A♯, ‘B♭ and B♭, ‘B and B, B and 'B, ‘*c* and *c*. Between these the interval is a Comma.

The scale may be seen to be roughly divisible into 53 steps or divisions; but these are not equal to one another; if they were equal, the logarithms would at each step acquire an equal increment; for the ratio between each tone and its predecessor would be equal throughout the scale. Roughly and for diagrammatic purposes it is, however, convenient to represent the interval between C and D by 9 steps, while that between D and

E is represented by 8: and the table is so arranged. A thoroughly accurate table of this kind would be one engraved on metal, the intervals between any two tones in column 3 being made directly proportional to the log. increment between them.

The intervals marked Pythagorean in the table are thus derived:—Start from *c* and go upwards by successive fifths, *c*, *g*, *d'*, *'a'*, *'e''*, *'b''*; going downwards we arrive at F, `B♭,, `E♭,, `A♭,,.

The following exercises will perhaps aid the reader:—

(*a*) If the violin be tuned , to correct fifths, starting with *a'*; show that these notes are respectively *e''*, *a'*, `*d'*, `*g*.

(*b*) The scale of "B♭ major" is obtained by transition from the key of C to that of F, and from that of F to that of "B♭." The scale of "G minor" has $g = la$, and thus $do = b♭$. Show that the respective descending scales are—

...	d'	t	l	s	f	m	r	d	....	"B♭ major."
...	`*b*♭,	*a*,	`*g*,	*f*,	`*e*♭,	`*d*,	*c*,	`B♭	....	
....			*g*,	'*f*,	*e*♭,	*d*,	'*c*,	B♭,	'A, G ..	"G minor."
....			l	s	f	m	r	d	t, l, ..	

The columns in the table, pp. 426–427, headed "Equally-tempered Scale," show the nature of the system of Equal Temperament, which is, as nearly as practicable, applied to the pianoforte and organ. The intervals are equal; the ratio between a tone and its predecessor and successor is in every case the same; between each pair of tones the logarithmic increment is equal: it is $\frac{\log 2}{12} = \frac{\cdot 301300}{12} = \cdot 025058\dot{3}$. The result differs widely from pure intonation; but we are accustomed to it. On a pianoforte equally tempered the fifths are not appreciably out of tune, though they are a little flat: but the thirds, three of which are forced to make an octave instead of extending only from C to B♯, are too sharp; and though this be not offensive on the pianoforte, to which indeed their sharpness lends somewhat of brilliancy, yet in slow sustained harmony these sharp thirds are really discordant, as may be well heard on a loud harmonium tuned in the usual manner, and on which thirds alone are played.

Loudness.—The physical Intensity of a sound depends initially on the square of the amplitude of the vibration of the sounding body; but the corresponding sensation of loudness depends not only upon peculiarities of sensitiveness of the ear, but also on the amount of physical disturbance of its drum, and

NATURAL SCALE OF C WITH TONES FROM SOME RELATED KEYS.								EQUALLY-TEMPERED SCALE OF C.	
Intervals with Keynote.				Frequency-Ratios relative to the Keynote.		Logarithm of the Frequency-Ratio.	Logarithmic Increment.		
Keynote . . .	d	C	... ...	1 : 1	1·000000	0·0000000		C	1·000000.
		'C	r_1 of Key B♭ . . .	81 : 80	1·012500	0·0053950	·0177288		
		—	... ...	...	...	...			
		C♯	l_1 of Key E . . .	25 : 24	1·041666̇	0·0177288	·0053950		
		'C♯	t_1 of Key D, r of Key B .	135 : 128	1·0546875	0·0231238	·0049049		
Minor second . .		'D♭	d when C is t_1 . .	16 : 15	1·066666̇	0·0280287		"C♯ or D♭"	$1·059463 = \sqrt[12]{2}$.
		D♭	r when A♭ is l . .	27 : 25	1·080000	0·0334237	·0177288		
		—	... ...	...	...	...			
Grave major-second .		'D	l_1 of Key F . . .	10 : 9	1·11111̇	0·0457575	·0053950		
Major second . . .	r	D	... ...	9 : 8	1·12500	0·0511525		"D or C𝄪"	$1·122462 = \sqrt[12]{2^2}$.
		—	... ...	...	...	...	·0123337		
		'D♯	l_1 of Key F♯ . . .	125 : 108	1·157̇40̇	0·0634862	·0053951		
		D♯	t_1 of Key E . . .	75 : 64	1·1671875	0·0688813	·0049049		
Pythagorean minor-third		'E♭	f_1 of Key 'B♭ . . .	32 : 27	1·̇18518̇5	0·0737862	·0053950	"D♯ or E♭"	$1·189207 = \sqrt[12]{2^3}$.
Minor third . . .		E♭	d when C is l_1 . .	6 : 5	1·200000	0·0791812			
		—	... ...	...	...	...	·0177288		
		—	... ...	...	...	...			
Major third . . .	m	E	... ...	5 : 4	1·25000	0·0969100	·0053950		
Pythagorean major-third		'E	r of Key D . . .	81 : 64	1·265625	0·1023050		"E"	$1·259921 = \sqrt[12]{2^4}$.
		F♭	f when A♭ is l . .	32 : 25	1·280000	0·1072100	·0123338		
		E♯	t_1 of Key F♯ . . .	125 : 96	1·302083̇	0·1146388			
		—	... .	...	...	...	·0122999		
Perfect fourth . .	f	F	... ...	4 : 3	1·33333̇	0·1249387	·0053950	"F or E♯"	$1·334839 = \sqrt[12]{2^5}$.
Acute fourth . . .		'F	s when G is l . . .	27 : 20	1·350000	0·1303337			
		—	... ...	...	...	...	·0123338		
Augmented fourth .		F♯	l_1 of Key A . .	25 : 18	1·388888̇	0·1426675	·0053950		

Acute augmented fourth		'F♯	t_1 of Key G, r of Key E .	45 : 32	1·406250	0·1480625	·0049050	"F♯ or G♭"	$1·414213 = \sqrt[12]{2^6}$.
Grave diminished fifth .		'G♭	s when 'A♭ is l . .	64 : 45	1·422222	0·1529675	·0053950		
Diminished fifth . .		G♭	s when A♭ is l . .	36 : 25	1·440000	0·1583625	·0123338		
		—	... ...	...	...	...			
Grave fifth . . .		'G	l of Key 'B♭ . . .	40 : 27	1·48148	0·1706963	·0053950		
Perfect fifth . . .	s	G	... ...	3 : 2	1·50000	0·1760913	·0177288	"G"	$1·498307 = \sqrt[12]{2^7}$.
Acute fifth . . .		'G	Fifth above 'C . .	243 : 160	1·51875	0·1814863			
Grave augmented fifth .		'G♯	Fourth above 'D♯ . .	125 : 81	1·54321	0·1884251			
Augmented fifth . .		G♯	m of Key E, t_1 of Key A .	25 : 16	1·56250	0·1938201	·0049049		
Grave minor sixth . .		'A♭	f of Key 'E♭ . . .	128 : 81	1·58024	0·1987250	·0053950	"G♯ or A♭"	$1·587402 = \sqrt[12]{2^8}$.
Minor sixth . . .		A♭	d when F is l_1 . .	8 : 5	1·60000	0·2041200	·0177288		
		—	... ...	...	...	...			
		—	... ...	...	...	...			
Major sixth . . .	l	A	... ...	5 : 3	1·66666	0·2218488	·0053950		
Pythagorean major-sixth		'A	r of Key G, s of Key D .	27 : 16	1·68750	0·2272438	·0123337	"A"	$1·681793 = \sqrt[12]{2^9}$.
		—	... ...	...	...	...			
Augmented sixth . .		A♯	m of Key F♯ . . .	125 : 72	1·73611	0·2395775	·0053950		
Acute augmented sixth .		'A♯	t_1 of Key B . . .	225 : 128	1·7578125	0·2449725	·0049050		
Grave or Pythagorean minor-seventh		'B♭	f of Key F . . .	16 : 9	1·77777	0·2498775	·0053950	"A♯ or B♭"	$1·781797 = \sqrt[12]{2^{10}}$.
Minor-seventh . .		B♭	d when G is l_1 . .	9 : 5	1·80000	0·2552725	·0123338		
		—	... ...	...	...	...			
Grave seventh . .		'B	f of Key F♯ . . .	50 : 27	1·85185	0·2676063	·0053950		
Seventh	t	B	... ...	15 : 8	1·87500	0·2730013	·0053950		
Pythagorean seventh .		'B	... ...	243 : 128	1·8984375	0·2783963	·0123338	"B or C♭"	$1·887748 = \sqrt[12]{2^{11}}$.
		—	... ...	...	...	...			
Augmented seventh .		B♯	m of Key G♯ . . .	125 : 64	1·953125	0·2907301	·0049049		
		'c	l of Key 'E♭ .	160 : 81	1·975301	0·2956350	·0053950		
Octave	d'	c	... ...	2 : 1	2·0000	0·3010300		"C or B♯"	2·000000.

if the sound be conducted to the ear by the air, it depends on the intensity of vibration of the air near the ear; and this varies not only (1) as the square of the amplitude of the original vibration, but also, in the open air, (2) inversely as the square of the distance of the sounding object.

To compare the relative loudnesses of two sounds of nearly the same pitch, place the sounding bodies at such distances that they become just inaudible, and no more: say that the one becomes inaudible at 10, the other at 50, yards: then the loudness of the one at 50 yards' distance is at the ear equal to that of the other at 10 yards: their initial intensities must be as $10^2 : 50^2$, or $1 : 25$.

If the sound be not propagated in free air, but be confined in a tube, the loudness of sound may diminish at a much less rate, for ultimately the waves become plane-fronted, and move down the tube without any loss of intensity other than what is due to such loss of energy as is brought about by friction against the sides of the tube or by the viscosity of the air itself.

Hence sounds can be carried along sewers, speaking-tubes, etc., to great distances without great diminution of loudness.

Similarly, if sound be propagated by parallel or convergent waves in the air, as when it issues from a wide aperture, or after reflexion from a curved surface, it may lose little of its intensity, or may even concentrate its intensity on some particular place.

The loudness of a sound also depends, if it be conveyed by a gaseous medium, on the density of that medium at the place where the vibration is imparted to it. The denser the medium the greater its inertia, and the more readily it is compressed against itself: the greater the compression, the greater the amount of energy imparted to the medium, and the louder the sound produced. A body vibrating *in vacuo* produces no sound: in rarefied air or hydrogen, or any other rare or rarefied gas, it produces a comparatively feeble sound; in carbonic acid it produces a louder sound than in air. A cannon fired on a mountain-top produces little sound; one fired beneath is heard distinctly and loudly from a balloon, even at a great height.

Concentration of sound-waves renders sounds louder, as in ear-trumpets and in those stethoscopes the auditory extremity of which fits into the ear.

Quality of Sound.—If a body vibrate so as to produce a

sound of the fundamental pitch C = 64, and if all the harmonics be present, the series is the following: —

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15, etc.
64	128	192	256	320	384	448	512	576	640	704	768	832	896	960
C	*c*	*g*	*c'*	*e'*	*g'*	*b♭'* −	*c''*	*d''*	*e''*	*f''* +	*g''*	*a''* +	*b♭''* −	*b''*, etc.
						*				*		*	*	

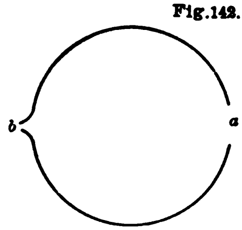
These are all tones of the scale of C, with the exception of the 7th and its octave the 14th, the 11th, and the 13th. The 7th and the 14th correspond to a very flat B of 112 vibrations, lying between A♯ and 'A♯; the 11th to a sharp F of 88 vibrations, lying between 'F and F♯; the 13th to a flat A, lying between A♭ and A.

Analysis of a sound into its components may be effected by several methods, of which we shall first consider one due to Prof. Mayer. As our example we take the sound produced by a vibrating organ-reed-pipe, a sound which we recognise as peculiar and characteristic. We are provided with a set of tuning-forks, one of which vibrates in exact unison with the fundamental tone of the organ-pipe, and the rest of which respectively vibrate 2, 3, 4, 5, etc., times as rapidly. As to the organ-pipe, a part of its wall has been replaced by a piece of inelastic thin morocco leather, or some similar substance, which vibrates exactly as the air within the pipe does. To one point of this is attached a bundle of silkworm-cocoon-threads, 40 inches or so in length: each of these is attached to one of the tuning-forks and tightened somewhat. The organ-pipe is caused to sound; the leather vibrates; the silk fibres are all set in motion, and each alternately tugs and releases its own tuning fork. If the vibration appropriate to any one of the tuning-forks be present in the original compound vibration, the corresponding fork is set in motion: if it be not present, that fork remains silent: if the vibration be ample, the fork sounds out loudly: if it be not, the sound is feeble. This arrangement analyses the sound into its components, for it can be seen which of the tuning-forks are set in vibration; and if the organ-pipe cease sounding, the forks go on sounding for some time, and by their joint action produce a compound sound closely resembling the sound of the reed-pipe which had been the means of setting them in vibration. This action is very exact: the slightest difference between the natural rate of any tuning-fork and that of the corresponding organ-pipe vibration

causes the fork to sound with comparative feebleness, or not to sound at all.

Resonators are extensively used as a means of analysis of sound. A resonator consists in its most usual form of a bulb, generally of glass or of brass, with a large aperture, *a*, at one side and a small one, *b*, at the other. The air within such a bulb has a natural period of vibration which depends upon the cubic contents of the resonator and upon the size of the orifices. This period can be found by the pitch of the sound produced on tapping the resonator with a soft substance, or by blowing brief blasts of air across its mouth. If the air convey a system of waves which agree in period, either absolutely or approximately, with the natural free vibration of the air in the resonator, the air in the resonator will absorb the energy of those waves, will be set in motion, and will act as a sounding body. If we be provided with a set of such resonators, the air in one of which freely vibrates in unison with an *a'* tuning-fork, and in the others respectively 2, 3, 4, 5, 6, 7, etc., times as rapidly, — then, on listening to an *a'* organ-reed-pipe, one ear being closed and the other adapted to each resonator in succession (this being done by fitting the nipple *b* of the resonator (Fig. 142) into the ear), we shall, if the proper sound of any of the resonators be contained in the complex sound to which we listen, hear that resonator loudly sing out its proper tone; while, if it be not present, we shall simply hear the ordinary sound of the pipe through the resonator, without any reinforcement. And further, if we fill our ears with the sound of the tone thus sung out by the resonator, and remember its pitch, we shall, when the note is again sounded out by the organ-pipe, have no difficulty, even without a resonator, in hearing the harmonic tone: and by dint of practice we may hear at will, or even independently of will, many if not all of those component harmonic tones which, by accompanying that fundamental tone to which alone in ordinary circumstances we are accustomed to listen, help to make up the note of the organ-pipe.

Fig. 142.



A very convenient form of resonator may be made of a common tall lamp chimney or a similar piece of tubing. If it be held vertical, as its lower end is immersed in water to various depths its natural pitch varies: and a tube thus gradually lowered into water is capable of resounding in succession to the different harmonics of a fundamental note, so that the ear,

placed near the tube, can recognise their several presences. In Wintrich's resonator an aperture at the side may be closed by the finger. By aid of the same resonator an observer is thus enabled to listen alternately to the grave-pitched muscle-sound of the heart and to the sharper valve-stretching sound. See Gscheidlen, *Physiol. Methodik.*

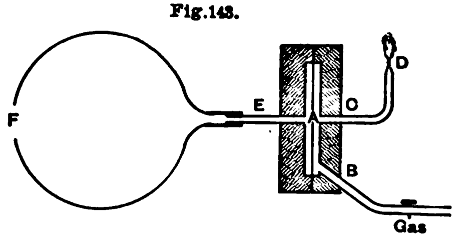
Resonators may be otherwise employed. If the small aperture *b* be stopped with wax, and if the resonator be brought near a sounding body, it will absorb the energy of any vibration corresponding to its own proper tone, and may then be removed and listened to: thus each one of a set of resonators may be made to select one tone out of the group of tones present in an ordinary musical sound, and to bring it to the ear to be listened to.

A resonator and a sounding body to which it is in response seem to be mutually repelled, in consequence of the stresses set up in the intervening air (Dvořak).

Resonators may further be used to transfer the energy, which they thus take up, to relatively-massive bodies such as tuning-forks. A resonator may be made in the form of a thin wooden box with open ends: a tuning-fork precisely in tune with it is fitted on its upper surface; a sound causes the air in the box to vibrate, the air acts upon the box, and the box upon the tuning-fork; if all be in exact unison, the energy accumulates in the tuning-fork, which comes to vibrate energetically and to produce a loud sound.

Again, the oscillation of the air in a resonator may be rendered visible by the following device:—A cavity in a block of wood (Fig. 143) is divided into two parts by a membrane, such as thin gold-beater's skin. The one moiety of the cavity is connected with the cavity of a resonator: the other is connected with a supply of coal-gas which enters at B and passes out at C on its way to be burned at the jet D. This contrivance is called Koenig's manometric capsule. When a sound is produced outside F, containing as one of its component tones the proper tone of the resonator, the air in the resonator oscillates in sympathy with that component, the diaphragm vibrates with it, and the flame at D is rendered alternately higher and lower

Fig. 143.



by the action of the vibrating diaphragm on the stream of gas. The flame obviously alters its character: and the change undergone by it can be studied by looking at it while the head is turned rapidly from side to side, the eyes being kept fixed relatively to the head; or by looking at the flame through an opera-glass, which is rapidly moved across the field of view; or, best of all, by looking not directly at the flame but at its image in a rapidly-rotating mirror: in all which cases the flame or its image appears to spread out, not into a uniform band of light, but into a band with serrated edges, or even into a chain of bead-like separate images.

A sufficiently-extensive set of resonators would thus enable us to effect the analysis of sounds of any degree of complexity: but resonators do not furnish us with as delicate a means of investigation as the means first described, unless indeed they be each allied with a tuning-fork; they respond in general with excessive readiness to any tone in proximity to their own natural tone.

Synthesis of Sound.—Von Helmholtz showed that any quality of sound may be built up by the superposed effect, upon the ear, of simultaneously sounding tuning-forks of the proper number, pitch, and relative loudness.

Complex Sound-Waves.—The pitch, the loudness, and the quality of a sound may be studied together by causing sound-waves to impinge directly upon some sensitive body without any intermediate process of selection or filtration. Thus, if instead of a resonator, as in Fig. 143, a cone be adapted to a manometric capsule, and if sound be produced at the mouth of the cone, sound-waves will impinge directly upon the membrane in A. The membrane will go through a complex motion somewhat resembling the original compound-vibration of the sounding body, and the flame will demonstrate this by its variations of height. The image of this oscillating flame will appear in a mirror, if the mirror be made to revolve, as a band of light, serrated by large teeth, whose outline is broken by subsidiary serrations; the number and size of the greater serrations indicate the frequency and amplitude of the fundamental vibration: those of the subsidiary serrations vary with the number and variety of the subsidiary vibrations. This experiment may be roughly carried out, if there be no revolving mirror at hand, by whirling the gas-flame itself (a rat's-tail jet at the end of a flexible tube) before the eye.

It is interesting to carry out this experiment by singing into the open end of the cone; even among notes of the same pitch sung to the same vowel, the association of different forms of the flame-image with different qualities of tone and different subjective sensations is very striking; and it is possible for a singer to attain to the production of very pure tone — such pure tone having, however, a somewhat hollow quality — by finding out for himself how to control the larynx so as to keep the serrations visibly open and simple.

If such a membrane have a small mirror attached to it, the mirror will share in the vibrations of the membrane, and, if it be jointed on a hinge, will reflect a beam of light in such a fashion as to produce a curve upon photographic paper uniformly rolled past the vibrating membrane; this curve will indicate the frequency, the amplitude, the complexity, of the vibrations of the membrane.

Sound-waves, however complex, may again be caused permanently to record the succession and variation of their own impulses. Léon Scott's Phonautograph is a conical vessel, closed at its narrower end by a membrane; to the membrane is attached a writing-point; the extremity of the writing-point is brought into contact with a smoked revolving-cylinder. So long as there is no sound, the writing-point describes a uniform line on the rotating cylinder: when sound-waves enter the cone the membrane is set in vibration, and the writing-point now describes an undulating line, which varies in its form according to the frequency, the amplitude, and the complexity of the original vibration.

It must be observed that membranes thus made use of do not exactly reproduce the original motion at any point of their surface: those components are exaggerated which approximately or exactly coincide in frequency with some normal mode of free vibration of the membrane itself.

Edison's Phonograph is a phonautograph whose writing-point is somewhat blunt; and it records the vibrations of its membrane by being driven through variable distances into a sheet of soft tin fixed on a rotating cylinder, or into the substance of a rotating cylinder or disc of wax: it leaves a permanently-deforming mark, a groove of varying depth. If the membrane, after having made such a mark, be raised from the rotating cylinder, and the cylinder turned back to its initial position; if the membrane be now readjusted in its former position, or, better, a little nearer the cylinder; and if the cylinder be again rotated in the former direction, with the same velocity as at first, — the depressions in the groove previously produced,

being of variable depth, cause the blunt writing-point, under which they pass, to move alternately towards and away from the cylinder; this compels the membrane to execute vibrations, and in so doing to set up vibrations and sound-waves in the air, which, being received by the ear, produce a sound similar to the original. Not exactly, however: the process is not perfectly reversible. Some consonants are not well reproduced, especially the explosives (b, p, t, d, k, g) and the sibilants (s, z, th); and further, it is generally found that there has been some exaggeration of some of the higher components in the course of transmission through the membrane, the effect of which is to render the sound reproduced one whose quality is somewhat metallic, nasal, or even squeaky and Punchinello-like.

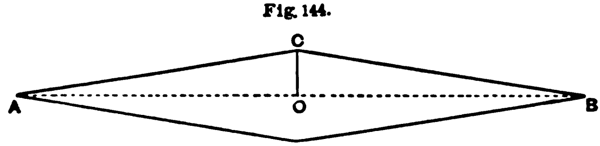
Laws of Vibration of Sounding Bodies.

These laws form properly a part of Kinetics; but the means of research into the phenomena of Vibration which lies most readily at our disposal is the observation of the pitch of the sound produced by vibrating bodies; for which reason some part of the consideration of these laws has been deferred to this place.

In general, any vibration of a vibrating or sounding body is a periodic motion, a Fourier-motion; though in particular cases we may find that the vibration is not a single Fourier-motion either simple or complex, but may be resolved into a number of such motions, simultaneous and superposed.

Transverse Vibrations of Strings.—If a string be stretched and drawn aside from its mean position, it tends to return to that position. In Fig. 144 let the string AB, subjected to a

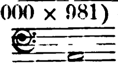
Fig. 144.



tension of T dynes (= the Weight of T/g grammes), be drawn into the position ACB, the particle C being supposed for simplicity's sake to have been initially at O, the centre of AB: the tension tending to bring back the particle C to O is the component of the total tension resolved in the direction CO: this varies directly as CO—that is, the restitution-force varies directly as the displacement—the criterion of Harmonic

Motion: and it can be shown as a consequence of the fact that the string is fixed at A and B, that the string will oscillate in some such manner that its aggregate motion can be analysed into a number of simple oscillations whose periods are commensurable: in other words, that the motion of the string is a Fourier-motion. According to the mode of disturbance — striking, plucking, bowing — of the string, or the duration of these operations, there may be an infinite variety in the relative amplitudes of these component simple oscillations. Some of these components may even be altogether absent: where, for example, a string is plucked at its centre, it is not possible that any of those components which have a point of rest at the centre of the string should be present, and the vibration of a string so plucked is one in which all the even components are absent. In general, a vibrating string does not present any component-oscillation any one of whose nodes would be at the point of disturbance.

Frequency of Oscillations. — The velocity of propagation of a transverse wave along a uniform stretched string, perfectly flexible, is $v = \sqrt{\mathbf{t}/\rho}$ (p. 268); here $\mathbf{t} = \mathbf{T}/\pi r^2$, where r is the cross-sectional radius of the string. Hence $v = \sqrt{\mathbf{T}/\pi\rho} \div r$. But the length of each wave is fixed by the condition of the string; the string is bound at each end, and if we confine our attention to the slowest component, the fundamental tone, we see that λ, the wave-length, is equal to 2AB or $2l$, twice the length of the string. Then, since n, the number of complete oscillations per second, is equal to v/λ, we have $n = v/\lambda = v/2l = \sqrt{\mathbf{T}/\pi\rho} \div 2rl$; or, if $m = \mathbf{T}/g$ be the number of grammes whose Weight stretches the string, $n = \sqrt{mg/\pi\rho} \div 2rl$.

Problem. — A wire of steel ($\rho = 7{\cdot}8$), 1 m. long and 1·2 mm. thick, is stretched by the Weight of 40 kilogrammes and set in transverse vibration: what will be the frequency of its fundamental vibration? What its pitch? — *Ans.* $n = \sqrt{mg/\pi\rho} \div 2rl = \{\sqrt{(40000 \times 981) \div (3{\cdot}1416 \times 7{\cdot}8)} \div (2 \times 0{\cdot}06 \times 100)\} = 105{\cdot}45$ vibrations per sec.;  when $c' = 253{\cdot}1$.

So for a perfectly-flexible string: the effect of rigidity of wire or string is to diminish the number of vibrations, and to cause the motion to assume the character of a number of superposed harmonic motions of incommensurable period.

The vibrations of a violin string differ much from those of a pianoforte string. In the violin the oscillating string sometimes travels in the same direction as the bow, sometimes away from it. When the bow and the string travel in the same direction, the bow drags the string with it, distorts it, pulls it out to an extent greater than that which it would have travelled if allowed freely to vibrate. When the string returns the bow fails to retain it, loses it, and as it is returning bites and catches

it again by means of some rough resinous particle at some other part of the bow.

The friction and, consequently, the adhesion between the string and the bow are relatively somewhat greater when both move in the same direction, for at low relative-speeds friction tends to increase.

The string is thus distorted and assumes successively a number of forms, of which no one is curved: and the form of the vibrating string at any instant presents an angle between two straight lines, a form differing considerably from the curve of sines. But it is periodic, and it is true Fourier-motion.

The mathematical problem is — What superposition of commensurate S.H.M.'s (compare Fig. 48) will produce a vibration-curve such that, for a certain distance, the flexures so balance one another as to produce a straight line, and then so aid one another as to produce an abrupt angle, again followed by a straight line? This can be solved, and the result is that the vibration of a bowed string must be composed of a fundamental vibration, of weak components 2d to 6th, and of ample higher components. This agrees with the result of resonator-analysis of the sound of a violin.

In the sound of a violin the upper harmonics are loud and piercing; the nearer harmonics are feeble, and the fundamental tone stands apparently alone, but rendered penetrating in quality by the high mass of harmonics. Purity of violin tone depends upon perfect periodicity of the peculiar motion of the string; this is difficult to attain, for a good elastic violin, uniform strings, a uniform bow, uniformly resined and evenly handled, are necessary: and any stumbling of the bow over the string, or any irregular movement of the string under the bow, is revealed by scratchiness of tone.

The sharper the angle made at the point of disturbance, the richer the tone in high harmonics. A string plucked with a quill, as in the old harpsichord, has thus a metallic tinkling quality, and its fundamental tone is relatively very feeble.

A string struck suddenly at one point has a form differing greatly from that of the curve of sines. Part of the string remains unaffected, while the part struck is distorted. This distortion travels along the string, and results in a periodic motion abounding in high components; the tone produced is tinkling. If the same cord or wire be struck gently by a soft elastic hammer, the blow being deliberate, and, as it were, gradually insisting upon the displacement of the string at the point struck, the disturbance is more evenly spread over the whole string, the fundamental component-vibration is more prominent, the higher components are relatively more feeble, and the tone is purer.

In a pianoforte string struck by an elastic soft-hammer the harmonics up to the sixth are present; the seventh is obliterated, or nearly so, by the hammer being made to strike the string at a spot one-seventh of its length from the end of the string—that is, at a spot which would have been a node of the seventh component if that component had been allowed to exist in the compound vibration: and the components beyond the seventh are feebly represented.

The **Monochord** is a box of thin light wood, containing air which communicates with the exterior air by lateral apertures. Upon this box rest two bridges ("banjo-bridges"), one near each end. Over the bridges is stretched a wire; of this, one end is firmly fixed to one end of the box, while the other is either passed over a pulley and made to support a weight, or else is connected with a tuning-peg, which may be turned by a tuning-key. The tension on the stretched wire may thus be varied.

Experiments with the Monochord.—For experiments it is better to use a form of monochord in which there are two wires, of which one is tightened by a peg, the other by the weight of a suspended mass; in the latter of the two wires the total tension on the wire can be directly measured, in the former it must be inferred.

1. Suppose a wire 1·2 mm. thick, whose free vibrating part is 1·2 metres long, to be stretched by the weight of 48 kilogr., and the pitch of the sound produced to be G = 96 vibrations. What weight ought to be added in order to raise the pitch to *d*?

The pitch is raised G : *d*—*i.e.* a fifth: the vibrations are rendered more numerous in the ratio 2 : 3; the tension must be increased in the ratio $2^2 : 3^2$, or 4 : 9; the stretching-weight must be increased from that of 48 to that of 108 kilogr.; the mass which would have to be added is 60 kilogrammes.

2. Two wires of equal thickness are stretched—one by the tuning-peg and the other by the weight of a heavy mass—so as to vibrate in unison. The weighted wire is removed and replaced by one of the same length, but of a different thickness, stretched by the same weight. A thinner wire gives a higher note, a thicker one a lower.

If in Ex. 1 a wire 1 mm. thick be employed, what will be the pitch of the sound produced? The frequency varies inversely as the radius: it therefore exceeds that of a wire 1·2 mm. thick in the ratio of 6 : 5; the note produced will be B♭.

3. A brass wire and a steel wire of equal gauge are equally stretched: they are free to vibrate in equal lengths. Brass has a density $\rho = 8{\cdot}38$, steel $= 7{\cdot}8$. The brass wire gives a sound lower in pitch than that given by the steel wire: the respective frequencies are in the ratio of $\sqrt{7{\cdot}8}$ to $\sqrt{8{\cdot}38}$ or $1 : 1{\cdot}03651$.

A catgut string, whose density is small, gives a higher note than a steel wire.

4. In order to vary the free vibrating-length of wire, a movable bridge is arranged under the string. If this be so placed that 60 cm. of the wire are free to vibrate instead of 120 as before, the sound produced will be the Octave; if 40 $(=\frac{120}{3})$, the Twelfth; if 30 $(=\frac{120}{4})$, the Fifteenth; if 24 $(=\frac{120}{5})$, the Seventeenth — and so forth — above the fundamental note emitted by the freely-vibrating string of 120 cm. length.

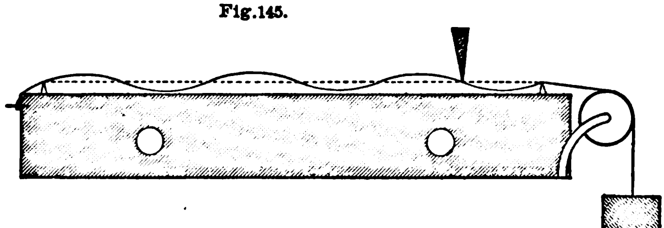
The number of vibrations of a string varies inversely as its length. Hence if we wish, with a string which sounds C, to produce the note D, whose frequency is $\frac{9}{8} \times$ that of C, we must allow the string to vibrate not as a whole, but only in $\frac{8}{9}$ of its length. To produce the scale on one string, the parts of the string which are allowed to vibrate are as follows: —

$$\left.\begin{matrix} \mathrm{d} & \mathrm{r} & \mathrm{m} & \mathrm{f} & \mathrm{s} & \mathrm{l} & \mathrm{t} & \mathrm{d}' \\ 1 & \frac{8}{9} & \frac{4}{5} & \frac{3}{4} & \frac{2}{3} & \frac{3}{5} & \frac{8}{15} & \frac{1}{2} \end{matrix}\right\} \text{etc.}$$

The application of this principle is familiar in violin playing.

5. Nodes and Loops can also be shown on the monochord. If the wire, 120 cm. in its vibrating length, be lightly touched

Fig.145.



at 20 cm. from the end, and if the twenty-centimetre-part of the wire be set in vibration by a bow, the whole wire is found to be in vibration from end to end; but not as a whole. It divides itself into segments or vibrating loops, separated by nodes or points of rest. Each segment is 20 cm. long: and the sound given out is that which might be emitted by half-a-dozen separate wires each 20 cm. long — that is, it bears to the note emitted by the whole string the same proportion as g' does to C

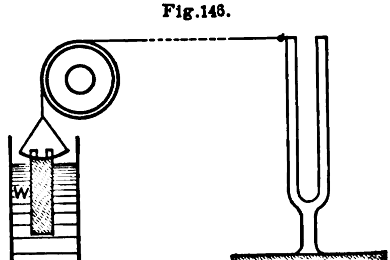
— two octaves and a fifth. Similarly for other fractional divisions of the wire or string. Lightly stopping the string has the effect of destroying or of checking the formation of all those modes of vibration which have not a node at the point touched; hence the 6th, 12th, 18th, etc., components of the vibration of the whole string are unchecked, while the other components are rendered impossible.

The nodes and loops of a string vibrating in this way are rendered manifest by paper riders placed, some at the nodes, some on the loops; when the string enters into vibration those riders which had been placed on the nodes will retain their place, while those on the loops will be jerked off.

Nodes and loops on vibrating strings may be illustrated on a large scale as follows:—Take an indiarubber tube 10 feet long, filled with sand, or a long spiral of iron or brass wire, and fix one end of this to a wall; hold the other end in the hand. On moving the hand gently, the natural period of oscillation of the cord can be easily found. Give with the hand a series of transverse impulses, so timed as to aid the natural oscillations: the tube or spiral will enter as a whole into wide oscillations. Now give such impulses twice as often: the cord will divide into two segments, pivoting in a striking manner round the central point. Do so three times as often as at first: the cord will divide into three segments or loops, pivoting on two nodes. By increasing the frequency of the movements of the hand, the cord can be made to oscillate in 4, 5, 6, 7, or even 8 or 9 segments, according to the dexterity of the experimenter. The experiment is a striking one; and it may be varied by causing the hand to move in a circle or an ellipse instead of a straight line.

Melde's Experiments.—A tuning-fork is provided with a little hook on one of its prongs; to this hook is attached a fine white silk thread. This thread is passed over a wheel and attached to a suspended mass partly immersed in water; the quantity of this mass can be coarsely adjusted by the addition or removal of sand; its effective weight can be finely adjusted by varying the quantity of water in W (Fig. 146).

Fig. 146.



The fork is set in vibration; waves appear to travel up and

down the thread; if the string be illuminated by a beam of light in a dark room, the effect is singularly beautiful. As the tension is increased, the segments vary: at length the thread vibrates as a whole, and seems to form an opalescent spindle. Its frequency of vibration is half that of the fork; the thread, when at its limit, is pulled back by the retreating fork into its mean position, but is relaxed and allowed to swing over upon the return of the fork; whence two oscillations of the fork correspond to one of the thread. If the tension be reduced to $\frac{1}{4}$, the vibrating part of the string must be shortened to $\frac{1}{2}$ in order to keep time with the fork, or else, if the string be not shortened, the string will divide into two equal segments or loops separated by a node: if the tension be reduced to $\frac{1}{9}$, the string divides into three loops with two nodes; and so forth.

If the tuning-fork be turned round through 90°, so as not now to tighten and relax the thread, but to give it a series of transverse impulses, a similar series of phenomena will be observed; but the fundamental vibration is now simply synchronous with that of the tuning-fork.

When the thread is suspended between two tuning-forks whose frequencies bear an aliquot ratio, the tuning-forks being placed at such distances from one another as to tighten the thread to the required amount, the motion of the thread becomes periodic, and presents a complex of beautiful loops and nodes which are obtained with comparative ease.

Transverse vibrations of cords may be studied with respect to the motion of each particle by casting a beam of light along a vibrating cord, and looking at a particular bright or brightened spot on the cord. The bright spot appears, when the cord is looked at end-on, to give in quick succession a large variety of such forms as we have already seen to be produced by the composition of S.H.M.'s. A bright spot on the cord may also be looked at through a microscope whose object-glass is borne upon a vibrating tuning-fork; the apparent motion of the spot produced by the motion of the object-glass (this being parallel to the length of the cord) is compounded in the eye with its real motion; the apparent up-and-down motion of the spot, as looked at transversely, is spread out into an open curve, and thus becomes more intelligible, for the eye can more readily comprehend open curves than simple up-and-down movements.

Longitudinal vibrations of a string may be excited by

drawing one point of a violin bow along the string: a very shrill tone is produced.

The velocity of propagation is $v = \sqrt{\mathfrak{g}/\rho}$; the wave-length is twice the length of the string, or $\lambda = 2l$; the number of fundamental vibrations per second, $n = v/\lambda = \sqrt{\mathfrak{g}/\rho} \div 2l$.

Problem. — A steel wire (elasticity $\mathfrak{g} = 2{,}520{,}000000\,g$, and density 7·8), of one metre in length, is clamped at the two ends and set in longitudinal vibration. What will be the pitch of the sound produced? — *Ans.* The frequency is $n = \sqrt{\mathfrak{g}/\rho} \div 2l = \{\sqrt{2520{,}000000 \times 981 \div 7{\cdot}8} \div 200\} = 2815$ vibrations per second $= f'''' +$.

As a rule the longitudinal vibrations of a string or wire are much more frequent than the transverse ones, and thus produce a much shriller sound, and further, they are not so much affected by tension applied to the string, for a variation of tension which would materially modify the frequency of transverse vibration would have little effect upon either the elasticity $\mathfrak{g}$, or the density ρ, upon which the longitudinal vibrations depend.

Before the transverse vibrations could be as frequent as the longitudinal ones, it would be necessary that the tension should be $\mathbf{t} = \mathfrak{g}$, the amount ideally necessary to double the length of the wire.

A violin e''-string gives, when rubbed longitudinally by one point of the bow, a sound in the neighbourhood of $('f\sharp)''''$; while, when let down so as to sound only e', it gives out a longitudinal vibration-sound not so low as $(f\sharp)''''$; the longitudinal vibration hardly falls a comma, while the transverse falls an octave. All the catgut strings of a violin may be observed to give out nearly the same longitudinal note, for this does not depend on their thickness.

From this we see how important it is to use the bow in such a way as to bring out transverse vibrations only, and by no means to wield it so that any component of its motion over the string can excite longitudinal vibrations, resulting, as these do, in shrill discordant tones.

By means of the monochord we may learn that a string, while vibrating longitudinally, divides into loops separated by nodes, just as it does while executing transverse vibrations.

Longitudinal vibrations of rods resemble those of strings or wires. A glass rod grasped by its centre and rubbed longitudinally by a resined cloth will enter into longitudinal vibration and will produce a shrill sound. A glass tube treated in the same manner may be made to vibrate so vehemently that it shivers into segments.

Transverse vibrations of rods obey the rule that if θ be the thickness of the rod, l its length, $\mathfrak{g}$ its Young's Modulus of elasticity, and ρ its density, then $n \propto \sqrt{\mathfrak{g}/\rho} \cdot \theta/l^2$, or $n = const. \times$

$\sqrt{g/\rho}\cdot\theta/l^2$. The constant varies according to the form of the cross-section and the mode of clamping or support; and for the different harmonics it presents values* which bear no simple numerical relation to one another.

As examples of rods free at both ends and vibrating transversely, we may take the common glass or metal harmonicon — plates of glass or metal supported by threads at the nodal lines and struck by hammers. As examples of rods clamped at one end and vibrating transversely, we may take reeds such as those of the harmonium or concertina. Their pitch is raised by filing off their substance towards their free ends; it is lowered by thinning them towards their base. Tuning-forks afford another example of vibrating rods; they are tuned in the same way.

In rods of the same thickness the frequency of vibration varies inversely as the square of the length, as the formula $n = c\sqrt{g/\rho}\cdot\theta/l^2$ indicates; but if the thickness θ and the length l vary together, so that different rods have the same shape, the frequency depends on the relative length only. Thus a tuning-fork 4 inches long and one 2 inches long, of the same shape, produce notes which differ by an octave: the same rule applies to reeds.

A rod does not vibrate as a whole, in halves, thirds, etc., but the component vibrations have incommensurable frequencies, and each component has its own rate of travelling through the solid, so that the periodic nature of the vibration is disturbed. Such a vibration of a rod is an extreme case of the vibration of a rigid or thick wire.

A rod of circular section can vibrate transversely with indifference in all directions. One whose section is oblong can oscillate more widely in a plane at right angles to the broad face than it can in a plane parallel to that face — *e.g.*, a vibrating reed, in which the latter oscillation is absolutely insignificant. If a rod of circular section be filed at one side, all component oscillations which tend to bend the rod upon the filed face are retarded; those which are at right angles to these are unaffected. Thus a rod of steel, grasped by a vice at a particular spot, may be so tuned that when it is set in vibration by a violin bow its point may execute vibrations in directions at right angles to each other, and bearing to each other any predetermined ratio of frequency.

Take a knitting-needle; fix it in a vice; mark on the needle the height at which it stands in the vice; touch the free tip of the needle with a little gum: scatter a little starch or powdered antimony over the needle tip; some will adhere. Suppose the ratio desired is 4 : 5; refer to Fig. 38. File the rod, always towards one aspect, until the movement of the tip of the rod, as revealed by the brilliant particles of starch or antimony, comes to present the curve sought. If the filing have been carried too far, a little metal may be removed from the needle at an aspect at right angles to that of the previous operation. If after a needle has been tuned in this way, so that its component vibrations have been rendered commensurate, it be grasped by the vice at a point a little above or below the original point, the intervals

* See Lord Rayleigh's *Theory of Sound*, Vol. I., Chap. viii.

cease to be commensurate, and the curves seen pass through a series of changes exemplified by those of Fig. 41.

A tuning-fork or vibrating reed, made to write its own vibrations on a rotating cylinder, describes a sinuous curve which is almost identical with the curve of sines. This shows that the motion is pendular. Again, if two harmonium reeds have their tips silvered so as to reflect light, and if they be arranged at right angles to one another; and if a lamp and lens be so placed that a beam of light falls first upon one reed-tip, then upon the other, and finally upon a screen; then if one reed be set in vibration, the spot of light opens out into a line, while if both vibrate the line opens out into some figure of the order of those shown in Figs. 35–40. This figure retraces itself and retains its form if the reeds be accurately tuned to an aliquot ratio of frequencies, while on the other hand, if the reeds be not so in tune, the figure undergoes rapid changes — changes painful to the eye, as the accompanying beats are to the ear.

Torsional Vibration of Rods. — When a rod is clamped by one end in a vice, and a violin bow drawn round it, it may be caused to execute vibrations in which it successively twists and untwists itself round its own axis; and it is found to do so with a frequency $\sqrt{\mathfrak{n}/\mathfrak{y}}\times$ that of the longitudinal vibrations in the same rod.

Vibration of Discs or Plates. — A disc of metal or of glass may be caused to vibrate by means of a violin bow drawn across its edges. The point of support of the disc is necessarily a nodal point; any number of points may be supported or fixed, and all these must also be nodal points. The disc or plate may, under such arbitrary conditions, adjust itself so as to vibrate, according to circumstances, with great variety of nodal lines and vibrating segments.

A disc of brass or glass may be fixed at its centre to a heavy stand. If the circumference be touched at any point while the whole is set in vibration by a violin bow, the point touched will be a nodal point; the spot where the violin bow is applied tends to become the centre of a loop; according to the relative situations of the points held fixed and of the point of application of the disturbing cause, will vary the manner and the pitch of the resultant vibration.

Different discs of the same shape and vibrating in similar ways have relative vibrational frequencies varying as $\theta/l^2\cdot\sqrt{\mathfrak{y}/\rho}$; the same law as holds good in the transverse vibration of bars.

The form of the nodal lines may be studied by strewing sand and lycopodium powder upon a vibrating disc or plate: the sand collects on the nodal lines; the lycopodium, by reason of the disturbance of the air, is blown towards the centre of each vibrating segment.

Contiguous sectors are in opposite phases of vibration. If the ear be placed immediately opposite the centre of figure of a

circular vibrating disc, there will be but little sound heard: the receding and the approaching sectors neutralise each other's effects upon the air and upon the ear. If the hand be held above a vibrating disc so as partly to cut off the effect of one of the sectors, the sound heard opposite the centre of the disc is enhanced; if two contiguous sectors be thus shaded from hearing, the sound is, as at first, very feeble; if two sectors not contiguous but vibrating in the same sense be thus covered, the sound produced is much louder. If a Koenig's manometric capsule be provided with a tube which bifurcates, and if the branch tubes each terminate in a cone, one cone may be placed over a vibrating sector, while the other may be moved about over the disc. As it passes round the circumference of the disc, it will be found that the gas-flame of the capsule is alternately much agitated and steady — agitated when both cones are over sectors vibrating in the same sense; steady, or nearly so, when they are over sectors vibrating in opposite phases. There is, also, always some oscillatory tangential twist of the disc.

Vibration of Membranes. — If a membrane be subject to a tension $\mathbf{t}$ equal over its whole circumference, such as that of a drum, it vibrates as a whole; its higher component vibrations are not commensurable with the slower ones, and the higher tones faintly to be heard in the sound of a drum discord with the fundamental tone. [$n \propto \sqrt{\mathbf{t}/\rho} \div r.$]

If the membrane be not equally stretched in all directions, it will, when set in vibration, so arrange itself as to vibrate feebly along the line of least tension, and strongly in the direction of greatest tension. Thus a square piece of thin indiarubber, clamped by the two opposite edges and stretched, will vibrate at the same rate as an indiarubber cord of the same free length and exposed to the same tension $\mathbf{t}$ per unit of sectional area; and it may be idealised as a collection of indiarubber cords, arranged side by side, attached to each other, and vibrating in unison.

Vibration of Bells. — A bell-shaped body, set in motion by being struck, or by being rubbed with the resined or wetted finger carried round the circumference, enters into vibration simultaneously radial and tangential. The bell divides into an even number of sectors; of these one half dilate, while the other half (individually alternating with the former) contract radially. At the same time sectors of the bell, moving tangentially, twist to-and-fro round the axis of the bell; alternate

sectors are opposed to one another in the direction of their twist; hence at some of the nodes which separate the sectors there is compression, at others dilatation of the substance of the bell. The loops of the radial motion are the nodes of the tangential motion; thus where there is least expansion or contraction there is the greatest amount of twist. In the circumference of a vibrating bell there are generally four loops corresponding to each motion.

The **effect of loading** a vibrating body is to lower its pitch; if the load be distributed uniformly, all the components are lowered; if it be suspended from points of the vibrating body, those component vibrations, if any there be, which have their nodes at those points remain unaffected.

The preceding propositions have related to the vibrations into which a body may enter when it is disturbed and then left to itself. The vibration in such cases is called *free vibration*, the period of which depends on the nature and the form of the vibrating body itself. If a body capable of vibration be acted upon by a series of impulses *ab externo*, the result depends upon the period of recurrence of these impulses. These may be so timed as to aid the natural free vibrations of the body, adding energy, and therefore increasing the amplitude at every oscillation; or they may be so timed as sometimes to aid, sometimes to thwart the natural oscillations, and thus to produce, on the whole, no effect so far as concerns the amplitude of these. In the former case the interval between two successive impulses *ab externo* is equal to the period of the natural vibration; while, when this interval differs materially from the period of the free vibration, the amplitude of vibration is not increased, but the energy communicated in the successive impulses is dissipated in heat.

A heavy bell has a natural period of pendular swing, and if a person gently pull the bell-rope, a very slight swing may be obtained, perhaps barely perceptible. If the pull be repeated while the rope is tending to slacken in the ringer's hand, the original small swing is increased, perhaps doubled; a succession of well-timed pulls causes ultimately a wide oscillation of the bell; and when the bell has been set fairly ringing the amplitude of the oscillation is kept up, and all loss of energy replaced, by a series of well-timed impulses. If, however, the impulses be so timed that they sometimes act in favour of the oscillations of

the bell, and are sometimes delivered against a tightening rope, the bell will either not ring at all or will do so very irregularly.

If a body capable of freely vibrating with n oscillations per second be placed in material communication with a body actually vibrating n times per second, the former will take up energy and enter into vibration. If a cylinder AB be fixed at the extremity of the rod CD; if another cylinder EF, of like material and dimensions, be fixed at the other end of CD; and if AB be set in longitudinal vibration,—then EF will be set in vibration of equal period. Here it must be observed—(1) that expansion of AB occurs at the same time with compression of EF, and *vice versâ;* and (2) that the centre of mass of the whole system remains unchanged in its position.

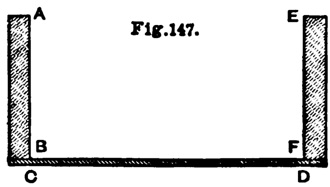


Fig. 147.

Two organ-pipes of exactly the same pitch, mounted on the same wind-chest, may fall into a similar opposition of phase, and retain it for long periods of time: so long as they do this, they can together emit but little sound, and the sound produced is rendered louder by silencing one of the pipes.

Two similar strings of equal length, equally stretched over the same solid framework parallel to one another, will so adjust their vibrations as simultaneously to approach or to diverge from one another.

A tuning-fork may be regarded as made up of two equal rods, connected with a common basis; the prongs simultaneously approach and diverge from one another when the fork is in vibration.

Two clocks which keep good time together will, when placed on the same table, beat synchronously.

A tuning-fork can be set in vibration by another tuning-fork of exactly the same pitch vibrating within the same room. The material communication between the two forks is effected by the air; the sounding-fork causes waves in the air; these cause well-timed impulses against the second fork; the effect of these is cumulative, and the second fork takes up the vibration of the first.

In the same way a mass of air in a vessel or flask, since it has a natural period of vibration, may, if air-waves dash against the open mouth of the vessel at such intervals as correspond to its natural vibration, be caused by the accumulated effect of these waves to enter into violent oscillation. This principle we

have seen made use of in resonators; and from the analogy of these instruments all phenomena of this kind may be called phenomena of **Resonance**; the law of which is, that any undulation or vibration is taken up — its energy is absorbed — by any body capable of freely vibrating synchronously with it, free to do so, and exposed to its periodic impulses. If the undulation or vibration which concusses the vibratile body be complex-harmonic, those components only which correspond to the natural period of the vibratile body are taken up by that body. On this principle resonators are used to detect the higher components of a complex sound.

Forced Vibrations. — Under certain circumstances a vibratile body may be compelled to surrender its own preference for a particular mode of vibration, and to vibrate with more or less accuracy in an arbitrary manner imposed upon it by external force.

Huyghens discovered that two clocks which did not keep time separately, kept time together when placed on the same table; the more rapid clock forced up the speed of the slower one, while it was itself delayed. Two prongs of a tuning-fork, slightly unequal in size, will force one another to agree in their periods of vibration. If the one prong be powerfully pulled to-and-fro by an external mechanism so predominant that its period cannot be altered by any resistance offered by the fork, the fork as a whole will be forced to vibrate at a rate determined by the exterior mechanism or outside force; and it will maintain this rate as long as it is compelled to do so, but no longer; this being the case of a tuning-fork controlled by an electromagnetic interrupter, which is found to return to its normal rate of vibration as soon as the electric current ceases. The nearer the rate of the forced vibration is to the rate of the free vibration of the fork, the wider is the oscillation.

A resonating chamber of air will also, on the same principle, resound to some extent under the influence of a tuning-fork of pitch slightly differing from its own natural pitch. Here we observe that a steel fork can force the less massive air slightly to alter the period of its vibration; towards the end of each swing of the particles of the air, they are in very slow motion, and can with comparative ease be constrained to shorten or to prolong the period of their oscillation. Air-waves, on the other hand, cannot force a massive tuning-fork in this way, and in order that a tuning-fork may take up a vibration conveyed to it

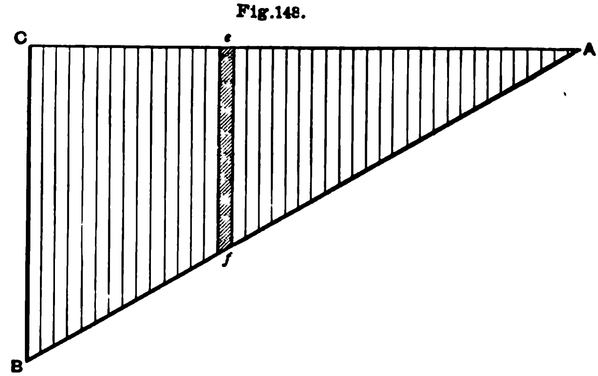
through air, the vibration of the primary sounding-body must be in exact unison with the free vibration of the tuning-fork acted upon.

There is difficulty in constraining a denser substance to take up a vibration communicated to it by a rarer one. The mechanical action of the ear involves, however, a problem of this kind; air-waves have to be communicated to the denser sense-organs; and this is effected by diminishing their amplitude and consequently increasing the force of their impulse — a principle now familiar to us.

Perhaps the best examples of bodies which can be forced into vibrations of any period are found among membranes; to some degree of amplitude any period of vibration can be forced upon a membrane, though all stretched membranes — as, *e.g.*, the orchestral drum — have natural periods of free vibration which can be modified by varying the tension. If a vibratile body which has a natural period of vibration be concussed by an external Fourier-motion, the resultant forced-motion of the vibratile body will still be Fourier-motion; but those components of the original motion which nearly coincide with the natural oscillations of the vibratile body will be represented in the resultant forced complex-vibration by components proportionately exaggerated — a principle we have already seen to affect the working of the Phonograph.

If a membrane be of unequal width and be stretched in one direction, a forced vibration imposed upon it will affect only certain parts of its area. In Fig. 148, ABC is a membrane,

Fig. 148.



triangular in form and exposed to a tension parallel to CB. Consider a single very narrow strip, *ef*, of this membrane; imagine it to be isolated from the rest of the membrane. Such a strip would have a certain length, thickness, tension, density; it would therefore, if it entered alone into transversal vibrations,

produce a note of a certain definite pitch. Let that note be sounded in the neighbourhood of the membrane; the membrane will vibrate strongly at *ef*, the disturbance rapidly shading off into rest on either side of *ef;* and, further, there will be some disturbance in those parts of the membrane whose length is 2*ef*, 3*ef*, and so forth. Each external sound is responded to by a different part of the membrane, which plays the part for the time being of an imperfectly-isolated string. At right angles to the direction of tension there is but little vibration.

If the external sound be complex, several such strips are set in motion.

Musical Instruments. — For the ends of musical art it is necessary that the vibrating body used as the source of sound should be capable, at the will of the performer, of producing several sounds. In the old Russian horn-bands each player had only one sound at his disposal, and by dint of practice and drill learned to produce his solitary note at the right instant; but this kind of orchestral music is quite exceptional.

We have already seen that all the notes of the scale may be produced on the monochord by varying the length of the free vibrating part of the string.

Some stringed instruments — the Lyre, the Harp, the Violin, etc., when played *pizzicato*, the Banjo, the Guitar — are played by plucking the strings. In some cases — the Lyre, the Harp — the number of sounds which can be produced is limited by the number of strings present; in others — the Banjo, the Guitar — each string is made to produce a number of sounds which depend upon the number of frets by which the finger of the executant is guided in shortening the string; in instruments of the Violin class there is no mechanical aid to the performer's fingers, and he is left to his own judgment as to the precise amount by which any given string should be shortened in order that it may emit the particular sound of which he has already formed a mental idea. All these instruments are provided with sounding-boards which increase the surface by which vibrations are communicated to the air; and when their strings are plucked, the sound produced is of short duration and rich in high harmonics, poor in lower ones.

The strings of the Harpsichord were plucked by quills which were actuated by hammers. The sound was poor in quality, being feeble in the fundamental tone and disproportionately strong in the higher harmonics; and it was feeble

in intensity as compared with the pizzicato notes of a violin, because the sounding-board was wanting in flexibility and had little effect on the air.

The Pianoforte contains very strong wire tightly stressed; the total stress on a Broadwood grand pianoforte exceeds the weight of 35,000 lbs., that on a Steinway is 72,000 lbs.; whence the modern pianoforte is, as regards its framework, necessarily a very much more massive instrument than its predecessors. The longer the string corresponding to a given note, and the greater the tension upon it, the more precisely will the harmonic tones be in tune with the fundamental, and the fuller and richer will be the sound produced. The sound of a pianoforte string struck in the usual way is rich in harmonics up to the sixth; the seventh is purposely prevented by the choice of the spot at which the hammer strikes; the eighth and those beyond are feebly represented. The sounds of the higher strings approximate in character to pure tones. The compass of the modern pianoforte is from $A_{\prime\prime}(=27{\cdot}1875)$ in the thirty-two-feet octave to $a''''(=3480)$, or even to c''''' $(=4176)$, the sound of an organ-pipe an inch-and-a-half long.

In a vibrating pianoforte-string those components disappear whose periods are $\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{7}$, etc., of the period of contact between the hammer and the string. Hence the varying quality of tone obtained from the same string by hammers of different degrees of repair or varying in the hardness or elastic softness of the leather, or by striking the pianoforte keys in different ways. The slower the stroke, the longer the contact, the greater the disappearance of higher harmonics.

The Violin, whose strings are tuned to $`g$, $`d'$, a', e'', has a compass ranging from $`g$ to about $e''''$; the Viola is tuned to $`c$, $`g$, $`d'$, a'; the Violoncello is tuned to $`C$, $`G$, $`d$, $a$; the Double-bass is tuned to $[E_{\prime\prime}]$, $A_{\prime\prime}$, $`D_{\prime}$, $`G_{\prime}$; these instruments give the strings of the orchestra an aggregate compass of from $E_{\prime\prime}$ or $A_{\prime\prime}$ to about e'''', or about seven octaves. In the Violin three strings are of catgut; their pitch depends upon their thickness; the fourth string is weighted by a spiral of silver wire — an arrangement which to a great degree obviates rigidity. The belly of the violin acts as a sounding-board. The air in the cavity aids in the resonance and improves the tone, for it is easily forced to take up the vibrations of the solid parts of the instrument, especially those component vibrations which are already the most prominent; and this action of the internal air is important, as may be found on covering the ʃ holes with tissue paper, for then the tone of the instrument is materially deteriorated.

The quality of the tone is found also to depend greatly upon the empirical form of the bridge.

Transverse vibrations of reeds are utilised in the Musical Box: reeds of various lengths are struck, vibrate in the free air, and produce sound of little intensity. In the Harmonium and Concertina the passage of air from a bellows into a resonating air-chamber is obstructed by reeds, which almost completely close the air passage. The pressure accumulates and bends the reed; the air escapes and the pressure is relieved; the reed returns and swings past its mean position, again to be driven outwards. The vibrations of such reeds contain, as harmonics, tones bearing to the fundamental ratios incommensurable, but approximating to $\frac{4}{1}$, $\frac{9}{1}$, $\frac{16}{1}$, etc. Such reeds vibrating in the open air produce very little sound: the air of the resonating cavity acts as an intermediary, and communicates the vibration by a broader surface to the external air.

In Mr. Baillie Hamilton's String-organ very various qualities of rich and full tone are produced by connecting in various ways a vibrating reed with a string stretched over a sounding-board.

In reed organ-pipes a reed vibrates at the bottom of a tube, the cavity of which it alternately opens to and closes against the access of air from the wind-chest: the air in the cavity of the organ-pipe resounds to the vibration of the reed, and does this most loudly when its own natural pitch coincides with the pitch of the reed. The Clarionet, the Oboe, the Bassoon, are instruments of this order; in them as in the organ-pipe the reed must be cut to a proper size: but each can produce several tones, for the vibration of a reed made of cane is more irregular than that of a metal reed, and as the size of the resonating air-cavity can be altered by manipulating the finger-holes and keys, to each altered size of the resonating cavity there corresponds a different tone or component of the motion of the reed, which the tube is capable of selecting, and to which it resounds.

In the ordinary Organ-pipe a sheet of air is blown across the *embouchure* of the pipe, and vibrates like an invisible reed just outside the pipe: the air inside is set in vibration. The pitch of the sound produced by it is not so high as we might be led to infer from the dimensions of the column of air in the pipe, its elasticity, and its density alone: the reason is that the column of air in the pipe is not an isolated mass of air. While vibrating it has, at its extremity, to thrust aside the surrounding

air at each expansion; the inertia of the surrounding air hampers the vibration, and, as it were, loads and retards the vibrating column.

Nodes in an organ-pipe may be demonstrated by Koenig's manometric capsules attached to the sides of the pipe. Near the centre of an open pipe there is a node, a maximum of variation of density, and hence the air at the node is alternately squeezed together and relaxed, the membrane of the manometric capsule placed at the node will vibrate, and the flame will either be extinguished or will present variations of height.

If an organ-pipe be strongly blown, the sound will rise slightly: if the force of the current exceed a certain amount, the sound suddenly breaks into the upper octave — a phenomenon familiar to flute-players — and near the centre of the pipe is now a loop, while there are now two nodes. Hence, in a pipe so overblown, the manometric capsule fixed at a point near the centre almost ceases to indicate variations of density, and the flame remains almost steady: never absolutely so in practice, for there is not even any one point, at the centre of any loop, at which all variations of pressure entirely vanish.

If a pipe be filled with hydrogen the sound produced is nearly two octaves above that produced by air at the same temperature. The velocity of the sound-waves is $\sqrt{k/c \cdot \mathfrak{k}/\rho}$ $=\sqrt{k/c \cdot p/\rho}$ (p. 368); in air at atmospheric pressure Π, this is $\sqrt{(1{\cdot}4058\,\Pi/\rho_{\text{air}})}$; in hydrogen, it is $\sqrt{(1{\cdot}4139\,\Pi/\rho_{\text{hyd.}})}$, for in that gas $k/c = 1{\cdot}4139$, the specific heats being 3·409 and 2·411; in the case of hydrogen $\rho = \dfrac{1}{14{\cdot}44} \times$ the density of air; whence the frequency of vibration in hydrogen is to that in air as $1 : \sqrt{14{\cdot}44 \times (1{\cdot}4139 \div 1{\cdot}4058)} = 1 : 3{\cdot}8107$ or : : C : 'b +.

If the barometric pressure vary, the pitch is unaffected: for $\sqrt{\mathfrak{k}/\rho} = \sqrt{p/\rho}$; but the density ρ increases or diminishes *pari passu* with the pressure p: thus p/ρ is constant, the velocity is constant, and the pitch is constant.

If the temperature τ vary, the velocity v, $= \sqrt{k/c \cdot \mathfrak{k}/\rho}$ $= \sqrt{k/c \cdot p/\rho} = \sqrt{k/c \cdot \mathfrak{K}\tau}$, and therefore also the pitch, varies as the square root of the Absolute temperature. A pipe which gives a sound $a' = 435$ at the temperature of 10° C. will vibrate 446·79 times per second at the temperature of 25° C., for as $\sqrt{283} : \sqrt{298} : : 435 : 446{\cdot}79$; the pitch is thus sharpened in the ratio $1 : 1{\cdot}027105$, or more than two commas; while the reeds are affected to a very much less degree, and thus an organ tuned

in a cool room becomes discordantly out of tune with itself when the room becomes warm.

Problem.—Of what length would an organ-pipe be which sounds $C_{,}$ at 0° C. and 760 mm. bar. pressure? $C_{,} = 32·625$ vibrations.—*Ans.* Since the frequency of vibration is $n = v/\lambda = \sqrt{k/c \cdot \mathfrak{k}/\rho} \div 2l = \sqrt{k/c \cdot \Pi/\rho} \div 2l$, the length $l = \sqrt{k/c \cdot \Pi/\rho} \div 2n = \sqrt{1·41 \times 1{,}013663 \div 0·0012932} \div 65·25 = 508·7$ cm. A pipe of this length really gives a sound somewhat lower than $C_{,}$; the wider the pipe the lower the pitch. A $C_{,}$-pipe is called a 16-foot pipe, though really somewhat longer.

The common Flute, the Fife, the Piccolo, the Flageolet, are modifications of the open organ-pipe: the size of the vibrating column of air is altered in these instruments by opening or closing lateral apertures. In the Flageolet the mouthpiece is so constructed that the stream of air cannot but pass in the direction empirically found to produce the best sound: in the Flute, Fife, and Piccolo the direction of the exciting stream of air, and to some extent the corresponding quality of tone, and even the pitch, are under the control of the player.

Brass instruments vary in shape from the Bugle (conical from mouthpiece to bell) to the Trumpet (slowly-widening tube and suddenly-widening bell), with the intermediate forms, the Cornet and the French Horn. The lips of the player are made to vibrate: the cavity of the instrument resounds. The scale of the instrument is confined to harmonics, which are obtained by varying the method and force of blowing and the tension of the lips; but since the size of the cavity of the instrument may be modified, any tone of the scale within the compass of the instrument may be brought in as a harmonic of some fundamental note. The size of the cavity may be modified by slides, as in Trombones — instruments which are capable of giving pure intonation; by pistons which lengthen the tube by predetermined amounts, as in the Cornet and Cornopean, the Euphonium, and Bombardon; by levers which shorten the tube, as in the Ophicleide. In some cases, however, as in the French Horn and Trumpet, the instrument is confined to one note and its harmonics; but its cavity may be altered in size by the addition of additional pieces of tubing or "crooks," whose dimensions are so adjusted as to cause the fundamental tone of the instrument to change by an interval predetermined for each crook; and the note produced may be to some extent modified both in pitch and in quality by the action of the lips, and of the right hand placed in the bell or the open mouth of the instrument.

The pitch of these instruments is that of an open pipe. Their tone is rich in quality and full of high harmonics; and though we are inclined to associate their peculiar quality with metal, it depends only on the form of the internal cavity and of the bell mouth, not on the material of the walls of the instrument, provided that these be rigid. A cornet sound will be produced by a cornet-shaped tube of guttapercha, if the tube be so thick as to be rigid.

Stopped Organ-Pipes. — A longitudinal column of air steadied at one end should vibrate at one half the rate of a column free to expand at both ends; but a stopped organ-pipe does not give, as this rule would indicate, the octave of the open pipe of the same length, but its seventh, or even its minor seventh. The air within it is even more loaded and hampered by the surrounding air than that in an open pipe. When overblown it breaks into harmonics. (See Fig. 73.)

Other Sources of Sound — Singing and Sensitive Flames. — A tube terminating in a blowpipe-nozzle is connected with the gas supply: the gas is ignited at the nozzle, and a small flame is thus produced. The nozzle is slowly passed up a wide glass tube, such as an Argand lamp-cylinder. At some particular position in the tube, the flame alters its character and begins to sound forth a note somewhat higher than the natural note (in cool air) of the tube itself; or, should it not spontaneously burst into song, it may be induced to do so by singing to it a note whose pitch is nearly that which it will emit when in action. When the action has commenced, the flame is found, upon observation with a rotating mirror, to be alternately extinguished and rekindled; its image appears to be broken up into separate beads. A flame of hydrogen is prompter in its action than a flame of coal gas; for it excites more disturbance in the column of cool air surrounding it. When the action has commenced, the vibrations of the flame and of the column of air react on one another.

The flame must be of a certain height adapted to each size of tube. If the apparatus be arranged so that the flame is put about one-quarter up the tube, and if the height of the flame be carefully altered by slowly turning the controlling gas-tap, it will be found that at a certain definite height of flame the action will spontaneously commence, doing so with small initial intensity, while, when the gas-flame is lowered to one-half of this effective height, the sound breaks into the octave above.

When a jet of gas is allowed to flow vertically upwards under a pressure of about $\frac{3}{4}$ inch of water, through a minute aperture, above which a sheet of fine wire-gauze is arranged horizontally at a distance of about 2 inches, the gas may be lit on the upper side of the wire-gauze, through which the flame will not descend. The distance of the jet from the gauze may be so adjusted that the flame is yellow at the tip, and at the tip only. The flame is now a *sensitive flame*, and responds by sinking down to the gauze when sound-waves strike it. If it be surrounded by a wide tube, it will sing spontaneously. If, while the flame is so surrounded, the gas be turned down until the singing just ceases, the flame becomes extraordinarily sensitive to all very high sounds, such as hissing, the rattling of keys, etc., and it sings loudly as long as these stimuli are maintained. A simple flame issuing from an exceedingly narrow steatite burner, under a very great pressure of gas ("10 inches of water"), is sensitive to sounds too acute to be perceptible to the human ear; it alters its form under their influence.

Trevelyan's Rocker. — A mass of lead so shaped as to rest upon two long but very narrow linear feet. Placed upon a hot body, the points of contact of the rocker with the hot body are suddenly expanded by heat; the rocker is jerked upwards; before it has fallen back, the heated points have cooled and returned to their normal dimensions. The process is repeated. The whole oscillates rapidly and makes a humming noise.

Radiophony. — When a beam of light or radiant heat falls upon a body capable of absorbing heat, that body becomes warmed, and expands. A flash of light produces an instantaneous expansion, which immediately dies away. An intermittent beam produces a succession of expansions and contractions; in other words, the surface of the body vibrates. The amplitude of its movement may, with beams of light of moderate intensity, exceed the ten-millionth part of a centimetre. Lord Rayleigh has shown that this amplitude is sufficient for the production of sound; and the power of converting the energy of an intermittent beam of light or radiant heat into that of sound has been shown by Prof. Graham Bell to belong to all matter, with a few doubtful exceptions.

If an intermittent beam be focussed upon a mass of lampblack, at each flash of light it becomes warm, and the air within it is dilated; if it be contained in a test tube the open end of which is connected with the ear by an indiarubber tube, as the

successive flashes produce successive dilatations and pulses in the air, these pulses are perceived by the ear as sound; if the lampblack be contained within a resonator, the frequency of whose natural vibration is equal to that of the frequency of succession of the flashes, the resonator emits a loud sound, audible at a distance.

Propagation of Sound.

Propagation of Sound occurs in all elastic media, and is effected by waves of alternate Compression and Rarefaction.

Propagation of Sound in Solids.—Sound being a vibration is propagated along the ground; we may put our ears to the ground to listen for distant railway trains, distant vehicles, distant firing or marching. It is conducted along wood, as in the ordinary stethoscope; or in the experiment of closing the teeth upon a long piece of wood, to the other end of which an assistant holds a vibrating tuning-fork; or of causing a long strip of wood to rest by one end against the panel of a door, while the other end is in contact with a vibrating tuning-fork; the vibration is, in this case, communicated to the panel, which acts as a sounding-board, and itself sounds out loudly. Sound is conducted by wires; taps on a telegraph wire are audible at a great distance to an ear applied to the wire, provided that there be no intervening tunnel or bridge to form a resonating cavity and to absorb the energy of the vibration. In the Wire Telephone the central points of two stretched membranes or boards of thin wood are connected by a long wire, which, if sufficiently heavy and tense, may be suspended from posts, or even stretched upon carpet or bent round corners, and may thus serve the purposes of domestic telegraphy. When sound-waves impinge upon one membrane of the wire telephone, as when it is directly spoken at, the complex motion of the air is transferred to the membrane; by the membrane it is transferred to the wire; by the wire to the second membrane; by that membrane to the air, and by this to the ear of the distant listener. If the membranes be of silk, and connected with the wire by synchronised spiral springs, the arrangement is very effective, and sound is carried to very great distances, even under unpromising circumstances (Mechanical Pulsion Telephone). A slender apparatus, consisting of two parchment membranes stretched on rings, with an intervening silk thread, is sold as a toy under the name of the Lover's Telegraph.

Propagation in Liquids. — Divers while under water hear the sound of waves beating against the shore. A tumbler of water standing on a resonance box will, if the handle of a vibrating tuning-fork be dipped in the water, convey the vibration to the resonance box, the air in which will resound. An inverted bell filled with water and set in vibration will cause the water to assume beautiful wave-forms — an experiment which may be performed with an inverted propagating-glass set in vibration by a wetted finger drawn round its edge, or with a capacious wine-glass set in vibration by a violin bow. In the latter case the interest of the experiment is increased by substituting for water some strongly-alcoholic liquid; the agitation breaks the surface of the liquid into drops which, by evaporation, lose some alcohol and dance on the surface of the vibrating liquid.

An organ-pipe may be blown by water under water; the water vibrates in the place of air.

Propagation in Air and other Gases. — In general, sound travels in air in concentric spherical waves. If it be restricted to tubes, the waves may become plane-fronted.

Though these waves are invisible their existence is beyond doubt, for when they strike any solid object they produce mechanical effects, and the phenomena of sound, so far as these depend upon propagation through the air, obey the laws of wave-motion.

The breaking of glass windows by the discharge of artillery, the destruction of the drum of the ear which has been known to be caused by the explosion of dynamite, the destruction of property by the explosion of a gunpowder magazine, are only exaggerated instances of that conveyance of energy by the air which is associated with the production of sound. In Edison's Phonomotor a membrane is stretched over a frame; at its posterior aspect it is connected with a broad hook which rests on the broad margin of a heavy wheel: the margin of this wheel is provided with small roughnesses so shaped that it is easy for the broad hook to slip over them in one direction, but in one direction only; on its return it is caught in the small teeth and tends to pull the wheel round. The membrane is spoken at; it trembles; the hook catches some of the teeth; on its return it gives an impulse to the wheel; continuous sound causes the wheel to rotate, and this with considerable speed and power; and if a little crank be fixed to the rotating wheel, the energy of the human voice may be made to perform obvious mechanical work.

Sound-waves in air are amenable to the laws of ordinary tridimensional Wave-motion already discussed.

In Figs. 43–48 we find curves whose forms depend upon the assumption that when two vibrations concur, the amplitude of the resultant is obtained by addition of the amplitudes of the components. To a first approximation this is true, but it leads to a curious result. If the amplitudes and periods of two vibrations be equal, the resultant vibration (Fig. 44), having twice the amplitude, will have four times the energy of either; the motions cannot be so superposed without a draft of energy from elsewhere. If two equal waves arrive in the same phase at the entrance of the same channel, there is found to be reflexion of a n e g a t i v e wave from the mouth of that channel.

Superposition of vibrations is familiar in Acoustics as a cause of **Beats.** Two tuning-forks or reeds are brought into exact unison; they emit jointly a smooth sound. Suppose their pitch to be $c'' = 522$. Load one with wax until it vibrates, say, 521 times per second. Once in the course of a second they will aid one another, and their action on the air at any given spot coincides; once in the course of every second they will thwart one another; their joint effect will be an alternate fading-away and swelling-out of the sound; but the pitch of that sound will be $521\frac{1}{2}$ vibrations per second. As the loading of the one fork increases, the beats increase in number until they become too rapid to be counted; but before this occurs the two notes have ceased to be blended into one note of average pitch, and the effect is that of a painful discord.

As differences of phase accumulate, the wave-form goes through periodic changes: and Lord Kelvin is of opinion that the ear can distinguish differences in the wave-form due to differences of phase, for between any two beats the sound heard has a certain rotatory character.

The easiest as well as the most accurate way of tuning a fork to a given note is to have at hand a standard fork which makes four vibrations less per second than correspond to that note; then adjust the fork to be tuned until it makes exactly four beats per second with that artificial standard.

Diffraction. — Sound-waves have in general the properties of waves whose wave-length is n o t s m a l l in comparison with the apertures through which they pass, the surfaces by which they are reflected, or the obstacles round which they flow. A sound-wave coming through a chink would suffer great lateral diffraction, as shown in Fig. 56; and the effects of obstacles

intervening in the path of a wave of sound may not be such as even to produce a sound-shadow. If, however, the apertures, or surfaces, or obstacles in question be very large in comparison with the wave-length, there may be true sound-shadows and limited beams of acoustic disturbance, reminding us of the shadows and beams of light met with in Optics. Sensitive flames may be used to detect such sound-shadows; and the optical effects of Diffraction may be imitated acoustically. The air-waves produced by the note C (= 64) have a length of (33,200 ÷ 64 =) 518·75 cm., or between 16 and 17 feet; those produced by the note c''''' have a length of about an inch and a half. When a mixture of long waves and ripples strikes an obstacle, the long waves may pass round it, while the obstacle may intercept the ripples, for relatively to these it may be wide enough to cast a sound-shadow. Thus the sound of a brass band suddenly changes in quality when the band comes round a corner into sight. It is plain that for acoustic purposes all the auditors at a concert, though they are not absolutely prevented from hearing by not seeing the performers, ought to be in full view of the orchestra.

It sometimes happens that a person at the same level as a source of sound, and in full optical view of it, hears nothing: the sound-waves, passing through air-strata of different density, curve upwards. Being very broad they present no diffraction, and pass over the head of the observer, who may again come within their range by elevating his position. This occurs when the upper strata are cooler; when they are warmer, the sound descends.

Reflexion of Sound follows the ordinary laws of the reflexion of waves. Sound can be reflected by a mirror; a high-pitched bell ringing round the corner of a house can be rendered audible by a sufficiently-large mirror placed at a proper angle. If a stretched sheet of tracing-paper be placed at the same angle it will reflect a proportion of the sound and transmit some of it. A dry handkerchief will transmit a considerable amount of sound; so will even a half-inch layer of felt; but if wetted these become better reflectors, while they become almost impervious to sound. The reflexive power of flame is nearly the same as that of tracing-paper; and hot air above a gas-flame can reflect sound almost as well as the gas-flame itself. In clear weather the air is rarely uniform; there are ascending and descending currents of hotter and colder air; at each surface of each of these, sound is partly

reflected, partly transmitted, and so, ere long, it is wholly dissipated. Sound is often heard better in foggy or even in rainy or snowy weather than in clear, for then the air is more uniform. Sound coming through a fog (vesicles or minute drops), or a shower of rain, or a shower of snow, must be to a certain extent lost by repeated reflexion; but this effect is often balanced by the increase of intensity arising from the concentration of the waves in the narrowed channels between the drops or flakes.

In many buildings there are whispering galleries or places where a faint whisper uttered at a particular spot is heard at a distant part of the edifice. This phenomenon may arise in two ways: — (1) Reflexion from the vaulted roof, which acts like a concave mirror and causes the waves received by it to converge after reflexion upon a particular focus — a phenomenon very common in ellipsoidal roofs, a whisper uttered at one focus of the ellipsoid being reflected to the other focus, and distinctly heard there; a similar phenomenon also occurs in elliptical rooms, where the sound is reflected by the walls; or (2) by the sound undergoing successive reflexions, and thus travelling round the walls.

Reflexion of sound is familiarly illustrated by the Echo. Sound striking a broad cliff or wall is reflected, the reflected waves sometimes travelling with singular absence of diffraction and precision of direction. If a person can utter ten syllables per second, and if he speak loudly at that rate in presence of a cliff or high wall directly opposite him and at a distance of 1660 cm. (55 feet), just as he is commencing the second syllable the reflected sound of the first syllable begins to arrive at his ear, and at the instant when he ceases to speak, the sound of the last syllable spoken begins to be heard; the sound travels to the cliff and back during the tenth part of a second. If the cliff were 3320 cm. distant, the speaker would hear two syllables repeated. When sound is re-echoed from cliff to cliff, or to-and-fro between two smooth walls directly opposite to one another, the result may be a multiple echo, which repeats a sound several times. The roll of thunder is partly due to multiple reflexion from cloud to cloud, partly to the varying distance of the points of disturbance, and the successive breaking on the ear of sound-waves produced along a long line.

When the distance of the reflecting surface from the source of sound is too small to produce a distinct and separate echo, the echo may be heard merely as a reinforcement of the sound

produced; whence the practice of placing sounding-boards behind and above pulpits and orchestras.

The action of the Ear-trumpet depends, in the first place, upon multiple reflexion; sound-waves on their arrival are reflected by the bell into the tube; then they travel, plane-fronted or nearly so, down the tube to the ear, narrowing in breadth and increasing in intensity as they do so.

Refraction of Sound. — If a lens be constructed of two large sheets of collodion cemented together at their edges and inflated with carbonic acid, sound-waves diverging from a watch placed at one side of this lens may, after passing through it, converge upon a focus on the other side of it; this shows that refraction occurs when the sound-waves enter and quit the denser gaseous medium, the carbonic acid.

Interference of Sound. — If A and B in Fig. 75 represent the position of two tuning-forks kept accurately in unison, being driven by the same interrupted electric-current, the ear, placed successively at a', b', c', etc., perceives alternate sound and silence. The same occurs when A and B in that figure represent apertures in the side of a padded box within which an organ-pipe or bell is caused to produce sound.

Velocity of Sound. — There is but little information as to the properties of sound-waves travelling in a substance whose elasticity is not the same in all directions.

In Scotch fir the longitudinal propagation is more rapid than that across the fibres of the wood in the ratio of 5 : 4; hence sound-waves in that wood must be spheroidal. This is ascertained by observing the form of the nodal lines in a vibrating plate of that wood.

In practice we meet with spherical waves such as those in the air, and plane-fronted waves such as those which run along wires.

The velocity $v = \sqrt{(\mathfrak{k} + \frac{4}{3}\mathfrak{n})/\rho}$ in tridimensional solids, $\sqrt{\mathfrak{k}/\rho}$ in non-viscous liquids, $\sqrt{k/c \cdot \mathfrak{k}/\rho}$ in gases; $\sqrt{\mathfrak{y}/\rho}$ along a rod or wire when the wave is plane-fronted, and $\sqrt{\mathfrak{t}/\rho}$ along a wire when the displacement of the wire is transversal.

The velocity of sound in Solids may be found by determining the pitch of longitudinal vibrations set up in long thin rods. The length of the rod is half a wave-length, $\lambda/2$; the pitch gives n, the number of vibrations per second; the equation $v = n\lambda$ gives v, the velocity of sound, in cm. per second.

The velocity of sound in Liquids may be determined by direct experiment, as by sounding a bell under water, and

by listening at a distant station for the arrival of the sound, the precise instant of the production of which is signalled; or by comparing the pitch of organ-pipes blown in different liquids.

The formula $v = \sqrt{k/\rho}$, for a non-viscous liquid, would give for water the velocity of 143,900 cm. per second, for k = (compressibility)$^{-1} = 2{\cdot}07 \times 10^{10}$ and $\rho = 1$; the observed value is 148,900 cm.

The velocity of sound in Gases — as, for example, air — has been directly determined by firing cannon at a known distance, and by observing the interval of time which elapses between seeing the flash and hearing the report. The objections to this method are, that such violent concussions as those of cannon produce aerial vibrations which can be shown to travel faster than disturbances of less intensity, and that the velocity is not equal in all directions round the cannon or at all distances from it.

The length of an organ-pipe producing a note of a given pitch has been taken as a means of measurement. The column of air in the pipe, if it vibrated alone, would be half a wave-length in length; but the air is not isolated; the sound actually produced is graver than that corresponding to an isolated column of air, and the velocity so measured is not even approximately correct.

On similar principles the length of a pipe closed at one end and subjected at the other to the aerial impulses derived from a vibrating tuning-fork — this length being so adjusted by a movable piston that the air in the tube resounds its loudest — is taken as $\lambda/4$, and the velocity of sound is again obtained — only approximately, however (Fig. 149).

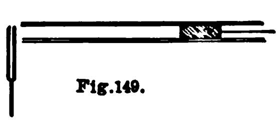
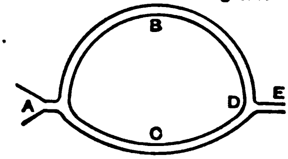


Fig. 149.

Fig. 150.



The velocity of sound in air may also be determined by methods based on interference. At A (Fig. 150) sound enters; the waves pass along the two channels B and C to E. B can be lengthened; it is lengthened until no sound is heard at E; B is now half a wave-length longer than C. At E may be placed the ear, a resonator, a manometric capsule, or any other indicator of sound-waves.

No energy passes down DE when no sound is heard at E: the waves are reflected at D and pass back to A, the B-waves returning along C, the C-waves along B; they arrive at A in the same phase. Even if they arrive at D in the same phase and pass on to E, there must be reflexion of a negative wave from D along B and C.

A glass tube vibrating longitudinally will emit a sound; the length of the glass is half a wave-length. If the tube contain air, the air will be set in vibration; but it must divide itself into segments, each half as long as an air-wave corresponding to the same tone. Finely-powdered silica placed in the tube will be distributed by the vibrating air in such a way as to accumulate at the nodes of the aerial vibrations. The number of air-segments thus indicated shows the comparative speed of waves in air and in glass. This is Kundt's method.

The theoretical value for the velocity in air is found from the equation $v=\sqrt{k/c \cdot \mathfrak{k}/\rho}$, where k/c can be found by thermodynamic experiments, $\mathfrak{k}(=p)$ and ρ by direct observation.

The best average value for the velocity of sound in air seems to be about 33,200 cm. per second; the extremes being 330·6 (330·7 Regnault) and 333·7 metres.

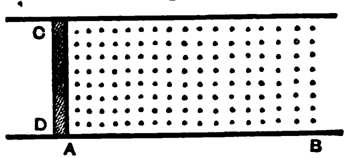
The velocity of sound in air is unaffected by variations of pressure, as we have already seen; it varies as the square root of the absolute temperature; it is affected by humidity, for damp air is lighter than dry air under the same pressure. It is also affected by wind, which not only retards the passage of sound to windward, but may also distort the waves and cause them to pass upwards. In general, the greater the intensity of sound the greater its velocity: sound therefore continuously slackens in speed of transmission as it becomes fainter.

It is still a moot point whether acute sounds are more or less rapidly conveyed through air than grave sounds: they are certainly not conveyed so far, for they are more affected by the viscosity of the air, and thus the higher components of an inharmonious sound are abolished by distance, and the whole effect is softened.

The velocity of sound being known, it becomes possible to measure certain distances by its aid. A lightning flash is seen, practically, instantaneously; if the sound take 5 seconds to travel, it must have travelled 1660 metres, or about a mile. Again, a stone is dropped down a well; the sound of the splash is heard in 6 seconds: at what depth is the surface of the water? The stone falls for t seconds through a space equal to $\frac{1}{2}gt^2$; the sound comes up for y seconds through a space equal to $33{,}200y$ cm. From the two equations $t + y = 6$, $\frac{1}{2} \times 981t^2 = 33{,}200y$, we find $t = 5{\cdot}543$, and the depth equal to 15,172 cm.

Propagation of Sound in gases according to the Kinetic Theory.* —Suppose AB to be a cylinder, CD an oscillating piston. The molecules which make up the gas in AB are represented by dots; for the nonce we shall suppose these molecules to be arranged in straight files, and we shall consider only one such file. The members of such a file remain in the same straight line, striking and rebounding from one another. We keep in mind the proposition that two perfectly-elastic bodies, when they enter into collision, rebound from one another with exchanged velocities; and that other, that a perfectly-elastic body striking a rigid obstacle returns with a velocity equal to the relative velocity with which it struck the obstacle. If the piston be at rest, it does not change the velocities of those particles which strike it, except in their direction. If it be moving towards them as they strike it, they rebound from it with a velocity greater than that with which they approached it; this increased velocity they communicate by exchange to others with which they come into collision: they then return to the approaching piston, and again rebound with increased velocity. These molecules thus act as carriers of energy; they borrow from the advancing piston a certain amount of energy, which they pass on to the molecules beyond; from molecule to molecule this energy is transmitted, and a transitory crowding together of the molecules, commencing at the piston, is propagated through the whole gaseous mass. Conversely, when the piston is in retreat, those molecules which overtake it rebound with a diminished velocity; they exchange this diminished velocity with those molecules which they tardily encounter, and which they do not turn back until these have travelled farther than they normally could have done; a diminution of velocity and an accompanying rarefaction are propagated throughout the gas. If the particles all lay in straight lines, the speed of propagation of sound would be the average speed at which the particles move; just as the rate of propagation of a message by couriers would be the average rate at which they ride. Nothing would be gained in point of speed by multiplying the number of such couriers if their horses were not susceptible to fatigue; and so it is a matter of indifference what the number of molecules is by the intervention of which the exchange and transmission of energy are effected, provided always that the collisions of the molecules occupy time inappreciable in comparison with the intervals spent by them in traversing their free paths, and further, that the size of the molecules be very small in comparison with the average length of the free path. Thus the velocity of sound does not depend, within the same gas, upon the density or on the pressure.

Fig. 151.



If the speed be changed, the case is different; an increased average speed causes an increased velocity of propagation. This may occur within the same gas when the temperature is altered; the absolute temperature measures the kinetic energy of the molecules; to the square root of this the mean velocity is proportional; the rate of sound-propagation would, if the particles all lay in straight lines, be equal to this mean velocity; whence

* See Tolver Preston, *Phil. Mag.* iii. (1877).

the rate of sound-propagation would vary as the square root of the absolute temperature.

On comparing two gases we find that the mean velocity of the molecules varies inversely as the square root of the molecular weight, and therefore as the square root of the density; whence the velocities of sound in two different gases would, on the above hypothesis, be inversely as the square roots of the respective densities.

We cannot, however, affirm that the particles of a gas lie in straight lines or files; they move on the whole with perfect symmetry with reference to every point. Professor Clerk Maxwell showed that, taking this into account, we ought not to expect the rate of propagation of sound to be *equal* to the average velocity of the particles, but *proportional* to it; and that, on the assumption that the particles were small as compared with their mean distances, and that each one was smooth and round, so as not to be set in rotation by impacts, then the rate of propagation of sound should bear to the mean velocity of the particles the ratio of $\sqrt{5}:3$, or $\cdot745:1$. Kundt and Warburg found exactly this ratio in the case of the vapour of mercury. In hydrogen the mean velocity is 184,260 cm. per sec.; the velocity of sound in hydrogen is, according to the mean of several observations, 126,917·6 cm. per sec.; the ratio is $\cdot6888:1$, less than that given above. The inference is that the molecules are to some extent set in rotatory as well as in translatory movement.

Döppler's Principle. — From a sounding-body, approaching or approached, sound-waves reach the ear in greater number than when the source of sound and the listener are relatively at rest; and conversely, if the sounding-body recede or be receded from, fewer sound-waves will reach the ear. To a person standing at a railway station while an express rushes whistling through, the pitch of the whistle seems suddenly to fall as the engine passes him. Even the puffs of an approaching goods-engine seem to the ear appreciably more numerous than those of a receding one.

Problem. — Show that with a velocity of 2000 cm. per sec. (say 45 miles an hour) the fall of pitch exceeds a major second, and is the same whatever may be the pitch of the note; the medium being air.

The Human Ear.

Aerial waves are communicated to the air in the external auditory meatus. This is short in comparison with the length of the average sound-wave. Its own proper sound is about g'''', and sounds in the neighbourhood of this tone are painfully reinforced by the resonance of the meatus.

The movements of the air in the meatus do not materially differ from those of a single point in the wave-front: the physical problem to be solved in the organ of hearing is one of the same kind as would be presented if the eye were called upon, by

the inspection of a single point on the surface of a multifariously-rippled sheet of water, to discriminate all the component undulations of the extended surface.

The movements of the air in the meatus are communicated to the drum of the ear, the *membrana tympani*, which is affected by the direct impact of the moving air-particles.

The drum of the ear may receive some vibrations by direct transmission from the bones of the skull.

We remark here — (1.) The natural note of so small a membrane is very high; but weighted as it is by the chain of bones of the internal ear, it can take up vibrations of a much less frequency than this note.

(2.) The vibration of the membrane is a forced one, and, as regards the relative amplitudes of very high components, does not precisely coincide in character with that of the air.

(3.) At the same time the form of the membrane is such that it vibrates more at its edges than at its centre, and the tendency of the membrane to set up vibrations of its own, or to alter those forced upon it, is mitigated.

(4.) The membrane is normally under tension: it is pulled inwards by the handle of the *malleus;* considerable pressures upon it cause very small inward movements, especially since its radial fibres have very slight extensibility.

(5.) It is easier for a rarefaction of the air in the meatus to cause an outward movement, which slackens the membrane, than for a condensation to drive the membrane inwards and thus to tighten it — a fact of importance in reference to combinational tones, of which hereafter.

(6.) When the membrane does move inwards, it pushes inwards the handle of the *malleus*, which is firmly attached to it: but only through a very small distance. This small amplitude of movement, about one-fortieth of that of the air in the meatus, implies that the handle of the *malleus* is wielded with considerable force — one step in the increase of the force of the aerial vibrations on their way to the internal ear.

(7.) The movements of the drum of the ear are astoundingly small. The greatest displacement seems to be about 0·1 mm. or 1-250th of an inch. A sound produced by an *f*♯ (= 181) pipe under an air pressure of 40 mm. of water can be distinctly heard at a distance of 115 metres. Töpler and Boltzmann calculated that at such a distance the movements

of the air must be reduced to ·000,04 mm.; but those of the more massive drum, with its appendages, cannot be more than ·000,001 mm. or the twenty-five-millionth part of an inch — an oscillation so minute as to be beyond direct microscopic observation.

The drum of the ear sets in motion the handle of the Malleus. The malleus is a small bone, somewhat resembling a hammer, with a head and a handle. It is so suspended by ligaments, head upwards, that when its handle is thrust inwards, its head is made to rotate to a limited extent. The head of the malleus is connected by a smooth joint of peculiar form with a second bone, the Incus. The action of the joint is such that when the handle of the malleus is forced inwards, the head, as it rotates, locks in the incus and forces it round; while, if the handle of the malleus be driven violently outwards, the head, rotating in a reversed direction, does not pull the incus with it, but glides over it, rotating through as much as 5° before the two bones again begin to move as one piece. If air be driven through the Eustachian tube from the mouth-cavity, as it always is during swallowing, it presses against the membrane from within; so, if it were not for this peculiar joint, there would be a decided risk of the chain of bones — malleus, incus, and stapes — being torn away from their connection with the internal ear. While the two bones are thus unlocked, as during swallowing, there is an impairment in their power of transmitting vibrations, and there arises a partial deafness, especially for loud sounds.

The incus has a process or long projection which, when the handle of the malleus moves inwards, moves inwards also. The point of this is attached to a little stirrup-shaped bone, the Stapes. Motion is thus communicated through malleus, incus, stapes; but the stapes moves only two-thirds as much as the end of the handle of the malleus — another step in the increase of force and diminution of amplitude of the vibration conveyed to the ear by the air.

Communication of sound through the bones of the head seems to be effected, for the most part, through the malleus, incus, and stapes. Communication of sound to the bones of the head may be facilitated by means of the audiphone. This is a plate of thin vulcanite, which is bent and kept by strings under a certain degree of tension, and the edge of which is placed in contact with the teeth. Even a piece of stiff brown paper, loosely doubled and grasped by its opposite edges between the teeth, will act as an audiphone and take up waves of sound from the air, and will convey them to the bones of the head. A certain proportion of the sound travels directly

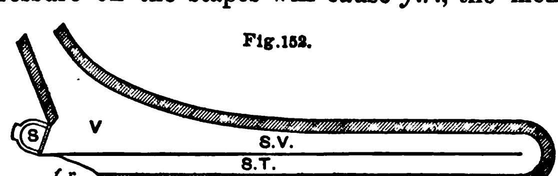
to the nervous apparatus embedded in the skull, which is directly shaken; sound is thus rendered to some extent audible to those whose auditory ossicles fail in their function.

The footplate of the stirrup is blended with a membrane occupying a small aperture — the *fenestra ovalis* — in the hard mass of the temporal bone. The footplate of the stirrup has itself a form closely resembling that of a footprint. If now the reader will place his foot on a soft carpet and forcibly drive into the carpet the outer edge of the foot, he will see that the inner edge of his foot is tilted upwards; this describes the motion of the stapes when driven inwards against the membrane of the *fenestra ovalis*.

Beyond this membrane lies the fluid of the internal ear, contained in membranous bags, which float in channels hollowed out in the temporal bone. This system of membranous bags containing fluid consists of the vestibule, the cochlea (in front), and the semicircular canals (behind). Here we have to do with the two former. The fluid lying in the vestibule is immediately behind the membrane of the *fenestra ovalis;* the vibrations of the stapes are communicated to it. This fluid is in direct communication with the fluid lying in a part of the cochlea called the *scala vestibuli.*

The structure of the cochlea seems at first sight somewhat complex. It is a snail-shell-like object; if unwound and laid flat it might be diagrammatically represented by Fig. 152. S is the stapes, blending with the membrane of the *fenestra ovalis;* V is the vestibule; S.V. is the *scala vestibuli,* a cavity extending to the tip of the cochlea; there it is continuous with S.T., the *scala tympani;* this ends at *f.r.* the *fenestra rotunda,* an aperture closed by a strong membrane. The fluid filling S.V. (the perilymph) is continuous with that in S.T., and hence strong pressure on the stapes will cause *f.r.*, the membrane of

Fig. 152.

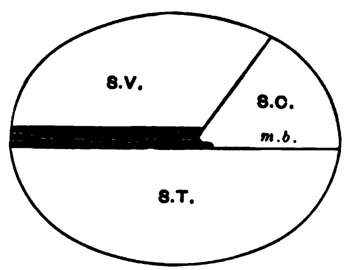


the *fenestra rotunda*, to bulge outwards. The *scala tympani* and the *fenestra rotunda* are perhaps a safety arrangement. The vibrations which are important to us are those of the fluid in the *scala vestibuli.*

Between the *scala vestibuli* and the *scala tympani* lies a partition, incomplete at the apex of the cochlea.' This partition is partly of bone, partly of membrane: the purely membranous part is called the *membrana basilaris.*

Transverse section of the cochlea shows us further that we have not only to do with the two scalæ and the intervening partition, but also with a third cavity. Such a section is shown diagrammatically in Fig. 153. The *scala vestibuli* rests only upon the bony part of the partition; the third cavity, the *scala cochleæ*, S.C., lies mainly above the membranous part, the basilar membrane, *m.b.* The *scala cochleæ* contains fluid (endolymph), and is practically a closed cavity. When the liquid in the *scala vestibuli* vibrates, the endolymph in the *scala cochleæ* is at each impulse forced into similar movement before the liquid in S.V. has had time to pass into S.T.; and it in its turn acts upon the basilar membrane.

Fig. 153.



The basilar membrane is triangular in form, being widest at the *tip* of the cochlea. It takes up vibrations of definite pitch, in response to which it vibrates not as a whole but locally (see Fig. 148); just as when we sing to a piano with the dampers down, only those strings respond which are in unison with the sound produced by the voice. It responds to vibrations of considerable slowness compared with its natural vibrations, for not only does it lie between two liquids, but it is also somewhat heavily loaded; upon it are mounted certain rigid structures arranged in two rows, the rods of Corti; and upon these are arranged a number of nerve-cells, the cells of Corti, each of which is connected with a single nerve-fibre. Each of these nerve-fibres can only be stimulated to sensation by the vibratile movement of that cell of Corti from which it runs; and it is deaf to every sound but that one to which the vibration of the particular underlying part of the basilar membrane corresponds. As there are from 16,000 to 20,000 such cells of Corti, and therefore the same number of nerve-fibres which, although they merge into a common strand — the *auditory nerve* — and enter the brain side by side, are isolated from one another, we may say that we have not one, but from 16,000 to 20,000 distinct Senses of Hearing, each with its own special Organ of Hearing; the Ear, as we have de-

scribed it, being merely a mechanical means for the transmission of vibration from the external world to these sense-organs, and its due distribution among them.

There is yet some difficulty in seeing how the rods of Corti, 3000 pairs in number, affect differently the half-dozen cells of Corti borne by each pair; hence some deny that more than 3000 different sounds can be perceived otherwise than by a process of comparison; even this would enable us to distinguish tones differing by less than the thirtieth part of a semitone. In some animals, also, the basilar membrane appears too unwieldy a structure to act well in the way described (Rutherford).

When an external sound does not coincide with any of the above 16,000 or 20,000, the cells most nearly corresponding to it will be disturbed; one of these cells will vibrate more than its neighbours. The phenomenon is now one of unconscious comparison of their relative disturbances. Within the middle range of hearing, sounds differing by one vibration in three seconds can be distinguished by some persons; when the notes chosen are very high in pitch, grave errors in the discrimination of pitch may, on the other hand, be readily committed.

When a compound sound is produced, the basilar membrane is set in motion in a number of limited regions at the same time, and the effect is mingled in the brain. The nature of the compound sensation, by which we thus recognise the different qualities of sound, is a question which passes the bounds of Physics.

The ear has a certain power of persisting in vibrations once set up in it; but only in small degree, and to an extent the greater the lower the pitch of the note. If the sound c' be broken up by alternate flashes of sound and silences of equal duration, when these each number 130 per second the sound seems continuous. Each pitch has its own duration of persistence; and Mayer has pointed out (see *Phil. Mag.*, 1876, vol. ii.) that in a mixture of sounds, alternately admitted to the ear and shut off from it by apertures in a rotating disc, some may be rendered evidently intermittent by the action of the disc, while others may appear continuous, and that thus we have a new means of analysing compound sounds.

As to the limits of hearing, sounds may be perceived by the human ear which range from 16 (Preyer), or 34 (von Helmholtz), to about 32,000 (Despretz), or even 40,000 (Appunn and Preyer); there being between different individuals curious differences in the power of perception, especially of high sounds.

A small number of abrupt clicks cannot, with ease, blend into a continuous sound; if they seem to do so, it is generally some high harmonic that is really heard. A very small number of true pendular vibrations seems to produce sound when sufficient surface is acted upon, or the ear otherwise firmly enough set in vibration; whence the very deep hum of a river-steamer slackening speed, or that sound of contracting muscle—19

vibrations per second — which is heard when the forefingers are pressed into the ears and the elbows pressed against the table.

The sensible loudness of sounds does not coincide very closely with their physical intensity. This arises partly from modifications in the form of the vibration induced by so complicated a transmission through the auditory apparatus, partly from causes purely physiological.

It is curious that, as Mayer has shown, high notes are heard with difficulty in the presence of lower ones. Hence sixteen violins in an orchestra produce by no means so great an effect as sixteen violins alone. A lower note tends to drown a higher one, especially if the higher note be thoroughly in tune, or form a correct acoustic interval with the lower. In a single musical sound the fundamental tone drowns the harmonics, even though the latter may, as in a tinkling piano, very greatly exceed it in physical intensity.

The sensitiveness of different ears to sound may be compared by measuring the relative distances at which a given sound becomes inaudible; as the squares of these distances, so are the sensibilities of the listening ears. If one ear can hear a certain sound at 3 feet, the other only at 3 inches, then the duller ear is $(36^2 \div 3^2) = 144$ times less sensitive than its fellow.

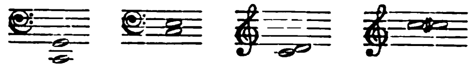
Even beyond the ear and within the brain there is some mechanism of which we are still ignorant. Professor Silvanus Thompson has shown that two sounds which beat with one another will, when conveyed separately one to each ear, produce beats which appear to jar the vertex of the brain; and further, that two sounds of the same pitch and phase, arriving separately in the ears, appear to be heard in the ears, while, if they arrive in opposite phases, the effect is as if the sound were heard not in the ears at all, but within the vertex of the cranium. The former phenomenon may be roughly shown by a tuning-fork held to each ear; the latter by a pair of telephones, one to each ear, one being provided with a commutator, by which the current in that telephone can be reversed at will.

Direction of sound can hardly be determined if the head be held fixed; we turn the head slightly while listening, and interpret unconsciously the consequent variations of intensity in the two ears.

Harmony and Dissonance.

Just as it is disagreeable to the eye to be exposed to flickering light, so it is painful to the ear to be exposed to audible flickering such as that produced by two sounds which beat when sounded together. The climax of unpleasantness or Discord is reached when the beats amount to about 32 per second. When the beats are still more numerous than this, the two notes which are sounded together become more distinctly separable by the ear, and the beats are less prominent to the sense of hearing.

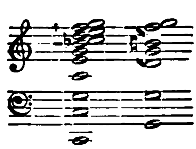
Were this the only cause in operation, the intervals



which all differ by 32 vibrations per second, would be equally painful to the ear. To some extent it is the case that an interval concordant between high notes is painful in the lower parts of the scale, as may be found on playing a major third in different octaves on a harmonium: but another cause affects the degree of painfulness of a discordant interval. When two contiguous sounds affect the basilar membrane simultaneously, the vibration of the basilar membrane is not limited to those fibres or narrow strips which exactly correspond to the two contiguous sounds heard; the result is a confused vibration of the membrane in a wider strip, and a sound is heard, compounded of all the notes within the region of that strip; this is interpreted by the ear either — where the two component notes are very close — as a single note of average pitch, or, where they are farther apart, as two separate notes coupled with a painful sensation. Discord is thus due partly to beats, partly to difficulty in identifying pitch.

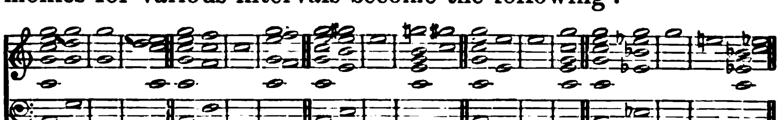
The beats produced by mis-tuned consonant-intervals correspond to curious vibrational curves, for which see Bosanquet, *Phil. Mag.* 1881.

When two notes are produced by separate sources of sound, the upper harmonics may possibly clash and beat with one another. This may be specially observed in the roughness of full chords of a brass band within an enclosed space. If, for instance, C and G be sounded together, the aggregate harmonics are the following: —



of which only g, g', and g'' are coincident.

If we eliminate all the harmonics beyond the sixth of the lower notes, the respective coincident and non-coincident harmonics for various intervals become the following: —



The most concordant interval is that in which the harmonics soonest coincide, and in which those non-coincident harmonics which discord with one another are as remote as possible from the fundamental tones.

This is not the only element to be taken into account in explaining the relative harmoniousness of certain intervals. When two notes are sounded together, there are produced other tones called Differential and Summational Tones, or, generally, **combinational tones,** which are heard along with the former.

Differential Tones. — When c' ($= 256$) and g' ($= 384$) are sounded together loudly and firmly, the listening ear can distinguish the sound c ($= 128$) humming at the same time. With c' ($= 256$) and e' ($= 320$) the differential tone is C ($= 64$). The vibrational number of the differential tone is equal to the number of beats which tend to be formed by the two prime tones. For this reason it was long thought that the combinational tones were produced by the blending, into a continuous sound, of Beats too numerous to reckon. These tones are, on the contrary, distinct from the beats. If two very high notes, differing by 64 vibrations per second, be sounded together, the discordant shiver due to the beats can be distinctly perceived, though the beats themselves cannot be reckoned, while at the same time the corresponding differential tone can be heard humming. If the sounds be separately conveyed, one to each ear, the beats may be distinctly felt within the head, but no differential tone is heard. And further, if the two sources of sound be markedly distant from one another, the beats may be apparent while the differential tone is feeble.

There are two causes for the formation of these tones. When a considerable disturbance has been produced in the air by one source of sound, the disturbance of the air produced by another source near the former is not simply added to the disturbance already existing, for Hooke's Law — as the force applied, so is the distortion produced — is not true except for very small disturbances: the amplitude of the compound oscillation falls short of the sum of the amplitudes of the components, and this is equivalent to the introduction of a new vibration whose fundamental period is that of the differential tone. Differential tones so produced — as by a harmonium — can be reinforced and investigated by means of resonators. The more effective cause of the production of combinational tones is, however, their origination in the drum of the ear: the drum of the ear moves more freely outwards than inwards; and von Helmholtz showed that if a transmitter possessing this peculiarity be acted upon by a disturbance which is the sum of two disturbances of frequencies p and q, the energy imparted to it is in part expended in producing new vibrations of frequencies $p - q$ and $p + q$;

these corresponding respectively to differential tones and to summational tones. Sounds so produced cannot be reinforced by resonators.

Summational Tones. — These tones were discovered by von Helmholtz. The notes *c'* and *g'* (256 and 384) will, when sounded together, produce a faintly audible tone *e''* (= 640 = 256 + 384). Tones of this kind are only to be heard with difficulty — a fortunate circumstance, since they mostly discord with their prime tones.

The summational and differential tones for some consonant intervals are represented in the following table, in which minims (𝅗𝅥) denote the prime tones and crotchets (♩) the combinational tones.

Harmony. — The chord *c'*, *e'*, *g'*, , has a differential tone between *c'* and *e'*, the tone C; between *e'* and *g'* the same; between *c'* and *g'* the tone *c;* together . Again, *c'* has a first harmonic *c'';* with *e'* and *g'* this makes the differential tones *g* and *c*. The note *e'* has a first harmonic *e'';* with *c'* and *g'* the difference-tones are respectively *g'* and *c';* and similarly the first harmonic of *g'* produces, with *c'* and *e'*, the tones *c''* and *b'*♭—. Altogether, these form the series

all the tones of which occur as partials in the note C, and therefore blend smoothly together. The minor chord *c'*, *e'*♭, *g'* in a similar way has the primary differential tones A,♭, E♭, *c*, and the secondary differential tones *c*, *b*♭, *d'*, *g'*♭—, *b'*♭, *c'';* and the primary chord is thus embedded in a mass of combination-tones comprising, among others, the discordant series —

The consonance of c', $e'\flat$, g' is therefore necessarily much harsher than that of c', e', and g'.

For further developments of this subject the reader must be referred to von Helmholtz's *Sensations of Tone*, translated by Mr. Ellis.

When just intonation is possible, as among glee-singers or quartette-players, each listens to his fellow-performers as well as to his own voice or instrument, and gives out that note which he feels to belong to the key in which the party is performing, and learns to do so in such a way as to avoid beats: thus, as is said, the performers rub off one another's asperities. In this way mathematically-exact ratios of considerable complexity are accurately attained without necessary knowledge of them on the part of the performers.

VOICE — VOWELS.

The voice is produced by vibrations of the larynx, especially of the vocal chords, in whole or in part. Above these is placed the mouth-cavity, which may assume various forms under the action of the various muscles which regulate the position of the tongue, the soft palate, the floor, and the sides of the mouth. This mouth-cavity acts as a resonator and reflector. According to the number of upper harmonics which are reinforced, and the extent to which they are severally reinforced, will vary the quality of the sound emitted. Upon this quality, and upon nothing else, depends that Character which we recognise as some particular Vowel; for every vowel is a particular Quality of Sound.

An elementary example of this is furnished by a common pocket tuning-fork; when set in vibration and the broad face of one of the prongs presented to the ear, the fork seems to emit the vowel *u* or *oo;* when its shank is pressed against a table the fork seems to say *ō;* now the octave becomes prominent. The reason is that the fork swings in circular arcs, and not in transverse straight lines; it consequently presses against the table at the end of each half-oscillation, and causes it to emit the octave as well as the fundamental tone. A tone almost pure gives the hollow sound of the vowel *ū;* one accompanied by its octave gives the brighter sound of the vowel *ō*. Each vowel gives a particular form of indentation in a phonograph.

Vowel sounds can be analysed by means of resonators; and when a particular vowel is sung in presence of an open piano (loud pedal down) that vowel is repeated by the strings: each component of the complex vibration is taken up by that string which is in unison with it. On the other hand, von Helmholtz

showed that by causing a number of resonators of a series whose frequencies were as 1:2:3:4:5:6:7, etc., to vibrate with independent intensities, he could at will produce by synthesis not only a great number of qualities of tone widely differing from one another, such as clarionet-tone, etc., but could also build up the different vowels themselves.

To each vowel corresponds a different form of the resonating mouth-cavity; to each such form corresponds a different natural pitch of vibration. When the larynx emits a complex sound containing as one of its components a tone of this natural pitch, this tone is strongly reinforced, and the quality of tone somewhat affected.

TRANSFORMATIONS OF THE ENERGY OF SOUND.

Sound being in its physical aspect a kind of motion, in the course of which work is done against elasticity and inertia, it is superfluous to speak of the conversion of the energy of sound into that of mechanical work. The transmission of sound is a transmission of energy, and the sound produced by a sounding-body is mechanically equivalent to a definite amount of work. When a heavy tuning-fork is attached to the piston of a little pump, as in Edison's harmonic engine, it can be made to do work; but then it produces somewhat less sound than when vibrating freely. The mechanical equivalent of sound may be estimated, as it has been by Mayer, by comparing the sound produced by a free tuning-fork with the sound produced by the same fork on equal excitation when its prongs are connected by a thin strip of indiarubber, and by finding the amount of heat developed in the rubber in the latter case.

Work may be, on the other hand, converted into sound. In general there are two methods of accomplishing this transformation, — firstly, by storing potential energy in an elastic body, which is then liberated; secondly, by transforming uniform into intermittent motion through the agency of friction. We have already studied the mode of excitation of a violin string. A pointed slate-pencil pushed across a slate at a certain angle produces a well-known shrill scream, and the mark produced by it will be found on close examination to consist of a train of separate dots; the action is not unlike that of the violin string. The scream of unoiled bearings in a machine may be accounted for in the same way, and in such a case much of the energy of rotation of the machine is wasted in the form of sound.

Heat may be, in some cases, transformed into the energy of Sound. Trevelyan's rocker and singing-flames we have already studied; the singing of a kettle is due to the rhythmical agitation produced by the formation and collapse of bubbles; the roar of steam issuing from a boiler is produced by the disturbance of the surrounding air by steam which, after thrusting aside the surrounding air, collapses into water-drops; the roar of a chimney is due to the oscillation to-and-fro, within the chimney, of heated columns of air or smoke which set the air within the chimney in vibration, of which the deep roar heard by us is generally a high harmonic: in all such cases the energy of the sound produced is obtained at the expense of the Heat supplied.

But, like other forms of energy, that of Sound is ultimately dissipated. When sound is produced in a room, every particle of the walls and contents of the room is set in vibration; there is, indeed, no way of protecting bodies surrounding a source of sound from this influence, except perhaps by placing them upon several alternate layers of caoutchouc and soft putty within a vacuum. At last the sound degenerates, after repeated reflexion within each object, into irregular molecular motion, and its energy is converted into Heat. So when a tuning-fork is set in motion and sounded in the open air, part of the energy which was initially communicated to the tuning-fork when it was first set in vibration is lost, in consequence of the viscosity of the fork, which becomes slightly warmer; while part of that energy is expended upon the external air, which, by reason of its own viscosity, gradually extinguishes the sound, beginning with the highest components, and the whole at length dies away, the energy of sound-motion becoming converted into the degenerated form of Heat, which ultimately becomes diffused throughout the entire Universe.

CHAPTER XV.

OF ETHER-WAVES.

IN this chapter a variety of phenomena fall to be considered which can be explained as phenomena of undulation in the all-pervading Ether, and may thus be said to be due to Ether-Waves.

It is necessary, however, to make a reservation of opinion, and to point out that all we are really entitled to affirm is that the phenomena in question are transferences of energy through the Ether, accompanied by variable disturbances of that medium — disturbances whose variations follow the same laws as those of wave-motion, but which may in themselves be due to changes not necessarily of position within the Ether, but possibly of its stress, of its electric condition, or of some other property of the interstellar medium as yet unknown to us. Their theory has been chiefly developed by those who considered the phenomena of Light, Radiant Heat, etc., as phenomena of Wave-Motion in the Ether; and, with this preliminary explanation, we shall in the sequel speak unreservedly of these phenomena as due to Ether-Waves.

NATURE OF RADIATION.

The all-pervading Ether can be set in vibration by the vibration of the molecules of ordinary matter. This local disturbance sets up waves; and by these waves energy may be transferred from one place to another. This process of transference of energy by Ether-waves is the process of **Radiation.**

The Radiation of Energy by the Sun amounts, according to the results of Prof. Langley's experiments on Mount Whitney, Southern California, to about 16500 horse-power per square foot of the sun's surface. Of this about one 2125,000000th part meets the earth; this, at the earth, amounts to about 150 ft.-lbs. per sq. ft. per sec., or 3 *ca* per sq. cm. per minute. Of this about one-third is always spent in heating the atmosphere; the rest may be more or less cut off from access to the earth's surface by clouds, dust, water-

vapour, etc. The energy of the waves comprised within a cubic mile of the Ether near the earth's surface, or, to use Lord Kelvin's phrase, the "mechanical value of a cubic mile of sunlight," is accordingly about 23000 ft.-lbs.

Ether-waves can also be produced by electric methods, for which see p. 741. These waves are, as yet, longer and of less frequency than those produced by the vibration of molecules.

Heat-waves and light-waves in Ether are not waves of compression and rarefaction, like those of sound in air. The propagation of an ether-wave is effected after a different fashion, somewhat difficult to realise. The analogy of a **transverse** vibration running along a cord, or of a wave of up-and-down oscillation running over the surface of water or over a thin membrane, must be extended to the Ether, with its three dimensions in space. At any point where the movement of the Ether is examined, it is found to be an oscillation at right angles to the direction in which the wave is being propagated, and therefore parallel to the wave-front.

The vibration of the Ether, when due to molecular vibration, is initially of the nature of a forced vibration; it is probably excited by the oscillation of a part of the Ether, which is in some way entangled within, or which envelopes, the vibrating molecule.

The molecular vibration which excites the ether-waves is a true vibration of the molecule, not a translational oscillation from place to place.

The molecules of ordinary matter must be supposed, in virtue of their small size, to vibrate very rapidly. We have already stated that the average diameter of molecules is perhaps the $\frac{1}{2,500000}$th part of a millimetre, and that they may perhaps consist of ether rolling within ether in vibratile vortices. A steel tuning-fork 2 inches (50 mm.) long may, if it be of the proper form, vibrate 240 times a second; if it were $\frac{1}{2,500000}$ mm. long, and of the same shape, it would vibrate 30,000,000000 times per second; if made not of steel, but of ether, its frequency would be greater in the ratio of the velocity of propagation in ether to that in steel, and would therefore amount to about 2600,000000,000000 oscillations per second. The vibration of a molecule may be more like that of a disc than that of a tuning-fork; but the rough analogy just mentioned may serve to show that it is, even *a priori*, possible that some such number might denote the average frequency of molecular oscillations,—an average modified in the direction of retardation by the formation of heavier molecules through the coalescence of smaller molecules, or perhaps by the reaction of the Ether which is set in forced vibration, or modified, on the other hand, in the direction of acceleration by the formation of higher-pitched vibrations, which may be, to use the musical analogy, dissonant with one another when the structural arrangement of the molecules is unsymmetrical. The molecule of sodium-vapour acts somewhat like a disc which is slightly unsymmetrical: such a disc would give out two tones very near one another in pitch: and a vibrating sodium-molecule gives rise to two sets of ether-waves which differ only slightly in frequency.

Frequency. — The ether-waves which are produced by the mechanical vibrations of molecules have frequencies which range between about 20,000000,000000 (Langley) and about 4000,000000,000000 oscillations per second — a range, to use a musical analogy, of about eleven octaves: but of these our eyes are sensitive to scarcely one octave — to those, namely, which range between about 392,000000,000000 per second (extreme red of the spectrum), and about 757,000000,000000 per second (extreme violet). Those ether-waves which have been produced by electric methods have frequencies ranging between about 500 per second and 7500,000000 per second.

Velocity and Wave-Length. — These waves all travel through the Ether of space at the same rate, namely, about 30057,400000 cm. (186,680 miles) per second. Ether-waves while traversing the Ether present no essential differences, except in respect of their wave-lengths; the wave-length λ is equal to v/n, where v is the velocity of propagation and n the frequency.

Those ether-waves which are produced by the oscillations of molecules vary accordingly in length, in a vacuum, from about $\frac{1}{670}$ cm. to about $\frac{1}{1{,}300000}$ cm., and those waves to which our unassisted eyes are sensitive, the waves of Light, have wave-lengths ranging between $\frac{1}{13042}$ cm. and $\frac{1}{25185}$ cm. These wave-lengths are usually specified in terms of "tenth-metres"; a tenth-metre being 1 metre $\div 10^{10}$, or 0·000000,01 cm. Extreme red and extreme violet have thus, in a vacuum, the respective wave-lengths of 7667 and 3970 tenth-metres. Those ether-waves which have been produced by electric methods have wave-lengths ranging between 60,000000 cm. (373 miles) and 4 cm.

Ether-waves do not traverse all substances with equal speed: hence their wave-lengths in different substances vary; if any particular kind of radiation have to be spoken of, it may be defined by specifying its wave-length in some specified medium, but it is better to state its numerical frequency. To do the latter implies, however, that we assume — and we are apparently justified in assuming — that all kinds of radiation pass through a vacuum — that is, through the ether of space — with equal speed.

Kinds of Radiation. — When a succession of ether-waves impinges on a mass of ordinary matter, the effect varies according to the nature and the condition of the body which receives their shock; if it be an ordinary opaque mass, that mass may be warmed, the energy of wave-motion being transformed into heat, and the waves which have impinged upon the opaque mass are

ex post facto called a beam of **Radiant Heat**; if they fall upon the eye, they may produce a sensation of light, and the wave-system is then called a beam of **Light**: falling upon a sensitised photographic plate, or a living green leaf, they may operate chemical decomposition, and the wave-system is then called a beam of **Actinic rays**. The word "rays" in the last phrase may be understood to mean, not imaginary lines at right angles to the wave-front, but k i n d s o f r a d i a t i o n; and hence we speak of Heat rays, of Light rays, of Chemical or Actinic rays; these names being given to one and the same train of waves according to the effects which it is found competent to produce. But while ether-waves are in course of traversing the ether, there is neither Heat, Light, nor Chemical Decomposition; merely wave-motion, and transference of energy by wave-motion. Hence none of these names can in strictness be applied to a train of waves while these waves are actually travelling through the Ether.

Ether-waves which differ in their frequency differ to some extent in their degree of power of producing the motion of heat or the sensation of light, or of doing the work of chemical decomposition. All ether-waves can produce heat, for their energy is converted into heat when they fall upon and are absorbed by such a substance as a thick layer of lampblack, which for the most part arrests and extinguishes them.

The long and slow ether-waves which have been produced by e l e c t r i c methods are not so readily arrested and extinguished as those produced by molecular vibration; they can, therefore, to a large extent traverse such an obstacle as a brick partition or a deal door; but the difference in this respect between them and ordinary radiation-waves is a question of degree, and by a sufficient obstacle their energy can be reduced to Heat.

Those ether-waves which take their origin in vibration of ordinary matter, and whose frequency is less than 392,000000,-000000 per second, are too slow either to affect the eye with the sensation of light, or, in the ordinary case, to impart to molecules an agitation brisk enough to shake them to pieces, and thus to operate chemical decomposition. Such slow waves, whose presence can only be recognised after their impact, by the conversion of their energy into Heat, are called **Dark-Heat-Waves**. If they fall upon an ordinary photographic plate they do not operate chemical decomposition; but if the molecules

upon which they impinge be specially heavy and complex, even these slow heat-waves may be found to toss and shake them with briskness sufficient to break them up.

Radiant Heat of sufficient intensity is thus found to operate chemical decompositions of a different order from those brought about by contact-heating; *e.g.*, the formation of olefines from paraffin-vapours (W. Young), instead of that of aromatic hydrocarbons with deposition of carbon.

A dark nebula in the Pleiades was first photographed and then with difficulty seen. Major Abney has been able to photograph with heat-rays down to a frequency of about 160,000000,000000.

The waves may, on the other hand, be so rapid—above 757,000000,000000 per second—as to produce no visual effect on the eye; the eye is normally, physiologically, blind to them, and is unable to feel their impact; but they may effect chemical decomposition; their successive impulses may aid the natural free vibrations of the molecule, which thus become increasingly ample: and just as a resonant tumbler into which its own note is steadily sung vibrates, shivers, and breaks into fragments, so a molecule, quivering under the steady, regular, and continuously well-timed blows of the rapid ether-waves, may yield and break up into its constituent atoms, or into groups of atoms, which constitute simpler molecules. Such rapid waves are called Invisible or Ultra-violet **Chemical Rays.**

According to Lubbock, ants and *Daphnia* seem to be able to see ultra-violet rays.

The power of operating chemical decomposition possessed by the more rapid waves depends more upon their frequency than upon their intensity.

The rays which are most active in decomposing carbonic acid by chlorophyll are visible, being the yellow and the red.

For waves of given length but different intensities, the quantity of chemical decomposition is, within wide ranges of intensity, the same when the product (Time of Exposure × Intensity of Light) is the same. For very feeble intensities this product must be increased; in some cases, as in star-photography, very considerably. This law is, however, the basis of calculations of "exposure" in photography.

The slower waves may thus produce heat, or perhaps chemical decomposition of heavy complex-molecules; waves of medium rapidity may produce heat, the sensation of light, or chemical effect; the more rapid ones may produce heat or chemical effect according to the substance upon which they fall.

The invisible chemical rays, though they can operate chemical decomposition, are yet of very feeble physical intensity;

their aggregate kinetic energy is, in the radiations from the sun, as we receive them filtered through our atmosphere, millions of times less than that of the slower red or dark heat rays: even those rays which are visible are effective not so much in virtue of their intensity, which is but small, as in virtue of the extraordinary sensitiveness of the eye to light — that is, to the impact of ether-waves of a certain range of frequency.

Colour. — Within the limits of visibility — 392 billions to 757 billions — there is an indefinite variety of integral and fractional numbers, each of which represents the frequency of a particular kind of radiation, a particular kind of light. Physically there are as many *kinds of light* as there are possible *frequencies* between the limits mentioned. These kinds of light, each physically characterised by the *number of waves* which strike the eye during a second, are recognised by the normal eye as being distinct, not as the result of any conscious process of counting the number of impulses suffered by the eye during a second, which would be absolutely impossible, but in consequence of the distinct and peculiar *Sensation* attending the reception, in the eye, of wave-motion of each particular frequency — a sensation known in each case as that of a particular **Colour.** Thus, when we look at a Bunsen burner, the flame of which is caused to emit a dingy-yellow light by contact with common salt, we recognise the sensation as one of yellow light. Colour is a sensation: it is not a material existence; but the physical basis and cause of the special sensation of yellow light is, in this case, the joint simultaneous impact on the eye of two kinds of ether-waves, which have the respective frequencies of 508,905810,000000 and 510,604000,000000 per second, or the respective wave-lengths in *air* of 5895 and 5889·04 tenth-metres.

Either of these trains of waves impinging singly on the eye would produce (see, however, p. 575) a sensation of yellow, the slower one giving a yellow very slightly more orange in its tint than the other does. The term "yellow light," which means primarily a certain sensation, means, secondarily, the physical cause of this sensation — that is, a train of ether-waves of a particular frequency. Any particular colour is best specified by a statement of the frequency of the single wave-motion, which can produce that colour when it enters the eye; and the analogy between light of any given Colour and a sound of any given Pitch is obvious.

When there fall successively, upon the normal eye, trains of light-waves which differ only slightly in their frequency, the respective colour-sensations produced by them may resemble one another generically, though not precisely. When, in gradual succession, luminous waves of all possible frequencies are caused to strike the eye, we obtain in successive gradation the sensations of all the colours of the spectrum. The slowest waves which can affect the eye produce a sensation of red, those somewhat more rapid a sensation of scarlet; then in succession we find, as the frequencies increase, that the sensations produced are those of orange-red, reddish-orange, orange, yellow-orange, orange-yellow, yellow, greenish-yellow, yellowish-green, green, bluish-green, greenish-blue, blue, blue-violet, violet. Waves of still greater rapidity than those which produce the sensation of violet are practically invisible; but it must be admitted that they are not perfectly so.

Even beyond the ordinary range of visibility some eyes are affected by ultra-violet ether-waves; a sensation of lavender-gray colour results: a spectrum is often seen, especially if the dispersion be small, to contain three bright bands of lavender-gray in the ultra-violet region. This light is, in intensity, about 1-1200th part of that which shines in the same region of the spectrum when it is rendered visible by fluorescence. Beyond the red there is, similarly, a crimson.

The table, page 485, modified from Ogden Rood's *Modern Chromatics* and Lord Kelvin's Royal Instit. Lecture, Feb. 2, 1883, gives the frequencies and the wave-lengths in air of the several undulations which correspond to the several leading colours of the spectrum, and to some of the so-called Fraunhofer Lines.

When a source of light is receding from the eye, fewer waves per second strike the eye; the light approximates towards red. Conversely, the light of an approaching luminous object is, as it were, sharpened in pitch. The characteristic lines in the spectrum are thus somewhat displaced; and by this application of Döppler's principle, the speed of relative approach or recession of the earth and many fixed stars has been estimated.

That which we call **white light** is, in the state in which we receive it from such a body as a white-hot bar of iron or, perhaps in its purest form, from the crater of the positive pole of the electric arc, a mixture of long and short waves; waves of all periods within the range of visibility are either continuously present, or, if absent for a time, are absent in such feeble proportions or for such short intervals that they are not appreciably missed by the eye. White light of this kind is comparable to an

utterly-discordant chaos of sound of every audible pitch; such a noise would produce no distinct impression of pitch of any kind; and so white light is uncoloured.

	Frequencies.	Wave-lengths in centimetres. (Ångström.)
Line A	395,000000,000000	·00007604
Centre of red	429,400000,000000	·00007000
Line B	437,300000,000000	·00006867
Line C	457,700000,000000	·00006562
Centre of orange-red	484,000000,000000	·00006208
Centre of orange	503,300000,000000	·00005972
Line D^1	508,905810,000000	·00005895
Line D^2	510,604000,000000	·00005889
Centre of orange-yellow	511,200000,000000	·00005879
Centre of yellow	517,500000,000000	·00005808
Centre of green	570,200000,000000	·00005271
Line E	570,500000,000000	·00005269
Line *b*	580,000000,000000	·00005183
Centre of blue-green	591,400000,000000	·00005082
Centre of cyan-blue	606,000000,000000	·00004960
Line F	617,900000,000000	·00004861
Centre of blue	635,200000,000000	·00004732
Centre of violet-blue	685,800000,000000	·00004383
Line G	697,300000,000000	·00004307
Centre of puce-violet	740,500000,000000	·00004059
Line H_1	756,900000,000000	·00003968
Line H_2	763,600000,000000	·00003933

If a parallel beam of light of one kind, one wave-length, one colour,—homogeneous or **monochromatic** light,—be caused to pass through a slit in an opaque screen, it may be received upon a white screen, and it will cast upon that screen a coloured image of the slit. If the light, on issuing from the slit, instead of being received directly upon a screen, be made to pass through a glass prism, the narrow edge of which is held parallel to the slit, it will be refracted by that prism, and the image of the slit will now be found in a new position on the screen. If a beam of **white light** be so dealt with, a number of coloured images of the slit will be formed, each in its proper place on the screen, each image overlapping its neighbour if the slit be of appreciable width; there will thus be formed a many-coloured band of light, in which the colours are marshalled in the order of the frequency of their waves,—the slowest waves, the red, being least refracted by the glass prism; the quickest

waves, the violet, being most refracted. This is the **spectrum**: every component of the original white-light is displayed in the spectrum, each in its distinct place; and thus the prism furnishes us with a means of analysing light — that is, of finding what its components are.

But the spectrum extends beyond the visible part of it; the more rapid invisible rays, being more refrangible than the violet, form an invisible part — an **ultra-violet region** — which we detect by the phenomena of fluorescence (p. 504), or by casting the whole spectrum upon a sensitive photographic-plate, upon which we afterwards find a record of a region of the spectrum invisible to the eye; and the slower dark-heat rays form an invisible part of the spectrum beyond the red, the **heat spectrum** or ultra-red region, not visible, but demonstrable by means of any apparatus, such as a thermometer or a thermopile (Fig. 212), which is sensitive to heat. If the prism used be made of quartz, or if the spectrum be produced by reflexion from a diffraction-grating (p. 549), it will be found that the ultra-violet region is, if the light analysed be that of the electric arc, from six to eight times as long as the whole of the visible part of the spectrum; while if the prism used be of glass, it absorbs to a remarkable degree these rapid ultra-violet waves. If the light analysed be that of the sun, the ultra-violet part of the spectrum is comparatively very short, on account of absorption by the atmosphere.

This effect of the atmosphere is of extreme importance. Sunlight is originally bright blue, and is extremely rich in the more refrangible rays, but filtration through two absorbent atmospheres — that of the sun and that of the earth — renders it a yellowish-white (Langley). The ultra-violet part of the spectrum is enormously brighter at high altitudes.

Compound Coloured-Light. — Let us now cast a beam of sunshine or of electric light, shining through a slit in an opaque screen, upon a piece of greenish-blue glass, and receive upon a white screen the light which passes through this coloured glass: by the aid of a lens we may obtain a greenish-blue image of the slit upon the screen. So far as we have yet learned, such coloured light, whatever be the mechanism of its production, is a single kind of light — perhaps due to waves of only a single frequency: whether this be so in the particular case may be tested by interposing a prism in the path of the coloured beam of light: if the greenish-blue light be homogeneous, we shall again have on the screen an image of the slit, altered in position,

but not in colour. This is not what we find: a short and imperfect spectrum is produced; the transmitted greenish-blue light is analysed by the prism into green light, blue light, yellow light, with perhaps some other colours, more or less faintly represented.

This phenomenon is very singular. It shows that two widely-differing physical causes are capable of producing exactly the same colour-sensation: the one being, as we have already seen, the impact of ether-waves of a single definite frequency, the other being the joint impact on the retina of a number of wave-systems, each of which is capable, if it were to act independently, of producing a distinct sensation; and the colour-sensation which is produced by the joint action of these wave-systems may differ from that which characterises any one of them. It is as if a listener to concerted music were to hear the strains of an orchestra compounded into some sort of loud melody of average pitch, he being wholly unable, by his unaided ear, to recognise the really compound nature of the sound heard by him. Then, whether the instruments all played in unison or diverged into precalculated harmony, the effect on his ear might remain the same.

Further, many such mixtures may produce the same apparently simple sensation; and, accordingly, such a phrase as "green light" or "orange light" is perfectly vague, unless it be accompanied by a specification of its physical cause.

Complementary Colours.—The greenish-blue glass, in the instance just alluded to, has in whole or in part prevented the transmission of violet light, of red, of orange, and of other kinds of light which are present in white sunlight; the complex of undulations thus denied transmission would, if collectively allowed to impinge on the eye, have produced a single sensation of r e d light. If this compound red-light had not been obstructed by the coloured glass, the transmitted beam would have been white; this compound red-light thus obstructed by the greenish glass, and the compound greenish-light transmitted by it, will pass together through a piece of clear glass, and will together produce the sensation of white light. To the eye it is a matter of indifference whether the red or the greenish light be monochromatic or compound; monochromatic red-light and monochromatic greenish-blue light, allowed to fall upon the same spot in the eye, will mingle, and, if they be of the proper hue, will produce the compound sensation of white light. These colours, red and greenish-blue, each of the proper hue, are thus **complementary** to one another; together they make up white light.

The following pairs of colours are, among others, thus complementary to one another:—Red and a very greenish blue, orange and cyan-blue (a rather greenish blue), yellow and ultramarine blue, greenish-yellow and

violet, green and "purple," the latter being a colour not in the spectrum, but formed by the superposition of blue and red.

The expression "white light," standing alone, is thus also wholly vague; physiologically it means light which produces the sensation of white; physically it may mean (1) a mixture of all possible light-waves, long and short, in certain proportions; or (2) a mixture of two complementary simple colours; (3) a mixture of two complementary compound colours; or (4) a simple colour blended with a complementary compound one of any degree of complexity.

The white light of sunlight at sea-level is made up (Vierordt and Rood) by a mixture (=1000) of the following coloured lights: — Red, 54; Orange-red, 140; Orange, 80; Orange-yellow, 114; Yellow, 54; Greenish-yellow, 206; Yellowish-green, 121; Green and blue-green, 134; Cyan-blue, 32; Blue, 40; Ultramarine and blue-violet, 20; Violet, 5.

RADIATIONS OF A HOT BODY.

The hotter a body, the greater the intensity of the aggregate disturbance which it sets up in the Ether; and further, the greater the frequency of the most rapid components of that disturbance. A white-hot iron ball is visible in a dark room; it emits dark heat-rays, light-rays, and also the rapid ultra-violet rays: it can be seen and photographed, and its warmth can be felt at a distance. If it be intensely hot it may emit so great a proportion of violet and blue light that it appears bluish; it is "blue-hot."

As it cools down, the more rapid vibrations die away; the ultra-violet waves cease to be formed; the mass becomes somewhat less easy to photograph by its own light. Gradually the violet rays cease to be emitted; the light radiated is now apparently tinged with yellow: the apparent colour becomes orange, then red; a body at a red-heat is difficult to photograph, though it continues perfectly visible in the dark. When its temperature sinks to a point below 525° C., it ceases to radiate light and becomes invisible in the dark; it continues, however, to radiate heat, as may be felt for some time by the cooler hand placed near it.

H. F. Weber points out that platinum at 390° C., gold at 417° C., and iron (not quite free from rust) at 377° C., become faintly visible, first fog-gray, then ash-gray, then yellowish-gray, then faintly red, then red-hot, and so on.

The luminous radiations of an Argand oil-lamp are ½ % of the whole: of a gas-flame, 0·3 to 1; a Welsbach incandescent gas-lamp, 1½; an electric glow-lamp, 5–6; a small electric arc, 5–10; a 5000-candle arc, at 3000° C., 25 %. Of the solar radiation, 25 % is luminous (Sir C. W. Siemens).

It never ceases to radiate heat; it could not cease to do so unless it were cooled down to absolute zero. Since the molecules of all bodies are in repeated collision with their fellow-molecules, as they rebound at each collision, they shiver and they vibrate. They must therefore continuously originate ether-waves — waves which, when the temperature of the body is below 525° C., are too slow to affect the eye.

Exchange of Radiations. — Two bodies placed opposite to one another, with intervening Ether, of which we cannot get rid, and with or without intervening air, may present the two following cases: —

1. Both may be of the same temperature, in which case the one loses by imparting to the other exactly as much energy as it takes up from those ether-waves which strike it, having been originated by the other hot body; whence two bodies equally hot exchange their energies by radiation, but do this to an equal extent, and there is thus no change in their relative temperatures.

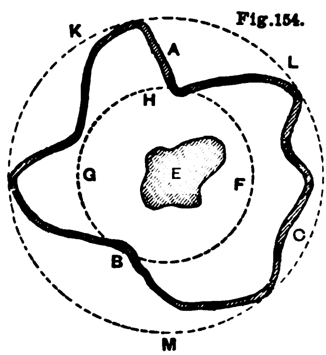
2. The one may, on the other hand, be hotter than the other. The hotter body sets up a more vehement system of ether-waves than the colder one can; in doing this it expends its energy to a greater extent than the colder one does; the hotter loses more energy than it gains; the colder gains more than it loses; in course of time their energies, and therefore their temperatures, become equal: when the temperatures have become equal, though the two bodies still go on imparting energy to each other, neither profits by the exchange, and their temperatures remain relatively equal.

The absolute amount of radiation of energy from a body does not depend on the condition or even on the presence of surrounding objects, but solely on the condition of the body itself. It is easy to see that the absolute physical brightness of the sun or of a candle is at any moment independent of the presence of illuminated objects; it is not, however, at first sight so clear that not only does a fire warm the walls of a room, but these walls also warm the fire; that the sun warms the earth while the earth — to a lesser extent, it is true — warms the sun; and that the warming of a colder body by a hotter one depends upon the difference of two similar but unequally-opposed actions.

When a lump of ice is placed near an object at the ordinary temperature, that object is cooled; it loses to the ice more heat than it gets from the ice: the ice apparently radiates cold.

When one body is surrounded by another, the body enclosed and the walls of the enclosure come to have the same temperature, if they be relatively at rest. A thermometer whose bulb is immersed in a cavity will come to indicate the temperature of the walls of the cavity, whether it be in contact with them or not. This equalisation of temperature by radiation is quite independent of the form of the walls of the cavity; a cavity of any form acts in the same way as a spherical cavity would do. In Fig. 154 the irregular hollow body ABC surrounds a body E; both E and ABC assume after some time a common temperature, and remain at an equal temperature.

Fig. 154.



The irregular hollow body ABC might be replaced by the hollow spherical body FGH, or by the hollow sphere KLM, or any other hollow sphere concentric with these. From this ensue the following propositions.

1. The amount of energy received by a receiving surface per unit of its area — the amount of heat received, the brightness of light there — varies inversely as the square of the distance from the source of radiation. The advantage of extensive surface possessed by the larger sphere KLM is exactly neutralised by its disadvantage of distance; its surface is greater, the radiation received by it per unit of area is less, both in the ratio of the squares of the radii, and the total radiation received by it is the same, whatever be its radius.

A candle at a distance of 1 foot can illuminate a printed page as brightly as a 25-candle gas-burner at a distance of 5 feet.

A bright wall is equally bright at all distances when looked at through a narrow conical tube. Close at hand it appears brighter, area for area, but less of it can be seen; at a distance it appears dimmer, but more of its surface can be seen; in all cases the amount of light falling on a given area of the retina is the same.

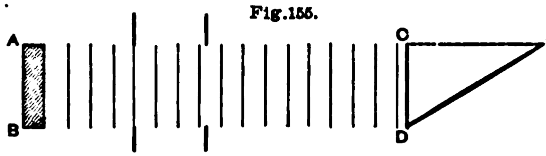
2. When a plane wave whose area is AB strikes squarely and simultaneously all parts of a surface whose area is also AB — the normals to the wave being also normals to the receiving surface — the receiving surface receives a certain number of units of energy per second. In Fig. 155 AB is a hot or bright body radiating ether-waves towards CD; CD receives e units of energy per second per unit of its area.

If the receiving surface be tilted, say into the position DE, the wave-front, striking obliquely, is now able to cover the larger surface DE; no more of the wave can now reach DE than would previously have reached

CD. The same quantity of energy is thus distributed over a larger surface: the quantity of energy communicated to it per unit of its area is diminished in the ratio $\frac{CD}{DE}$, or is equal to ($e \times \cos CDE$) units.

In accordance with this, the intensity of sunlight at noon is greater than during the earlier and later portions of the day, when the surface of the earth is presented obliquely to the sun's radiation.

Fig. 155.



3. Let AB receive energy from CD or DE; then, whether the surface be the smaller CD vertically facing it, or the larger DE arranged obliquely, is a matter of indifference; in either case there will be radiated towards AB the same amount of energy. DE therefore radiates towards AB, in the direction DB, less energy per unit of its surface than CD does in that direction when equally heated, and that in the ratio of cos CDE:1.

Were this not so, and did a hot surface radiate equally in all directions, then a body placed within a hot enclosure might become hotter than the walls of that enclosure.

This principle explains the apparent uniformity of brightness of the sun's disc. Towards the margin of the sun's apparent disc, areas which seem equal to similar areas near the centre are in reality much larger; but we see them obliquely; their larger superficial area exactly compensates the effect of their oblique aspect.

4. Radiation reflected from a mirror to a focus can never make an object placed at the focus radiate more energy per sq. cm. than the Source does; the temperature of the object cannot exceed that of the Source; but the object may, if sufficiently small, come to the same temperature as the source, after which there is between it and the source an equilibrium of radiation. Whence a thin wire in the focus of a very large mirror in sunlight ought (atmospheric absorption, etc., apart) to come up to the Sun's Temperature (3000° C., Siemens, but this is apparently too low a figure), but not to exceed it. Ericsson's Sun-motor is practically a huge parabolic mirror, in whose focus a high temperature is attained, which is utilised by an engine.

The law just stated — that bodies are always radiating and receiving energy — that the amount of radiation depends on the temperature of the radiating body — that at constant temperatures bodies radiate as much energy as they receive — is known as **Prevost's Law of Exchanges.** From this it follows that good radiators are good absorbents; and conversely, good absorbents are good radiators.

If a hot-water vessel be intended to retain its heat for a comparatively long period in the open air, it must be polished externally; a polished surface, being a good reflector, is a bad absorbent, and is therefore a bad radiator; while a blackened surface, being a good absorbent, is a good

radiator, and heat is with comparative rapidity lost through a coating of lampblack, provided that it be not so thick as to impede conduction of heat to the surface.

Prevost's Law is not only true of the aggregate energy gained or lost by a body through radiation; it is also true, as Balfour Stewart pointed out, with regard to each particular form or kind of radiation by means of which energy may be conveyed between neighbouring objects.

If a piece of yellow glass be placed within a hot shell of iron, the glass and the iron may both shine by their own light, and the glass may be looked at through a minute aperture in the wall of the hollow shell. Yellow glass absorbs ultramarine light, and a white-hot object, looked at through it, appears yellow, provided that the glass be colder than the source of the white light; but when the yellow glass is itself as hot as the source of white light, as it must be in this instance, in which we look through the white-hot glass at the white-hot wall of the iron shell, the glass seems perfectly transparent to the whole white light, — a phenomenon which may be interpreted as showing that while the glass only transmits yellow, it itself radiates blue light; the aggregate radiations, the transmitted yellow and the radiated blue, produce in the eye an aggregate effect of pure white. If the yellow glass be hotter than the source of light behind it, it seems relatively blue. The conclusion is, that as yellow glass absorbs blue light, so when itself heated it radiates blue light.

Stokes's Law. — A body which absorbs any particular kind of radiation will in general, when heated, become a source of radiation of the same kind; just as a resonator will, when it vibrates, impart to the air the same kind of sound of which it may rob the air when it, the resonator, is relatively at rest.

If a screen of strings tuned, say to the note of *a*, be arranged between a sounding *a* organ-pipe and a listener, the latter will hear comparatively little of the sound produced by the pipe; by resonance the strings have taken up the energy, and have converted part of it into Heat. If a mixed sound were produced on the farther side of such a screen, the sound of *a* would not be transmitted to the listener; the rest of the mixed sound would be heard by him.

When mixed ether-waves strike a system of molecules of which some are tuned to particular frequencies, those molecules will take up the energy of vibrations of those frequencies: the body will appear to be opaque to the corresponding waves.

From the reciprocity of absorption and radiation it follows that if a given substance be divided into portions, of which the one, A, is hot, while the other, B, is comparatively cool, radiations from A will be absorbed by B; the cooler portion, B, is

opaque to radiations from the hotter portion, A. Thus, if carbonic oxide be burned, its flame contains hot carbonic-acid; the radiations from such a flame cannot pass through pure, comparatively cool carbonic-acid, and are checked in very large proportion by air containing even a very small percentage of that gas or, curiously, of CS_2-vapour.

A hydrogen flame contains hot aqueous-vapour; the heat radiated from this — very slow dark heat-waves — cannot pass through comparatively cool aqueous-vapour: the result is, as Prof. Tyndall showed, that while the sun's light and heat can reach the earth's surface through the humid atmosphere, their effect is to warm the earth and cause it to produce slow waves of dark heat; these resemble in frequency the waves produced by hot aqueous-vapour in a hydrogen flame, and they cannot pass away through the aqueous vapour of the atmosphere. The atmosphere thus acts as a kind of heat-trap, and the surface of the earth is preserved from extremes of cold produced by excessive radiation. Prof. S. P. Langley has shown that, besides this, the atmosphere is itself, independently of aqueous vapour, remarkably opaque to certain heat-waves of great length, which are radiated outwards from the soil, but which, being absorbed by the atmosphere and spent in warming it, are trapped by it; that these same heat-rays, on their way from the sun, are absorbed by our atmosphere and never actually reach the earth; and that were it not for the atmosphere the earth's temperature would be below $-200°$ C., even under the vertical rays of a tropical sun. Thick glass has also a remarkable effect of this kind.

Burning sodium-vapour emits a particular yellow light; if looked at through a mass of sodium-vapour, it can hardly be seen; sodium-vapour absorbs the light given out by hotter sodium-vapour. Even though light, of that particular kind, do not happen, in any particular instance, to have been emitted by burning sodium, if the attempt be made to transmit it through sodium-vapour the sodium-vapour will be found opaque to that kind of light. If an electric lamp produce a beam of light which contains amongst others this particular kind of light, and if a spirit lamp have salt (NaCl) or, better, NaBr, placed in its wick so that it gives out this particular yellow light (this denoting that the spirit-lamp flame contains incandescent sodium-vapour); and if the electric arc be looked at through the spirit-lamp flame, then the colour of its light would appear, if the eye were sufficiently sensitive, to be altered; it is bluer; the sodium-yellow light of the electric arc is absorbed as it passes through the comparatively cool spirit-lamp flame, which, by its own comparatively-feeble radiation, does not repair the damage done by it, and the light which has passed through the spirit-flame is comparatively (not absolutely) wanting in that particular kind of yellow. The beam may be made, after passing through the

sodium-vapour, to traverse a slit and a prism, and thus to form a spectrum on a screen. It will be found, if this be done, that the spectrum is discontinuous; at the place where the particular yellow light ought to have been found, and would have been found had no spirit-lamp flame intervened, we find a **dark line** — a dark image of the slit, which, if the slit be fine and the focussing accurate, is found to be a double line; a line not absolutely lightless, but shining with the comparatively-feeble rays of the spirit-lamp, and therefore dark in comparison with its environment. If the temperature of the spirit-flame be increased, the dark line brightens up; if the temperature of the absorber be equal to that of the source, there is no dark line; if the temperature of the absorber be higher than that of the source, more of the particular light is emitted than is absorbed by it, and the line is relatively bright. The prism, which resolves any compound light into differently-coloured linear images of a slit, — images which stand side by side so closely as to blend into one another, but any defect or redundancy of brightness in any one or in any group of which can be at once detected, — offers a more delicate means of investigation than the eye can afford. In **Spectrum Analysis** a prism or a diffraction-grating is used, to disperse into a spectrum the light which passes through a narrow slit from a luminous body; by inspection of the spectrum we can at once see what kinds of light are emitted, and what kinds are not emitted, by a luminous body. But the kinds of light emitted by incandescent substances are generally (since they depend on the vibrational frequencies of the molecules of the substances) distinctively characteristic of each chemical element, and, to a certain extent, of each physical state — of each degree of temperature — and even of the chemical constitution of the incandescent substance.

The spectrum of the limelight is continuous; that of the sun is not. It presents dark lines; among others, the double sodium-line: the presence of this indicates a bright central source of light, a hot region of the sun's atmosphere, containing incandescent sodium-vapour, the light from which is absorbed by the cooler sodium-vapour in the upper and cooler regions of the same atmosphere. These lines, discovered by Fraunhofer, and named after him, are distinguished by letters; and the best-marked of the numerous Fraunhofer-lines are known as A, B, and C in the red, D (a double line) on the orange side of yellow, E in the green, F in the blue, G at the beginning and

H_1 and H_2 near the end of the violet. The position of any colour is often roughly specified by stating its proximity to one or other of these Fraunhofer-lines.

The lines C, F, and G pertain to hydrogen, the double line D to sodium, and E to iron.

The vapour of Helium, an element found in solar eruptions, and whose spectrum is a single line D_3, a little to the violet side of the sodium-lines D_1 and D_2, seems to have *no* absorptive power; this is perfectly exceptional.

Particular wave-lengths do not occur among the radiations from ice; whence ice presents a dark-heat spectrum with invisible dark lines.

When the body radiating energy consists of a gas, each molecule, as it proceeds in its free path, executes free vibrations, like a vibrating tuning-fork thrown through the air; and the mass thus vibrating may impress upon the Ether only one kind of vibration, or perhaps, if the structure of the vibrating molecule be complex, a large though not an indefinite number of simultaneous oscillations whose frequencies may or may not be commensurable. Thus a white-hot vapour may emit only a few distinct kinds of light, and may produce a **line-spectrum** — a spectrum consisting of a few isolated, linear, diversely-coloured images of the slit.

A very rare gas may emit very little heat or light, even at such temperatures as 1500° C.: gases are bad radiators. The outer shell of a flame is non-luminous, and may sometimes cause the formation of dark axes in the bright lines of the spectrum. Vapours are nearer their points of liquefaction than gases are, and are better radiators.

When the particles are so close together as to have no free path, or but a small one, they very frequently collide and rebound, and thus vibrate in an irregular manner; no rate of vibration is long enough absent for the eye to detect its absence. From the radiations of an incandescent solid or liquid, no kind of radiation appears to be absent, up to the most rapid which is given out by the incandescent body; and the spectrum of such a body is **continuous,** so far as it extends. It is not, however, necessarily equally bright throughout; didymium and erbium oxides give well-marked bright bands in the spectrum of the light which they emit while incandescent.

If a heated gas or vapour be compressed, the shocks between its molecules become proportionately more numerous: if its temperature be increased, the energy of each shock becomes greater; in either of these cases the vibrations of the molecules tend towards irregularity and complexity; and there may, in addition to the main free-vibration of the molecules — which is well-

marked if there be any appreciable free path — be a number of additional vibrations of all or of many frequencies: a condition which is indicated by the broadening of the lines in a linear spectrum of a gas into the bands of a **band-spectrum.** As pressure is relieved, the spectrum merges into that of an ordinary incandescent gas, or, on the other hand, as the pressure is increased, into the continuous spectrum of an incandescent liquid or solid.

Even the flame of hydrogen, at high pressure, is luminous, with a continuous spectrum (Lockyer).

Continuity between the gaseous or vapourous and the liquid states is thus indicated on an independent ground.

Light from incandescent solids or liquids travels from some distance within the surface; for it is polarised at right angles to the plane of incidence; this shows that it has been refracted on its outward passage through the surface of the incandescent body and into the rarer surrounding medium. Light from incandescent gases is not polarised; sunlight is not polarised; hence sunlight is due to incandescent gas or vapour.

Variations in the light emitted by one and the same substance under different conditions, and therefore in the spectrum of that light, serve to indicate molecular changes in the substance which radiates light. Salts, if undissociated, have a different mode of vibration, and therefore a different spectrum, from their component elements; heat, or, if ordinary heat fail, the extremely high temperature produced by a discharge of high-tension electricity will break them up into their elements. Even the elements are reduced to comparatively-simple forms of aggregation by high temperatures; their continuous spectrum breaks up into one of bands; a still higher temperature, such as that of a high-tension electric spark if other means fail, converts the spectrum into a line spectrum, — the line spectra being perhaps due to atoms, the band spectra to molecules. The spectra of the same substance at different temperatures are often remarkably dissimilar.

At temperatures beyond our reach, such as those of some of the fixed stars, or the lower levels of the sun's atmosphere — the high temperature of which may be inferred from the great amount of the highly-refrangible rays emitted by them — the elements themselves appear to be broken up and reduced to simpler forms of matter. This lends probability to the belief that the various elements are modifications of one kind of matter — a belief somewhat strengthened by numerous coincidences between the lines of the spectra of different elements.

Transmission, Reflexion, and Absorption.

When ether-waves fall upon a **transparent** body, they pass through it: they are propagated through the ether which lies between the molecules. When a body is thus pervious to light it is specially said to be transparent; when pervious to dark heat, as rock-salt is, it is said to be diathermanous,—no special term being used to denote transparency to actinic radiation. A body impervious to light is opaque; one impervious to dark heat is adiathermanous.

A perfectly-transparent body is invisible. Colourless thin glass, with a dustless, polished, clean surface, approaches this character: objects are seen beyond it, and, as we say, through it: they appear, if the glass be thin, inappreciably distorted. Light may be reflected from the polished surface of glass, and the presence of the glass may thus be rendered manifest to one standing in a particular position; the sun shining on the windows of a distant house makes the window-glass visible.

Glass is almost invisible in a mixture of 6 vols. essence of cloves and 1 vol. essence of turpentine.

If glass be roughened at its surface, it presents numerous facets which reflect light so as to make the glass visible in all directions; and light passing through it is irregularly turned out of its path in all directions; objects beyond cannot be seen distinctly, though light can pass through the whole mass, and roughened glass, though not perfectly transparent, is translucent.

When glass is powdered, the powder presents so many facets and reflects so often the light which falls upon it that the whole is practically opaque: it is a powder which reflects in every direction the light incident upon it—in white light a white powder, in red light a red powder.

When ether-waves of any kind impinge upon a body impervious to them their progress is arrested; in part they are reflected or scattered; in part they are absorbed by the impervious body; the Ether loses energy, ordinary matter gains it, and the impervious body is heated to an extent corresponding with the amount of energy absorbed—this heat being first communicated to the superficial layer of the body.

If ether-waves impinge upon a body which is transparent and diathermanous, that body is not heated, for the etherwaves pass through it and are not absorbed. Thus, clear moun-

tain air is not heated by the sunshine which streams through it; in the shade it may be very cold. Sunshine may stream through clear ice, or even through hoar-frost, without melting it. If there be any particles of dust in the air or in the ice, these, being opaque, will become heated, and the air is then, by conduction, rendered warm, or the ice is melted.

Some bodies are impervious to all kinds of radiation; others, having a power of Selective Absorption, are impervious to some kinds only.

Thus radiant heat can pass, while the more rapid light-waves cannot pass, through a thin piece of black vulcanite, or through a strong solution of iodine in bisulphide of carbon: while a crystal of alum is, on the other hand, transparent to light, but is almost adiathermanous, impervious to heat-rays. Lampblack, again, is very transparent to the slowest heat-waves, and air very opaque to some of them.

A soap-bubble film is remarkably adiathermanous, cutting off about half the heat of an incident beam.

Glass is transparent and diathermanous, but is somewhat opaque to the ultra-violet rapid ether-waves; a quartz prism or lens allows a great amount of ultra-violet radiation to pass through it which a glass prism or lens would extinguish. On the other hand, very long ether-waves go readily through a stone wall; and silver-leaf, just thick enough to be opaque, transmits ultra-violet rays.

The absorptive power of a substance may not be so extensive as to enable it to absorb and extinguish light-rays or heat-rays of all kinds; it may arrest some only. A piece of green glass can only allow a certain number of kinds of light to pass through it; by their joint impact on the retina these produce the sensation of green. Sunlight contains other waves than these; they have been absorbed; the green glass is opaque to them. These waves would together have produced a sensation of purple-coloured light. If this purple light had alone fallen upon the green glass, it would not have been transmitted; the glass would have appeared to be opaque. When sunlight is directed first through purple glass and then through green, the eye perceives blackness: the two pieces of glass are together opaque, though each of them is transparent to its own kind of light.

Very dark-red glass and green glass together produce a similar effect of blackness: pale-red glass allows some green light to traverse it, and so, when it is combined with green glass, the result is dark-green light.

Nickel nitrate absorbs red and violet, and is therefore green when in solution. Cobalt solution is red. A mixture of strong solutions of the two metals is black: diluted it becomes, however, almost colourless.

Copper, when it receives the impact of white light, emits orange light, together with superficially-reflected white light. Electrically-deposited cop-

per, while immersed in a solution of sulphate of copper, which does not allow the transmission of orange light, looks as white as plaster-of-Paris does in the same liquid.

The colour of a coloured object, as seen by transmitted light, is produced by subtraction of the light absorbed from the light incident upon the object.

The kind of light transmitted may vary with the thickness of the absorbing medium. In such a case the medium is said to be **dichromatic, or dichroic.** A solution of chloride of chromium, in a thin layer, absorbs much yellow, orange, and yellowish-green light; in a thicker layer it absorbs all but the red and some green and blue; in a still thicker layer the only colour transmitted is red. Thus a wedge-shaped layer of this solution appears to vary in colour, according to the thickness, from a greenish-blue, through purple, to red. Chlorophyll appears green in thin layers, red in thick. Iodine vapour transmits a blue group and a red group, as also ultra-violet rays; together these produce an impression of purple: in thicker layers the blue rays alone are transmitted, and the vapour appears blue.

Each wave-length has its own Coefficient of Transmission through each transparent substance; if half the intensity be lost by transmission through a layer 1 cm. in thickness, the proportion actually transmitted is 50 per cent., and the coefficient of transmission is 0·50. The next cm. of thickness will transmit $0{\cdot}50 \times 0{\cdot}50 = 0{\cdot}25$; the tenth cm. will only transmit $0{\cdot}50^{10} = 0{\cdot}00098$ times the original intensity. Small differences in the coefficients for the several wave-lengths make considerable differences in the composition of the aggregate light transmitted through a thick layer of a selectively-absorbent substance.

When a strong solution of blood is interposed in the path of a beam of light, no light but red is transmitted; dilute the solution gradually, and successively the solution appears more and more yellowish, and of increasingly paler hue.

The special absorptions of absorbent bodies are most thoroughly studied, not by means of their visible colours, but by the prismatic analysis of the light which passes through them. It is then found that some substances absorb several distinct kinds of light, belonging to different regions of the spectrum.

Transparent coloured-objects, through which light is filtered, give dark bands across the spectrum — the so-called "**Absorption-bands,**" which indicate what kind of light has been stopped and extinguished by the absorbent object. These bands vary in breadth with the degree of concentration of the absorbent solution employed, and they vary in position with its nature.

When a strong solution of blood is interposed in the path of a beam of light which is on its way to form a spectrum on a screen, all the spectrum, with the exception of the red part of it, disappears. As the liquid is diluted the spectrum lengthens out: orange, yellows, greens, blues, are successively added; but there always remain two relatively-dark absorption-bands in the spectrum, in the yellow and in the green, between the Fraunhofer-lines known as D and E.

If the blood be treated with sulphide of ammonium, it will be reduced; its oxyhæmoglobin will become reduced hæmoglobin; the chemical constitution changes, and with it the absorbent power; the absorption-band is now a single band placed between the two preceding.

If absorption-bands be numerous and pretty uniformly distributed throughout the spectrum, or if they be in complementary regions, the absorbent substance may present no distinctive colour, *e.g.*, benzene.

On the evidence of absorption-bands Major Abney has brought to light the existence of traces of benzene vapour between the earth and the sun, and Prof. Langley has shown that there are very peculiar gaps in the heat-spectrum, which are probably due to absorption by the upper regions of the solar atmosphere.

The kind of light absorbed by a body may also vary with its molecular constitution.

It is supposed (von Helmholtz) that each absorption depends on the presence of a particular kind of molecule, differing from the simple chemical molecule. Chlorine has many absorption-bands in its spectrum, and it must either arrange itself in many kinds of molecules, or else its ordinary molecules, considered as vibrating bodies, must be extremely complex, and have many free periods of vibration.

By changes in the absorption-bands we may learn that substances change their molecular constitution when heated. Iodine vapour gives an extensive absorption; when highly heated, the absorption-spectrum becomes reduced to a few bands; when the vapour is still more highly heated, some of the absorption-bands disappear, and one of them is replaced by a group of fine lines.

Sulphur-vapour changes its absorption-spectrum when its density changes at 1000° C.; N_2O_4, when it becomes NO_2, changes its spectrum, though it does not do so when it becomes a liquid; iodine, on the other hand, when dissolved in carbon disulphide, has the same absorption-spectrum as when it is in the state of vapour.

In a red solution of cobalt — the chloride, for example — when heat is applied to it, the salt enters into a different state of hydration; its molecular structure is changed; the solution becomes blue.

If there be no molecular difference between a substance incandescent and the same substance absorptive of light from a hotter object, a condition probably realised in the case of didy-

mium, erbium, and terbium compounds, the incandescence- and the absorption-spectra will be mutually complementary; the one presenting bright lines where the other presents dark.

The **Colour** of a coloured object seen by reflected light is also generally due to absorption. An object seen by reflected sunlight does not appear to be coloured in any degree unless there have been absorption of some of the components of the incident white-light, and the colour of a coloured object is complementary to the colour which would have been produced by these absorbed components had they jointly impinged on the eye.

Some of the light incident on a piece of coloured glass is reflected at its surface; there is no absorption; if the incident light be white, the light reflected is also white. If a piece of green glass be laid upon black paper, and if it be looked at in such a direction that daylight is not directly reflected from it into the eye, it will be nearly invisible, and will be devoid of colour; it will appear black. If coloured glass be ground to powder, the powder is white; white light is reflected at every facet, while the light reflected from the lower surfaces of the fragments, and again issuing into the air, has nowhere traversed a layer of sufficient thickness to cause the extinction of all the absorbable components of the incident sunlight. The finer the powder, the whiter it is; the coarser it is, or the more energetic its absorption, the more marked is its colour. For the same reason, froth is white. If the upper surface of a sheet of green glass be ground, it will appear almost white; if the ground surface be looked at through the glass, it will appear green, for the light issuing from the glass is white light, which has undergone a certain amount of absorption.

If the green powder be immersed in water or oil, there is less superficial reflexion at the several facets; there is deeper penetration of the light into the mass, and consequently more absorption; the colour appears to deepen. Hence the value of oil as a medium in painting.

A solution of chloride of copper placed in a deep black-walled vessel will not appear to have any colour; it will seem black; it reflects no light except from its surface. If powdered chalk be mixed with it, light is now reflected from the white particles of chalk, and passes out in every direction, through every part of the surface; so much of the reflected light is absorbed that it appears green when it reaches the eye, — the milky mass appears green. In a similar way a piece of malachite

is penetrated by light to a very small depth; internal reflexion occurs; absorption of all the outpassing light takes place, with the exception of certain kinds, which jointly appear green; the malachite is green. A piece of polished gold reflects white light at its surface; it also reflects interiorly, and from within the substance of the gold at a very small depth there is reflected in all directions a quantity of light which, by absorption before reaching the surface, has become of an orange colour.

If the layer of gold be very thin, that part of the light which would be absorbed by a thicker layer may, in part, pass through and issue into transparent media before its energy is wholly converted into heat. A thin piece of gold-leaf thus appears transparent and allows a greenish-blue kind of light to pass through it, which, if the leaf be rendered very thin by the action upon it of a solution of cyanide of potassium, may become violet, for both green and violet light then find their way through.

The object-glass of an astronomical telescope may be covered with a thin layer of silver, which will reflect the heat and some of the light, allowing a pleasant greenish light, and also some actinic rays, to pass.

When a beam of light enters the eye after undergoing repeated reflexion from gold to gold, it is of a deep-orange colour; this is the true colour of gold. As we ordinarily see gold, the orange light coming from its deeper particles is mixed with much white light irregularly reflected from its surface. The true colour of copper is scarlet, of silver a yellowish-bronze colour, of brass a rich golden-red. By reason of such repeated reflexion, a deep metal-vase, equally polished within and without, appears to be of a much richer colour internally than it is externally, and silk-velvets appear of a richer colour than silks, for light undergoes repeated reflexions between the vertical fibres which constitute the outer aspect of the former.

When there is little opportunity for reflexion from the inner particles of a body, as where light falls exceedingly obliquely upon a gold mirror from a white object and is reflected into the eye, the image of the white object in the polished gold-mirror appears not gold-coloured, but white.

Some metals can be rendered transparent, not by being reduced to thin films, but by being reduced to the liquid state: potassium and sodium can be dissolved in anhydrous liquid-ammonia; the solution is blue, and the true colour of these metals is therefore a copper colour.

If the incident light be already coloured, it may be that the whole of it is absorbed. An object, blue or red in daylight, if illuminated by a sodium-flame, may absorb all the light that falls upon it; if it do so, it appears black; a bunch of flowers, looked at in such a light, where it is not yellow appears black; it must either reflect some or none of the light which falls upon it. A piece of red cloth illuminated by the red regions of the spec-

trum glows with a bright red; when moved into other regions it becomes black, for it absorbs the incident light.

The **blue colour of opalescent bodies**, which in general present a multitude of reflecting particles embedded in a uniform matrix, and of which we may take as a type the sky-blue liquid obtained by adding to water a very small proportion of milk, is not primarily due to absorption. The principle is an established one (p. 523, near top), that where there is most refraction of light there is the greatest proportion of reflected light. A beam of mixed light falls upon a colourless transparent-body: all the rays are both refracted and reflected; the blue and violet are the more sharply refracted, and a greater proportion of them is reflected than of the less-refrangible rays. Even after one reflexion the image of an object in a mirror is bluer than the object itself. After multiple reflexion, light may become distinctly blue. Multiplicity of reflexion is favoured by smallness of the individual particles. The light which is not reflected is wholly, or in part, absorbed; the sun, looked at through a thin layer of dilute milk, appears yellow; through a thicker layer, orange or red; through a still thicker layer it cannot be seen. Similar phenomena are presented by water into which a little very dilute alcoholic-solution of resin or mastic has been dropped with stirring, by salt water into which a few drops of a very dilute solution of nitrate of silver have been stirred, by a thin haze, by smoke; all these appear blue by reflected, yellow or red by transmitted light. Even the Sky itself is a haze of this kind, the air being rendered visible, against the dark background of black space, by sunlight reflected from its fine suspended dust- or water-particles; while the light transmitted is always more or less yellowish, and, in the afternoon and evening, when sunlight comes to us through a greater thickness of the more dusty layers, verges towards orange or even red. Such a dust-haze is more opaque than adiathermanous.

When the particles of a haze increase in size they jointly offer a greater resistance to the entry of light into the fog: light is reflected more promptly, and the reflected light presents a large proportion of white light. This phenomenon is familiar to the smoker; the thick clouds of smoke produced by vigorous smoking are obviously different from the thin fine blue columns which ascend from a cigar laid aside for a moment.

The **colours of metals** may be partly accounted for in a similar way. Steel and zinc have a normal refraction; the violet is most refrangible; they appear blue. Bell-metal, brass, Au, Cu, Ag, have abnormal dispersion; the red end is most refrangible and most reflected; they appear red or reddish. Speculum-metal refracts red more than green, but also violet more than green; on the whole it is reddish (Jamin).

Those rays which are absorbed in the greatest proportion by any substance are reflected by it in the least; when a beam of sunshine falls on a green leaf, the actinic rays are absorbed and spent in doing chemical work; the light reflected from such a leaf is feeble in actinic rays, and foliage is consequently not easy to photograph. Light which is absorbed is generally converted into Heat; this may presently be radiated away; shorter, quicker light-waves strike the body; longer, slower waves of dark heat leave it.

Fluorescence, Phosphorescence, and Calorescence.

Fluorescence and Phosphorescence. — The molecular disturbances of the interior particles of a body impinged upon by light may, however, give rise to other waves which are not so slow as to be invisible; the ether-waves absorbed may thus give rise to Light. In this case the body may not only reflect light, but it may also seem to emit light from within; it is **fluorescent.** The particles down to a very small depth, being set in agitation, originate a new set of ether-waves, which are propagated from each particle in every direction.

The phenomena of Fluorescence may be shown by a solution containing æsculin and fraxin, which may be very simply prepared by stirring some horse-chestnut twigs in water; a beam of light is caused to pass through this solution, and then for some distance within the solution the liquid seems self-luminous and shines in a dark room with an opalescent shimmer along the track of the beam of light. This effect is partly due to the impact of the light rays, but is principally due to the rapid invisible ultra-violet waves. If a piece of paper be wetted with this solution, and if this paper be then used as a screen on which the image of a slit is thrown through a quartz prism, the ultra-violet part of the spectrum is rendered visible; a compound blue light radiates from the paper over an area six or eight times as long as the ordinary visible coloured spectrum; the light refracted by a prism may, with the same effect, fall on the walls of a glass vessel containing the fluorescent solution. Quinine chloride or disulphate, on paper or in solution, gives a blue light — that blue which is seen about the edge of the upper surface of a solution of quinine in a phial; petroleum or shale oil a green; turmeric solution in alcohol, or much better in castor oil, a green; uranium compounds, especially uranium glass, a green light; chlorophyll in solution, or lying undissolved in the cells of leaves, a red; an alcoholic solution of soot or one of *datura stramonium*, a greenish blue. Among fluorescent substances we find also such compounds as eosin (tetrabromofluorescein), fluorescein (resorcin-phthalein), anthracene, fluor-spar (especially chlorophane, which, when heated by conduction or by radiant heat, shines with an emerald-green light), many sulphides, especially those of barium and calcium, and, to a slight degree, the cornea and the crystalline lens, and the rods and cones of the retina.

Very frequently a body goes on vibrating for some time after ether-waves have ceased to strike it; this is familiar when the waves given out by it are Heat-waves. Sometimes, however, the body thus vibrating produces Light, and such a body — Balmain's luminous paint, for example — which goes on visibly shining or fluorescing for some time after ether-waves have ceased to impinge upon it, is said to be **phosphorescent.**

Among such bodies we find barium and calcium sulphides, diamonds, chlorophane, dry paper, silk, sugar, teeth, the alkalies and alkaline earths and their salts in general, and compounds of uranium.

These substances may be placed in a Geissler tube in a dark room; an electric current passes; the solids commence to fluoresce in the light produced by the discharge, but the observer's eyes are kept shut; the current is stopped, and the eyes are at once opened to look at the tubes; the solids are seen shining in the dark room.

For substances the duration of whose phosphorescence is very small Becquerel's Phosphoroscope may be employed. In rapid succession a phosphorescent body is exposed to bright light and brought against a dark background before the eye of an observer situated in darkness. Most objects are found by this means to be to some extent phosphorescent; and apparently all are markedly so when extremely cold.

The compound nature of the light produced by fluorescence or by phosphorescence can be ascertained by means of a slit and a prism.

It is a very singular fact that the red rays of the spectrum and the invisible heat-rays have the effect of accelerating the exhaustion of a phosphorescing body. If a body, phosphorescing after exposure to white light, or better, to violet and ultra-violet rays, have a spectrum instantaneously thrown upon it, the body thereafter phosphoresces more brightly in the area occupied by the ultra-red part of the spectrum; if the exposure to the spectral image be relatively prolonged, the phosphorescence becomes exhausted in those regions on which heat-rays had fallen, and now the Fraunhofer dark lines in the invisible part of the spectrum are rendered manifest by the survival of local phosphorescences in those parts of the screen which have not been affected by the impact of heat-waves (Becquerel).

A similar action of these rays has been long known: they often reverse the chemical action of the actinic rays.

As a rule a fluorescent or phosphorescent body emits for a longer or shorter time, on exposure to light, or, specially, on exposure to actinic rays, the same kind of light which, when light falls upon it, it absorbs; and thus, in some instances, the light emitted by fluorescent and phosphorescent bodies presents bright bands where the absorption-spectrum of the same substance presents dark bands; but the whole series of phenomena of fluorescence is one full of anomalies; we do not fully know the laws of the molecular groupings of different substances, simple and compound, their necessary modes of vibration, or their relations to the Ether.

A mixed beam of sunlight which has passed through a fluorescent solution cannot affect another solution of the same kind; fluorescent solutions rapidly absorb those rays which are the effective cause of their luminosity.

We sometimes find transformation of *slower* waves into more *rapid* ones. When a solution of naphthaline-red has been shone upon by a beam of deep-red light, it emits by fluorescence an orange-yellow light. Chlorophyll presents an analogous phenomenon; it fluoresces with a red light, even though it

be shone upon by a slower red-light. In the case of chlorophane, the impact of slow radiant-heat waves is competent to set up an emerald-green light.

Calorescence. — When a beam of light is filtered through a solution of iodine in bisulphide of carbon, so that dark heat-rays can alone pass through, these heat-rays may be brought to a focus by a lens, and absorbed by a piece of platinum placed at the focus; this will become luminous and give rise to ether-waves of all kinds; if its light be examined by a prism it will be found to give a continuous spectrum. This phenomenon was called by Tyndall the **calorescence** of heat-rays.

SOURCES OF ETHER-WAVES.

Vibrations of Molecules. — Light, Heat, and Chemical Radiation being primarily due to the vibration of particles of ordinary Matter in the midst of Ether, the energy of ether-waves is derived from the kinetic energy of vibrating particles; and whatever increases the Kinetic Energy of these vibrating particles increases their vibratory movement, and gives rise to increased radiation. When by any action a given amount of energy is liberated in or communicated to a system of material particles, the rapidity of their resultant vibration, and therefore that of the ether-waves set up by them, depends on the rapidity with which that action occurs. When energy is slowly imparted to or liberated among them, the vibrations of the particles may remain relatively slow, and radiant heat may alone be the result; while if the particles be suddenly set in violent commotion, their vibration will be complex and irregular, the particles will become incandescent, and they will at once originate not only heat, but also light or even actinic waves.

When a flash of lightning or an electric spark rushes through the air it jars the particles of air, and renders the air incandescent and luminous; and it even originates actinic waves, for an electric spark can be photographed as well as seen. When the electric discharge through the air is continuous or rapidly intermittent, its light is, to the eye, apparently continuous, and we have the Electric Light. When a flint and steel are struck together, the concussion agitates the molecules of those particles of steel which are knocked off, and a luminous spark is produced; so also when a bullet strikes a target there is a flash of light. Within a gas-flame, molecules of a hydro-carbon are robbed of part of their hydrogen by a process of destructive distillation; the residues are heavy, almost purely-carbonaceous molecules, and these, in virtue of the energy supplied by the combustion of the hydrogen, become strongly agitated and incandescent, oscillating within the gas-flame, and therein act-

ing as sources of light until the current takes them into the zone of perfect combustion in the outer region of the flame; there they become completely oxidised into gaseous carbonic acid, and thereupon lose in great part their radiative power. The brightness of a gas-flame is favoured by external pressure, or by a relatively small internal pressure and velocity of outflow, by the long continuance of carbon particles or other solid particles (which in a candle-flame can reflect light and cast a shadow in sunlight) within the flame in which they are incandescent, and by heating the gas before it reaches the flame.

When a crystal is cleft it often emits a flash of light; work is done in splitting the crystal: the energy of part of this work appears as that of ether-waves.

When salts suddenly crystallise out of a liquid menstruum it not unfrequently happens that the formation of crystals is attended with a flash of light; the salt leaves the water and coheres with particles of its own substance; the agitation attending this process causes ether-waves to be set up.

Even the application of moderate heat, falling far short of such a temperature as might produce incandescence, may cause a body to become luminous, as in the case of the fluorescence of fluor-spar (chlorophane) and the diamond, which shine when heat is imparted to them by conduction.

We have already seen that light may result from the impact of ether-waves upon a body.

Chemical union is often attended with both heat and light: as when we drop copper filings or powdered antimony into chlorine gas, or in the ordinary phenomena of combustion. Even slow combustion, such as that of eremacausis or decay, may cause light, as in the luminosity of decaying wood; or the green luminosity visible on the surface of some fish when in a state of incipient decay; or the slow oxidation of a piece of phosphorus in the air at ordinary temperatures, or of sulphur or the metal arsenic at higher temperatures. Even during the life of organisms they may become luminous either abnormally, as when the skin of the human body evolves phosphuretted hydrogen; or normally, as in the glow-worm, in the *noctiluca*, in medusoids, and in many other invertebrate animals; and the production of light may even be under the control of the animal, as in the fish *photichthys*, which can temporarily illuminate its prey. Light is in these cases produced at the expense of the animal heat which might otherwise have been evolved.

Vibrations communicated to the Ether.—In the cases discussed, the origin of the light plainly is in the agitation of

ordinary matter, but there is a certain deficiency of knowledge in respect of the next step in the transference of energy. How is any ether-wave set up in the Ether by the motion of any particle of ordinary matter within it? A full answer to this question would involve a full knowledge of the constitution of the Ether, and of the relation of the Ether to the particles of ordinary matter which are embedded in it — a question still under discussion.

Some hold that the Ether is entirely independent of ordinary matter, being unaffected in density by its presence; others hold that it is of various densities in various substances, these densities being in different transparent substances inversely proportional to the squares of the velocities of light within them. Some hold that it is so independent of ordinary matter that a moving solid body moves freely through ether like an ideal net through ideally-frictionless water; in which case it would be difficult to understand how a vibrating molecule could set up vibrations in it. If this were so, the most rapidly-moving solid transparent object would allow the transmission of light through the ether which permeates it, as if it were itself at rest. The contrary view seems probable; a ray of light is said to be retarded a little by being made to pass up a running stream of water; the effect, quite perceptible in the case of water circulating at the comparatively-slow rate of two metres per second, is, however, imperceptible in a current of air.

A beam of light was found by Fizeau to be retarded when made to pass through a cylinder of glass, rotating in such a direction that the rotation of the glass tended to carry back the light while in the act of passing through it.

The consequence of such an adhesion between the Ether and the matter embedded in it is, that the earth must to some extent drag the Ether with it as it rolls through space; yet Aberration (p. 511) tells somewhat against this.

The whole subject is as yet one of the most recondite in Physics.

Propagation of Waves through the Ether.

At present it is usual, in discussing the propagation of ether-waves, to assume the wave to have been effectually set up; the wave-motion is studied as it diverges from a small wave-front formed in the immediate neighbourhood of the vibrating molecule; and in discussing the transmission of ether-waves of different wave-lengths through different transparent bodies, we

shall have to take for granted that the interaction of the Ether and the ordinary Matter — an action which cannot be very great, for, if it were, Transparence would be impossible — is such as, in different media, unequally to retard ether-waves of different wave-lengths. This retarding effect depends somehow upon the nature of the transparent body; and this holds good not only with regard to light in general — as where a diamond is found to transmit light much more slowly than water does — but also with reference to each particular kind of light. Each transparent substance has its own rate of transmission for ether-waves of each particular frequency; and this is found for each case only by experiment. A denser substance may sometimes transmit ether-waves more rapidly than a rarer one does: light passes more rapidly through water, for example, than through alcohol or oil of turpentine. A substance through which light travels more slowly is said, however, to be optically denser.

On the assumption that the density of the Ether is different in different substances, it would follow that all wave-lengths must be diminished or increased in equal proportions, that all kinds of waves must be equally retarded or accelerated, and all kinds of light, heat, or chemical rays therefore equally refracted, on passing from one medium into another — a conclusion contradicted by the simplest experiment with a prism. Cauchy, on the arbitrary assumption that the Ether consisted of separate particles of an average size extremely minute as compared with the average distance between them, found that the amount of retardation was affected by the frequency of undulation (much in the same way as the speed of propagation of a wave along a cord is modified by stiffness of the cord), and that thus prismatic dispersion became explicable; an assumption which more modern writers — unwilling to admit that the Ether, which is not found to be capable of having waves of compression and rarefaction set up in it, and whose parts yet preserve or tend to preserve their mean positions, can be a fluid composed of separate molecules — have converted by interpretation into the following, namely, that there is some kind of discontinuity in the relations between the Ether and the ordinary matter which it permeates; a discontinuity which is held to show that while the Ether may be considered to be a homogeneous jelly-like solid, which can yield to powerful stresses after the manner of a fluid, the matter, apparently homogeneous, which is embedded in it, is not truly homogeneous throughout.

The index of refraction, β, varies with the wave-length, λ, being connected with it by the law $\beta = k\lambda^2 + A + B/\lambda^2 + C/\lambda^4$ (Briot), where k, A, B, and C are constants to be determined by experiment.

The Ether is analogous to a very weak solution of gelatine: to relatively-great momenta it acts as a fluid, and it closes up behind moving particles; to small stresses it acts as a solid, and it suffers tangential strain, without change of volume, under the influence of a tangential stress.

Ether-vibration transverse. — When any part of the Ether is displaced by a vibrating molecule, the displaced portion always

tends to return to its normal position; in doing so it sets up waves. These are waves of transverse vibration, like those of an elastic string or rod plucked laterally.

Some have held that the Ether is absolutely incompressible, and that it is impossible to form waves of compression in it; according to others, waves of compression are at first formed, but very rapidly die out.

If it be assumed that the Ether is analogous to an elastic solid; that the resistance to compression, $\mathfrak{k}$, is very great in comparison with the rigidity, $\mathfrak{n}$, to transverse distortion; then (Green) it can be shown that the compressional waves will travel with extreme velocity, but will die out after a few wave-lengths. In explaining Double Refraction on this basis, Green found it necessary, however, to assume that the vibrations occur parallel to the plane of polarisation, an assumption which is now considered inadmissible (p. 522): and there are also other difficulties in working out this theory. If, on the other hand, it be assumed (Lord Rayleigh, Lord Kelvin, and Mr. Glazebrook) that the resistances of the Ether to compression and distortion are the same in all media, but that in crystals the matter present acts so as to make the Ether behave as if it were itself different in density in different crystallographic directions; that the Ether has a certain *negative** resistance to compression, which means that it would *dilate on pressure*, and is infinite in extent or bounded by a rigid boundary; and that the vibrations are at *right angles* to the plane of polarisation:— then there can be no compressional or dilatational waves, and the results of mathematical calculation agree in the main with the facts, so far as these are ascertainable by experiment external to crystals, and also agree with the results deduced from Clerk Maxwell's theory.

According to Clerk Maxwell's view the Ether is a homogeneous body, a non-conductor of electricity: periodic electric stresses applied to this produce waves which travel at the rate of about 300,000000 metres per second; these waves are waves of transverse vibration, and there is no vibration longitudinal or normal to the wave-front. These waves, due to electric displacement, are quite competent to explain the ordinary phenomena of light, and this theory explains on mathematical grounds that absence of the normal or compressional vibration which has been a source of great perplexity in most of the mechanical theories of light. According to this view, each particle of a body through which light is shining is in rapid succession exposed to alternately-opposite electric stresses: at each half-vibration it becomes oppositely electrified; but the ordinary effects of electricity are not generally observed when light shines through or on a body, for the electrification produced by any one half-vibration simply reverses the effect of that produced by the previous half-vibration.

The **Velocity of propagation** of ether-waves through the *Ether of space* is found by two astronomical methods.

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* Any compression-and-rarefaction waves formed will promptly die out if their velocity, $\sqrt{(\mathfrak{k}+\frac{4}{3}\mathfrak{n})/\rho}$, be very great, or if it be very small, in comparison with the velocity, $\sqrt{\mathfrak{n}/\rho}$, of transverse-distortional waves. Green assumed their velocity to be comparatively very great, whence $\mathfrak{k}$ would be very great in comparison with $\mathfrak{n}$; the other physicists named have assumed their velocity to be comparatively very small, whence $(\mathfrak{k}+\frac{4}{3}\mathfrak{n})$ very nearly $=0$, and $\mathfrak{k}$ would be *negative*, and nearly equal to $-\frac{4}{3}\mathfrak{n}$.

1. Jupiter's Satellites.—These pass out of sight behind the mass of Jupiter and again reappear: when the earth is nearest to Jupiter the eclipses and reappearances appear to take place 8¼ minutes earlier, when the earth has wheeled round to the opposite side of its orbit and is at its farthest from Jupiter 8¼ minutes later, than they would have appeared if the earth had been at the centre of its orbit. The suddenly commencing or ceasing light takes 16½ minutes to cross the earth's orbit, a distance of 299,270,000000 metres: it therefore travels 302,300000 metres per second. According to the latest determinations, the velocity is 299,336000 metres per second.

2. Aberration.—No star is seen in its true place: every star seems to describe a little ellipse in the heavens, and seems to travel round the ellipse once a year. The reason is, that as the earth wheels onward in its orbit, bearing the observing telescope with it, rays of light, coming from distant stars, on their way down the telescope tend, short though the telescope tube be, to verge towards the hinder side of that tube: for which reason, in order to see the star in the centre of the field, the eyepiece must be tilted appreciably backwards in a direction opposed to that of the earth's orbital motion: the telescope, when the star is seen in the centre of its field, is therefore directed not towards the true position of the star, but towards a point in advance of it. In the course of a year, therefore, as the earth bowls round its elliptical orbit, the successive points to which it is necessary to direct the telescope are found to have been situated on the circumference of an ellipse. The size of this ellipse indicates the amount of tilting of the telescope: from this can be inferred the proportion between the length of the telescope and the distance traversed by the ocular during the time spent by the ether-waves in passing down the telescope tube; the speed of the waves of light can be calculated from these data, and is found to be 299,300,000 metres per second.

Such is the simple theory of Aberration: but the amount of aberration is the same whatever be the transparent medium—*e.g.*, water—with which the telescope is filled. Hence it would appear that a diminution of the relative motion of the earth and the ether exists, and may be explained by assuming that the water carries the contained ether, wholly or partly, along with it. This is confirmed by Fizeau's experiments on the bodily transference of ether-waves in a stream of water, like that of sound-waves by wind; but Michelson and Morley find that the earth as a whole drags the surrounding ether with it in a way which is difficult to reconcile numerically with the ordinary theory of Aberration.

Do waves of different frequencies travel through the Ether of space at the same or at different rates? If their rates were different, then a suddenly-appearing satellite of Jupiter, or a suddenly-brightening variable star, would be first rendered visible by that light which first arrives at and enters the eye, and it might consequently appear violet or blue; and when it disappeared it would continue for the longest time visible by that component of light which is slowest in travelling, and therefore might appear red before vanishing; or again, aberration of light would necessarily have the effect of giving us an image of

each star drawn out into a spectrum. Nothing of the kind is observed; all kinds of ether-waves must therefore travel through the ether of space at the same rate.

Terrestrial experiments for ascertaining the velocity of light are based upon one of two principles.

1. Fizeau's principle.—A ray of light is rendered intermittent by flashing between the teeth of a rotating cogwheel. It travels to a distant mirror; each flash is there reflected along its former path. Before a flash can again reach the cogwheel, the cogwheel may have rotated so far that one of its cogs now obstructs the returning ray; if a sufficiently-increased speed be imparted to the cogwheel, the light is allowed again to pass between the teeth of the wheel through a neighbouring notch, which has now come to occupy the position at first occupied by that notch through which the light had flashed on its outward journey. Given, then, that the light has travelled to a certain distance and back, and that in the meantime the cogwheel has been rotated through a certain angle, it is, in principle, easy to find the speed of propagation of the light. Fizeau found this to be 314 million metres: Cornu, by similar experiments, obtained the value 300,400,000 metres *in vacuo.*

2. Foucault's principle.—A beam of light starts from a source S; it strikes a mirror M, and is reflected to a distant mirror R, on which it is focussed by a lens between S and M: it is there reflected and retraces its journey: it is again reflected from M and returns to S. If, however, the mirror M have, in the meantime, been rotated perceptibly before the beam of light has had time to return from the distant R, the light can no longer be reflected from M towards the original point S; it illuminates some other point T. The distance between S and T can be measured; the amount of rotation of the mirror M in the time taken by the light to go from M to R and back can be inferred from this; the speed of rotation of the mirror M can be read off on a speed-indicator attached to the rotating apparatus: the distance traversed by light in one second can be ascertained by calculation from these data. There is no need to use instantaneous flashes of light from S; the steady beam from S reflected from the rotating mirror M only encounters the small fixed mirror R for an instant once in the course of each revolution, and is thus rendered practically instantaneous. Michelson put the lens between M and R, and thus obtained greater brightness, which enabled M and R to be much farther apart, and a greater deflection to be produced.

By this means, with a mirror rotating 1000 times in a second, Foucault demonstrated that light takes a measurable time to pass through a distance of 7 or 8 yards.

Lord Rayleigh has shown that these different methods cannot be expected to give the same results, for it is not precisely the same thing which is observed in all these cases. In some (aberration method) the speed of single waves is observed; in others (Fizeau, Jupiter's satellites) the speed of propagation of groups of waves, which is not the same as that of a single wave, unless the velocity of the wave be independent of the wave-length; in others (Foucault) these are blended. From the concordance of the results obtained by the different methods it would appear that the wave-velocity is, at any rate for wave-lengths between blue and red, not dependent on the wave-length.

As a mean result it may be stated that the velocity of ether-waves in a vacuum — that is, in the Ether of space — is 300,574000 metres, or 30,057,400000 cm. per second = 186772 miles per second.

From this it follows that $\mathfrak{n}$, the rigidity of the Ether, and ρ, its density *in vacuo*, are definite in amount, and bear to one another the relation $\mathfrak{n} = \rho \times (30057{,}400000)^2$; for $v = \sqrt{\mathfrak{n}/\rho}$ centimetres per second. The mean velocity in air is less than that *in vacuo* in the ratio of 1 to 1·000294.

It is generally believed that light of all colours travels with equal velocities through air, though some doubt has been cast on this result by the experiments of Forbes and Young (*Phil. Trans.* 1882), who conclude that blue light travels more rapidly in air than red light does, in the ratio of 1018 to 1000. If this were so, however, Foucault's experiment would give drawn-out coloured images; which it does not do.

By a modification of Foucault's method, above described, the relative speeds of light in two different transparent media, or in the same medium at different temperatures or under different pressures, may be compared. The light between M and R has to traverse a space in which a certain thickness of the medium, whose retarding power is to be examined, may be laid in the path of the beam: the beam may be exposed, by having to pass through this medium, to a retardation, which is rendered manifest and measurable by an alteration of the position of the image T.

The **Physical Intensity** of light at a place is measured by the energy transmitted through that place, per unit of cross-sectional area, in a second of time; for light of constant colour, this intensity is also proportional to its brightness as perceived by the eye.

Hence there are two methods of measuring the intensity of a beam of light:—1. Calorimetrical: allow the beam to fall upon a thermopile, and estimate the intensity of the light by the amount of heat into which it is converted upon absorption; the beam in this case having undergone a preliminary sifting through some adiathermanous medium. 2. Photometrical: two sources of light are placed at such distances from an illuminated body that they appear to produce the same effect, such as equal shadows, or equal illumination of the two sides of a disc; but this method is only accurate when the two lights to be compared are of exactly the same colour. The intensity of actinic radiation may be estimated by observing the depth of tint produced in a piece of photographic paper exposed for a given time. The total radiation may be measured calorimetrically.

As a unit of photometric intensity the Paris Electrical Standards Committee recommended (May 1884) the light emitted by 1 sq. cm. of melted platinum at its solidification-temperature. The twentieth part of this is the normal "decimal candle" (International Electrical Congress, 1889).

For many purposes of mathematical calculation it is more advantageous to measure the intensity of radiation by the average energy in ergs per cub. cm. See p. 142.

MODE OF PROPAGATION — POLARISATION.

Waves of light have the peculiarities of propagation characterising waves whose wave-length is generally small in comparison with the breadth of their wave-front. They do not usually diverge laterally from the directions mapped out by the normals to their wave-fronts; or, as it is commonly expressed, Light travels in straight lines; they can only so diverge when they are made to pass through apertures or round obstacles not very much greater in breadth than their own wave-length.

The light from a single luminous point is propagated in spherical waves; that from such an extended object as a candle-flame, in waves which, at some distance from the source, are approximately spherical. If light from a wide source be made to pass through a narrow tube, or successively to traverse equal apertures in two opaque screens, at such a distance from the source that the wave passing through the second screen has a plane front (see Fig. 57 *a*), then on the farther side of the second screen there may be an unwidening or parallel beam of light. Such a parallel beam of light, as it traverses space, may be compared to a bundle of vibrating strings of ether, isolated in the ether, vibrating independently, and practically unaffected by the ether situated laterally with respect to them. Each such imaginary individual cord may enter into transverse vibrations of different kinds, analogous to the vibrations of strings.

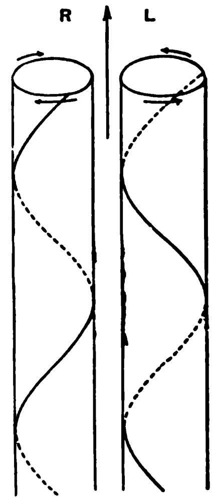
1. It may transversely vibrate simply up-and-down, or from side-to-side, or in any other single direction, — its vibrations are restricted to one plane; the whole beam is then called a beam of **Plane-Polarised** Light.

2. Its vibration may be resoluble into equal simultaneous transverse-vibrations in two planes at right angles to one another.

(*a.*) These may be of equal period, and the vibration in one plane may be ¼ period behind or in front of that in the other; looked at endwise, *any part* of the ether in such a beam would necessarily be seen — if it could be rendered visible like a bright point on a vibrating string — to execute small circular vibrations. Such a beam of light is said to be **Circularly-Polarised.** Looked at from one side the vibration would apparently progress like a screw.

A common corkscrew is a right-handed spiral. A simple experiment with a string, one end of which is fixed to a wall, while the other is held in the hand, will show that, in order to impress upon the string the right-handed spiral form, we must rotate the free end in a direction opposed to that of the hands of a watch. Such is the movement in a so-called **right-handed circularly-polarised beam** of light. When the rotation is in the opposite sense, the circularly-polarised ray is **left-handed**. Fig. 156 shows the direction of propagation and of rotation, and the forms assumed by the vibrating ether in a right (R) and in a left-handed (L) ray respectively.

Fig.156.



(*b*.) The circle may, by a difference of phase other than ¼ period, be converted into an ellipse. A beam, the ether in which rotates in ellipses, is one of **Elliptically-Polarised** light; this again may be right or left-handed.

(*c*.) The periods of vibration in the two planes may not be equal, but may be commensurable. A beam of light of this kind would present movements which, looked at end-on, would present, for each portion of the vibrating ether, figures like those of Figs. 35–40, etc.

(*d*.) The periods of vibration in the two planes may not be commensurable or even constant, and further, the vibration in each plane may be variable in its amplitude. In such cases the vibrations would rapidly run through a great variety of figures, circles, ellipses, figures of eight, and non-reëntrant complex harmonic curves of every kind. This is the condition of a beam of **Common Light.** No single plane has any advantage. If for a moment the amplitude in any particular plane preponderate, this is but momentary: and since the most irregular transverse-vibration can be resolved into a vibration up-and-down, and one side-to-side, or may be resolved in any other two planes arbitrarily chosen at right angles to each other, a beam of common light may be held to be the result of the superposition of two simultaneous irregular transverse-vibrations, each plane-polarised, each possessed of half the energy of the whole vibration, and both propagated with the same velocity through the Ether.

The doctrines of composition and resolution of harmonic motion are applicable to each small portion of the Ether within such a transversely-vibrating beam, just as they are to transversely-vibrating strings.

A beam of common light encountering an object which is selectively transparent to vibrations in one plane, but opaque to vibrations in a plane at right angles to this, will have the latter vibrations extinguished; it will lose half its energy; the beam to which the object is transparent — the transmitted beam — will have half the energy of the original beam; and all its vibrations being executed in one plane, it will be a beam of Plane-polarised Light. A body which acts in this way on a beam of common light is called a **Polariser.**

A beam of plane-polarised light falling on a polariser will, should the plane in which its vibrations are executed happen to coincide with the plane of those vibrations to which the polariser is transparent, be found to pass freely through it: if the plane of vibration be, on the other hand, a plane at right angles to the plane of the freely-transmitted vibrations, no vibration can get through, no light is transmitted, and to plane-polarised light vibrating in such a plane the selectively-transparent polariser proves perfectly opaque. If the condition be intermediate — that is, if the plane of the actual vibrations and the plane of free transmission through the polariser be neither coincident nor at right angles to one another — then the actual vibrations of the plane-polarised light must be resolved into two sets of component plane-polarised vibrations at right angles to one another; these, looked at end-on, carry out the principle of Fig. 42; and of these components — the one in the plane of free transmission, the other at right angles to that plane — the former is transmitted, while the latter is extinguished by absorption, its energy becoming converted into heat.

When ordinary light has had its vibrations in a given plane quenched in a certain proportion, while in the plane at right angles to this they are not quenched at all, or not quenched in equal proportion, — it is in a state of partial polarisation, and is called **Partially-Polarised** Light.

A plane-polarised beam of light may not only be resolved into two at right angles to one another and coincident in phase, but also (see Figs. 46 and 47) into two circularly-polarised beams, the one left-handed, the other right-handed. If it be supposed that a transparent body or a region of space is so peculiarly constituted or stressed that a left-handed circularly-polarised beam travels more rapidly through it than a right-handed one can, then, on passing a plane-polarised beam of light through such a region, the left-handed circular component emerges with its phase less advanced than the right-handed one;

but the plane-polarised light equivalent to the synthesis of two such circularly-polarised beams can no longer be due to a vibration in the original plane; the plane has been turned round a longitudinal axis in the centre of the beam; and the farther a plane-polarised beam travels through such a body or region of space, the greater, in a direct ratio, will be the rotation of the plane of polarisation of that beam — a result observed in many cases, and to be described under the head of the so-called **Rotatory Polarisation.**

When the left-handed component is relatively accelerated in its transmission, or retarded in its phase of emergence, the plane is rotated to the left — *i.e.*, to an observer stationed at the source of light the plane of polarisation of the receding beam is seen to rotate in a direction opposed to that of the hands of a watch: the more-rapidly-travelling left-handed component is at any point less advanced in phase than the more-slowly-travelling right-handed component at the same point; the contrary-to-clock rotation of the right-handed ray prevails over the less-advanced clockwise rotation of the left-handed ray.

Reflexion and Refraction.

When a ray of light, travelling in a rarer medium, strikes the surface of an optically denser transparent medium, some light is reflected, some refracted; and if there had been neither absorption nor scattering, the energy of the reflected ray, together with that of the refracted ray, would have been equal to that of the incident ray.

Lord Rayleigh finds that there is always some reflexion, even when the optical density is the same in the two media.

The incident ray and the reflected ray are in one plane: this is called the **plane of incidence.** The plane of incidence is at right angles to the reflecting surface at the point of incidence and reflexion.

The vibration of the incident light may be either at right angles to the plane of incidence, — *i.e.*, parallel to the reflecting surface, — or it may be in that plane, in which case the vibrating ether will not brush, but will strike the reflecting surface: or it may be in any intermediate direction or sequence of directions.

Fresnel, in investigating this subject, made the following assumptions, viz. — (1) that of the conservation of *vires vivæ*,* or, as we would now say, the Conservation of Energy; (2) that the movement of the incident ray merges continuously into that of the refracted ray; and (3) — a very arbi-

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* *Vis viva* $= \Sigma mv^2$; energy $= \Sigma(\frac{1}{2}mv^2)$.

trary assumption — that differences of velocity of ether-waves in different substances are due to differences of density of the ether, whose elasticity remains unaffected.

From these postulates he showed by mathematical reasoning —

(1.) If the incident beam be a plane-polarised beam, vibrating parallel to the reflecting surface, the refracted and the reflected beams are also plane-polarised beams whose vibrations are parallel to the original direction.

(2.) The frequency of vibration is unaffected by reflexion and refraction: the colours of the incident, reflected, and refracted rays are the same.

(3.) The angle of incidence is equal to the angle of reflexion.

(4.) The sine of ι, the angle of incidence, bears to the sine of ϱ, the angle of refraction, a constant ratio, β, the "Index of Refraction;" the numerical value of β depends upon the nature of the two media. Sin $\iota = \beta \sin \varrho$.

(5.) The amplitudes of the three rays are related to one another in the following way: —

The angle of incidence is ι; that of refraction is ϱ. Then $a_{\prime}$ the amplitude of the reflected ray, is equal to $\left(\frac{-\sin(\iota-\varrho)}{\sin(\iota+\varrho)}\right)$, while $a_{\prime\prime}$ the amplitude of the refracted ray, is equal to $\left(\frac{2\sin\varrho\cos\iota}{\sin(\iota+\varrho)}\right)$, times the original amplitude a. From the former formula we learn —

(*a*) That the greater the angle of incidence ι, the greater is $a_{\prime}$ the amplitude of the reflected ray.

(*b*) That when the incident ray is so nearly parallel to the surface of the glass as simply to graze it, the reflected ray is equal to the incident one, and the amplitude of the refracted ray $= 0$.

(*c*) That when ι is greater than ϱ, $a_{\prime}$ is negative; while when ι is less than ϱ, $a_{\prime}$ is positive: or in words, when a ray strikes the surface of a denser medium, the reflected ray is a direct continuation of the incident ray, changed in direction; while when light travels in a denser medium, half a wave-length is lost on reflexion at the surface of a rarer medium. This conclusion is independent of any hypothesis as to particles, such as that by which we have already illustrated the same propositions on pages 124 and 125.

(6.) The respective intensities of a reflected, a refracted, and the incident ray, in ergs per cub. cm., are in the ratios of

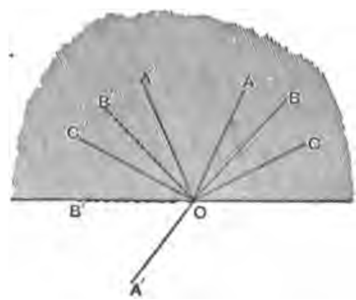
$$\frac{\sin^2(\iota-\varrho)}{\sin^2(\iota+\varrho)} : \frac{\sin^2 2\iota}{\sin^2(\iota+\varrho)} : 1.$$

(7.) When light, travelling in a denser medium, strikes the surface of separation between the denser and a rarer medium at such an angle ι that $\sin\iota$ is greater than β, both reflexion and refraction are possible; but if the angle of incidence be such that

$\sin\iota = \beta$, then $\sin\varrho = 1$, and the ray refracted into the rarer medium grazes the reflecting surface, for $\varrho = 90°$; and any light falling still more obliquely upon the surface will be **totally reflected,** the reflected ray possessing the whole energy of the original incident-ray. In the last case $\sin\varrho, = \sin\iota./\beta$, would be greater than 1, and ϱ, the angle of refraction, an impossible angle; there is therefore no refraction.

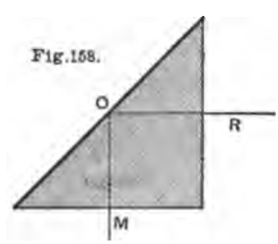
In Fig. 157 the ray AO is partly reflected to A″, and partly refracted to A′; the ray BO is partly reflected to B″, and partly refracted to B′, the refracted portion grazing the refracting surface; the ray CO is wholly reflected to C″, and is not refracted at all into the rarer medium.

Fig. 157.



As examples of Total Reflexion we may take a tumbler of water held above the head; it will give a clear mirror-image of the objects on the table below it; a bubble of air in water, or a test tube containing air immersed in water, will, when looked at under a certain angle, appear to have as bright a mirror-surface as that of mercury. In Fig. 158, light entering a total-reflexion prism at M is totally reflected at O, and travels towards R, which it reaches without refraction. Total reflexion is also exemplified by the silvered glass bars used by surgeons and microscopists to transmit light.

Fig. 158.



(8.) The energy of a ray totally reflected is equal to that of the incident ray; there is, however, a slight retardation of its phase.

These laws all apply to vibrations **executed at right angles** to the **plane of incidence,** and were deduced by Fresnel from the fundamental hypotheses already mentioned.

Let us now turn to the reflexion and the refraction of plane-polarised light whose vibrations are at right angles to these, and are thus executed **in the plane of incidence.**

In the refracted and reflected rays the vibrations will still be in the plane of incidence, but they cannot, after encountering the refracting surface, remain parallel to their original direction. The consequence deduced

by Fresnel from this is, that while the ordinary laws of refraction and reflexion are obeyed by such plane-polarised light so far as directions are concerned, the relative amplitudes and intensities of the incident, the reflected and the refracted light, are not the same as they were in the preceding case, but are now respectively in the ratio

$$1 : \frac{\tan(\iota - \varrho)}{\tan(\iota + \varrho)} : \frac{2\cos\iota \sin\varrho}{\sin(\iota + \varrho)\cos(\iota - \varrho)} \quad \text{(Amplitudes.)}$$

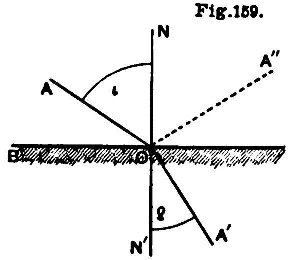
$$1 : \frac{\tan^2(\iota - \varrho)}{\tan^2(\iota + \varrho)} : \frac{\sin^2 2\iota}{\sin^2(\iota + \varrho)\cos^2(\iota - \varrho)} \quad \text{(Intensities.)}$$

incident : reflected : refracted.

According to Fresnel's formulæ, the intensity of the reflected light is, in a particular case, $=0$; that is, when $\{\tan^2(\iota - \varrho) \div \tan^2(\iota + \varrho)\} = 0$, or when $(\iota + \varrho) = 90°$. In fact, however, it does not entirely vanish: it only attains a minimum.

In Fig. 159 the ray AO, whose vibration is *in* the plane of incidence, falls at such an angle ι that it is refracted along OA′; if it had been reflected at all it would have been reflected along OA″; at one particular angle of incidence, AON, the refracted ray OA′ and the reflected ray OA″ (if there had been such a ray) would have been at right angles to one another. When the angle of incidence and the angle of refraction together make up a right angle, there is no reflected ray; no vibration effected *in* the plane of incidence is reflected at all, and a plane-polarised ray of this kind falling at the appropriate angle of incidence, however bright it may be, will fail to be reflected from a mirror. This is a case of **Total Refraction**; the whole of the energy of the incident ray is in the refracted ray. When the incident light grazes the refracting surface, the reflected beam also grazes it, and there is no refraction.

Fig.159.



When light whose vibration is in the plane of incidence is totally reflected, it undergoes a slight retardation of phase, less than that observed in the case of light whose vibration is at right angles to the plane of incidence.

From these results it is easy to pass to the case of light whose vibration may be considered to be the result of the composition of vibrations, parallel to the plane of incidence, with others at right angles to that plane. Such light may be plane-polarised, vibrating in some plane neither the plane of incidence nor at right angles to it; it may be circularly- or elliptically-polarised light, or it may be common light. In all these cases each component is reflected and refracted according to its own laws. In

this way the reflected and refracted rays may come to differ in character from one another and from the original ray. As an extreme case, let a beam of common light fall upon a piece of glass at such an angle that $\iota + \varrho = 90°$ (Fig. 159); of that part of the incident beam which is due to vibrations executed at right angles to the Plane of Incidence, there is reflected a certain proportion; of that part of the incident ray which is due to vibrations parallel to this plane, there is reflected none. The light reflected from glass at such an angle has its vibrations thus restricted to a plane at right angles to the plane of incidence; it is plane-polarised light. A piece of glass held in the course of a beam of common light at the proper angle may thus be used as a simple means of obtaining a beam of light Plane-Polarised by Reflexion.

The precise angle of incidence ι, for which $\iota + \varrho = 90°$, is called the **Angle of Complete Polarisation.** It depends upon β, the refractive index of the refractive substance, and the angle ι is such that $\tan \iota = \beta$; for $\sin \iota = \beta \sin \varrho = \beta \cos \iota$. Polarisation is, however, never complete on one reflexion, except in substances whose refractive index is 1·46; in others it is only a maximum at the angle whose tangent is β. In metals the departure from completeness is most marked. There is also no angle of complete polarisation for the refracted ray.

Such are the consequences deduced by Fresnel from his hypothesis, that the Ether is condensed around the particles of ordinary matter while its elasticity remains unaffected.

Neumann and MacCullagh, from the contrary hypothesis — that the density of the Ether is the same in all substances, while its elasticity or rigidity is different in different substances — deduced a set of conclusions precisely similar to those above given, so far as regards all the results which it is possible to verify by experiment, but with this remarkable difference, that the properties attributed by Fresnel to plane-polarised ether-waves whose oscillations are effected at right angles to the plane of incidence were, by the latter writers, found to be associated with plane-polarised light whose vibrations are parallel to that plane, and *vice versâ*. The fundamental postulates of the two theories are closely associated with these consequences.

We must now turn to a point of terminology. When a beam falls upon a mirror at the angle of complete polarisation, the reflected ray, if there be any, is plane-polarised; it is said to be **polarised in the plane of incidence,** and the plane of incidence is called its **plane of polarisation.** According to Fresnel's view, the vibrations in this beam are supposed to be executed in a direction at right angles to the plane in which the beam is thus said to be polarised.

The question between the followers of Fresnel and Cauchy on the one hand, and those of Neumann and MacCullagh on the other, may thus be stated: Are the vibrations of plane-polarised light executed at right angles or parallel to the plane in which the light is said to be polarised — a plane which by convention is called the **plane of polarisation**? On the one hand, there appears, on Fresnel's assumptions, to be a serious objection to his view; this objection is based on certain mathematical difficulties arising from the admission of his second hypothesis, that the movement of the ether in the second medium is continuous with that in the first; for this hypothesis is found to lead directly to the conclusion that the density of the ether in the two media must be the same, and is therefore one which is incompatible with his third hypothesis.

Professor Stokes, on the other hand, reasons in favour of the movement perpendicular to the plane of polarisation, and the following is a sketch of his argument. The vibrations in a beam of light are admittedly transverse to the direction of propagation. Consider a polarised reflected beam; the vibrations are admittedly symmetrical with regard to the plane of reflexion; they must be either parallel to it or at right angles to it. Now, suppose a horizontal beam to strike a haze and then to be reflected vertically upwards into the observer's eye. The reflected light is undoubtedly polarised in a plane passing through the source of light, the point of the haze looked at, and the observer's eye; that is, it is polarised in the plane of reflexion. If the vibrations be parallel to this plane in the ascending beam, they must either have been originally parallel to the direction of propagation of the incident beam, which is impossible; or else they must have been changed in their vibratory direction by impact against obstacles smaller than their own wave-length, which is improbable. If, on the other hand, the vibrations be at right angles to the plane of reflexion, the general direction of vibration is the same after reflexion as before it. The latter view is preferable: and according to it, the vibrations are at right angles to the plane of polarisation. He has also shown that certain phenomena of Diffraction afford a crucial test, the experimental answer to which is in favour of the proposition that the vibrations are perpendicular to the plane of polarisation.

Though Neumann and MacCullagh's hypothesis works out in many respects better than Fresnel's, it is therefore inadmissible; and besides, it would lead (Lord Rayleigh) to the conclusion that there are two angles of polarisation, which is not the case.

When ordinary monochromatic-light is reflected at any angle other than that of complete polarisation, the reflected and the refracted beam must both be partially polarised, and each will be polarised to an equal extent, though in contrary senses. In the reflected beam, light polarised in the plane of incidence preponderates until the incidence is a grazing one: in the refracted ray, light polarised at right angles to that plane preponderates to an exactly equal extent, so far as the energy of the vibration is concerned.

When mixed light, such as white light, falls upon a refracting surface, then, since β, the index of refraction, is different for each kind of light, the proportions of each coloured light present in the reflected and the refracted

rays respectively are different; white light, when reflected from a normally refracting surface, always becomes bluer, the refracted light redder; and we have seen this to account for the blue colour of haze.

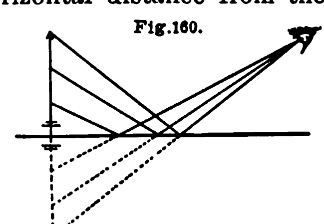
The intensity of a reflected ray is represented by $\{\sin^2(\iota-\varrho)+\sin^2(\iota+\varrho)\}$. If we pass to a ray of greater refrangibility we alter ϱ to $\varrho_{,}$, and our intensity becomes $\{\sin^2(\iota-\varrho_{,})+\sin^2(\iota+\varrho_{,})\}$, which is always greater than the former. Where the actual refraction is greater, the corresponding angle of refraction is less; wherefore, in this case, $\varrho_{,}$ is a smaller angle than ϱ.

Near the incidence of total reflexion some colours may be totally reflected, others in part refracted; near the incidence of total refraction or complete polarisation an analogous result is obtained. The slower waves of heat have a lower refractive index, and must therefore strike a refracting surface at an angle somewhat more vertical or nearer the normal than those of light, in order to become completely polarised.

We thus learn that refraction and reflexion may materially modify the character of light which strikes on a refracting surface. If, however, we attend only to the Direction of the respective rays, and not to their states of polarisation, or to their colour, etc., we may study the effect of mirrors or lenses in modifying the direction of an incident beam of light, whether this be plane-fronted, convergent, or divergent. We shall first consider the case of monochromatic light.

Mirrors. — Plane mirrors reflect light in such a way that the reflected waves are, as regards their direction, precisely such as might have come from an object or source of light situated behind the reflecting surface, and at a distance behind it equal to the distance between the object and the mirror. This is illustrated in Fig. 61, and it is a matter of familiar knowledge in the use of looking-glasses, and in the appearance presented by the inverted image of objects on the shore when these are seen reflected on a surface of smooth water. The images then seen are apparently at the same horizontal distance from the eye as the objects themselves; while the image of a slightly-clouded sky, as reflected in very smooth turbid water, appears extremely deep.

Fig. 160.



The apparent inversion of an image in a mirror is a natural result of the fact that the image of each point is apparently situated behind the mirror. Fig. 160 explains this result.

The reader may construct a diagram to show how it is that a mirror

about half a man's height, and placed opposite the upper half of his body, will give him a full-length image of himself.

The use of mirrors fixed at an angle of about 45°, in order to cast the light of the sky into a room, or in order, when fixed outside a window, to enable a person within a room to see the passengers in the street outside, is sufficiently intelligible.

In medicine the same principle is utilised in the Laryngoscope. Light falling from a lamp upon a concave mirror is cast upon a small plane-mirror, held by means of a long handle at an angle of 45° within the pharynx of the patient; the light is reflected and passes down towards the larynx, which is illuminated and becomes a source of light; rays returning from this, passing upwards, strike the small pharyngeal-mirror, and are diverted by it so that they traverse, horizontally, the cavity of the mouth, and pass through a small aperture in the centre of the concave mirror into the eye of the observer, who is thus enabled to see the larynx and windpipe.

In a mirror, the object and the image are interchangeable; so that, for example, a person cannot look at another in a mirror without the person looked at being also able to see him in the mirror, unless indeed the first observer be in darkness.

The brightness and distinctness of an image depend upon the polish of the mirror and on its not scattering the light which falls upon it. A smooth clean mirror is, itself, almost invisible.

A piece of polished platinum reflects light as well when it is white-hot as when it is cold.

The image of an object in a mirror can never be brighter than the object itself, however smooth the mirror be. Hence, if a candle be held between two parallel mirrors, the long series of images produced by multiple reflexion grows fainter as the images seem to grow more distant.

When we have multiple reflexions of light between two polished plates, if the plates be parallel and the incidence oblique, the reflexions are more numerous the nearer the plates are to one another. If the two plates be inclined to one another at an angle of 60°, the images of a point lying between them, the image of which is multiplied by repeated reflexion, are so situated that the first, second, . . . sixth, form together a symmetrical Kaleidoscope-image of the point, a group of images ranged round a central axis, while the seventh and further images coincide with their predecessors. Similarly for such angles as 90°, 45°, 30°, 24°, and other aliquot parts of 360°.

When a mirror is rotated, a beam of light reflected from it is deflected through an angle equal to twice that of the rotation of the mirror itself. In Fig. 161 S is a source of light; AB a mirror; SM an incident ray; MR a reflected ray. If the

mirror be turned into the position A′B′, the reflected beam is now MR′: the reflected ray has swept through the angle RMR′, which is equal to twice the angle AMA′.

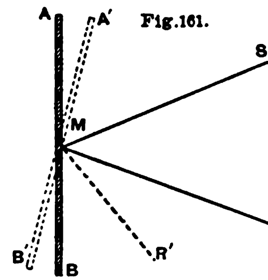


Fig. 161.

We have already (p. 122) considered some cases of the reflexion of waves at parabolic and elliptical mirrors, and on segments of spheres. Parabolic mirrors are used when it is desired to bring the plane-fronted light of a distant star accurately to a focus, or to produce a parallel beam of light: in the latter case the source of light is placed at the focus of the paraboloid. Spherical mirrors may be considered, if we restrict our attention to those rays which fall very near the centre of the mirror, to have an approximate focus at the tip of their "caustic by reflexion" (Fig. 64).

When the beam is broad the rays do not converge accurately to a focus, and the image of a point is a circle, brightest towards its centre. This is the **Spherical Aberration** of a mirror, which renders its definition, especially of somewhat broad objects, very bad. In consequence of this, it is often necessary to cut off the lateral rays by a diaphragm, which increases the clearness of definition, though it diminishes the brightness of the image.

When light coming from a source at a positive distance, d, is reflected by a concave spherical-mirror, it is reflected back to an approximate focus at a positive distance d'. The distance d of the source, the distance d' of the approximate focus, and r, the radius of a spherical mirror, all measured positively, are connected by the law $1/d + 1/d' = 2/r$.

Here r, the distance of the centre of curvature of the mirror, is to the right, positive.

When the source is infinitely distant, $1/d = 0$, and $d' = \frac{1}{2}r$; the case of Fig. 64, page 123. The focus to which the light converges in this case is called the Principal Focus of the mirror, and its distance d', $= \frac{1}{2}r$, is written f, the **Principal Focal Distance**. If the course of the light be reversed, and $d = f = \frac{1}{2}r$, $d' = \infty$, and the light is reflected to a focus at an infinite distance; it is approximately parallel-rayed or plane-fronted.

When the source is at a definite distance, beyond the centre of the sphere, the focus is between the principal focus and the geometrical centre of the sphere; conversely, when a source of light is between the principal focus and the centre of the sphere, the reflected light converges upon a point at a definite point beyond the centre.

Light radiating from the centre of a spherical mirror, after reflexion again converges upon the same point. When the source is between the principal focus and the mirror, *i.e.* when d is less than $\frac{1}{2}r$, d' is negative, and the reflected rays seem to diverge from a point on the other side of the mirror, and the **Image** of the point is imaginary or **Virtual**.

Pairs of points at the respective distances d and d', as defined by this formula, are called **Conjugate Foci.**

Conjugate foci are found in one of the four following relations:—

(*a.*) Coincident, both being at the geometrical centre of the mirror.

(*b.*) One between the principal focus and the centre; the other beyond the centre.

(*c.*) One at the principal focus; the other at an infinite distance beyond the centre.

(*d.*) One between the principal focus and the mirror itself; the other, a virtual focus, apparently behind the mirror.

If the source of light be not in the axis of the mirror, the light is not brought to a focus in the axis, but at some point situated laterally. In Fig. 162 the rays proceeding from A converge upon A′, and an eye situated in the direction of B looking towards the mirror will receive rays which appear to diverge from a **Real Image** at the point A′, a point which the waves really traverse. If the source of light be an extended object AZ (Fig. 163), there will be formed an inverted real image Z′A′. After the reflected rays from the mirror have reached this real image, they diverge from every point of it, as if it were a real object suspended in space. In Fig. 163 the object AZ produces a diminished and inverted real image at Z′A′; or, conversely, an object at Z′A′ will present an inverted and enlarged real image to the eye placed beyond AZ, an image which appears nearer than the object itself.

Fig. 162.

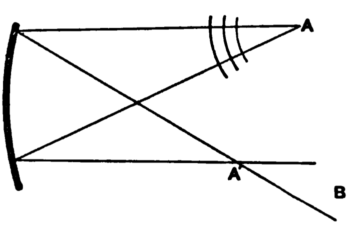
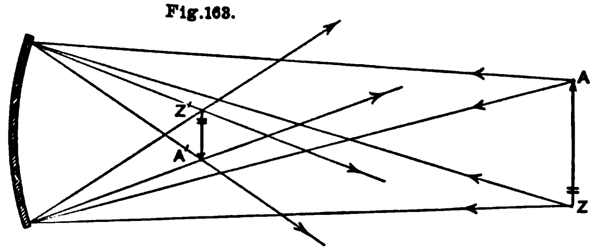


Fig. 163.



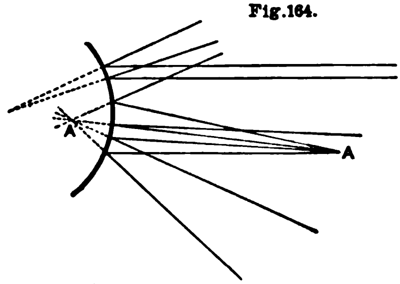
Suppose that the object AZ is the face of a person looking at a concave mirror, it is plain that the eyes in that face are eyes situated beyond Z′A′: a real image of the face is seen, diminished and inverted, and apparently situated at Z′A′, between the observer and the mirror. As the observer approaches the mirror, the image approaches him: it appears to grow larger. When his eye is at the geometrical centre of the mirror, the inverted image

coincides with his face, and he sees nothing; when his eye is between the centre and the principal focus, the inverted image is (or would be) behind his eye, and again he sees nothing. When his eye passes between the principal focus and the surface of the mirror, the rays reflected seem to come from a virtual image behind the mirror; this virtual image is erect, and is larger the nearer the eye is to the mirror.

The student will have little difficulty in drawing the diagrams appropriate to each case, and in verifying the results even by means of such a simple concave-mirror as the inside of a watch-case, or of a common large spoon.

Convex Spherical-Mirror.—In Fig. 164 we see that a beam of light, plane-fronted or parallel-rayed, and therefore coming, apparently or really, from an infinite distance, is so reflected that it appears, after reflexion, to diverge from the principal focus of the convex mirror, this point being the tip of its caustic; while a beam diverging from any point A appears, after reflexion, to diverge similarly from some point A′ situated between the mirror and the principal focus.

Fig. 164.



The formula is $1/d+1/d' = -2/r$.

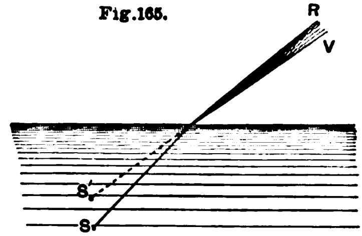
This formula is really the same as the preceding; but r is now negative.

When **mixed coloured-light** falls upon a mirror, all the reflected rays, whatever be their wave-lengths or relative intensities, are reflected in the same directions.

When a mirror is flexible and has a variable curvature, as the form of the mirror is made to vary irregularly, intermittently, or in accordance with some harmonic law of simple or complex variation, so the intensity of light at any point near the focus varies irregularly, intermittently, or harmonically.

Refraction of Light.—Light-rays are bent when they reach the surface of separation between an optically rarer and an optically denser medium. In Fig. 165 a body situated at S seems to be situated approximately at S′. In this way, if a coin be placed in a basin, and the eye placed in such a position as just not to see it, water poured into the basin will bring the coin into view. The sun is seen before he has astronomically "risen," and continues to be seen after the true "sunset." A stick placed in water appears bent, for the image

Fig. 165.

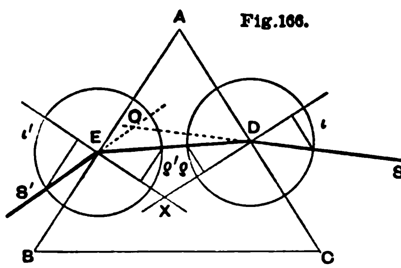


of each point of its surface appears raised within the water by an amount proportioned to its depth beneath the surface of the water; each point of the image appears indistinct, being brought only to an approximate focus, and the image of the whole is blue on one side, red on the other, for, as shown in Fig. 165, the different colours of white light travelling from any point S are unequally refracted into the air.

When light passes through a number of parallel-sided transparent plates of different densities, the total angular-deviation produced is the same as if the last of them had alone stood in the course of the transmitted light.

Prisms — Monochromatic Light. — If we confine our attention to a single ray of light impinging upon a prism surrounded by a single medium, we may trace the course of the corresponding wave-front, as in Fig. 166. The light travelling from S strikes at the angle of incidence ι; it is refracted at the angle ϱ; $\sin \iota = \beta \sin \varrho$ (i.). It strikes the second face at the angle ϱ'; it leaves the glass at the angle ι'; $\sin \iota' = \beta \sin \varrho'$ (ii.). Further, the angles $\varrho + \varrho'$ can be proved together equal to the angle A;* $\varrho + \varrho' = A$ (iii.). Lastly, if the angle between the incident ray SD and the deviated ray ES′ be δ, then $\iota + \iota' = \delta + A$ (iv.).† From these four equations, which involve β together with the six angles, ι, ι', ϱ, ϱ', A, δ, we may, if we know any three of these angles (say A, ι, and δ), determine, for the particular monochromatic light employed, the Index of Refraction β of the material of the prism; this numerical quantity β expresses, as compared with unity, the relative slowness of ether-waves in the prism as compared with that in the medium surrounding the prism.

Fig. 166.



Thus, if a particular glass prism have the index 1·5 in air for a monochromatic yellow light, that light travels 1·5 times as fast in air as it does in the glass out of which the prism has been cut.

* The normals to the faces, drawn at the points of entrance and exit of the ray, cross one another at X; the quadrilateral AX has its internal angles equal to four right angles; two of these (at D and E) are right angles: the remaining two (A and X) are therefore equal to two right angles. In the triangle DEX, the angles $\varrho + \varrho' + X$ are again equal to 180°; whence $\varrho + \varrho' = A$.

† The angles $\varrho + \varrho' = A$; $\varrho + \varrho' + \delta = \delta + A$; $\delta = OED + ODE$; $\therefore \varrho + \varrho' + OED + ODE = \delta + A$; but $\varrho + ODE = XDO = \iota$; and $\varrho' + OED = XEO = \iota'$; $\therefore \iota + \iota' = \delta + A$.

To find the Refractive Index of a Liquid, the amount of refraction must be directly observed. Two telescopes arranged radially on a vertical circle or alidade; light traverses the one, and is rendered parallel by it; it then impinges on the level surface of liquid in a certain vessel, penetrates it, leaves the vessel, passing normally through the glass bottom of the vessel, and enters the second telescope, which it traverses. The relative angles made by the two telescopes with a fixed bar indicate the angles of incidence and of refraction, which being known, the index β is known. The angle of total reflexion may also be observed, and the index of refraction calculated from this; $\sin \iota = \beta$.

The Refractive Index of a Gas is found by processes which depend on the amount of retardation suffered by light in a long column of that gas, as will be seen under "Interference," and as has already been explained under "Velocity of Light" (the rotating-mirror method, page 512).

If instead of a prism filled, say, with water in air, we use a prism filled with air and submerged in water, the deviation of a ray travelling in the water will be equal, but of opposite sense to that of a ray travelling in air and refracted by the water-prism.

The refractive index of organic substances is found to have a close relation to their chemical constitution. If an organic liquid contain carbon, hydrogen, and oxygen, and if its density be ρ, it is found that the numerical quantity $\{(\beta - 1)/\rho \times \text{molecular weight}\}$ is constant, even though the density be varied by changes of temperature, and it is called the Molecular Refractive Power of the substance. Instead of $\beta - 1$, the factor $\{(\beta_\infty^2 - 1) \div (\beta_\infty^2 + 2)\}$ gives very good approximations to fact, applicable to the same substance in both the liquid and the gaseous state. The molecular refractive power depends on the chemical composition and on the chemical constitution of the substance; it is the sum of definite numbers, one for each atom of the element, these numbers (or Atomic Refractions) being different, for O, N, C, S, etc., according to the chemical part which each atom plays in the molecule.

If in the last figure (166) light start from S′, it will retrace the line S′EDS; and the same result will follow if a mirror at S′ turn back the light coming from S: on its direction being reversed, light will retrace its path—a proposition applicable to waves in general.

Minimum Deviation.—It is only when the prism is turned into such a position that ι, the angle of incidence, becomes equal to the angle ι', that light diverging from a source S (Fig. 166) can converge upon a single focus lying towards S′; and this is

the position in which the incident ray of light is, on the whole, least deviated by the prism.

If **mixed coloured-light** be passed through a prism, each colour has its own index of refraction, β, its own path through the prism; the whole light is broken up by **Dispersion** into a bundle of monochromatic rays, each travelling in a separate direction, and diverging the more widely from one another the farther they travel; and, received on a screen, these form a **Spectrum.**

Each kind of light is differently deviated by the prism; and for each kind of light the minimum deviation of the prism is different; whence an image of a slit looked at through a prism, or cast upon a screen, cannot be sharply in focus in all parts of the spectrum at once; only one colour can be accurately in focus at any one time; and to put any particular colour of the spectrum into focus, the prism must be rotated one way and another in the beam of light until that position is found for it, which corresponds to the greatest approach of the particular colour towards the red end of the spectrum: this is, for that colour, the position of minimum deviation and of most accurate definition.

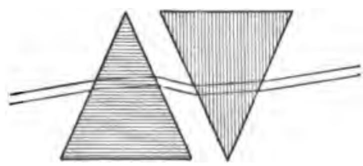
The **Rainbow** is produced by reflexion and refraction of sunlight in the drops of water which make up falling rain. Parallel sunlight falls from behind the spectator; in each drop the light is dispersively refracted, and then reflected from the farther face of the drop; it travels back through the drop, and emerges in a state of chromatic dispersion. Drops which for the moment are situated at a certain angular height send violet light into the eye of the observer, but the red light from them misses his eye, for it strikes too low. Drops at a greater angular height send red light into the observer's eye, but violet light proceeding from them is, as it were, aimed too high, and does not enter his eye. Drops in intermediate positions send intermediate-coloured light into his eye. Drops above or below a certain range of angular height do not send light into his eye at all. The whole phenomenon is symmetrical round an axis containing the sun and passing through the observer's eye, and the bow is, according to the height of the sun in the heavens, a greater or lesser portion of a circle whose parts are equidistant from that axis. The rainbow as seen by the one eye is not formed by the same water-drops as the rainbow seen by the other eye.

Repeated reflexions and refractions in raindrops frequently give rise to secondary, tertiary, etc., rainbows, which, under experimental conditions, have been observed to the number of eighteen: while interference between rays which have traversed different distances within the drops gives rise to spectral fringes or supernumerary bows, which sometimes reduplicate the tints of the rainbow, and whose breadth is greater the smaller the drops.

The **halo** seen round the sun when it shines through a frozen cloud is due to refraction of sunlight through the crystals. Conceive a circle of prisms round the sun, arranged in such a position as to send a maximum of sunlight into the eye: a circular spectrum would be seen, red internally; among the particles of ice in the cloud some must be in the favourable position of these prisms.

Recomposition of White Light. — If a bundle of coloured rays, emergent from a prism, be received on a second prism, similar to the first but reversed in position, these monochromatic rays are again recombined, and again form, on emergence, a beam of the original mixed coloured-light, parallel to its original direction; on the whole there is neither dispersion nor angular deviation.

Fig. 167.



Deviation without Dispersion. — If white light fall upon a flint-glass prism of such an angle as to make on a screen, at a distance of, say, 10 feet, a spectrum whose length between two definite colours or Fraunhofer lines is, say, 3 inches; and if another prism of crown glass which is able to produce a spectrum of an exactly-equal length between the same definite colours or lines be so placed as to reverse the dispersive action of the former prism, according to the principles of the last paragraph, the light leaving the prism is approximately white. It is not, however, parallel to its original course; for when we pass from prisms of one substance to prisms of another, we find a phenomenon known as the **Irrationality of Dispersion**; we find that the relative amounts of mean deviation and the relative amounts of dispersion produced by two given prisms are independent of one another; and hence, to reverse dispersion is not necessarily to reverse deviation, if this be effected by a prism of a second, a different substance.

A flint-glass prism and a crown-glass prism, thus combined, may produce deviation without producing any dispersion, and the emergent light is approximately white; and thus, for any two kinds of light, a flint-glass prism may be **achromatised** by a second prism of crown glass. The recomposition of the colour is not perfect, except for the two colours (or lines) chosen; it would have been perfect were the spectrum of crown glass precisely similar to that of flint glass in respect to the proportionate lengths of the coloured areas in it; but it is not so; in the crown-glass spectrum the orange and yellow are proportionately more refracted, and are spread over a proportionately greater area than they are in the flint-glass spectrum, and the blue and violet less so; the former are for accurate recomposition too much, the latter too little, refracted by the achromatising crown-glass; the issuing beam, white at the centre, is yellowish on one side, bluish on the other. Three prisms may be combined so as to blend three colours in the emergent ray; and so forth.

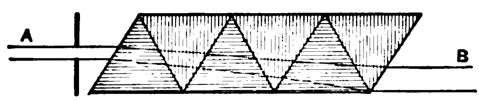
Reflexion at the surface of a mirror may be said to furnish an example of deviation without dispersion. Sometimes a

reflecting prism (Fig. 158) is preferred to a mirror, especially in lantern projection-apparatus.. Light enters normally at one face of the prism, is totally reflected at the second, and passes normally through the third. The image produced by such a prism is inverted, and if the incident beam be parallel there is no refraction, and therefore no chromatic dispersion, while the loss of light is very small.

Dispersion without Deviation. — If crown-glass prisms and flint-glass prisms be alternated, they can be made to produce dispersion while the issuing rays are parallel to the original direction of the entering light.

This principle is applied in the Direct-vision spectroscope, which simply consists of a train of such prisms, to which

Fig. 168.



the light is admitted by a slit at A (Fig. 168), and from which the light issuing at B is caused to pass through a lens which can be so adjusted as, for each colour, to give the eye a clear image of the slit; and the eye accordingly receives the impression of a continuous spectrum, situated at the focus of the lens.

Abnormal Dispersion. — The amount of deviation of each kind of coloured light can be directly measured when a spectrum is formed; so can the angle of the prism and the angle of incidence; thus for each transparent substance, and for each wave-frequency of incident ether-waves, the index of refraction may be calculated and recorded.

The spectra produced by similar prisms of different substances differ not only in their absolute lengths, but also in the proportionate length of each colour, and even in their arrangement. A hollow glass prism filled with iodine vapour refracts red light most, and violet least; it gives a spectrum the order of succession in which is ultra-violet, violet, blue, then a dark band, then red, the reverse of the ordinary order; the intermediate parts of the spectrum are lost by absorption. A weak alcoholic-solution of fuchsin in a hollow glass prism refracts the blue and violet less than it refracts the yellow and red; the spectrum thus presents the following order: — Fraunhofer lines, F to H — *i.e.* green, blue, violet — then A to D, red, orange, and yellow, not quite up to the E line: the green from E, nearly as far as F (E inclusive), being absent on account of absorption. Aniline violet, aniline blue, indigo carmine, give a green-blue-orange spectrum. A concentrated solution of cyanin in a hollow glass prism gives a spectrum consisting of — first, green-blue, then a dark band, then red, and traces of orange.

In general all bodies which, like many of the aniline dyes or crystals of permanganate of potash, act upon some kinds of light as metallic reflectors,

and present, when in the solid state, superficial colours differing from the body-colours (the colours best reflected being those absorbed on transmission), are found to give, when their solution occupies a hollow glass prism, an abnormal-refraction spectrum, produced in the same way as any ordinary absorption-spectrum, but in which the order of the colours is not that of the spectrum as produced by a prism of solid glass. On the redward side of an absorption-band the index of refraction is found to increase so rapidly as to throw the part of the spectrum near the redward side of the absorption-band over towards the violet; and conversely, the part of the spectrum situated near the violetward side of an absorption-band is thrown back towards the red. The effect may be simply to render the absorption-band narrower than it would have been had there been no effect of this kind, or to cause overlapping of different parts of the spectrum; or, as in the case of fuchsin and cyanin, actually to throw over the redward and the violetward parts of the spectrum into each other's places, and to make red light more refrangible than violet. These phenomena, experimentally systematised by Kundt, were discussed by von Helmholtz on the supposition that the absorbed light being in tune with the molecules of the body, these molecules are set in motion, which motion is modified by intramolecular friction so as to react on the transmitted light. The question is still obscure.

The heat region of the spectrum is very much shortened or compressed by the use of glass prisms.

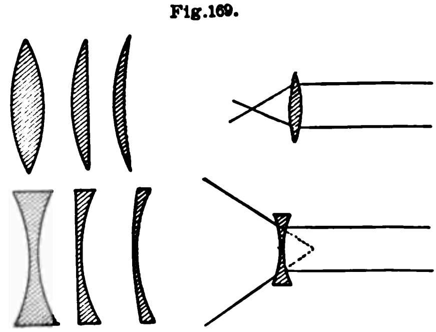
Lenses. — Simple lenses are of two main kinds: —

a. **Thin-edged,** thick in the centre; either convex on both sides, plano-convex, or convexo-concave, with a shallow concavity; a parallel-rayed beam, falling upon one of these lenses from either side, is made to **converge** upon a **Real Focus** on the opposite side of the lens.

b. **Thick-edged,** thinnest in the centre; biconcave, plano-concave, or concavo-convex, with a deep concavity: a plane-fronted beam, incident on either side, is made to **diverge** so as to seem to come from a **Virtual Focus** on the same side of the lens as the source of radiation itself.

Fig. 169 shows these different forms of simple lenses, and their action on a parallel beam of light travelling from right to left.

Fig. 169.



Every lens has a **Principal Focus**; this is the point to which a parallel beam of rays is caused to converge, or from which it is apparently caused to diverge, as the case may be.

The distance of this principal focus from the lens — or rather, from a point in or near

the lens which will be more precisely defined afterwards, — is called the **Focal Length,** f, of the lens; and the parallel rays being, by convention, taken as coming from the right, f is negative (to the left) in convergent, positive (to the right) in divergent lenses.

It is convenient for many purposes to assume that a lens has no thickness, and may be replaced by a refracting film, the proper position of which, as replacing the lens, will depend upon the form of the lens. This convention enables approximate formulæ to be used, but the results obtained by its aid are only approximate. The focal length will then be the distance between the principal focus and this film: and we shall first deal with the matter on this understanding. The point of this imaginary film which corresponds to the axis of the lens is called the Optical Centre of the lens. In a symmetrical biconvex or biconcave lens this Centre is at the centre of the lens itself.

If r and r' be the distances of the centres of curvature of the right and left faces of the lens respectively, both distances being measured from the centre of the lens positively — that is, towards the right on a horizontal line; and if a parallel beam come from the *right;* then, neglecting the thickness of the lens, $1/f=(\beta-1)(1/r-1/r')$, where β is the index of refraction of the lens in air, and f the positive distance of the focal point to the right, in air. If f be found negative, then the focal point is to the left, beyond the lens, a real focus for rays coming from the right.

This general formula applies without alteration to concavo-convex lenses, the convexity of which is turned towards the left, and both of whose centres of curvature are therefore situated in a positive direction. In a biconvex lens r is negative, and $1/f=(\beta-1)(-1/r-1/r')$; in a biconcave one r' is negative, and $1/f=(\beta-1)(1/r+1/r')$; in a convexo-plane lens (convexity to the left) $r=\infty$, and $1/f=(\beta-1)(-1/r')$; in a concavo-plane lens (concavity to the right) $r'=\infty$, and $1/f=(\beta-1)(1/r)$.

The expression $1/f$ measures the increase of divergence produced by a lens; it is called its Power. In thin-edged lenses, which are convergent, it is negative; in thick-edged, positive. Thus in an achromatic combination, consisting of a convergent lens (focal length $=-f$) with a divergent one ($f_{\prime}$) in contact with it, the power of the combination is, neglecting the thickness, $1/F=-1/f+1/f_{\prime}$; whence F can be found approximately.

Reversibility of Lenses. — Let us reverse a lens, still neglecting the thickness. Then the new $r=-r'$; the new $r'=-r$; the value of $1/f$ remains unchanged.

To find, roughly, the focal length of a thin-edged lens, find by experiment the distance at which it will make a clear image of the sun upon a screen: light coming from the sun is practically plane-fronted, and is caused to converge upon the principal focus.

Lenses, like mirrors, have Conjugate Foci at distances d and d', + or − according to their direction with reference to the "optical centre" of the lens; rays coming from an object placed at one conjugate focus are caused to produce an image, real or virtual, at the other.

Let d be the distance of the Object from the lens or from its optical centre, measured as a positive distance in front of the lens or its centre; let d' be the distance of the Image, similarly measured; and let f be the focal length (which is − in thin-edged, + in thick-edged lenses). Then the general formula, applicable to all lenses, is $1/d' = 1/d + 1/f$.

Examples. — 1. A convergent (thin-edged) lens of 25 cm. focus ($f = -25$); object at 75 cm. ($d = 75$) in front of the lens; $1/d' = 1/75 - 1/25$; $d' = -37\frac{1}{2}$; that is, the image is $37\frac{1}{2}$ cm. *behind* the lens; a Real Image.

2. The same lens; object at 20 cm. ($d = 20$); $1/d' = 1/20 - 1/25$; $d' = +100$; that is, the image is 100 cm. in *front* of the lens; a Virtual Image, beyond the object.

3. A divergent (thick-edged) lens of 25 cm. focus ($f = +25$); object at 75 cm. ($d = 75$); $1/d' = 1/75 + 1/25$; $d' = +18{\cdot}75$ cm.; that is, the image is 18·75 cm. in *front* of the lens; a Virtual Image, between the lens and the object.

4. The same lens; object at 20 cm. ($d = 20$); $1/d' = 1/20 + 1/25$; $d' = +11\frac{1}{9}$; that is, the image is $11\frac{1}{9}$ cm. in *front* of the lens; a Virtual Image, between the lens and the object.

Thin-edged Lenses. — Since f is negative, the equation becomes $1/d' = 1/d - 1/f$, and from it we find:—

A beam comes from an infinite distance; $d = \infty$; then $d' = -f$; light converges really upon the principal focus on the opposite side of the lens.

If the source of light be at a distance less than infinity, but greater than twice the focal length, the image is real, and is on the other side of the lens, at a point between the principal focus on that side and a point twice the focal length from the lens; that is, if $d = +(2f + x)$, $d' = -(f + \overline{f^2/f + x})$, which is greater than f, less than $2f$: and the image is then smaller than the object, in the ratio $d' : d$, linearly.

If the object be at a distance $2f$, the image is also at a distance $2f$, on the other side of the lens. The object and the image are then equal in size. Hence another method of finding the focal length of a thin-edged lens. Adjust an object, the lens, and a screen, so that the image on the screen is equal in size to the object: the screen and the object are now both situated at distances equal to $2f$ from the optical centre of the lens; one-fourth the distance between them is the focal length.

If the object be at a distance greater than f, but less than $2f$, the image is real, at a distance greater than $2f$, and is larger than the object, in the ratio $d' : d$, linearly.

If the source of light be at a principal focus, $d = f$; $\therefore\ d' = -\infty$; the image is real, but at an infinite distance beyond the lens.

Hence the focal length of a thin-edged lens may also be formed in the following way. Focus a telescope upon a very distant object; it is then in

focus for parallel rays. In front of the telescope, so adjusted, fit up the lens, well centred with respect to the telescope. In front of the lens adjust the position of a spider-web or other delicate object, until this is seen sharply defined in the telescope. The rays divergent from the object are then rendered parallel by the lens, and the object is, accordingly, then at the principal focus of the lens.

If the object be between the lens and a principal focus, the rays are not made sufficiently convergent to cross at any place; they seem to come from a virtual image beyond that principal focus, and farther from the lens than the object; but the virtual image rapidly gains on the object as the object approaches the lens.

Thick-edged Lenses. — Since f is positive, the equation remains unchanged, and from it we find: —

When d is of any value > 0, $< \infty$, d' is less than f, and also less than d.

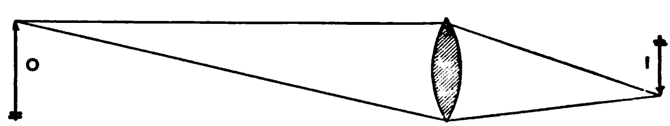
When $d = +f$, $d' = +\frac{1}{2}f$; Virtual Image at half focal distance.

If rays converge upon a point behind the lens, so that d is negative, then, so long as d is not numerically greater than f, the lens will make them converge upon a point at a greater negative distance. If $d = -f$, the convergent rays are rendered parallel. If d be negative and numerically greater than f, the convergent rays are made to diverge as if from some positive distance d', $< \infty$, $> f$. When $d = -2f$, $d' = +2f$.

The focus of a thick-edged lens is most conveniently found by coupling it with a thin-edged one, found by trial among a sufficiently extensive series, so that together they shall produce no change in the apparent size of an object seen through them.

As to the inversion or erectness of the image produced by a **thin-edged** lens, an object at O (Fig. 170), at a distance exceeding twice the focal length, produces a smaller inverted

Fig. 170.



image, a **real image**, at I, and an eye placed beyond I — that is, at a sufficient distance from the lens — will perceive the real image of a distant object, inverted, smaller than the object, and apparently situated in space between him and the lens; the contrary being the general impression. The eye must be so far beyond I that the real image in space can be looked at in the same way as any ordinary object of vision. In all cases, an object and its real image are interchangeable, so that an object at I will produce a real image at O.

If the distance be greater than the focal length, but less than twice that length, the image is still real and inverted, but is larger than the object.

When, however, the object is brought so near the lens as to lie at a distance from the lens less than the focal length, then, to an eye situated at any distance on the other side of the lens, a **virtual image** will be apparent, erect, magnified, and more distant than the object. Hence these lenses are commonly used as magnifying-glasses (Fig. 171). The relative linear sizes of object and image are in all cases proportional to their respective distances from the "optical centre" of the lens.

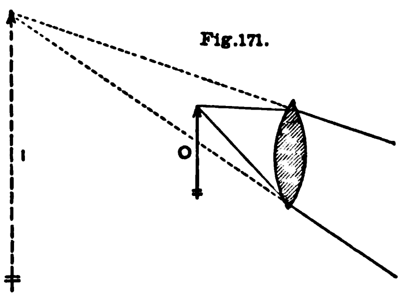


Fig. 171.

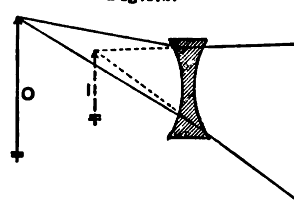


Fig. 172.

In divergent or thick-edged lenses, Fig. 172 shows that the image of a real object is erect, diminished, virtual, and nearer to the lens than the object itself; and, since there is no subsequent crossing of the rays beyond the lens, there is no inversion of the image. The virtual image of such an object is always at a distance from the lens less than the focal length. Lenses of this kind may be used as diminishing-glasses.

For some purposes flexible lenses may be used in which the curvature may be slightly varied. Cusco's ophthalmoscopic lens consists of two pieces of thin microscopic cover-glass fixed in a frame: water fills the cavity between them; by forcing more or less water into the cavity the curvature may be varied.

Even when the light transmitted through convergent lenses is monochromatic, the focussing can never be exact if their surfaces be spherical; each point of an extended object forms a slightly-blurred image. This effect can be reduced somewhat by the use of Diaphragms, which allow only the central part of a beam to pass through the centre, and the marginal rays to pass through the marginal part of a lens; they thus diminish the Spherical Aberration of the lens (as in the pupil of the eye), but this can only be brought to a minimum by modifying the curvatures of the lenses used. This could be done, for parallel incident rays, or for rays coming from a predetermined distance, by making the anterior face of a single

lens ellipsoidal or hyperboloidal and the hinder face spherical; but such surfaces cannot well be produced. If lenses could be produced diminishing in density towards the centre, the same effect might be attained. Both these refinements are present in the human eye. What the optician does is to combine lenses, which have spherical and plane surfaces, so as approximately to bring about this desired result, and then to correct the curvatures by a process of systematic trial and error. Some combinations of lenses are so devised as to bring all the points of an extended image into the same plane, and thus to produce a flat field; others to bring points differing in distance to foci which differ very little from one another, and thus to secure penetration. The calculation of the various curvatures necessary for these ends often involves considerable mathematical skill.

In general a lens with spherical surfaces is equivalent to a series of prisms whose angles vary with the varying distances from the axis. In a thin-edged lens the marginal rays are generally more refracted than the axial; thus a square object yields a Distorted Image, the corners of which appear squeezed in, and the boundaries of which are convex. Similarly, a thick-edged lens draws out the corners and produces concave bounding lines. This tendency is obviated by using lenses in pairs, symmetrically disposed, so that the distortions produced by the one lens may be reversed by the other. Again, if the screen on which the image is received be not parallel to the object, the image is apparently so distorted that lines parallel in the object appear to diverge or converge in the image; whence the use of the Swing-back, maintained vertical, in photographic cameras.

The general principle underlying calculations relating to systems of lenses is that the image formed or tending to be formed by the first lens is taken as the object (real or virtual) of the second, and so on. The upshot is, that for every arrangement of any lens-system, there is always an image formed somewhere, corresponding in size (but not in its position) with that which might have been produced by an Equivalent single Lens. The adjustment of a system of lenses (*e.g.*, the focussing of a telescope or microscope) is for the purpose of causing the image to be formed in a place where it will be convenient to inspect or to use it.

The action of a system of lenses is, approximately, equivalent to the formation of an image by a simple lens *plus* a determinate shifting of the image formed. This shifting being allowed for, it is often convenient to

represent a system of lenses by an equivalent lens. For instance, the Eye may be ideally reduced for many purposes to a single lens composed of aqueous or vitreous humour, having its back coincident with the retina, and its anterior aspect a spherical surface of 5·1248 mm. radius, situated at its most anterior point 2·3448 mm. behind the actual anterior surface of the cornea. Such a lens would refract incident light and bring images of distant points to a focus upon the retina in the same way as the actual Eye does.

Gauss's Method. — Gauss, followed by Listing, starting from this consideration as to the equivalence of any system of lenses to a single lens *plus* a determinate shift, found that every possible system of lenses could, if well centred, be reduced to a Region of Space to be traversed by the incident light, and presenting six characteristic or Cardinal Points, ranged along the axis of the system. These are the Incidental Focus F, the Incidental Principal Point P, the Incidental Nodal Point N, the Refractional Principal Point P′, the Refractional Nodal Point N′, and the Refractional Focal Point F′. The incidental points become refractional, and *vice versâ*, when the direction of the rays is reversed. The rays are assumed to travel all near the axis.

All rays proceeding from F become after refraction parallel to each other and to the axis; all parallel incident rays, parallel to the axis of the system, pass through F′. An object at P, or in the same plane (at right angles to the axis of the system) with it, forms an equal and erect image at P′ or in the same plane with it; there are only two such points. Any ray apparently making for N before refraction is, after refraction, parallel to its former course, but appears to be coming from N′; there are only two such points. The distance PF is the Incidental, while P′F′ is the Refractional Principal Focal Distance.

These six points, all in one line, are closely related. The distance FN is equal to the distance F′P′; and N′F′ = FP. Therefore PN = P′N′ = FP − F′P′; and PP′ = NN′. Further, if β be the ratio between the index of refraction of the medium nearer the source and that of the medium beyond the lens, $FP = F'P' \cdot (1/\beta)$; in the case of a lens in air, these two principal focal distances are equal, and further, P coincides with N and P′ with N′.

Planes passing through F and F′ at right angles to the axis of the system are called its Focal Planes.

Rays diverging from any point in one of these focal planes (of which rays one might be towards the corresponding nodal point) emerge parallel to one another; and since the ray from the divergence-point to the corresponding nodal point would have emerged parallel to its original direction, all the rays must necessarily emerge parallel to the original direction of that ray. Thus they retain parallelism with that ray, the direction of which is, on its emergence, determinate.

Rays parallel in the first medium converge on some point in the second focal plane. That ray which travels towards N emerges as if it had come from N′ parallel to its former course. Hence a line drawn from N′ parallel to the originally parallel rays will cut the second focal plane in a certain point; towards that point in the second focal plane all the rays originally parallel must converge.

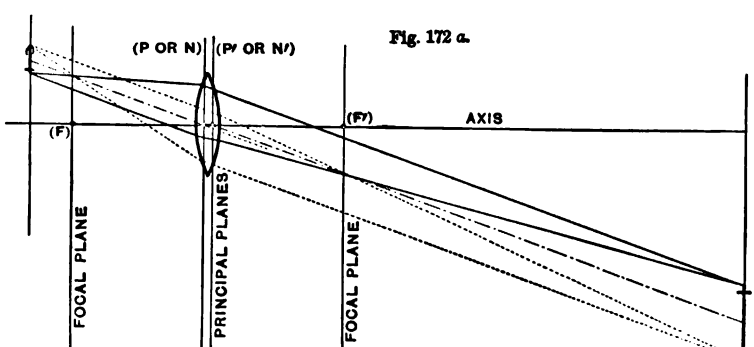
The artifice of Gauss's method (for which see his *Collected Works*, or, for an elementary exposition, von Helmholtz's *Physiol. Optik*, and Clerk Maxwell, *Qu. J. Mathem.*, 1858, p. 233; or Pendlebury, *Lenses and Systems of*

Lenses) is, so to speak, the identification of an incident set of rays *as they cross the first Focal Plane;* the rays are then traced until they arrive at the second plane; from the data thus obtained their subsequent course can be ascertained. Mathematical difficulties are thus minimised, for the problem becomes mainly one of finding these cardinal points for a lens-system of any given form and of any degree of complexity.

In the case of a single lens surrounded by a single medium, such as air, let the radius of the right-hand surface be r ($+$ if the centre be towards the right, $-$ if towards the left), and let that of the left-hand surface be r' (similarly $+$ or $-$); and if β stand for β_i/β_0, the relative refractive index of the lens as compared with that of the medium; and if A and B be the left- and right-hand axial points of the lens, so that $AB = t$, the axial thickness of the lens: then, all measurements being reckoned on the footing that $+$ AX is a distance from A towards the right and $-$ AX the same distance towards the left, we have $AF = (-\beta rr' - (\beta - 1)\, tr') \div (\beta - 1)\, [\beta\,(r - r') + (\beta - 1)\, t]$; $BF' = (\beta rr' - (\beta - 1)\, tr) \div$ the same divisor; $AP = -tr' \div [\beta\,(r - r') + (\beta - 1)\, t]$; $BP' = -tr \div [\beta\,(r - r') + (\beta - 1)\, t]$; $PF = -P'F' = -\beta rr' \div (\beta - 1)\, [\beta\,(r - r') + (\beta - 1)\, t]$. From these formulæ the position of the principal planes with respect to the lens, and the distance PF or $P'F' = f$ between the principal planes and the corresponding focal points (which is the *true* Focal Distance) can be found for a single lens of any form and refractive index, immersed in any medium. For example, in a convexo-concave (thin-edged) lens, let $r = +12$ cm., $r' = +9$ cm., and $t = 0{\cdot}8$ cm.; and $\beta = 1{\cdot}5$. Then, by the above formulæ, $AF = -67{\cdot}59$, $BF' = +64{\cdot}16$, $AP = -1{\cdot}47$, $BP' = -1{\cdot}96$, $PF = -66{\cdot}12$, and $P'F' = +66{\cdot}12$. Whence P is outside the lens, 1·47 cm. to the left of A; P′ is also outside the lens, 1·16 cm. to the left of A; and F is 67·59 cm. to the left of A, while F′ is 64·16 cm. to the right of B.

From these formulæ it will be found that in thin-edged lenses the points F, F′, P, P′, mostly lie on the axis in the order FPP′F′, while in thick-edged lenses they mostly lie in the order F′PP′F; and that unsymmetrical

Fig. 172 a.



lenses are only truly reversible when the principal focal planes are made to exchange places; which, since in the case of thin-edged convexo-concave lenses the principal planes mostly lie outside the convex face and in that of thick-edged convexo-concave lenses outside the concave face, often involves considerable shifting of the lens in its setting.

If the object O be at any distance $d = PO$ from the incidental principal

plane, the distance of the image I from the other principal plane, $d' = \mathrm{P'I}$, is determinable numerically by the equation $1/\mathrm{PO} + 1/\mathrm{P'I} = 1/\mathrm{PF}$, or $1/d + 1/d' = 1/f$, f being taken as + in divergent, − in convergent lenses.

If we put the thickness of the lens out of view, making $t = 0$ in the formulæ, we arrive at the usual lens-formulæ. For example, in the convexo-concave lens discussed above, $\mathrm{AF} = -\beta rr' + \{(\beta - 1)\cdot\beta(r - r')\} = \{(\beta - 1)(1/r - 1/r')\}^{-1} = -72$; $\mathrm{BF'} = +72$; $\mathrm{AP} = 0$; $\mathrm{BP} = 0$; $\mathrm{AB} = 0$; that is, the focal distances measured from the imaginarily coinciding principal planes or, in other words, from the optical centre of the lens, are each equal to 72 cm. It will be seen how widely these values depart in this case from the true values as given by the Gauss-formulæ above.

The accompanying diagram (Fig. 172*a*), in which the obliquity of the marginal rays is exaggerated, may serve to illustrate the method as applied to a biconvex lens in air: in this case the focal distances are equal and the principal points, which coincide with the nodal, are within the lens.

Chromatic Aberration. — When mixed coloured-light is passed through a thin-edged lens, violet light is most refracted, and comes to a focus sooner than the red rays do; beyond the red focus is the heat-focus; between the violet focus and the lens is the region of the photographic focus.

If a beam of white light be passed through a single convergent-lens, a screen placed at the violet focus will give an image with a red border — the red rays not having yet converged; if it be placed a little farther off, at the red focus, the image is now surrounded by a violet border, for the violet rays are already divergent. Consequently no clear definition can be obtained by the use of such simple lenses, and it is necessary to render them Achromatic. A biconvex lens of flint glass, more convergent than is necessary, is coupled with a biconcave lens of crown glass of proper curvature; the latter destroys the dispersion, by bringing two colours to the same focus, without wholly doing away with the deviation; the couplet acts on the whole as a single lens, producing a somewhat smaller refraction than either of the lenses. This arrangement may be seen in the object-glass of any common telescope. For still further accuracy three, four, or even a greater number of lenses may be combined, by which three, four, or more colours are brought to the same focus; as in the achromatic objectives of microscopes.

Makers of photographic lenses have shown much skill in making the photographic and the visual focus coincide; for special photographic work, such as Rutherford's lunar photography, lenses have had to be constructed whose curvatures are calculated with reference to the focus of the highly-refrangible actinic rays alone; and, while nothing can be distinctly seen through such lenses, photographs of extraordinary clearness have been taken by their aid.

Radiant Heat may be shown to be reflected and refracted like Light, by concentrating rays of dark heat upon a Thermopile by means of a lens or a mirror, or by refracting them by means of a prism into a new path, in the course of which the thermopile must somewhere be placed before it will indicate the impact of Heat-waves: by photography of the infra-red region of the spectrum; by Langley's Bolometer (p. 717); by Becquerel's Phosphorescence-effect, p. 505.

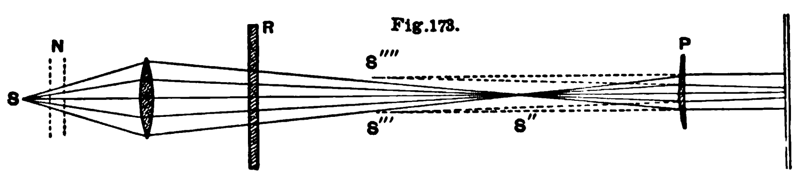
INTERFERENCE.

Ether-waves are capable of Interference. Two systems of equal waves, arriving at the same point in opposite phases, will produce at that point no effect, either of light or of heat or of photographic action: at that point the ether will be at rest; and thus light added to light may produce darkness. In Fig. 75 the two points A and B are centres of wave-motion, and at the points b', d', f', on the screen MN, there is no disturbance, while at intervening points, a', c', e', the amplitude of disturbance is doubled.

Interference of waves thus affects the distribution of energy in a system of undulations, and such a screen produces a system of n e g a t i v e reflected waves from a', c', e', etc.

Let us now consider a monochromatic beam of plane-polarised light. Such a beam may be divided into two parts by reflexion from a silvered or platinum mirror bent in the middle at an angle very nearly equal to 180°, or else by refraction through a biprism whose angle is very nearly 180°. The last case is shown in Fig. 173. S is a source of light; the light from it

Fig. 173.



is transmitted through a polariser N: it is now a polarised beam. The rays are received by a convergent lens, which makes them converge upon S″. In its course it is passed through a piece of glass R, coloured red with suboxide of copper: it is now to a rough approximation monochromatic. It is then passed through the b i p r i s m P, which refracts it in such a way that it seems to come from two equal and equally-distant foci at S‴ and S⁗.

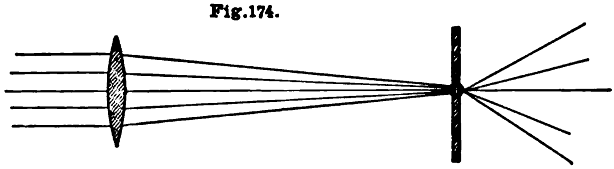
The light may then be received, either on a screen, or directly in the observer's eye placed in the onward path of the beam. A series of dark and bright fringes will be seen, corresponding to the alternate fringes of rest and disturbance of Fig. 75. The two beams apparently travelling from S''' and S'''' are polarised in the same plane, and any irregularity of amplitude characterising the one is participated in by the other. Hence they are in a position to interfere fully and regularly with one another. If, on the other hand, they had been polarised in planes at right angles to one another, they could not have extinguished one another at any point. When common light is used, it may be at once filtered through a piece of red glass and then passed through a convergent lens.

Light from two different sources cannot show interference-phenomena well; any irregularities in the one vibration ought to be participated in by the other; and hence even light from the same source, if one of the beams have been very much delayed, may be rendered unable to show these phenomena, through the irregularities having had time to produce a difference between the two beams of light, originating from the same source at different times.

To procure monochromatic light it is better to project a spectrum upon a screen in which there is a slit, and then, behind the screen, to make use of that part of the spectrum whose light falls upon and traverses the slit.

It is very easy to procure a bright spot which may represent a simple luminous point, by making a small hole in a metal screen, and in this inserting a drop of glycerine. This acts as a powerfully-convergent lens, and if sunlight be concentrated upon it there will appear on the dark side of the

Fig. 174.



screen an intensely bright little spot of light which may be used as a source of light for many experiments; with such a source of light Fresnel discovered the laws of diffraction. More elaborately, the same result may be better attained by means of the electric light made to converge by an achromatic lens of exceedingly short focus, a high-power microscopic objective.

When monochromatic common light, proceeding from a luminous point, is passed through a biprism, its vibrations in each of two planes, at right angles to one another, produce the

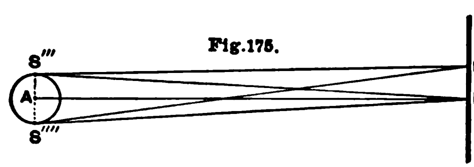
effects of interference independently of one another, but produce their respective fringes and bands in coincident positions on the screen. When mixed coloured-light or white light is treated in this way, the red fringes do not coincide with the violet fringes; the violet fringes are more numerous than the red fringes, and are closer together. This will be understood from Fig. 75; if the wave-length be increased, the points a', b', c', d', e', must become farther distant from one another. A violet fringe is seen near the axial line of the beam; it is overlapped by a blue, the blue by a green, and so on: each coloured fringe produced by the interference of white light presents a complete spectrum. The number of such spectra is limited; at a little distance from the axial line of the beam the fringes overlap one another so as to produce what appears to the eye to be simply white light, but the spectrum of which shows a series of alternately dark and light bands: all the colours being equally encroached upon by dark bands, the result seems white.

Michelson has been able to observe 200,000 fringes.

A bent mirror used instead of a biprism produces, by reflexion of white light upon a screen, alternate fringes of white light and darkness.

Measurement of Wave-length.—If S‴S⁗ be the apparent position of the two images or apparent sources of light, which must be monochromatic; N the position of the central fringe, illuminated by the joint action of S‴ and S⁗; the angle S‴NS⁗ $= 2\delta$; N′ the position of, say, the fourth bright fringe; S‴N′ is shorter than S⁗N′ by four wave-lengths; this difference is very nearly equal to $\{NN' \times S'''S'''' \div AN\} = NN' \times 2 \tan \delta$.

Fig. 175.



The angle 2δ can be measured with a theodolite; the distance NN′ can be measured with a micrometer; the value of the four wave-lengths, and therefore of one wave-length, can be determined from these data.

Fig. 75 shows that the line of propagation of these fringes in space is hyperbolic; the foci of these hyperbolas being the two apparent sources.

The bands vanish when one-half of the biprism or mirror is covered.

If the light from one of the sources be retarded by being made to pass through a layer of a substance in which light travels more slowly than in air, the whole of the fringes will be shifted somewhat towards the side on which the retardation

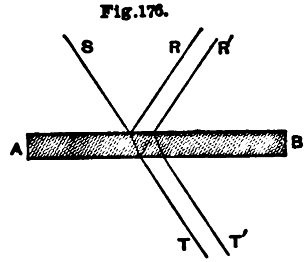
takes place. From the amount of this shifting may be calculated the amount of retardation; and by means of this the relative velocities of light in (and therefore the refractive indices of) such things as hot air, cold air, hydrogen gas, normal glass, compressed glass, compressed liquids, and so forth, may be estimated.

Colours of thin films. — Thin films of transparent substances, such as oil upon the surface of water, iron oxide upon the surface of tempered steel, oxides deposited upon metals by the galvanic battery, soap bubbles, glass blown out to an extreme tenuity or exfoliating under the influence of slow decomposition, present curious colours when shone upon by a comparatively bright light.

Such films may be rendered permanent; a solution of bitumen and a little caoutchouc in a mixture of benzene and oil of naphtha, dropped upon water, forms films which solidify and may be caused to adhere to a sheet of paper.

In Fig. 176 monochromatic light from S is incident upon a thin transparent-film AB of uniform thickness. A part of the light is at once reflected to R from the first surface of the film. Another part is refracted to R′ after having undergone one reflexion at the second surface. If the path of the beam in the film be an *even** number of half wave-lengths, the beam travelling to R′ is opposed in phase to that travelling to R, and an eye placed at RR′ (these points being supposed very close together) will receive no impression of light; or, rather, it will receive but a feeble impression, for the ray to R′ cannot be quite equal in intensity to that travelling to R. Again, an eye placed at TT′ will perceive but a feeble impression of light; not absolute darkness, for the ray to T is considerably more intense than that to T′, and is not completely neutralised by it.

Fig. 176.

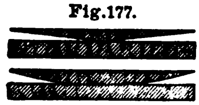


There is, however, complete interference for any one wave-length if multiple reflexion be taken into account.

* This seems strange; we might have expected a retardation of an *odd* number of half wave-lengths to produce a difference in phase of half a period; but it will be remembered that the beam reflected at one of the surfaces of the film — that surface, namely, which separates an optically denser from a rarer medium — suffers a loss of half a wave-length, which is independent of the thickness of the film.

Let the film be of variable thickness; a film of air between a glass plate and a biprism, or between a convex lens and a plate of glass, varies in thickness with the distance from the centre; in the former case the thickness of the film of air varies as the distance, in the latter approximately as the square of the distance from the central point. Monochromatic light reflected from such a system presents the appearance of alternately dark and bright bands or circles — bright where the directly-reflected light and the light reflected from the second surface of the film are similar in phase — dark where they are opposed. In the case of a lens pressed against a plate they are known as **Newton's rings.** The less the curvature of the lens the greater the distance between two consecutive rings. The distance between the consecutive rings is, approximately, inversely proportional to the radius, so that the external rings are most crowded together. If such a substance as water be used between the lens and the glass, the rings are closer together; the width of the rings varies inversely as β, the refractive index of the substance thus employed; for a shorter distance in an optically-denser medium is equivalent to a longer distance in air. On inclining the incidence of the light, the rings become dilated. By transmission, a second system of rings is produced, complementary but dimmer. If mixed coloured or white light be employed, the dark and bright rings of the several components cannot coincide, and the result is a series of circular spectra, in each of which the violet circle is the narrowest. These spectra overlap one another at a little distance from the centre, and blend into what appears to the eye to be white light.

Fig. 177.



A series of dark rings or fringes may be obtained by rubbing a film of soap on black glass, drying it, and breathing gently upon one point of this through a glass tube; this, done in the sunshine, gives rise to bright colours.

It is not possible actually to obtain monochromatic light; even that emitted by incandescent sodium-vapour, in which some five hundred rings can be seen, is not quite monochromatic.

The centre of Newton's rings is dark if there be approximate contact; perfect contact there never can be, for a dustless surface it is impossible to obtain; even when there is no appreciable thickness of film traversed, the fact that one ray is reflected from the upper, and the other from the lower surface, the one at the bounding surface of an optically-denser, the other at the surface of an optically-rarer medium, causes the one to lose, while the other does not lose, half a wave-length on reflexion; they thus become

opposed in phase, and the centre is dark. If, however, both reflexions be made to take place from the surface of an optically-denser medium, as in Young's experiment, — in which light travelling through a lens of crown glass was reflected first from the upper surface of a film of oil of sassafras, lying between that lens and a plate of flint glass, sassafras being intermediate in its refractive power between crown and flint glass, — there is no such relative retardation, and the centre of the system of rings is bright.

The Iridescence of mother of pearl and of objects with a finely-grooved or striated surface, such as butterfly's scales, is an effect of interference. Sunlight falls upon their surface; some of this is reflected from the ridges, some from the grooves, and in this way a difference of path is set up among the reflected rays, which causes differences of phase among them, and, in the case of some of them, opposition of phase and extinction. When the incidence of the reflected light is very oblique, the ridges alone may reflect, the differences of phase and of path produced will be very small; there will be little iridescence and very considerable reflexion.

The propagation of light "in straight lines" within the same isotropic medium is itself a result of interference. From it is derived the power of making a geometrical **Shadow**. In Fig. 178 a real focus at F acts as a source of light. It casts a sharply-defined shadow of an object O upon a screen. If the source of light be an extended one, not a mere point, the shadow consists of two regions, a central umbra and a marginal penumbra. In Fig. 179 the sun, S, shines upon the earth, E: the earth being smaller than the sun, there is formed a cone of darkness behind the earth; if the moon travel wholly or partially into this cone of shadow, it will be wholly or partly unillumined, and we have a total or a partial eclipse of the moon.

Fig.178.

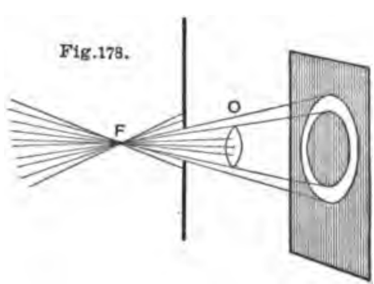
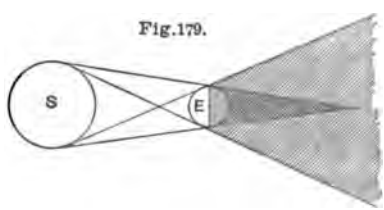


Fig.179.



But outside this shadow there is a penumbral region, in which a body, or any point of a body, will be in "half-shadow," not fully illuminated, because able only to see a portion of the illuminating body.

When light radiating from an extended object passes through a small aperture, the waves arriving at the aperture from the object traverse the aperture, and there cross each other; they then diverge, and a screen placed on the opposite side of the aperture receives an inverted image of the object, whose size varies with the distance of the screen, as in the well-known **Camera Obscura.**

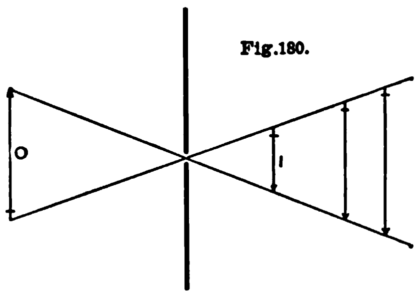


Fig. 180.

An aperture of no appreciable breadth would, at whatever distance the screen might be placed, give a perfect image in the natural colours, an image of which no part would be out of focus; one of $\frac{1}{25}$-inch diameter will give on a screen at 40 inches distance an image which, though wanting in brightness, is as perfectly defined an image as any possible lens placed at the aperture can produce: one of $\frac{1}{10}$-inch will produce the same definition at a distance of 250 inches; or, in general (Lord Rayleigh), if λ be the wave-length, r the semi-diameter of the aperture, d the least distance of good definition, $d = (2r^2/\lambda)$. When the screen is nearer than this, each point of the object makes on the screen an image which has the same shape as the aperture, and the superposition of these makes a blurred image. When the diameter is $\frac{1}{51}$-inch (No. 10 steel sewing-needle), $d = 8{\cdot}57$ inches (pin-hole photography); but with smaller apertures than this, Diffraction begins to confuse the result.

Light thus travels in straight lines, and is incapable of passing round corners under ordinary circumstances, and as examined by our ordinary senses.

A closer examination of the subject shows, however, that light does to a certain degree pass even round corners. The phenomena of **Diffraction,** in which this is observed, are explicable on the ordinary principles of interference. Let S (Fig. 77) be the source of light; waves diverge from this as a centre. These waves impinge upon a screen AB. Fig. 77 shows that beyond the screen AB there is a series of fringes within the geometrical shadow; that even in the part directly in view of the source of light there are bands of relative darkness; that the central point of the shadow may be nearly as brightly illuminated as if there had been no screen AB; that the broader the object AB, the narrower will be the fringes; that the forms in space of the regions of approximate darkness are hyperboloids; while if the source of light be removed to an infinite distance, the hyperbolic lines of relative rest in the illuminated region are practically

reduced to straight lines, but sweep past the obstacle without touching it.

When the obstacle is circular — a minute circle of tinfoil pasted on a piece of clear glass — the shadow cast upon a screen, or received in the eye directly or by the aid of a lens or telescope focussed on the obstacle, is seen to be surrounded by a series of dark and bright rings; or, if the light from S be mixed-coloured or white light, by a series of spectra; while the shadow is also modified by a series of such bands or spectra, and its centre is bright. A similar construction for a little circular aperture in an opaque screen at AB will show that the bright spot produced on a screen beyond AB will have fringes blurring the sharpness of its edges, and that at certain distances of the second screen from AB the centre of the bright spot will be dark.

When the obstacle or chink is linear and parallel-sided, the fringes or spectra are parallel to one another; when it is not so they assume a curved form; when it is angular the fringes may assume a great variety of remarkable and beautiful forms.

The phenomenon of diffraction can be roughly observed by looking at a distant gas-flame, edge on, with the half-closed eyes; the sun shining on the eye-lashes will also produce a similar effect; the morning sun, shining on twigs of trees situated between the sun and the eye, causes the shadows of some of them to become bright in the centre, and a curious silvery appearance results.

The image of any point seen through a telescope or microscope has its clearness of definition interfered with by the diffraction of rays of light round the edges of the diaphragm, or round the edges of the lens. This effect is generally insignificant in terrestrial telescopes; it is very noticeable in astronomical telescopes, where the source of light, a distant star, ought to appear reduced to a point, but is apparently enlarged into a perceptible disc surrounded by rings; and in the microscope it sets a limit to the powers attainable, for high powers involve small lenses and small apertures, and these bring diffraction in their train. The limit of microscopic definition is about $\frac{1}{3000}$ mm. with white, and about $\frac{1}{5000}$ mm. with blue light.

If a very large number of parallel equidistant lines be ruled upon glass or metal, plane-fronted light issuing from a slit or from the image of a slit will, if transmitted through or reflected from this so-called **Diffraction-grating,** and focussed

upon a screen or in the eye, be found to be resolved into a central bright image of the slit, on each side of which is a dark space, and then a series of successive spectra, overlapping or separated by dark spaces, according to the fineness of the grating: these spectra have their violet ends turned towards the central bright image (see Fig. 77*a*). By multiplying the number of lines in the Diffraction-grating, as in Prof. Rowland's gratings, which have 43,000 equidistant lines to the inch, the spectra may be rendered almost perfectly pure, so that Fraunhofer's lines may be easily seen in them.

A microscopical preparation of muscular tissue will often be found to act as a more or less efficient diffraction-grating; the striations of the muscular fibres take the place of the grooves engraved on the glass.

The value of diffraction-spectra is that the deviation in the successive spectra depends directly upon the wave-length; their disadvantage the mechanical difficulties of uniform grooving of the grating, and of making clean-cut grooves.

If any kind of light have, in air, the wave-length λ centimetres, and if N be the average number of lines per centimetre engraved on the grating; and if δ be the angular deviation of any particular coloured light (or, better, of any particular Fraunhofer line), — then $\sin \delta$ is equal to $N\lambda$ for the first spectrum, to $2N\lambda$ for the second spectrum, and so forth; and since N and δ can be measured, λ can be accurately found. At the spot where light of wave-length λ appears in the third spectrum, that of wave-length $3\lambda/2$ in the second and that of wave-length 3λ in the first spectrum coincide.

The definition in the diffraction-spectrum is best in the position of minimum deviation (p. 141); and the normal spectrum is produced when the angle of incidence is so regulated that the angle of diffraction is zero (see p. 141).

The Twinkling of Stars is another effect of interference: light, coming to the eye from a star so distant as to be practically a single luminous point, arrives in rays which have traversed slightly unequal distances in an irregularly-refracting atmosphere and thus enter the eye in irregularly-unequal phases. Now one colour is extinguished, now another; the eye perceives coloured light complementary to that momentarily lost. No two persons can, as a rule, see any star twinkling in precisely the same manner. The planets twinkle only at their edges: their discs present many points or sources of light, whose scintillations, on the whole, mask one another.

If a planet and a twinkling star — say Jupiter and Sirius — be severally looked at through an opera-glass which is rapidly whirled across the field of view, the image of the planet will appear to be drawn out into a continuous

streak, while that of the star will be broken up into a chain of unequally-bright and differently-coloured spots of light.

The colours of light from a bright point twinkling through a dusty haze, or shining through a piece of glass covered with lycopodium; the Corona (red externally) which surrounds the moon as it shines through an atmosphere charged with particles of condensed aqueous vapour; the coloured rings seen when particles float in the vitreous humour of the eye,—these are all different diffractive effects of interference; and the smaller the size of the particles which produce them, the greater the breadth of the coloured rings. Each particle acts as a partially opaque small screen.

The interference of Actinic Rays may be shown by photography; of Dark Heat, by passing a delicate Thermopile, a Tasimeter (p. 636), or a Bolometer (Langley's Thermic Balance, p. 717) through an invisible diffraction-fringe system of dark heat-waves, obtained by treating rays of dark heat with a bent mirror or a biprism; under these circumstances the instrument employed will alternately indicate and cease to indicate the impact of heat-waves.

Double Refraction.

If a transparent medium have the same properties in all directions it is homogeneous, or, optically, **isotropic.** A wave of mechanical disturbance starting from a single point of disturbance in it will be spherical. The properties of the ether-waves within transparent substances are, in some fashion, correlated with the molecular structure of the substance, and thus any ether-waves propagated from centres within homogeneous or isotropic substances are themselves also spherical.

Substances in which the propagation of light is in spherical waves are either amorphous, or else belong to the cubical system of crystals, the system in which the three crystallographic axes of the crystal are equal.

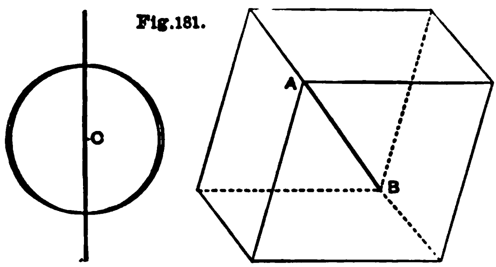
In some crystalline substances one of the crystallographic axes differs from the other two; the crystal is then symmetrical in reference to this axis only, and is said to be **uniaxial.** A mechanical disturbance is propagated in such a crystal in the form of an ellipsoid.

A slice cut out of such a crystal in such a way that its faces are parallel to this principal axis, is said to have been cut parallel to the **Principal Section** of the crystal.

The propagation of an ether-wave in a uniaxial crystal is peculiar. Fig. 181 shows an equal-sided rhombohedron cut out of a crystal of Iceland spar by splitting it along its natural cleav-

age-planes; its axis AB joins the opposite obtuse-angles. Let a point C on this axis be a centre of optical disturbance. Then two concentric sets of waves are produced; the one spherical just as in glass, the other ellipsoidal; one of the axes of the ellipsoid coincides with the axis of the crystal, and is equal for a given interval of time to the diameter of the sphere developed in an equal time; the other two axes, which, to avoid circumlocution, we shall here call the **extraordinary** axes, are equal to one another, and are either longer or shorter than the former, according to the nature of the crystal. The next question is, — Which part of a general disturbance at C is propagated in the spherical, and which in the ellipsoidal wave?

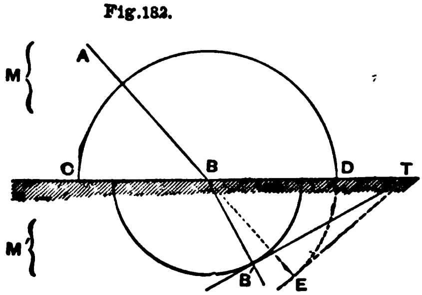
Fig. 181.



It may roughly be stated that just as we have seen beams of polarised light differently affected by simple reflexion and refraction according to the plane of their polarisation, so in double refraction the behaviour of a beam of light depends upon its state of polarisation.

On referring to Fig. 59 we find that the construction there given for the course of a refracted plane-fronted wave may be reduced to the following construction (due to Huyghens) for a single ray refracted at the surface of an ordinary isotropic medium. AB is an incident ray travelling through the medium M; CD a circular arc, drawn from centre B, with radius proportionate to the velocity of light in the medium M. Continue the arc CD into the second medium M′; produce AB until it cuts that arc in E; from E draw a tangent line (or plane) cutting the refracting surface in T. From B as centre draw a semicircular arc in the medium M′, with a radius proportionate to the velocity of light in M′. From T draw a tangent to this arc; the tangent touches the arc at B′; join BB′. BB′ is the refracted ray.

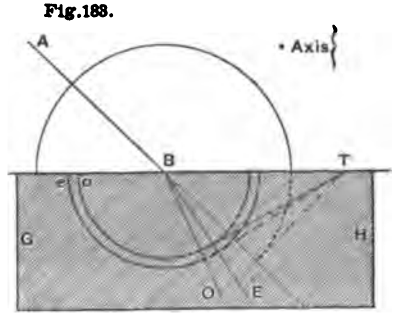
Fig. 182.



A series of somewhat similar constructions will enable us to study a certain number of cases of double refraction.

Suppose a block to be cut out of a crystal of Iceland spar in such a way that one of its cut surfaces is parallel to the axis; and suppose an incident beam to fall upon that surface in a direction at right angles to the axis. Fig. 183 shows that if GH represent such a block, and if the incident beam be in the plane of the paper, the axis is in such a case looked at end-on; and then we find that the incident ray is divided into two parts, which travel at different rates, the slower one, BO, in the central sphere, the more rapid one, BE, in the outer ellipsoid, which, looked at in this aspect, has a circular section; the former, BO, the **Ordinary Ray** (which obeys the ordinary law $\sin \iota = \beta \sin \varrho$), being more refracted than the latter, BE, the **Extraordinary Ray**. Both these rays are in this case in the plane of the paper, like the original incident-ray. The relative radii of the two circles may be found from the respective amounts of refraction of the two rays at this kind of incidence.

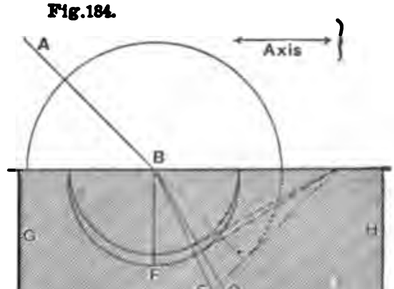
Fig. 183.



For the light emitted by sodium-vapour, the ordinary index and the extraordinary index of Iceland spar are respectively 1·65850 and 1·48635; the reciprocals of these numbers represent the relative velocities of the ordinary and the extraordinary rays in Iceland spar as compared with that of light in air, this being reckoned as unity. In such crystals as those of Iceland spar the ordinary ray is more retarded than the extraordinary.

Let us now turn the block of spar round so that its axis is brought into the plane of the paper — that is, into the same plane with the incident light; the incident light now travels in a principal section of the crystal. One of the extraordinary axes of the ellipsoid, being at right angles to the axis of the crystal, is at right angles to the refracting surface; its semi-axis, BF in the sectional figure (Fig. 184), bears to the

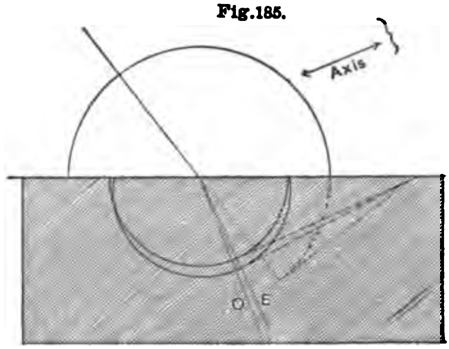
Fig. 184.



radius of the circle the ratio of 1·65850 to 1·48635 if the light used be that emitted by sodium-vapour.

If the block of spar be cut by a plane at right angles to the principal sections, but not parallel to the axis, we obtain the result shown in Fig. 185. The incident light is in the plane of the paper; the axis of the crystal is also in the plane of the paper.

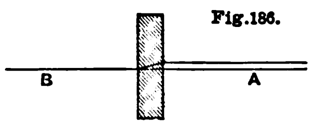
Fig.185.



When the surface which receives the incident beam has been cut at right angles to the axis, and the light falls upon it normally (that is, at right angles to the surface, or parallel to the axis), there is no double refraction; the ordinary and the extraordinary rays coincide.

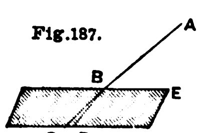
A parallel-sided slice of Iceland spar cut in any other direction than at right angles to the axis will divide an incident ray into an ordinary and an extraordinary ray, except in the case in which one of the rays is so refracted as to become parallel with the axis, in which case the other ray coincides with it. In Fig. 186 the incidence is normal, and an observer at A will see two images of a spot at B, of which one, the ordinary, is produced as it would have been by ordinary glass: while if he turn the slice round, the extraordinary image will rotate round the ordinary one. This can be readily observed with an ordinary crystal of Iceland spar.

Fig.186.



Light striking on a plate or a common crystal of Iceland spar is thus split into two rays, and a single point or a page of print looked at through such a crystal gives a double image. Conversely, a pair of points, C, D, if looked at by an observer at A, will have their images blended, and by finding for various distances between C and D the angle ABE, at which these points appear to blend, the two refractive indices may be found: the rays CB, DB, and BA being caused to lie all in a principal section of the crystal. BD represents the ordinary and BC the extraordinary ray.

Fig.187.



When the incident ray is oblique to the principal section, the extraordinary ray is no longer in the same plane with the incident and the ordinary refracted ray, but is deflected to one or the other side: the tangent plane to the ellipsoid does not touch it in the plane of incidence.

The above figures are drawn for crystals such as Iceland spar, beryl, emerald, mica, ruby, sapphire, tourmaline, the ordinary index of refraction of which is greater than the extraordinary, and in which the ordinary ray travels more slowly than the extraordinary, and lies between the extraordinary ray and the axis; such crystals are called **Negative Crystals.** In others, such as ice, quartz, boracite, the extraordinary ray lies between the ordinary ray and the axis; such crystals are called **Positive Crystals.** In the latter, the extraordinary axes of the ellipsoid are shorter than the diameter of the sphere, which thus encloses the ellipsoid: the extraordinary index of refraction is in them greater than the ordinary index.

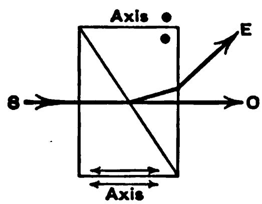
The two rays, the ordinary and the extraordinary, are found to be polarised in planes almost exactly at right angles to one another. The **ordinary ray** is **polarised** in a plane containing both the incident ray and the crystalline axis.

If the incidence be that of Fig. 184, the incident ray, the reflected ray, and both refracted rays are in the same plane, the plane of the paper, and the axis is parallel to that plane; the ordinary ray is said to be polarised in that plane; light polarised in such a plane of incidence passes through the spar as an ordinary ray. The **extraordinary ray,** when the whole three rays thus travel in a principal section of the crystal, is found to be polarised in a plane exactly at right angles with the plane of polarisation of the ordinary ray.

The second face of the block of crystal may be so cut that it receives the ordinary and the extraordinary rays at such an angle as to transmit the one, but totally to reflect the other. In **Nicol's prism** a long rhomb of Iceland spar is cut in this way, and the portions are so cemented by Canada balsam that when common light enters the Nicol it is divided into two rays, of which one, the Ordinary, is totally reflected when it meets the cemented surface, while the Extraordinary ray is transmitted and emerges (the faces of the prism having been, in order to permit this, cut down to the proper angle) in a direction parallel to that of the incident ray. The whole arrangement is thus capable of acting as a **polariser**; and if polarised light be sent through it in one rotational position, the Nicol will

transmit it freely; while if the Nicol be rotated through 90° in either direction, on either side of the most favourable position, it will transmit none of it. It can thus serve as a means not only of producing polarised light, but also of detecting polarised light, and of finding in what plane it is polarised; and when it does this duty it is called an **analyser.**

Fig. 187 *a*.



Foucault's prism is shorter than Nicol's, and an air-film replaces the balsam. In Rochon's prism two similar pieces of quartz make up a parallel-faced block (Fig. 187*a*): the ordinary ray emerges without deviation: the extraordinary is sent away to one side.

In tourmaline there is double refraction; but one of the rays, the ordinary, is absorbed, and the extraordinary alone passes through. Thus a thin plate of tourmaline acts as a polariser of common light incident upon it, and another plate rotating in front of it may act as an analyser,—a convenient arrangement, were it not that tourmaline is always dark in colour, and absorbs much of the light incident upon it. For this reason Nicol's prisms are commonly used as sources of polarised light.

Crystals of sulphate of iodo-quinine act like tourmaline, but are useless because they are dark, small, and brittle.

We may here recall the different modes of obtaining a beam of plane-polarised light.

1. Reflexion of ordinary light from glass at the angle of complete polarisation.
2. Transmission through a pile of glass plates with parallel sides; the angle of incidence being the angle of complete polarisation, or an angle approximating to it.
3. Separation of the ordinary from the extraordinary ray produced by double refraction; this being done
   (*a*) by tourmaline, which extinguishes the ordinary ray;
   (*b*) by a Nicol or a Foucault prism, which turns aside the ordinary ray;
   (*c*) by a Rochon prism, which turns aside the extraordinary ray.

Some crystals, such as topaz and arragonite, have two axes, and are called **Binaxial Crystals**: in these the wave-surface is very complex, and they have three indices of refraction.

In general, in these crystals, the wave-front is oblique to the rays, and there is no ray which obeys the ordinary law of refraction that $\sin \iota = \beta \sin \varrho$; but that ray which does so most nearly in general, and which does so perfectly when the incidence is in one of the principal sections, is called the ordinary ray; while the other of the two rays, into which a ray of incident

light is divided on non-axial incidence, is called the extraordinary ray. In such crystals the positions of the optic axes, which have no invariable relation to the crystallographic axes, are variable; they vary with the temperature of the crystal, and with the kind of light employed; and in some cases a crystal is found to be binaxial for one, uniaxial for another kind of light;

Fig. 187 *b*.

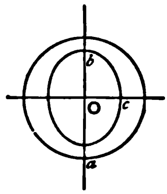


Fig. 187 *c*.

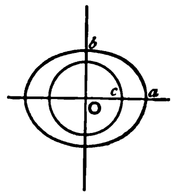
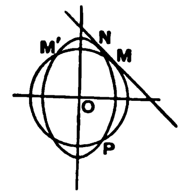


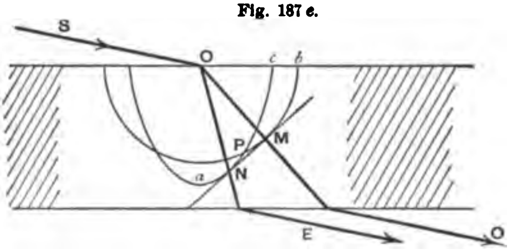
Fig. 187 *d*.



Glauberite (native sulphate of soda and lime), for example, being binaxial to red, uniaxial to violet light.

If we take a crystal of arragonite, which happens to have its three indices of refraction in directions at right angles to one another, we find that in the three principal sections the circles and ellipses, which represent the simultaneous propagations of the different parts of the wave from a central point O, are differently related to one another. Let $v_{,}$ be the greatest velocity (the velocity in air being taken as equal to unity), $v_{,,,}$ the least, and $v_{,,}$ the intermediate, these three velocities determining the three indices of refraction for any particular colour; then the propagation of a disturbance from a central point O results in the formation of a complex wave-surface which may be understood by looking at Figs. 187 *b*, *c*, and *d*, which represent the three principal sections: or better, by studying the models of this kind of surface which are now procurable. One of these principal sections (Fig. 187 *d*), that of greatest and least elasticity, presents the peculiarity that the ellipse is partly inside, partly outside the circle. If a line NM be drawn, touching both ellipse and circle at N and M, it will be seen that the disturbance from O reaches M and N at the same time; and after successive intervals of time, as ellipse and circle expand, the successive tangent-lines M′N′, M″N″, etc., remain parallel to MN. The portions of the wave-front at M and N

Fig. 187 *e*.



therefore move forward, with respect to the direction OM, with equal velocity; and this direction is one of the Optic Axes of the crystal, for the particular colour employed; there being another, OM′, in the same principal section.

If a single ray of natural monochromatic light, SO, Fig. 187 *e*, fall upon a plate of arragonite at such an angle that the ordinary ray travels along the optic axis OM, the common tangent-plane NM advances parallel to itself;

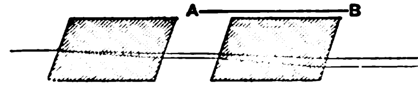
and a separate ordinary and extraordinary ray, O and E, might be expected to emerge, producing two images of S.

Internal Conical Refraction. — So far as the diagram 187 *d* can show, we would not expect more than these two images: but if we refer to a model of the wave-surface, and if, instead of applying a mere tangent-line MN, we apply a tangent-plane such as a piece of flat glass, we shall find that the tangent-plane is in contact with the wave-surface along the whole periphery of a circle of contact. Each and every point of this circle satisfies the same conditions as the points M and N; and if non-polarised monochromatic light reach the point O (Fig. 187 *e*), there to be refracted so that the ordinary ray tends to go along the optic axis OM, and the extraordinary along ON, the result is that the two images of S, visible at other angles of incidence of SO, open out into a complete circle of light, which has been produced by the splitting of the incident ray of natural light, SO, into a hollow cone of rays, ONM, within the crystal; the rays composing this cone all pass through the above-mentioned circle of contact. The light at every point of this circular image is polarised, and at opposite points it is polarised in planes at right angles to one another.

External Conical Refraction. — In Fig. 187 *d* we see a point P — there being four such points — at which rays from O in both waves arrive at the same time. The direction of vibration in the two simultaneous rays is not, however, the same. If the light from O emerged at P into air, the rays OP would be refracted, so far as the diagram can show us, in two directions. But on referring to a model of the wave-surface we would find that at P there was a conical dimple, into the bottom of which a hollow tangent-cone might be fitted, with its apex at P. Every radial line in this tangent-cone would determine a different refraction of a pencil of light travelling along OP and emerging at P into air. Light travelling in the direction OP therefore opens out, when it emerges into the air, into a hollow cone of light; and conversely, a solid cone of rays concentrated by a lens may in part be collected and made to run in the common direction PO.

When light has passed through a crystal of Iceland spar and been divided into an ordinary and an extraordinary ray, if it be caused to fall upon a second crystal whose faces are parallel to those of the first, the two rays pass through, suffering no further division; the ordinary ray emerging from the first crystal is still the ordinary ray in the second crystal, which acts like a mere prolongation of the first. If the second crystal be turned 90° round a longitudinal axis parallel to the line AB in Fig. 188, there is still no division of the rays; but the ordinary ray on emergence from the first crystal

Fig.188.



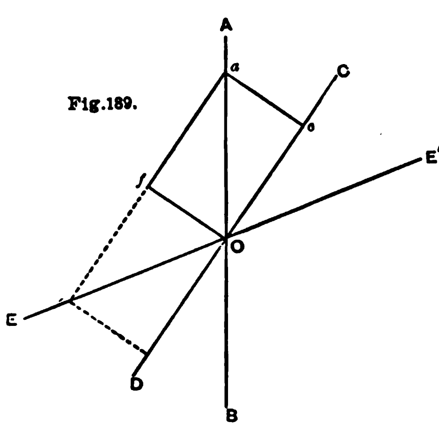
is an extraordinary ray relative to the second crystal, and is refracted as such in that crystal; and the converse applies to the extraordinary ray emerging from the first crystal. If the second crystal occupy any rotational position intermediate between

these, each ray incident on it is decomposed into an ordinary and an extraordinary ray. There are thus, in the ordinary case, four images of a bright point seen through a pair of crystals arranged end to end, at a distance from one another, and these images blend into two when the crystals are, by rotation, placed parallel or at right angles to one another.

Interposed Lamina. — When a polariser and an analyser of any kind are arranged at right angles, so that a plane-fronted beam incident on the system is wholly cut off or deflected by it, an eye placed beyond the analyser can perceive no light; but if a thin film of mica, or other double-refracting substance, uniaxial or binaxial, of uniform thickness, be caused to intervene between the polariser and the analyser, the field may become filled with light, coloured or white, according to the position of the interposed film.

In Fig. 189 the line AB represents a plane vertical to the paper, and cutting the paper in AB: we call this the vertical plane, or the plane AB. Then let us by any convenient means produce a beam of plane-polarised monochromatic light, polarised in the plane AB, and let us suppose this beam to be seen end-on, travelling away from the observer's eye. Interpose a thin plate of some birefringent substance in the path of the beam: let the axis of this lie in the plane CD. The beam AB is broken up by the interposed plate into two: one in which the plane of polarisation is parallel to CD, one in which it is at right angles to that plane. The former is transmitted through the interposed plate as an ordinary ray, the latter as an extraordinary. The lines Oa, Of, Oc, indicate the relative amplitudes of vibration in the incident polarised beam, in the extraordinary, and in the ordinary transmitted beams respectively. The interposed plate may be so thin that although the incident beam is divided into two transmitted beams, these have not perceptibly separated from one another, and on emergence are not only parallel, but are also practically coincident. In a wide-fronted wave-system this coincidence may be held to be absolute except at the edges of the beam. Though the two beams coincide in direction, their undulations do not coincide in phase; in positive crystals the extraordinary, in negative crystals the ordinary ray is more retarded than its companion. Let us suppose that the more retarded ray has lost one wave-length: then the result of superposition of the two emergent rays will be a **plane-polarised** beam similar to that which had originally fallen upon the interposed

Fig. 189.



plate, and in the same plane AB; if half a wave-length ($=\frac{1}{2}\lambda$) be lost, the result will be an equal **plane-polarised** beam, polarised in the plane EE′.

If, again, CD coincide with AB — that is, if the principal plane of the interposed crystalline plate be parallel to the plane of polarisation of the incident light — there is no extraordinary beam O*f;* and the light, having been transmitted through the interposed film as an ordinary ray, emerges as it entered, **plane-polarised** in the original plane AB. If, again, CD be at right angles to AB, the incident beam is wholly transmitted as an extraordinary ray, and emerges polarised in the original plane.

Let us now suppose that CD is inclined to AB at an angle of 45°: if one of the rays be retarded by some even multiple of $\frac{1}{4}\lambda$, the result is plane-polarised light, either polarised in the original plane (when the retardation may be measured in whole wave-lengths), or in one at right angles to it (when the retardation is some odd number of half wave-lengths), for EE′ is at right angles to AB when CD makes 45° with it. Again, if the retardation be some odd multiple of $\frac{1}{4}\lambda$, the extraordinary and ordinary rays are compounded into a **circularly-polarised** ray of light; and if the retardation be of any value other than some multiple of a quarter wave-length, the result is an **elliptically-polarised** beam, the ellipse being, according to the amount of retardation, some one of those indefinitely numerous ellipses which may be described within the rectangle E*a*E*b*′.

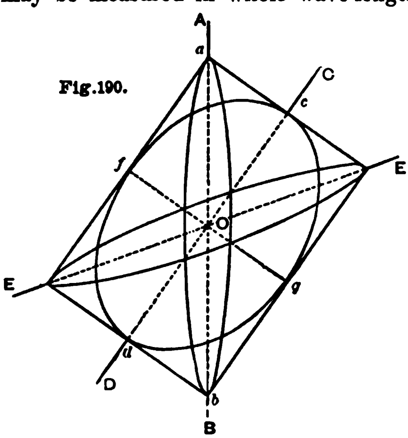


Fig.190.

In the general case, AB (Fig. 190) being the plane of the incident beam, CD the principal section of the interposed plate, the angle AOC having any value, and O*a*, O*c*, O*f* being respectively the relative amplitudes of the incident ray polarised in the plane of AB, and of the ordinary and extraordinary rays emergent from the interposed plate; compounded, their result is an **Elliptically-Polarised** beam, of which the limits are: —

(*a*) A **plane-polarised** beam, whose plane of polarisation is AB and whose amplitude is represented by O*a*.
  (1) When CD coincides with AB.
  (2) When CD is at right angles to AB.
  (3) When the relative retardation of *cd* and *fg* is 0, or an even multiple of $\frac{1}{2}\lambda$.

(*b*) An equal **plane-polarised** beam whose plane of polarisation is EE′; the angle AOE′ being equal to twice AOC: this is the result when the relative retardation is an odd number of half wave-lengths.

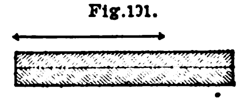
(*c*) A **circularly-polarised** beam when the angle AOC is equal to 45°, and the relative retardation is some odd multiple of $\frac{1}{4}\lambda$.
  (1) Right-handed (rotation contrary to hands of a watch) when the component polarised in *fg* loses (together with any number of whole wave-lengths) one quarter wave-length or gains three quarters relatively to that in *cd*.
  (2) Left-handed when it relatively gains one quarter or loses three.

Elliptically-polarised light is produced in every other relative position of CD. This is right-handed if the relative retardation of the extraordinary ray *fg* transmitted through CD lie between 0 and $\frac{1}{2}\lambda$ or between $n\lambda$ and $n\lambda + \frac{1}{2}\lambda$, where n is any whole number; left-handed if its relative retardation lie between $\frac{1}{2}\lambda$ and λ, or between $n\lambda + \frac{1}{2}\lambda$ and $(n+1)\lambda$. If the plane CD lie so that the angle AOC lies to the left of AB (the observer being, as hitherto, supposed to be stationed near the source of light), these conditions of left- and right-handedness respectively are reversed.

A plate of birefringent substance of such a thickness, that when it is interposed in the path of a beam of plane-polarised light of a particular colour, with its principal section at an angle of $\pm$ 45° to the plane of polarisation, it converts that plane-polarised light into circularly-polarised light, is called a **quarter-undulation plate.**

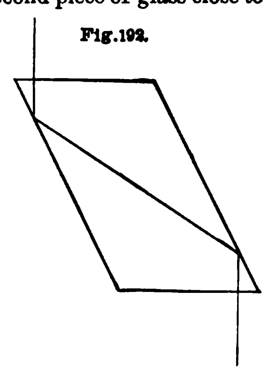
Quarter-undulation plates are of two kinds: (a) Where the thickness is just such as to cause a relative retardation equal to $\frac{1}{4}\lambda$, or to $(n\lambda + \frac{1}{4}\lambda)$; ($b$) Where the plates are thicker, but are opposed in their action. In Fig. 191 two plates cut out of a doubly-refracting crystal are shown fitted together; the one is cut so that its axis is parallel to the plane of the paper; the other has its axis at right angles to the paper. Incident light arrives already polarised; it is divided by double refraction into two rays, an ordinary and an extraordinary; then, since the second plate has its axis at right angles to the axis of the first, the ordinary ray of the first plate is refracted in this as an extraordinary ray, while the extraordinary ray of the former passes through as an ordinary ray. On emergence both rays are parallel and practically coincident; and the amount of relative retardation is equal to that produced by a thin plate equal in thickness to the difference between the thicknesses of the two plates.

Fig. 191.



When light, plane-polarised, is totally reflected from glass, it is found to be elliptically-polarised, unless it had been originally polarised in the plane of incidence, or in a plane at right angles to this. Reflexion from metals presents this peculiarity at all angles of incidence. The vibratory movement actually extends beyond the surface of the glass into the rarer medium beyond, as may be proved on bringing a second piece of glass close to the totally reflecting surface, when interference-colours will be seen. As a result of this, a difference of phase is set up between the two components (polarised in and at right angles to the plane of incidence) into which the incident light may be resolved. A similar result occurs in metallic reflexion, for some of the light penetrates to a slight depth below the reflecting surface. A wave cannot have its direction abruptly changed; and during the gradual change of its direction, its phase becomes altered to a slight extent: and this effect differs in amount according to the direction of vibration of the incident waves. When the angle of incidence is such that the difference of phase set up corresponds to a relative retardation $\frac{1}{8}\lambda$, two such total reflexions would convert a plane-polarised ray into a circularly-polarised one. If a rhomb of glass be cut in such a form that a ray of light may pass

Fig. 192.



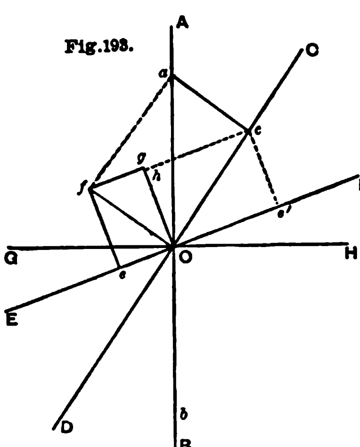
normally through one surface, strike a second surface at the appropriate angle of incidence and be there totally reflected, strike the third surface at an equal angle, and pass out normally through a fourth surface, a ray so travelling through it will, on emergence, be found to be circularly-polarised. Such a rhomb is known as a **Fresnel's Rhomb**, and acts as a quarter-undulation plate for every kind of light, while a film of mica, real or virtual, can only act as such towards light of one kind.

If plane-polarised light pass successively through two similar quarter-undulation plates, similarly placed, the emergent light is plane-polarised in a plane at right angles to the original plane of polarisation; whereas, if the two quarter-undulation plates be opposed in their action, the light is restored by the second to its original state, plane-polarised in the original plane. A second quarter-undulation plate of known action affords us a means of distinguishing right- from left-handed elliptically- or circularly-polarised light.

In metallic reflexion there is always a particular angle of incidence, at which circularly-polarised light is converted by reflexion into plane-polarised light.

The two vibrations which make up the circular or elliptic vibration of the ether in a circularly or elliptically-polarised beam of light are not in a condition to interfere with one another on account of their difference of phase, because they are executed in planes at right angles to one another. If a beam circularly or elliptically polarised by an interposed lamina be received upon a birefringent analyser, it is split into two parts, one an ordinary ray, the other an extraordinary ray, and each of these is plane-polarised. In Fig 193 AB is the plane of original polarisation, CD a principal section of the interposed lamina, EE′ a principal section of the analysing crystal. Then a plane-polarised ray whose amplitude is represented in magnitude by the line O*a*, and whose plane of polarisation is AB, is resolved by the interposed lamina into two, O*c* and O*f*, which are upon emergence compounded into a plane- or an elliptically- or circularly-polarised beam, according to their relative retardations. When this strikes the analyser, its components O*c* and O*f* are themselves resolved each into a pair of components parallel and at right angles to EE′; these are respectively O*e*′ and O*h* from O*c*, and − O*e* and O*g* from O*f*. In the plane EE′ we have therefore two vibrations, O*e*′ and − O*e*; in the plane at right angles to EE′ we have the vibrations O*g* and O*h*. But O*e*′ and − O*e* differ in phase; so do O*g* and O*h*. These are therefore in a condition for interference. The ordinary ray, passing through the analyser, is made up of the mutually-interfering components, O*e*′ and − O*e*, and the extraordinary of O*g* and O*h*; the effect of interference is to cause a distribution of energy such that the ordinary ray gains or loses as much energy as the extraordinary loses or gains, and thus the energies of the

Fig.193.



ordinary and the extraordinary rays are, taken together, equal to the energy of the incident plane-polarised ray. The amount of relative retardation caused by the interposition of the doubly-refracting lamina, when measured in wave-lengths, depends upon the particular kind of light employed. Hence when the original plane-polarised light is a white light, each colour obeys its own law; each colour, if strong in the ordinary, is weak in the extraordinary ray, and *vice versâ;* thus the extraordinary ray and the ordinary are coloured, and their colours are complementary.

The following are the limiting cases: —

1. There is no extraordinary ray when —
   (*a*) AB, CD, and EE′ (Fig. 193) coincide.
   (*b*) AB and EE′ coincide, and CD is at right angles to them.
2. There is no ordinary image when —
   (*a*) AB and CD coincide, and EE′ is at right angles to them.
   (*b*) CD and EE′ coincide, and AB is at right angles to them.
3. The two images are equal for every colour, and are therefore white —
   (*a*) When AB and CD coincide, and the angle AOE′ = $\pm 45°$.
   (*b*) When AB and CD are at right angles, and the angle AOE′ = $\pm 45°$.
   (*c*) When CD and EE′ coincide, both being at an angle of $\pm 45°$ with AB.

In every other position the two images are complementarily coloured.

Determination of the character of a Beam of Light. — A crystal of Iceland spar capable of rotation round a longitudinal axis may be used as an analyser, and will enable one, with the intervention of a doubly-refracting lamina, to determine the character of a beam of light falling upon it.

Plane-polarised light: as the prism is rotated, the ordinary and the extraordinary images appear and alternately wax and wane, disappearing and reappearing. In this instance the doubly-refracting lamina is dispensed with.

Elliptically-polarised light and partially-polarised common light: the two images never entirely disappear, though they become alternately brighter and dimmer.

Circularly-polarised light, and natural light: the two images do not vary in their relative intensity with the rotation of the prism; they continue nearly equal.

Elliptically and circularly-polarised light on the one hand, and common light unpolarised or partially polarised on the other, are distinguished by the respective actions upon them of a quarter-undulation plate, interposed between the source and the analyser; the former are converted by this plate into plane-polarised light, the latter are not; and the former then produce only one image in some positions of the analyser, while the latter always produce two.

Colours produced by interposed film. — When a polariser and an analyser are so placed that the latter quenches the light which the former transmits, the interposition between them of a plate of mica or selenite, or any other doubly-refracting substance, will cause light again to reach the eye, provided that the principal section of the interposed substance be neither par-

allel nor at right angles to the principal sections either of the polariser or analyser.

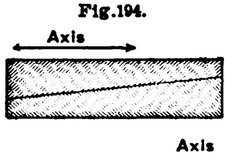
In Fig. 193 above, let the angle AOE′ be made a right angle; Og and Oh come to coincide in direction with AB; Oe and Oe' with GH, at right angles to AB. The polariser allows ab to pass: the analyser cuts off all components polarised in the plane AB; whence crossed prisms produce perfect darkness.

But the intervention of the doubly-refracting substance resolves the light which cannot traverse the analyser into two rays, of each of which there is some part that can traverse that obstruction. If the doubly-refracting substance interposed be uniform in thickness, the whole field under crossed prisms becomes filled with uniform coloured light; if the polariser, or the analyser, or the interposed film, be turned round, the light first becomes white, and then passes into the complementary colour.

The colours produced by a given film depend upon the amount of relative retardation produced by it in light of each kind. This depends upon (*a*) the substance of the film and its refractive indices; (*b*) its thickness; (*c*) the inclination at which the ray traversing it strikes it; (*d*) the relation of its optic axis or axes to the plane of its surface.

When an irregular film of mica or selenite, flaked off with a penknife from a large mass, is interposed between crossed prisms, the eye, looking through the analyser, sees the darkness of crossed prisms transformed by the interposition into a series of gorgeously brilliant colours; and as the analyser is turned round, these fade away into white light, and reappear in complementary hues. If the film be a very thin wedge, each thickness of it produces its own colour, and a kind of spectrum is thus produced. A double wedge of quartz, known under the name of **Babinet's compensator**, and shown in Fig. 194, acts as a virtual film of graded thickness, and gives a series of fringes or spectra. This chromatic property of a doubly-refracting film and an analyser may be made use of to detect polarised light: if the light looked at through such a system be wholly or even partially polarised, the phenomena of polarisation-colours come into view; and while, for example, natural light in such a case gives two nearly equal white images when a crystal of Iceland spar is used as the analyser, circularly-polarised light, on the other hand, gives two complementary coloured-images of almost exactly equal intensity — equal, that is, from the physical point of view, though to the eye these coloured images may not seem equally bright.

Fig. 194.



When a divergent or a convergent beam of white light passes normally through an interposed film cut at right angles to its axis, the centre of the ordinary image is, when the analyser is parallel to the polariser, found to be bright and colourless, while round this there is a series of annular fringes or spectra, the local colours of which depend upon the local relative retardations; the whole being traversed by a colourless cross, whose branches are parallel and at right angles to the plane of polarisation. At the same time, the extraordinary image presents the complementary appearances — a black centre, a black cross, and complementary colours. When the analyser is turned round through 90°, so that the ordinary image becomes an extraordinary one, it reverses its appearance.

This cross is really a coincidence of two crosses, one parallel and at right angles to the primitive plane of polarisation, and the other parallel and at right angles to the principal section of the analyser.

When a lamina is interposed whose axis is not at right angles to its surface, the coloured (or isochromatic) lines are modified into hyperbolic curves, or even into lines nearly straight.

When the lamina used has been cut from a binaxial crystal, the isochromatic lines are converted into a series of curves known as lemniscates, and the dark or colourless crosses are represented by a pair of hyperbolic curves.

The doubly-refracting power of a body may be detected when it is placed between crossed prisms, and by this means it is found that substances which are ordinarily isotropic become doubly refracting when they are exposed to compression, or to dilatation, or flexure, or torsion, or vibration (especially at the nodes), or to molecular stress, as where they are heated and then suddenly cooled, or to electrical stress; and crystals ordinarily isotropic become double-refracting when exposed to mechanical stress, or when they crystallise irregularly or are not homogeneous. Organic tissues are by this means for the most part found to be double-refracting, and they seem, when placed between crossed prisms, to shine by their own light against a dark background — a circumstance favourable to definition, for there is no diffraction of light round the fibres, but practically of little utility, for it is difficult to get prisms of Iceland spar sufficiently clear to be interposed in the path of the rays coming from a high-power objective.

It has been proposed to make use of a dynamometer which measures forces by the compressions exerted on glass which is interposed between crossed prisms, these compressions being estimated by the colours produced:

the greater the compression, the greater the difference of phase set up between the ordinary and the extraordinary rays, and the greater the wave-length corresponding to that colour which is cut out of the emergent light.

It has also been found that slices of different minerals placed between crossed prisms act in very characteristic manners, and are thus, in many cases, easily identified.

Andrews proposed as a test for sodium to make sodium-platinum chloride, which produces, when placed between crossed prisms, colours so vivid and characteristic that the millionth part of a grain of sodium can be detected by this means.

ROTATORY POLARISATION.

When natural white-light is passed through a polariser, then through a film of mica or selenite cut parallel to the axis, and lastly, through an analysing prism of Iceland spar, it gives, as we have seen, two colourless images of the source of light. If now we replace the mica or selenite by a slice of **quartz** cut parallel to the axis, the two images produced are complementarily coloured.

If their light be examined with a prism, it is found that the spectrum of the light of the extraordinary image is lacking in a particular region, which presents a dark band, while that particular region is bright in the spectrum of the ordinary image. Further, as the analyser is turned round, the dark band in the spectrum of the extraordinary ray seems to travel up or down the spectrum; and if the piece of quartz used be very thin, this dark band may traverse the whole spectrum while the analyser is rotated through an angle of less than 180°. That particular kind of light which is absent in the extraordinary ray leaves the quartz plate in a condition of polarisation in a plane parallel to the principal section of the analyser.

Each position of the analyser cuts off a distinct kind of light in the extraordinary ray: hence light of each colour must have become polarised in a special plane, and the plane of polarisation of the light incident upon the quartz has been rotated, that of each component colour to a specific extent.

Rotation is easier to detect with polarised than with common light; but common light is similarly rotated, as interference-experiments may be made to show.

Biot found that a, the amount of angular rotation of the plane of polarisation of each colour, was, very roughly, proportional to the square of its wave-frequency, or inversely proportional to the square of λ, the wave-length. Boltzmann showed that the true law is that $a = (A \div \lambda^2) + (B \div \lambda^4)$: in quartz, for example (Stefan), $a = [(7{\cdot}07018/10^6) \div \lambda^2] + [(0{\cdot}14983/10^{12}) \div \lambda^4]$, where λ is the wave-length in mm., and a the rotation produced by a slice 1 mm. thick, this rotation being measured in degrees of angle.

We have seen that a plane-polarised beam is equivalent to two equal and opposite circularly-polarised beams; but quartz allows a right-handed circularly-polarised beam to travel faster througn it than a left-handed one; at any given point the right-handed component is therefore not so advanced in its phase as its left-handed companion: this is equivalent to a relative gain of phase by the so-called left-handed component (see definition, p. 515); this causes the plane of the plane-polarised ray gradually to turn to the right, in the same direction as the hands of a watch when the ray is looked at from

behind, from polariser towards analyser. Ward thinks the periods are altered, not the velocities.

A piece of quartz 1 mm. thick thus turns the plane of polarisation of yellow rays about 22°; a piece about 16·36 mm. thick will turn it through 360°, for the amount of rotation is proportional to the thickness of the rotating medium. For the Fraunhofer line B the specific rotatory power of quartz (1 mm. thick) is 15°·55; for line D, 21°·67; for line H_1, 50°·98.

A substance which acts in the same sense as quartz is said to be **dextro-rotatory** or positive; one which, causing a relatively-slow propagation of right-handed circularly-polarised light, rotates the plane of polarisation to the left, is **lævo-rotatory** or negative. This property is not confined to crystals. The following list comprises a few examples of bodies of each kind:—

Dextro-rotatory.—Some samples of quartz; cane sugar, grape sugar, camphor; many essential oils, such as oil of orange, oil of caraway; cinchonine, quinidine; castor-oil.

Lævo-rotatory.—Some samples of quartz; oil of anise, oil of mint, oil of turpentine; quinine; sugar of fruits, starch; albumin.

The rotatory powers of different substances are compared by means of two constants. (*a*) The real specific rotatory power; the rotation, for a given colour or Fraunhofer line, produced by a layer 1 mm. thick of the substance itself. The symbol α_D denotes the real rotation for the Fraunhofer line D.

(*b*) The apparent specific rotatory power, $[\alpha]$, for a given line or colour ($[\alpha]_D$, that for the line D); the rotation produced by a substance in a state of dilution. It is equal to $\alpha/\epsilon l\rho$, where α is the observed rotation (measured in degrees), ϵ the quantity of active substance per gramme of solution, l the length of the column employed, and ρ the density of the solution.

The apparent specific rotatory power is slightly increased by rise of temperature and modified by the nature and proportion of the diluent substance.

With these variations, for cane sugar $[\alpha]_D$ is about 67°; for milk-sugar —α-lactose 80°, β-lactose 54°·5; for crystallised grape-sugar in 7·68% solution $[\alpha]_D = 52°·89$, in 82·6% solution $[\alpha]_D = 57°·8$.

A column 20 cm. in length of solution of cane sugar, containing in each 100 cubic cm. 16·350 grms. of cane sugar, is equivalent in rotatory power to a plate of right-handed quartz 1 mm. thick. This fact, coupled with the fortunate circumstance that the rotatory dispersion for quartz is the same as that for cane sugar and glucose, enables the strength of solutions of sugars to be approximately determined by means of a Saccharimeter.

Essential oils are found to retain their rotatory power unimpaired (due allowance being made for proportionate dilution) when in dilute solution, or even when in the state of vapour, provided that they undergo no chemical change. When substances lævo- or dextro-rotatory are mixed with each other or with indifferent substances, and if there be no chemical change, the rotatory effect of the whole is found by multiplying the rotatory index of each substance by the proportion in which it is present, and finding the joint effect of the components of the mixture by a process of simple addition. If a rotatory substance assume the crystalline form, its rotatory action is very often masked by double refraction: whence solids, such as camphor, are

generally best examined in solution; exceptions to this being found in some cases, such as those of benzile and chlorate of soda, where the rotatory power depends upon the crystalline structure, and in which the crystals are generally hemihedric, or, as it were, distorted towards one side.

Rotatory polarisation is thus due either to crystalline arrangement of molecules or to the structure of the molecules themselves; and it has been shown (van 't Hoff) that bodies gifted with the molecular power of rotation have, in their chemical graphic formulæ, a marked want of symmetry. It is thought that wherever there is such asymmetry of the molecule there ought to be rotatory power, but that this is masked by different molecules possessing opposite asymmetries: this is illustrated by dextro-rotatory and lævo-rotatory tartaric acid, whose crystals possess opposite asymmetries, but which, when mixed in solution, form non-rotatory racemic acid.

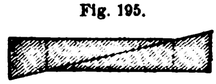
We are now in a position to understand the pieces which make up a **Soleil's saccharimeter.** 1. A Nicol's prism, achromatised by a properly shaped prism of glass through which the transmitted extraordinary ray passes: the achromatic Nicol, thus acting as a polariser, is so placed that the light transmitted by it is polarised in a vertical plane.

2. A double-quartz plate, or Biquartz; two semicircular plates of quartz joined by a vertical cement-line, and thus forming a circular disc of uniform thickness: the two halves have opposite rotatory power, and their thickness is so adjusted that they respectively deviate through 90° in opposite directions the plane of polarisation of incident plane-polarised greenish-yellow light; they therefore both deviate greenish-yellow light, incident upon them and polarised in a vertical plane, into the same horizontal plane.

3. A Liquid-holder; a tube fitted with clear glass at each end, in which is placed a layer of the liquid to be examined, 10 centimetres in length, such being the distance between the terminal glass-plates.

4. A Compensator. This is in its effect a quartz plate of variable thickness. It consists of two pieces of quartz of a wedge shape. One of these can be made to slip over the other; the central thickness is thus variable at will. The amount of movement can be measured by means of a vernier connected with the one wedge, and a scale connected with the other. When the zero of the vernier coincides with the zero of the scale, the thickness of the compensator is such that it exactly neutralises the rotatory effect of one of the halves of the biquartz, while it doubles that of the other, the effect being, in both cases, to bring the light back to the original vertical plane of polarisation.

Fig. 195.



5. An Analyser: this is generally a Nicol's prism.

6. A Lens to be focussed on the biquartz.

To use the instrument:—Fill the liquid-holder with water, and put it in position; focus the lens 6 so as to obtain a clear image of the biquartz; make the vernier and the scale of the compensator to coincide; turn the analyser round until there is observed to fill the field a particular hue, lying between the red and the blue, and called the *teinte de passage;* this hue being chosen out of the many which will successively come into view, on the ground that as the instrument is constructed, the appearance of this colour denotes that the principal section of the analyser is parallel to the plane of polarisation of the yellow; if the yellow could have passed through the analyser it would have been transmitted as an ordinary ray: but a Nicol prism cuts off the ordinary ray; the yellow is therefore cut off; the extraordinary ray,

which alone passes through the analyser, is thus represented in colour by white daylight, minus its bright yellow: the remainder produces in the eye the effect of a dim lavender gray, which, with great sensitiveness, merges into red on the one hand, or into blue on the other, when the analyser is slightly rotated. Both halves of the quartz plate appear of the same colour, because, from them both, the yellow light issues polarised in the same horizontal plane.

If now the water in the liquid-holder be replaced by the liquid to be tested, and if that liquid have rotatory power, the two halves of the quartz will cease to appear of the same colour: the liquid aids the rotatory effect of the one, and is opposed to that of the other. The effective thickness of the compensator is now varied until the rotatory effect of the liquid is neutralised: the vernier shows by its displacement how much the thickness has been increased or diminished: the graduation of the vernier is arbitrary, but a displacement of one step on the scale amounts generally to a difference of one-tenth of a millimetre in the thickness of the quartz; and as the vernier reads to tenths, the positive or negative alteration of thickness of the quartz, found necessary to restore the uniform coloration of the field, may be measured to the hundredth of a millimetre. If the thickness of the compensator have to be diminished, the liquid has rotatory power similar to that of the quartz used in making the compensator; if it have to be increased, its action is contrary to that of the quartz. It is necessary to know of what kind this quartz is; this being known, it can be stated that 100 mm. of the liquid are equal, positively or negatively, to so many millimetres of dextro-rotatory or lævo-rotatory quartz, as the case may be; and thus the rotatory power of the liquid can be specified with precision.

Thus a layer of water 10 cm. thick, containing diabetic sugar in solution in the proportion of 10 grammes per litre, is equivalent to a thickness of 3·42 mm. of right-handed quartz: the thickness of a dextro-rotatory compensator of quartz would have to be diminished by an amount corresponding to 34·2 divisions of the scale on the interposition of a solution of that substance of the given thickness and the given strength; while if the solution were weaker or stronger, the amount of change of thickness of the quartz, as shown by the amount of displacement of the vernier, would be approximately proportional to the strength. If the liquid to be examined be coloured, a Nicol and a quartz in front of the saccharimeter will frequently enable this to be corrected, by filling the field of view with the complementary colour.

For other Saccharimeters in use, see Watt's *Dictionary of Chemistry*, Suppt. iii. p. 1198.

Transformations of the Energy of Ether-waves.

We have already seen this energy transformed into molecular work in the processes of photography, and it is now merely necessary to remark that whatever increases the absorption of light by a set of molecules, increases the chemical work done by the incident ether-waves; if, for example, a spectrum be cast upon a photographic plate prepared with collodion in which chlorophyll has been dissolved, the *local* of the chlorophyll

absorption-bands comes out most strongly in the resultant photograph of the spectrum.

The impact of ether-waves upon some substances gives their molecules a new arrangement: selenium is thus so acted upon by light that it becomes a better conductor of electricity than it is in the dark; and hard rubber is superficially acted upon by light, so that when the incident beam is intermittent or harmonically variable in intensity, the rubber emits a sound which reproduces in its pitch or its complexity the peculiarities of the incident light.

It has been proposed to call the last-mentioned property of hard rubber the s o n o r e s c e n c e of that substance.

As to the mechanical or molar work, the pressure exerted by the impinging ether-waves, though small, is definite. The energy in one cub. cm. of sunlight at the earth's surface is about $\frac{416}{6,0000000}$ erg, and the pressure of direct sunlight per square cm. is about $\frac{416}{6,0000000}$ dyne, or, roughly speaking, about the weight of 4 lbs. on each square mile of ground.

This would, if the earth were a rigid obstacle, press upon its whole surface with an aggregate force of about 884×10^{11} dynes. This force, acting upon the earth's mass (614×10^{25} grammes) would produce an outward acceleration of $1 \cdot 44 \times 10^{-14}$ cm.-per-sec. per second, or a yearly acceleration of 14·33 cm.-per-annum. The effect due to this, say 10,000 miles in 15,000 years, would be small in comparison with the uncertainties which have existed as regards the earth's true distance from the sun; but it would produce other retardations which do not appear to have occurred. The earth cannot, however, be regarded as a rigid obstacle; it is to a great extent pervious to the ether in which it travels.

The mechanical effect of ether-waves is rather to be looked for in their heating effect than in direct pressure. They may heat absorbent gases, such as ammonia, and cause them to do mechanical work, or to produce sound if the incident beam be intermittent or harmonically variable.

OPTICAL INSTRUMENTS.

The Eye, considered as a simple lens, brings parallel rays incident upon the cornea to a focus upon the retina. Hence, when it is at rest, as when one meditatively contemplates space, it is adapted for vision of infinitely-distant objects, or, as the phrase goes, it is accommodated for infinity. To look at nearer objects requires an effort for each — an effort of accommodation. This is effected by increasing the convexity of that part of the eye called the crystalline lens, which is normally flattened.

The range of accommodation provided by our power of varying the form of the crystalline lens is the same as if we were provided with a set of lenses of all focal lengths between infinity and about ten centimetres.

The front of the eye has the form of a prolate spheroid, and parallel rays tend, after refraction thereat, to converge upon one point, without spherical aberration. The external rays are cut off by the iris, which acts as a diaphragm whose aperture can be automatically adjusted according to the brightness of the incident light, and thus again tends to counterbalance spherical aberration. The crystalline lens of the eye varies in density from surface to centre, and thus again spherical aberration is reduced. By these means the image formed on the retina is made as clear as usually need be: but spherical aberration is never completely absent.

The eye presents several other faults, as we find when we expose it to severe tests. Its several parts are not truly centred. Its surfaces are never truly symmetrical round an axis. It is often too long in the bulb, so that rays are brought to a focus before arriving at the retina, and produce, instead of clear images of the several points of an object, a number of overlapping diffusion-circles, and there is consequently produced a blurred image of the whole; this condition requires the use, in front of the eye, of thick-edged lenses, in order somewhat to diverge the incident beam. The bulb may, on the other hand, be too short, so that converging lenses are necessary. The images of equally-distant coloured objects can never appear equally distinct, for they are not in focus at the same time. The front of the cornea has frequently a somewhat cylindrical form, in consequence of which horizontal and vertical objects do not come to the same focus. The field of vision is extremely limited, and the most sensitive part of the retina is excentric. Yet for all this we are for the most part insensible to these defects; we have the power of adjusting the eye with extreme rapidity for all the parts of an extended object, and we have been educated by experience to use both our eyes, and thus, by blending the separate pictures provided by the two eyes, to form judgments as to the solid form and distance of remote objects, — a power which we discover to have depended greatly upon binocular vision when we try, shutting one eye, suddenly to touch any given object at arm's length, though it can be cultivated even with one eye; just as microscopists who have long used a monocular, and cultivated a habit of keeping the fine adjustment in action, find no perspective advantage in the use of a binocular microscope. With age the power of accommodation wanes; for near objects the image cannot be brought to a focus on the retina; and then, in order clearly to see near objects, the aid of convergent lenses must be sought.

The Microscope. — An ordinary thin-edged lens is called a simple microscope or magnifying glass. The simplest compound microscope is formed of an objective — a combination of lenses which converges the rays, divergent from the object, into an inverted real image, achromatic and aplanatic (*i.e.* devoid of the effects of spherical aberration), in a plane in space between itself and the eyepiece — and of a convergent eyepiece, also compound, and corrected for spherical and chromatic aberration, which magnifies this inverted real image, and produces an inverted virtual image at an apparent distance from the eye, not less than that of the nearest distinct vision. The real image formed by the objective must be at the focus of the eyepiece; hence, when a more highly-magnifying eyepiece is used, in order to throw the real image up to the focus of the eyepiece the objective must more closely approach the object examined. In practice the eyepiece consists essentially of two parts; that nearer the object is the field-glass, which catches the rays that were on their way to form a real image far up

the tube, and brings them to another focus, so that they form a real image at a point which coincides with the focus of the upper part of the eyepiece —that is, at the place where the diaphragm within the eyepiece is situated. To make this real image visible, remove the upper lens of the eyepiece, and drop a disc of oiled tissue-paper down upon the diaphragm within the eyepiece. The field-glass brings the whole image of the object within the field of view of the eye-glass.

The rays from the objective, instead of being received by an eyepiece, may, the eyepiece being removed, be allowed to fall upon a screen; they will there form a real image of any size, which may be traced by hand, if the illumination be sufficient; if the screen be a sensitised photographic plate, a photograph may be produced. In this case, the image being more remote, the object must be nearer the lens than it is in the ordinary use of the instrument.

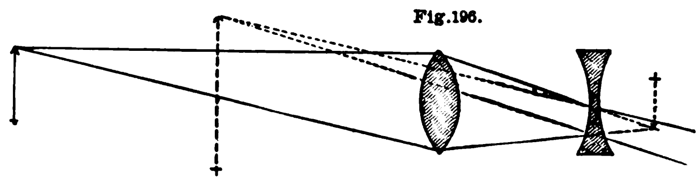
The minute virtual image of surrounding objects produced by reflexion from a globule of mercury is one of the most trying tests for an ordinary microscope.

In the **astronomical telescope** parallel rays from a distant star are made to converge and form a small real image; this is examined by a simple achromatic eyepiece. The image is inverted like that in the microscope.

In the **terrestrial telescope** rays nearly parallel are made to converge and form a small inverted real image; this small image is magnified and reinverted by an arrangement of lenses equivalent to a compound microscope.

In the **opera glass** a convergent lens directs incident rays towards an inverted real image, but before this is formed the rays meet a divergent lens,

Fig. 196.



which causes them, instead of converging towards a real inverted image, to diverge as if from a virtual erect image, as is shown in Fig. 196. This combination of lenses — Galileo's doublet — is one of the simplest and most useful.

In the **ophthalmoscope**, as used for the observation of an erect image of the fundus of the eye, the principle of Galileo's doublet is *sometimes* utilised. In the first place, light is made to fall upon the fundus of the eye by means of a concave mirror held in the hand or fixed upon the forehead of the observer. The fundus is thus illuminated, and becomes a source of light. Rays from it pass towards the eye of the observer through a central aperture in the mirror, placed opposite the eye of the observer. These rays from the fundus are, if the eye observed be myopic (too long in the bulb), rendered convergent by the media of the observed eye itself, and a thick-edged lens, placed near the eye observed, causes them to enter the eye of the observer as if they had proceeded from an enlarged erect virtual image. The convergent lens of Galileo's doublet is thus represented by the observed eye itself, while the biconcave lens employed makes up the pair of lenses. If the eye observed be normal, and accommodated for an infinite distance, rays

proceeding from any point of its retina emerge parallel, and a second lens is not absolutely necessary if the observing eye be normal, for the rays come to a focus on the observing retina, if the observing eye be also accommodated for infinity; if the observed eye be too short in the bulb the rays are, on emergence from it, still divergent, and in this case a convex lens is necessary.

The ophthalmoscope may also be used in such a way as to give an inverted image, not so much magnified as in the preceding case, but more extensive in its field, brighter, and more easy of attainment. A beam of light reflected from the mirror converges upon and passes through a focus; it then diverges on its way towards the eye, but encounters a thin-edged lens which causes it rapidly to converge into and then to pass through a focus within the eye, and, after traversing this focus, to illuminate a wide area of the fundus of the eye. Light from the illuminated fundus is collected by the biconvex lens before mentioned, which forms a real image; the rays from this image pass on, through the aperture in the mirror, into the eye of the observer, who then perceives an inverted and magnified image of the fundus of the eye, — an image which may be still further enlarged by means of a second convergent-lens placed behind the aperture of the mirror.

Visual Perception.

The retina is not a uniform surface, but is made up of elements whose average distance from one another, in the yellow spot, is about ·005 mm. Distant points, whose angular distance is such that their images on the retina are less than ·005 mm. from one another, seem to blend into one, and thus two stars, whose angular distance is less than 70″, appear to the eye as a single star.

The stimulation of nerves is associated with chemical work in the nerve-ends; and this with absorption. In this respect it is interesting to find that the retina, which is particularly sensitive to yellow and green light, absorbs green and yellow light, and in white light appears purple. It has been pointed out that the blindness of the eye to heat-waves and actinic waves is of advantage: for the energy of heat-radiation is relatively so great that everything would appear intensely bright, and our ordinary vision of objects would be impossible if the rays of dark heat were visible; while if the ultra-violet rays were visible, the image of every point would be shrouded in a haze due to chromatic aberration.

Prof. S. P. Langley estimates that the amount of energy which is necessary to produce vision ranges from $\frac{1}{1000}$ erg for the extreme red to $\frac{1}{100,000000}$ erg for the green of the spectrum.

When two colours affect any one element of the retina at the same time, the resultant sensation is that of a single colour,

not the same as either of the components. Thus, when the stimulations which would separately give rise to red and yellow are superposed, the resultant impression is one of orange; red light and yellow light together make orange light. In the same way yellow and green make yellowish-green or greenish-yellow; and in general, colours near one another in the spectrum give rise, when compounded, to an average or intermediate sensation. But in the same way green and purple-red in different proportions, that is, of different intensities, will produce all hues of purplish-red, red, orange, yellow, greenish-yellow, and yellowish-green; green and indigo-blue, all hues of greenish-blue and blue; indigo-blue and purple-red, all hues of violet-blue, violet, and purple, up to the purple-red employed.

Methods of Mixture of Colours. — 1. A source of light: a prism: a screen upon which a spectrum is formed: two slits in the screen, so placed as to admit the passage of two selected colours of the spectrum: achromatic lenses behind the screen converge the two coloured beams towards a common crossing point: a screen there placed indicates the mixed colour.

2. A V-shaped slit in a screen (von Helmholtz); a prism behind this: the two spectra produced overlap each other and produce a very extensive series of combination-colours.

3. Maxwell's discs: *e.g.*, a disc of red- and one of green-painted cardboard: each disc is slit down to the centre, and cut out at the centre so as to be fitted upon a rotating top: the one disc being slipped through the other, the relative proportions of red and green in view can be modified at will: the whole is rotated at such a rate that the successive impressions of red and green enter the eye at least from twenty-five to fifty times per second: each local impression of red in the retina is still vivid while that of green has already commenced, and *vice versâ;* the colours blend in the eye, and various shades of orange-red, orange, yellow, or yellowish-green are produced, according to the relative proportion of the colours blended.

4. Parallel rays are caused, by a lens, to converge upon a focal point; the light traversing different portions of the lens is, by the interposition of transparent coloured screens, diversely coloured; all comes to the same focus; the eye, placed axially at the focus, receives mixed rays; the colours blend in the eye (Aitken).

But a colour on one side of the green, when blended with one on the other side of it, produces always a certain amount of white light, which dilutes the resultant colour: and if we try to blend yellow with blue, we obtain nothing but white light. Yellow and blue are said, then, to be **complementary** to one another: and to every colour there is some complementary colour. The complement to green is purple-red, which does not happen to be in the spectrum, but is intermediate between the spectral red and violet.

That yellow light and blue light make white light is contrary to the general impression, which is that yellow and blue make green: when yellow and blue pigments are mixed, the yellow and the blue lights reflected from the mixture destroy one another, forming white light; and the residual green, never absent even from the purest pigment-reflected blue or yellow light, is perceived, somewhat wanting in brightness, and diluted by the white light produced by the complementary colours.

The phenomena of Double Refraction enable us to produce an indefinite number of complementary colours.

Any pure colour or hue may, by means of Maxwell's discs, be diluted with varying proportions of white or of black. Pure red thus treated passes through pink tints to white, or through brown shades to black. Every one of these tints or shades will be complementary to the colour (greenish-blue) which was complementary to the original pure red; but the result of the mixture is not, in the case of the darkened shades, a pure white, but a lighter or a darker gray; and in all cases the proportion of the pure red to the pure greenish-blue (to keep by our example) remains fixed and independent of the percentage of white or of black added to the original pure red. Any colour in nature can be matched by finding out a proper angular proportion (including the case of complete omission of one or more) for a set of five Maxwell's discs, viz., white, black, vermilion-red, emerald-green, and ultramarine-blue.

From this point of view, green would appear to affect the eye as a **primary colour,** the others being a purple-red and an indigo-blue.

The primary colours are, according to von Helmholtz, a slightly purplish red, a vegetation-green, slightly yellowish (wave-length about 5600 tenth-metres), and an ultramarine-blue (about 4820). Young's original statement was red, green, and violet; Clerk Maxwell concluded they were vermilion, emerald-green, and ultramarine-blue; and Fick, red, green, and blue. Hering, on the other hand, contends that there are four primary colours; Helmholtz's purplish-red, and a green, complementary to it, between the Fraunhofer lines *b* and F; a yellow (about 5750 tenth-metres), and a blue, complementary thereto (about 4830).

The explanation of this peculiar blending power of the eye has been, that every element of the eye which is broad enough to perceive white light consists of three ultimate elements, each of which is capable of perceiving one of the physiologically primary colours; and that relatively varying degrees of stimulation of the respective nerve-ends give rise to blended sensations of intermediate character. Then it was supposed that those who were "colour-blind" had lost their sensitiveness to one (or more) of these primary colours. But it now appears, as Pole and others have shown, that whenever sensitiveness to green is lost, that to red is lost also; while that to yellow and that to blue generally remain, but may also be lost together, so as to leave no colour-sense at all. Hence Hering's view seems preferable to the trisensational theories.

A spectrum formed by light travelling from a waning source is found to modify its tints as the light fades; the orange-red seems to become more purely red, the yellow-green more purely green, and so on; at length the faint spectrum is approximately restricted to red, green, and violet or violet-blue bands; of each group of nerve-ends, one is feebly stimulated by a given

colour, the others are inappreciably so, though if one be stimulated the others can never remain wholly unaffected. On the other hand, if a coloured light be rendered exceedingly bright, the other nerve-ends participate in the excitement: very bright red seems somewhat orange; violet very easily passes over into whiteness when its brilliancy is excessive.

A black colour is due to the absence of stimulation of any of the nerve-ends; and between bright white and black there is a gradation of weak whites which are called grays. The purest black (Chevreul's black) is obtained on looking at a comparatively small hole in the lid of a deep black-velvet-lined box.

Fatigue of the retina causes it to become insensible to a colour long looked at: when white light is then looked at, it appears of a hue complementary to that colour, the sense for which has been temporarily exhausted.

When some of the nerve-ends of the retina are stimulated, the stimulation spreads to some degree: a very narrow white-hot wire appears, especially from a little distance, to be much wider than it really is; this phenomenon being named **Irradiation.** In consequence of this, the crescent moon appears larger than that part of the moon which is illuminated by light reflected from the earth; a candle- or gas-flame appears continuous, though its incandescent particles are by no means in contact with one another; and the glowing filament of an electric incandescent lamp appears much thicker than it really is.

Perception of Form. — The two eyes receive images of different form; these are blended by a mental operation into a compound image, which experience has taught us to associate with the distance of the several parts of the object. This is applied in the Stereoscope; two pictures of images taken from different photographic standpoints are formed, one in each eye, and the effect is that of outstanding relief. This may be exaggerated with singular effect where the photographs are taken from standpoints situated at a mutual distance of several feet: mountain scenery is thus brought into perspective. The same exaggerated effect may be observed when a landscape is looked at through a pair of telescopes, parallel but at several inches' distance from one another, the light traversing each being brought into the corresponding eye by an arrangement of reflecting prisms.

The images in the two eyes may often differ in brightness: when this is the case, there is a struggle between the two fields of view, which causes the impression known to us as that of **Lustre**; this effect being specially well marked in the case of metals.

One of the most curious things in the action of the eyes is that a single mental image is not formed in binocular vision unless the images be formed on **Corresponding points** of the two retinæ: if we displace one eye we see two images; and the relative positions of the eyes are adjusted by a system of muscles, so as to secure this correspondence.

CHAPTER XVI.

ELECTRICITY AND MAGNETISM.

ELECTRICITY and MAGNETISM are not in themselves forms of Energy; neither are they forms of Matter.

They may perhaps be provisionally defined as properties or Conditions of Matter; but whether this Matter be the ordinary matter, or whether it be, on the other hand, that all-pervading Ether by which ordinary matter is everywhere surrounded and permeated, is a question which has been under discussion, and which is now held to be settled in favour of the latter view.

At first sight it would appear that the electricity of an electrified body is a condition of that body itself. When a small piece of resin and a small piece of glass are rubbed together, it is found that after they are pulled asunder, the resin and the glass attract one another with a definite and measurable force; and that this force varies (beyond a certain small distance) inversely as the square of the distance between them. This attraction across an intervening space has been by some held to be due to a so-called Mutual Action at a Distance; but when the bodies are pulled away from one another, work is done upon them, which will be restored when they are allowed to approach one another; and it seems probable that this work has been done, not upon two isolated bodies mutually acting at a distance, but upon a system, which consists of the two bodies together with the Ether between them. This Ether has been stressed by their separation; the tendency of the two bodies to approach one another is the elastic tendency of the Ether to recover its original condition; and these phenomena of electric attraction and repulsion may be explained as phenomena of **Ether-stress.**

Two masses of resin rubbed on glass are found to repel one another; two masses of glass which have been rubbed with resin

also repel one another; in other words, two masses in a similar electric condition generally repel one another.

According to the nature, the size, the dryness, of the pieces of material exposed to mutual friction, and according to some other circumstances, it is found that after friction and separation the force of mutual repulsion or attraction of two electrified bodies varies. One body may thus be more or less highly electrified than another; it is said to possess or be **charged** with a greater or a less **quantity** of electricity.

Two bodies are said to be **equally charged** or to be charged with equal Quantities of Electricity when (being of the same size and form) they can precisely replace one another in their action upon other electrified bodies.

When two equally electrified bodies, at a mutual distance of one centimetre **in air,*** repel or attract one another with a force which balances one dyne, they are each said to be charged with a quantity equal to one C.G.S. Electrostatic **Unit of Electricity.** If one of these bodies, thus said to be charged with a unit of electricity, be brought to an exact centimetre's distance, in air, from a body charged with an unknown quantity of electricity, the force between the two electrified bodies may be measured directly; and if it be equal to Q dynes, the body tested is shown to bear a charge of Q units of electricity. Further, if a body bearing Q units be brought to the same distance from a body charged with Q′ units, the force between them, in air, will be equal to $Q \times Q' = QQ'$ dynes.

Quantity of Electricity is thus treated of as if it were analogous to Mass, or Quantity of Matter; but only as a means of expression. The facts observed can, to a great extent, be stated and systematised by means of the device of attributing the phenomena to the existence of Electric Matter, which may be variously distributed; but it will soon be seen that this is purely a device, and that the electric matter, with whose quantities and actions we deal, is **imaginary** merely.

A piece of glass, after being rubbed with resin, is said to bear a charge of **vitreous** electricity; the resin, on the other hand, is said to be charged with **resinous** electricity. If any body become electrified in any way, it must become either vitreously or resinously electrified.

Similarly-electrified bodies **repel** one another; **dissimi-**

* We shall, in the meantime, assume that the medium surrounding the electrified bodies is air in all cases.

larly-electrified bodies **attract** one another; these statements being, when the bodies are very near one another, subject to an exception hereafter to appear (p. 601).

When a jet of water issues from a metallic nozzle connected with an electric machine, the particles of the issuing stream, being similarly electrified, repel one another, and the jet is broken up into spray. When the nozzle has a capillary orifice, the surface-tension at the aperture is overcome by the electric self-repulsion, and the liquid rapidly issues as if its viscosity were greatly diminished.

If a body charged with resinous electricity and one equally charged with vitreous be brought into contact, the charges of both apparently disappear and the bodies resume a **neutral state.** Vitreous and resinous electricities are thus found to bear to one another the same relation as positive and negative quantities in algebra, and by a purely arbitrary convention charges of *vitreous* electricity are said to be **positive,** and *resinous* **negative.**

The above statements are comprised within the statement that if F be the mechanical force of **repulsion** between two charges of electricity, these charges being Q units in one body and Q′ units in another, and d the distance between them, $F = k \cdot QQ'/d^2$; and if our units of quantity be so chosen that $k = 1$ when the medium between the charges is air, then $F = QQ'/d^2$. If Q and Q′ be both positive or both negative, the product QQ′ is positive, and the stress is expansive or repulsive; while if one of the charges be resinous and the other vitreous, QQ′ is negative, and the stress is such that the bodies appear to attract one another.

When an electrified body presents a charge of Q units uniformly distributed over a superficial area of A sq. cm., its charge per sq. cm. is $Q/A = \sigma$, the so-called **Superficial Density** of the Electric Charge.

If the distribution be not uniform, the density over any minute area may be expressed as the ratio of the charge borne by that area to the area itself.

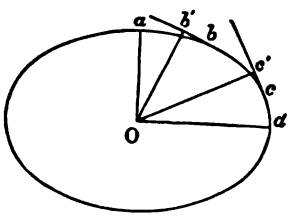
The superficial density of a given charge may be increased by diminishing the free surface of the charged conductor.

A piece of tinfoil charged and connected with a gold-leaf electroscope (p. 604) will cause a divergence in the leaves of that electroscope, which increases when the tinfoil is partly rolled up. When charged aqueous particles coalesce to form raindrops, their free surface diminishes, and the density of their charge increases.

The superficial density of a charge borne by a conductor varies from point to point, according to the form, but independently of the material, of the conductor. Only upon a sphere, in free space, is it uniform.

A charge borne by an ellipsoid assumes at the points a, b, c, d (Fig. 197) densities proportional to Oa, Ob', Oc', Od; bb', cc' being tangents at b, c, and Ob', Oc' at right angles to these. The density at the extremities is thus greater than it is elsewhere. A needle-point resembles the extremity of a very elongated ellipsoid, and the density of a charge borne by a needle tends to be extremely great at the apex.

Fig. 197.



The density varies over a surface of any form in the same way as the thickness of a hollow shell of the same form would vary if that thickness were so adjusted as to produce, under the law of inverse squares, no interior effect.

A body cannot bear an indefinite charge of electricity; if the density be very great over the surface or at any part of the surface of a conductor, sparks will fly, generally from the point at which the density is greatest, either to surrounding objects or into the surrounding air.

If two bodies equally and strongly charged with opposite electricities be brought sufficiently near one another, a **spark** will pass between them, their electricities will combine, and they will be discharged and return to the neutral state. Sparks will pass, as a rule, when two bodies differing greatly in their electrical condition are brought sufficiently near one another.

The thickness of air across which a spark can leap is known as the Striking Distance in air; and it depends in general upon the nature of the substance through which the spark passes, and the nature, condition, and shape of the surfaces, as well as, in particular cases, upon the density of the charges at the points from which the sparks leap. The hotter the charged bodies, the greater the striking distance.

An electric spark is disruptive in its effect, and, in air, it tears its way from one dust-particle to another; it heats air; it produces sound and light; in water it may jar the liquid and shatter the containing vessel; it can pierce glass, and will scatter but not ignite gunpowder.

When a body is very highly charged, the air in its immediate neighbourhood becomes similarly charged, is repelled, and masses of it are torn off and repelled in constant succession, an electrical wind or stream of electrified air being produced.

When a body is charged with electricity, there is always an equal charge of the opposite kind of electricity somewhere: every distribution of electricity has a corresponding **complementary distribution** of an **equal** amount of electricity of the **opposite** kind. In the case of the mutually-rubbed pieces of glass and resin previously adverted to, the charges borne by the

two masses are equal and opposite: when a single object, electrified by friction, stands within a room, the walls of the room are, over their whole inner surface, oppositely electrified, and bear a charge numerically equal to that of the electrified body. When an object is electrified in the open air, the earth itself (together with the heavenly bodies) takes up an equal and opposite charge; and thus the **algebraical sum** of the positive and negative electricities in the Universe is constantly equal to **zero.**

This doctrine is (Lippmann, Silvanus Thompson) called the Law of the Conservation of Electricity: whatever charge one body gains, others lose.

To feign, as we have already done, that there are, in cases of electric phenomena, distributions of positive or negative **imaginary electric matter,** is convenient for many purposes of calculation. For example: a metallic sphere or hollow globe, when electrified, presents no electric phenomena within its substance or its cavity; its electrified condition is manifest only externally, and is uniform all round its surface; whence it may be imagined that a uniform film of electric matter covers or coincides with the surface of the metallic body, and this imaginary film repels or attracts equally imaginary films of matter distributed over the surfaces of neighbouring electrified bodies, and does so with forces whose amount may be calculated in accordance with the propositions of the section on Attraction, p. 188.

The law of the resultant force resembles that of Gravitation. Every particle of this imaginary electric matter in the Universe **repels** every other, existent at any given moment, with a Force F proportional to the product of the Quantities, and varying inversely as the Square of the Distance between them. [In air, $F = QQ'/d^2$.]

From this ensue the following propositions:—(1.) An electrified metallic sphere acts upon all external electrified particles as if its charge were concentrated at its centre; and therefore its repulsion of (or its attraction for) a Unit of Electricity, external to the sphere and situated at a point at a distance d from the centre—*i.e.*, the resultant "**Electric Force**" there—is, in air, $Q/d^2 = \phi$, where Q is the charge on the sphere.

(2.) The density of the surface-distribution on the sphere = charge/area $= Q/4\pi r^2 = \sigma$; and at a point immediately, but distinctly, *outside* the sphere, so near to it that d, the distance from its centre, can be taken as equal to r, the local Electric Force, ϕ, $= Q/d^2 = 4\pi r^2\sigma/d^2 = 4\pi\sigma$. This result is independent of the shape of the conductor, provided that the distribution of its charge be such (see Fig. 197) as to produce no interior effect.

(3.) A superficial distribution of electricity presents from point to point such variations of density that it has no action upon particles within it. A metallic chamber of any form may be electrified until sparks fly from its outer surface; yet no electrical effect will be perceived internally. This is the strongest proof of the law of the inverse square that can be imagined: no other law of repulsion or attraction could result in a force imperceptible internally, the distributions being such as those actually observed.

(4.) Every element of the imaginary superficial shell or film itself is subject to repulsion from its fellow-elements. This repulsion of the film itself amounts, per unit quantity, not to ϕ, but to $\frac{1}{2}\phi$*; hence it is, per sq. cm., $\frac{1}{2}\phi \cdot \sigma = 2\pi\sigma^2$ dynes. A soap-bubble when electrified expands; the atmospheric pressure is resisted by a self-repulsion or so-called Electric Tension over the surface, whose outward resultant, $f = p_0$, is equal to $\frac{1}{2}\phi \cdot \sigma = 2\pi\sigma^2$, all in dynes per sq. cm. A soap-bubble may be electrified by blowing it on a metallic pipe, and connecting the pipe with an electric machine.

From this we must conclude that the surface of every electrified body is in a state of expansive tension, whose resultant is at right angles to the surface, and that the film of air in contact with it is subject to a disruptive tendency, which varies as the square of the superficial density. Sparks fly from the surface of a charged conductor into the surrounding air, when $p_0 = \frac{1}{2}\phi \cdot \sigma = 66{,}708$ dynes per sq. cm.

Since the Self-repulsion or outward Electric Tension $p_0 = \frac{1}{2}\phi \cdot \sigma$ per sq. cm., and the Electric Force $\phi = 4\pi\sigma$, it follows that $p_0 = \phi^2/8\pi$; and this is equal to the Mechanical Force $f = \mathbf{t}$, in dynes per sq. cm., across the field, in the direction of the lines of force. That is to say, the space surrounding the conductor is under tension, and pulls upon the conductor with a force or traction $\mathbf{t} = f = p_0 = \frac{1}{2}\phi \cdot \sigma = 2\pi\sigma^2 = \phi^2/8\pi$, all in dynes per sq. cm. of the surface of the conductor.

Let it be borne in mind, however, that what is actually observed in electrostatic phenomena is the measurable repulsion or attraction, which is found to vary as if it were due to positive or negative charges under the law of inverse squares. The actual movements or tendencies to movement are equally explainable under the assumption that they are due to Ether-stress, which implies that there shall be two localities between which the Ether may be stressed. Accordingly, if an electrified body be "insulated" by being placed on a dry glass stand within a room, the walls of the room are oppositely electrified, and bear a complementary charge, numerically equal in the aggregate to the charge of the insulated body. The space comprised between the electrified body and the oppositely-electrified walls of the room is a **Field of Force,** permeated by **Lines of Force** and **Equipotential Surfaces.** The lines of force traversing such a field quit the free surface of the insulated body at right angles, and strike the walls of the room, again at right angles. They

* For a gravitational analogy, see problem 10, p. 189.

are, in general, of a curved form. A certain number of lines of force may be grouped within a bundle or **Tube of Force,** whose cross-sectional area increases as the lines of force diverge from one another, or diminishes as they converge; and ϕ, the resultant local **Electric Force, or the mechanical force on a unit of electric quantity** placed at any point within any such tube, must vary inversely as the local cross-sectional area of the tube. The **Intensity of the Field,** or the **Number of Lines of Electric Force per sq. cm.** of cross-sectional area of any Tube of Force, at any point, is also equal to the local value of ϕ.

The force ϕ upon a unit-charge brought within distance d of a charge Q would be $\phi = Q \times \text{unity}/d^2$. The Intensity of the Field, or the local value of ϕ, is therefore Q/d^2. This, taken numerically, will be the Number of Lines of Force per sq. cm. cross-section of a Tube of Force cut across at distance d cm. from the charge Q. But the area of a sphere of radius d cm. is $4\pi d^2$ sq. cm.: hence the total number of lines of force cut by such a sphere, *i.e.* the Total Number of Lines of Force radiating from a charge Q, $= \phi \times \text{area} = 4\pi Q$.

If the tubes of force be constant in cross-sectional area, the lines of force are parallel to one another, and the equipotential surfaces are equidistant and plane; the field is then a **Uniform Field of Force,** the Ether in which is exposed to a uniform intensity of stress.

Such a field we find in the central part of the space between two parallel plates insulated from one another and brought to different potentials.

The imaginary electric matter or imaginary superficial film appears only at the free ends of the lines of force; and on a conductor its imaginary local "superficial density" is always $\sigma = \phi/4\pi = \sqrt{f/2\pi}$, where f is the actual mechanical force or traction **t** per sq. cm. in the field, and ϕ the actual force upon a unit electric quantity put in the field, close to the charged conductor.

The lines of force tend to shorten themselves and, if they run in the same general direction, to repel one another. The Lines of Force are, it will be borne in mind, merely convenient means of representing to ourselves the actual forces or stresses within the field of electric force.

Dust floating in the air between two charged surfaces finds its way along the lines of force to one surface or the other, and if sticky, it agglomerates (Lodge). "Thunder clears the air."

The conception of **Potential** (Chapter VII.) is one of the highest importance in the theory of Electricity: but it must be

remembered that "Potential" is not in itself a physical state, nor is it an explanation of electric any more than it is of gravitational phenomena; it is a scientific concept; it is an aid to calculation, and it enables us to see and to make use of gravitational analogies; it is based upon the law of the repulsion and attraction of the imaginary electric matter, which law is itself merely a mode of statement of the observed forces in the field of force.

The **Absolute Electrical Potential** at a point is a mathematical expression, possessing a numerical value: it measures the tendency which the existing electric forces would have to drive an electrified particle away from or to prevent its approach to the point in question, if such a particle, bearing a charge equal to one unit in quantity, were situated at that point or were brought up to that point; and it is numerically equal to the number of ergs of work that must be done in order to bring a positive unit of electricity from a region where there is absolutely no electric force — *e.g.*, from a region at an infinite distance from all electrified bodies — by any path up to the point in question; provided always that the transfer of the positive unit of electricity be supposed to have no effect whatsoever upon the distribution of the electricity of other bodies in the neighbourhood of that point.

Difference of potential between two points. — If VQ ergs of work must be done in order to remove a quantity Q of electricity from the point A to the point B against electric repulsion, then the two points A and B are at potentials which, considered absolutely, may be unknown, but which differ numerically by V: and B is said to be at a higher potential than A, by V units of potential.

When there is a difference of potential between any two points in space, a body bearing a charge of positive electricity, and placed at the point at which the potential is greater, is driven towards the point of less potential, just as in the corresponding gravitation-problem, a mass tends to fall towards a lower level; and if free to move it will follow the track of the lines of force, travelling thus from each equipotential surface to the next one, indefinitely near it, by the shortest path. The path between the two points is not necessarily the shortest, for the lines of force are often curved (see Fig. 234).

A positively-charged particle, placed in a region of positive potential, will be repelled along the lines of force into a

region of less or of zero or of negative potential: a negatively-charged body under the same circumstances travels in the opposite direction. In other words, the Lines of Force correspond at each point to the Direction in which the Potential most rapidly decreases.

The Mean Electric Force ϕ acting upon a Unit-Charge of electricity within an electrical field is equal to the difference between the potentials of two points within that field and situated at a mutual distance of one centimetre, that distance being measured along the lines of force: for if $V_{,}$ and $V_{,,}$ be the potentials of two points whose mutual distance is d, the Work done in moving a unit of electricity from the point of lower to the point of higher potential is $V_{,} - V_{,,}$; but this Work done is also equal to ϕd, where ϕ is the mean force resisting the transfer; whence $\phi d = V_{,} - V_{,,}$, and $\phi = (V_{,} - V_{,,}) \div d$. When $d = 1$ cm., $\phi = V_{,} - V_{,,}$; and when $(V_{,} - V_{,,}) = 1$, $\phi = 1/d$.

But this Electric Force on Unit Quantity, $\phi = (V_{,} - V_{,,}) \div d$, is the Potential-Slope or **Potential-Gradient**; and in a uniform field this is uniform, for the potential diminishes equably throughout the field. Hence in a uniform field the Electric Force, which is $4\pi\sigma$ near one of the charged conductors, remains equal throughout the field up to the opposed conductor; and the superficial density σ' of the opposed charge must be equal, for $\phi = 4\pi\sigma = 4\pi\sigma'$. In a non-uniform field, the gradient varies as the equipotential surfaces lie nearer or farther apart; and as Tubes of Force widen out or become narrower, the Electric Force ϕ — *i.e.* the number of lines of force per sq. cm. of cross-area of the tubes of force — is inversely proportional to their area; and when they reach the opposed conductor, the tubes of force which have left an area A engage an opposing area A′; hence $\phi' = 4\pi\sigma \cdot A/A'$; but this is also equal to $4\pi\sigma'$; whence $A'\sigma' = A\sigma$, and the charges on the two conductors which determine a field of force must be equal, as well as opposite.

Since under the law of inverse squares the potential due to repelling mass Q at distance d is Q/d, and at distance d' is Q/d', the difference of potential at points situated at distances d and d' respectively from the repelling mass Q is $Q/d - Q/d' = Q(d'-d)/dd'$: and since the difference of potentials of any two equipotential surfaces is numerically equal to the Work done in transferring a unit-charge from the surface of lower to the surface of higher potential, it follows that the Work done by or on a unit-charge, on its moving or being moved from distance d to distance d' from a charge Q, is $Q(d' - d)/dd'$, positive or negative as the case may be.

If a body charged with electricity be not free itself to move along the lines of force, we find this most remarkable phenomenon — that in a field of force, the points of which corresponding to the extremities of the body are at different potentials, the **electrical condition** of the body tends to **travel**: one aspect of the charged body — the aspect, namely, which looks towards that direction in which the charged body would itself travel if it were free to do so — tends to become more strongly charged or to acquire a greater superficial density of charge; the oppo-

site aspect tends to become less strongly, or, it may be, even oppositely charged. This redistribution, if it take place, has the effect of equalising the potential throughout the body placed within the field of force; and it reminds us of the readjustment of **level** and accumulation of **water** towards the lower end of a tank laid on a sloping surface or tilted up at one end, during which readjustment a difference of level produces a flow of water. And further, it does not matter whether a body in the field were originally charged or not: if an electric field be set up round an uncharged body, that body tends to become positively charged in the region of lower potential, and negatively charged in that of higher potential. The new distribution, once assumed, is permanent so long as the field of force which immediately surrounds the body, and which tends to determine a difference of potential between its opposite aspects, remains unchanged; but while that distribution is being assumed we have a brief **Current of electricity.**

A Difference of Potential, in whatever way it may be set up within a body, produces a tendency to prompt equalisation of potential throughout that body, and thus to the establishment of a momentary Current of Electricity; a permanent difference of potential, in whatever way kept up, tends to produce a continuous current.

A lightning conductor is the seat of a continuous current so long as the earth at its base and the air at its apex are at different potentials.

If we employ the analogy of air drawn up by a fan through a mine, down one shaft and up the other, while air flows by a return path from the upcast to the downcast shaft to equalise any differences of pressure set up in the atmosphere, we may see that all displacements and currents of electricity must take place in closed circuits. For the most part we attend only to potential-differences and flows of electricity between two points or places. But it is important to remember that every current in a conductor is accompanied by a simultaneous Displacement-Current in the surrounding Ether. This displacement-current, along with the Conduction-Current along the conductor, makes a closed circuit; and it comes to an end when the Ether has been put into a condition of Electric Stress (or want of stress), such as shall correspond to the final difference of potential between the ends of the conductor. When this has been done, a condition of equilibrium between the conductor and the surrounding medium is reached, and Energy is stored in the medium, up

to a certain definite amount. When the conductor returns to a neutral state, this energy is restored by the Ether, which regains its normal condition.

Even within a conductor there may be this storing up of energy, due to the imperfect conductor itself acting more or less as a dielectric. The true current will accordingly consist of the Conduction-Current *plus* this Displacement-Current, which only ceases when a uniform state, whether of quietude or of steady conduction, has been attained.

Difference of potential is analogous to difference of level or Head of Water in hydraulics; and when it determines a flow of electricity, it is often called electromotive force or E.M.F. — a term which, in this sense, might with advantage be abandoned, and instead of which we shall use the phrase electromotive difference of potential or E.M.D.P. In place of this phrase the reader who has any reason for doing so will easily read the words Electromotive Force. The objection to employing this expression, when that which is meant to be spoken of is in fact a potential-difference, is simply that a difference of potential, like a difference of water-level, is not itself a Force, and does not even completely specify the Electric Force ϕ which determines an electric flow: to determine this the form and the dimensions of the electric conductor must be known, as well as the difference of electric potential between its extremities, just as the dimensions of a water-pipe must be known, as well as the available head of water, before we can calculate the local falls or slopes of pressure and the forces producing flow.

We have already seen that $\phi = (V_{,} - V_{,,}) \div d$; and this is Clerk Maxwell's "Electromotive Intensity" or true Electromotive Force, the Electric Force ϕ acting upon a unit-charge of electricity at any point referred to. This is, in other words, the **Potential-Slope** across the point in question, in the direction of the Lines of Force.

The Cause of D.P., whatever that may be, is also called E.M.F.: and this operates in a direction **opposed** to that in which the resulting E.M.D.P. tends to act; just as the upward pressure of a force-pump driving water into a cistern is opposed to the downward pressure and flow obtainable from the cistern when filled.

This may sometimes operate from point to point, so as to heap up the resultant D.P.: and when we sum up the effects of all the local E.M.F.'s, we may arrive at the aggregate resultant D.P. produced.

Difference of Potential is often spoken of by electrical engineers as Electric Pressure: thus we hear of high-pressure and low-pressure currents. It is also known as **Voltage**, when measured in the particular units known as Volts.

If two bodies be at different potentials, when they are con-

nected by a metallic wire the charge over them will be readjusted by a momentary current along the wire, and they will come to the same potential.

Two bodies are said to be at the same potential when electricity has no tendency to travel from one to the other, even **though** they be brought into communication by a metallic wire.

Difference of potential is thus also analogous to difference of temperature, and "electromotive force" ϕ to temperature-gradient.

The **earth** itself is arbitrarily assumed to be at **zero** potential: and bodies in such a condition that when they are placed in contact or in metallic communication with the earth their electric condition is unaltered, have a potential whose value is equal to this arbitrary zero.

The arbitrary or conventional potential — or, briefly, **The Potential** — of a point in an electric field of force is, numerically, the number of ergs of work necessary to bring a unit of electric charge up to the point in question from a region of nominal zero-potential — *e.g.*, from the surface of the earth.

Between a positively-charged body within a room and the negatively-charged walls of the room there must lie, in the intervening field of force, one equipotential surface which has a Zero Potential, its potential being the same as that of the earth outside the room. Within this equipotential closed surface there is a region of Positive Potential; exterior to it there is a region of Negative Potential. The potential of the inner region is greatest at the surface of the electrified body; the potential in the negative region is most negative on the surface of the walls.

If the walls of the room be at the same potential as the earth, then the whole field of force is at positive potentials, relatively to the arbitrary or conventional zero of potential. The stress in the field, the forces and potential-difference across it, are not affected by this. The walls and the earth have still been negatively charged; but a change has been effected in the absolute value of the nominal zero of potential.

The potential cannot be a maximum or a minimum at any point within a field of force, if that point be not upon the surface of one of the conductors whose surfaces bound the field.

Conductors and Non-conductors. — In the familiar case of a lightning-conductor we see a marked distinction between the conductive copper along which a continuous current of electricity can flow, and the air or an unprotected building which can only be traversed by a disruptive discharge. A **conductor** is, when a charge is borne by it and retained by it in equilibrium, a substance throughout the whole volume and over the whole surface

of which the potential is uniform; while if inequalities of potential were set up within it, the conducting material of a perfect conductor would offer no resistance to the readjustment of potential by means of a current. A perfect non-conductor or **dielectric** would, on the other hand, be a substance the different parts of which may, after an electric disturbance, remain, without any process of readjustment and for an indefinite period of time, at potentials differing to any extent. There are no bodies which are absolute non-conductors; all conduct electricity more or less slowly. There are no bodies which are perfect conductors; all offer more or less resistance to the flow of electricity. Bodies which conduct extremely badly are called **Non-conductors** or **insulators**: bodies which offer comparatively small resistance to the passage of electricity are in practice called **Conductors.**

The ether in an insulator can stand exposure to a moderate stress; that in a perfect conductor, for some reason, cannot; that in an ordinary conductor yields continuously, with a greater or less quasi-plasticity.

When a charged body is placed upon an insulator, such as ebonite, guttapercha, indiarubber, dry glass, sealing-wax, quartz, it is said to be **insulated**; its potential cannot become equal to that of the earth for a long period of time; it is said to retain its charge for a long period.

Air at a high pressure is almost an absolute insulator: cold air, damp or dry, at the ordinary pressure is one of the best insulators: but even within cold air, bodies charged with electricity gradually lose their charge; a partial vacuum is a good conductor; a good vacuum is again an extremely good insulator. Ice insulates, water is a bad conductor; obsidians and lavas insulate when hot, and steatite even when red-hot; glass when dry is an insulator, but when very hot is a conductor. A body charged and supported upon a dry-glass stem within a vacuum or a very dry cold atmosphere will retain its charge for a very long period; but if the air be damp, so that the insulating glass stem condenses upon its surface a film of moisture from the air, that film will slowly conduct the charge to earth. The insulating character of air seems (J. J. Thomson) to indicate that free molecules of air cannot be charged, but that there must be free atoms, or that dust or extremely minute water-drops must be present, before this can be done. A metal or a phosphorescent substance, negatively charged and exposed to ultra-violet light, readily loses its charge; and it appears that the surface of the conductor is then to some extent broken up into dust and repelled. In a high vacuum ($\frac{1}{1,000,000}$ atmo.), silver negatively charged will (Crookes) evaporate in this way so vehemently as to assume a superficial glow, so intense is the agitation of its particles.

A sufficient difference of potential will cause a spark to fly between two charged conductors across the intervening dielectric: in the case of turpentine, paraffin, and olive oil, the striking distance is when the discharge is continuous regularly, when the discharge is interrupted irregularly propor-

tional to the difference of potential; in air the striking distance increases faster than the difference of potential, and the curve indicating the ratios of striking distances to differences of potential is a parabola. A steeper potential-slope, ϕ, is required for thin than for thick layers: but a potential-difference less than 1 to $1\frac{1}{3}$ electrostatic unit will not, in air, produce a spark at all.

Kinds of Conductors.—There are two kinds or classes of Conductors: first, those which act as conductors without any apparent displacement of their own substance, such as metals and other substances ordinarily known as good conductors; and second, those in which the transfer of electricity is accompanied by a relative displacement of particles within the conductor. If a current of electricity be sent through a solution of common salt in water, it will be found that those particles of Na and Cl, into which the salt is likewise on other grounds believed to break up on its being dissolved in water, are displaced within the solution, and that they travel, the Na towards the electrically negative and the Cl towards the positive extremity of the solution, each kind of particle with its own specific velocity. Each such particle or sub-molecule or *ion* parts at its extremity of the solution with a certain definite quantity of electricity, positive or negative, and no more. Conductors of this kind are called *Electrolytes*; and they consist mainly of solutions of salts and of chemically-strong acids or bases, in which the whole or a part of the substance dissolved has been broken up into these sub-molecules or ions, but in which the water plays no part as a conductor.

The phenomena of electricity present themselves *within* a conductor only while a current is actually passing through it; for then only are there any differences of potential within the conductor. And even this can only occur when the conductor is an imperfect one; for within a perfect conductor there never could be any difference of potential set up. With a perfect conductor, or when there is no current, but a more or less permanent condition of Statical Equilibrium of the charge, electrical phenomena are restricted to the Field of Force—that is, to the non-conductor or Dielectric external to the conductor; for within non-conductors alone, not in conductors, can any electrical stress or difference of potential, permanently or for any length of time, be maintained.

If the air had been as good a conductor as copper we would probably never have known anything about electricity, for our attention would never have been directed to any electrical phenomena.

Phenomena of electricity in a state of equilibrium, associated with more or less permanent differences of potential and evinced within a dielectric, are said to be **electrostatic**; those evinced during adjustment of electric potential by the passage of a current along a conductor are said to be **electrokinetic.**

If electrostatic phenomena be due to stresses in the Ether, electrokinetic are due to movements of the same; and a momentary current of electricity in a copper wire is a throb due to a readjustment of the stresses in the Ether or dielectric surrounding that wire; the throb is accompanied by a readjustment of the Lines of Force in the field surrounding the wire; these lines, as it were, slip along the wire, carrying Energy with them **in the Ether or other dielectric**, not in the wire itself, except in so far as the imperfectly-conducting character of the wire may lead to its acting to some extent as if it itself were a dielectric or non-conductor. The lines of force in that case traverse, and slip along within, the substance of the wire itself.

"Free" and "Bound" Charges.—A distinction is frequently made between a free and a bound charge of electricity. The former is understood to be a charge borne by an insulated body, and independent of surrounding objects, while the latter is such a charge as is held in position by the presence and attraction of a charge of the opposite character upon a neighbouring body. In truth, however, all charges are bound charges; the complementary distribution must be somewhere; the field of force may be great or small, but it must have its limits. It may be small, as when a little electrified body is suspended within a metal flask which is not insulated; it may be great, like the field of force between a thundercloud and the earth: in the former case the complementary charge is distributed over the inner surface of the flask; in the latter it travels about, and is at its densest upon the surface of the earth beneath the travelling thundercloud, or else upon adjacent clouds. Even when an electrified body is placed at an extremely great distance from all surrounding objects, it cannot be held to have a free charge, for its charge is bound by the complementary distribution upon the far-distant objects; and a particle isolated in otherwise vacuous infinite space, if such a thing were possible, could not become charged with electricity at all, for the complementary charge could, in such a case, have no locus.

If the Ether be stretched or compressed, it must be stretched or compressed between at least two points, which may be near

or far from one another. Bearing this in mind, however, it is undoubtedly convenient in many respects to permit ourselves the use of such expressions as "a body freely charged with Q units of + electricity," and in so doing to omit, provisionally, all consideration of the complementary charge, which is supposed sufficiently distant.

This mode of expression is also in accord with the convention that the potential of the earth is always zero.

Division of Charge. — When a conductor charged with electricity is brought into contact or into metallic communication with another at a different potential, the electric potentials of the two conductors become equalised; and if the two bodies be of the same form, size, temperature, and chemical nature, and if they be symmetrically arranged, they will, after separation, each bear a charge equal to one-half of the algebraical sum of the original charges of the two bodies.

This change of distribution involves a readjustment of the lines of force and of the equipotential surfaces throughout the surrounding dielectric, and an alteration of the distribution of the complementary charge over the opposite boundary of the field of force.

When the bodies are unequal in size, etc., or are unsymmetrically arranged, the division of the charge between them is not equal. Two similar but unequal spheres are found, after being brought into communication by a long thin wire, which is then removed, to bear charges proportioned to their radii.

Electrostatic Capacity or Permittance. — When a conductor has a charge of electricity imparted to it, the potential of its surface and of its whole volume is raised, positively or negatively (*i.e.*, lowered), as the case may be. When a body, insulated in air, requires a charge of C units of electricity to be imparted to it in order to raise its potential by one unit — that is, from zero to unity, or from V to V + 1 — it is said to have a Capacity or Permittance of C units. When this body, so insulated, has its potential raised by the amount V, the Charge of Electricity imparted to the conductor is $Q = VC$ units.

When a series of conductors — whose electrostatic capacities are $C_,$, $C_{,,}$, $C_{,,,}$, etc., and which are charged to potentials $V_,$, $V_{,,}$, $V_{,,,}$, etc., so that their several charges are respectively $C_,V_,$, $C_{,,}V_{,,}$, $C_{,,,}V_{,,,}$, etc. — are connected by a wire, the potential thereupon assumed by the whole system is equal to —

$$\frac{\text{The whole charge}}{\text{The whole capacity}} = \frac{V_,C_, + V_{,,}C_{,,} + V_{,,,}C_{,,,} \ldots}{C_, + \quad C_{,,} + \quad C_{,,,} \ldots} = \frac{\Sigma VC}{\Sigma C}.$$

The electrostatic capacity of a conductor is the same whether it be solid or hollow: the merest film of gold leaf supported on a wooden ball has as great a capacity as a solid metallic sphere.

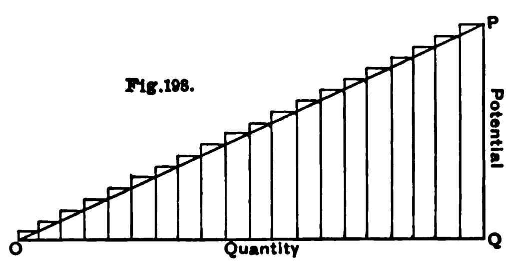
Electrostatic stress can only persist within the field of force, the dielectric, which is limited by the surface of the conductor; beneath this surface it is a matter of indifference what the metallic thickness may be, since within a conductor there can be no permanent difference of potential, no permanent electrostatic stress. The quantity C is not really a capacity of the conductor for electricity at all, but is a measure of the elastic yielding of the Field of Force involved.

The Electrostatic Capacity of a Sphere.—A sphere of radius r, within an unlimited air-space containing no other charged bodies, is charged with quantity Q; this quantity, uniformly distributed over the surface, acts as if it were gathered at the centre, and therefore at a distance r from the surface. The potential at the surface of the sphere must therefore be $V = Q/r$. The capacity $C = Q/V$; this is $Q/\overline{Q/r} = r$; the Electrostatic Capacity of a Sphere, insulated in air, is therefore, in C.G.S. electrostatic units, numerically equal to its Radius.

The **Work** spent in charging any body is equal to half the product of Q, the Charge imparted to it, into V, the rise of Potential produced in it.

To bring a quantity, Q, of electricity from a place of zero potential to a place of constant potential, V, involves the expenditure of QV units of work, by our definition of Potential. To bring a charge Q by successive instalments into a region whose potential is at first zero, but steadily rises as the successive instalments arrive, work must be done which is equal to **half** the product of the final rise of potential into the whole charge brought up. In Fig. 198 the small rectangles represent the work done in bringing each successive instalment up to the corresponding potential; these rectangles increase, and their sum, which represents the total work done in bringing up the whole charge OQ to the final potential QP, is represented by the triangle OQP, whose area $= \frac{1}{2}OQ \times QP = \frac{1}{2}$ whole quantity $\times$ final rise of potential $= \frac{1}{2}QV$.

Fig. 198.



The work done in charging a body is equal to the electrical energy stored up in the field of force surrounding that body; and since this is equal to $\frac{1}{2}QV$, we see that the **energy** of an electrified body depends not only upon the Quantity of

electricity borne by that body, but also upon the Potential; just as the potential energy of a mill-pond depends not only upon the Quantity of water contained in it, but also upon the average elevation of that water above surrounding objects. For which reason a mere Quantity of Electricity is not, in itself, a quantity of Energy; and therefore Electricity, as measured by Quantity of Electricity, is not, like Heat, itself a form of Energy.

The energy of a charged conductor of any kind is measured by $\frac{1}{2}QV$; but this is equal (since $Q = CV$, where C is the electrostatic capacity of the conductor) to $\frac{1}{2}CV^2$ or to $\frac{1}{2}Q^2/C$.

The energy of a system of connected conductors is equal to $\frac{1}{2}V^2 \cdot \Sigma C$, or to $\frac{1}{2}Q^2 \div \Sigma C$, where ΣC is the aggregate capacity of the whole system.

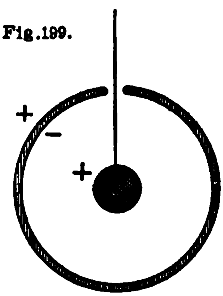
Suppose now that two conductors, of which the one is charged to potential V while the other is at zero potential, and of which the respective capacities are $C_{,}$ and $C_{,,}$, are placed in metallic communication; on contact they form a joint conductor whose capacity is $(C_{,} + C_{,,})$. The energy of the single charged conductor was $\frac{1}{2}Q^2/C_{,}$; that of both taken together is $\frac{1}{2}Q^2/\overline{C_{,} + C_{,,}}$, a smaller quantity. There is therefore an apparent Loss of Energy equal to $\{\frac{1}{2}Q^2/C_{,} - \frac{1}{2}Q^2/\overline{C_{,} + C_{,,}}\} = \{(\frac{1}{2}Q^2/C_{,})(C_{,,}/\overline{C_{,} + C_{,,}})\}$, or $(C_{,,}/\overline{C_{,} + C_{,,}})$ times the energy of the original charge. If $C_{,} = C_{,,}$, half the energy of the original charged conductor is apparently lost by partial discharge, being transformed into Heat.

Wherever there is a readjustment of electricity in the form of a running-down of electricity from a place of high potential to a place of low potential, there is a loss of energy of electrification; just as when a full pond is allowed partly to discharge itself into an empty one, the average level of the whole is lowered, and the energy of position partly disappears, to reappear in the form of Heat. In general, where electrified conductors are connected by metallic wires, if there be a current, the potential energy of the system sinks to a minimum; heat and — if a spark pass — light, sound, and mechanical effect being produced. Where the components of an electrified and insulated system are allowed to approach or to recede from one another in obedience to the electric forces, the energy of electrification becomes in part converted into mechanical work, and therefore falls in amount; while if they be pulled asunder or made to approach against the electric forces, the mechanical work done upon the insulated system from without is converted into energy of electrification. In the former case the energy left in the system is that of the same charge at a lower potential; in the latter case it becomes that of the same charge at a higher potential.

Electrostatic Induction. — When an electrified body or

system is placed within a hollow metallic shell (Fig. 199), with which there is no communication except through non-conductors, the shell becomes charged by 'Induction' across the intervening dielectric. If the bodies placed within the shell be positively charged, the inner surface of the shell becomes negatively, the outer positively electrified. The opposite charge thus induced on the inner surface of the shell, the similar charge induced upon its outer surface, and the original inducing charge on the internally-suspended system, are all equal in amount, if the shell completely or, practically, even if it very largely surround the electrified body suspended within it. Thus the positive and the negative charges called into existence by Induction are together algebraically equal to zero.

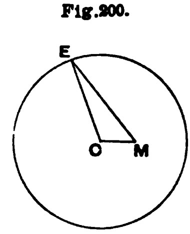
Fig. 199.



The Difference of Potential between an inducing sphere and an induced spherical surrounding shell is $Q/r - Q/r'$, where Q is the charge on the inducing sphere, r its radius, and r' the radius of the hollow spherical shell.

The distribution of the induced charge on the interior surface of a completely-surrounding shell is such that on external points it produces an effect equal and opposite to that of the interior insulated charge; the two inner charges therefore produce together no effect upon external bodies, and the induced charge on the outer surface is the only charge which can affect particles situated in the outer air. The two interior charges are bound to each other, for they are of opposite character, and there is a field of force between them; the outer charge is said to be free.

Fig. 200.



The distribution of electricity over the inner and outer surfaces of the shell is, if the shell be spherical, governed by the law that the superficial density σ at any point E is $\sigma = (CE^2 \sim CM^2) \cdot Q/\overline{4\pi \cdot CE \cdot ME^3}$, where C is the geometrical centre of the shell, and M the point at which the charge Q is situated. Then the inner charge Q at M, and the opposite charge $-$ Q on the surface (the inner surface of the shell), produce together no effect on surrounding particles. The charge + Q on the outer surface acts as if it were all at M.

The lines of force or of induction radiating from M are not displaced by the interposition of the insulated shell; neither are the equipotential surfaces: but the shell divides the field of force round M into two regions, of which either can be destroyed without affecting the stress or the potential-slope in the other.

At page 583 we saw that the number of lines of force from a charge Q

was $4\pi Q$. Similarly, the total **induction** in the inductive field is said to be also $4\pi Q$, or Q/d^2 per sq. cm.; and every 4π Lines of Induction passing out of a dielectric into a conductor are said to be able to induce one unit of charge on the bounding surface of that conductor; so that the Induction per sq. cm., $\mathfrak{i}$, $=4\pi\sigma$, where σ is the superficial density of the induced charge. Then, since the Electric Force ϕ is also equal to $4\pi\sigma$, it is said that the Lines of Force and the Lines of Induction coincide in air, the standard medium.

If the outer surface of the shell be connected with the surface of the earth, the shell and the earth become one extended conductor, and the positive charge on the outer surface of the shell is repelled to the earth's surface; it now blends with and neutralises the negative charge previously borne by the earth and surrounding objects in consequence of the original positive electrification of the inducing body. As a result of this we have, within the shell, a purely local field of force, restricted to the space between the internally-suspended body and the interior surface of the shell, and giving rise to no phenomena outside that cavity.

The potential of the whole system is lowered by this annihilation of the outer region of the original field of force; but the potential-slope within the interior field remains unchanged.

If, on the other hand, the insulated body within be made to touch the enveloping shell, the internal field of force will be destroyed; but the outer induced charge will remain, distributed over the outer surface of the shell.

Any quantity of electricity may thus be wholly transferred to the outer surface of a hollow insulated conductor, if a charged body be made to touch its internal surface.

A sheet of tinfoil charged, and separated from a second sheet by an intervening layer of air or glass or mica or waxed paper, will act inductively across the dielectric. The nearer surface of the second sheet is oppositely, the farther surface similarly charged; and if the second sheet of tinfoil be connected to earth, the similar charge escapes and the field of force is now almost wholly limited to the thin space between the two sheets or plates.

A layer of dielectric intervening between two conducting surfaces constitutes an Electrostatic Accumulator or **Condenser.** In this dielectric layer, a limited Field of Force may be set up, the lines of force through which stretch across from one conducting surface to the other; and the Permittance or electrostatic Capacity of such a field is greater than that of the field

set up when either of the two conducting surfaces is separately charged in the open air.

From the four equations, $Q = A\sigma$ (i.); $\phi = (V_{,} - V_{,,})/d$ (ii.); $\phi = 4\pi\sigma$ (iii.); and $Q = VC$ (iv.); we find that $Q = (V_{,} - V_{,,})\cdot A/4\pi d$, and $C = A/4\pi d$, where A is the opposed area of the plates, and d the thickness of air between them.

Suppose a condenser to be made up of an inner conducting sphere (solid or hollow) of radius r and, concentric with this sphere and separated from it by a layer of air, an outer shell whose inner spherical surface has a radius r'. Let the inner electrified sphere bear a charge Q; this charge acts as if it were concentrated at the centre. The potential-fall or -difference across the field of force is, as at Fig. 199, $Q/r - Q/r'$. The mean area A is $4\pi rr'$. The expression $C = A/4\pi d$ becomes $C = rr'/(r' - r)$. This is greater than the capacities of either of the two component spherical surfaces, the capacities of these being respectively equal to r and r'.

The *thinner* the *dielectric* between the two metallic surfaces of a condenser, the *greater* is its electrostatic *capacity*; and correspondingly, the less will be the potential to which a given charge will raise it. Starting from a given original charge, with its corresponding field of force, the thinner the dielectric of the condenser the smaller will be the part of the original field left when the second plate of the condenser is put to earth; but, the charge on the first plate remaining the same, the potential-slope in that part remains unaffected: the difference of potentials between the two plates is therefore less than the original potential-difference, in the larger original field, under the same potential-slope ϕ due to the given charge. If the opposed plates be made to approach one another, the *potential-difference* between the opposed plates falls proportionately, because $\phi = 4\pi\sigma$ remains constant if σ be not altered; and in order to maintain a given potential-difference constant, it would be necessary to increase the charge on the insulated plate: or conversely, if the potential-difference be determinate (*e.g.*, where the opposed plates are each connected with one terminal of a galvanic battery), on mutual approach of the plates the charges upon them will proportionately increase.

In this, it is assumed that any "free charge" on the farther surface of either conducting film is so small that it may be left out of account. But see p. 630.

The **nature of the dielectric** between the plates of the condenser is not a matter of indifference. It is found—and this proves that in the phenomena of electrical force the dielectric plays an important part—that the permittance or capacity of a condenser varies with the nature of the interposed dielectric, and is proportional to a constant special to each substance (or in some instances even to particular directions within the same substance) and called the **Specific Inductive Capacity** or the **Permittivity**, K, of that substance. The sp. ind. cap. of air being taken as a standard and equal to unity, that of sulphur is 3·2. Sometimes the sp. ind. cap. of a vacuum, which differs little from that of air, is taken as unity, in which case the

dielectric is the Ether itself. The sp. ind. cap. of glass rises slightly when the temperature is increased (between 12° and 83° C. a rise of $2\frac{1}{2}$ per cent). All gases have very nearly the same inductive capacity, whatever their chemical constitution, their temperature, or their density. If, however, their pressure be increased or diminished, the minute difference between their sp. ind. cap. and that of a vacuum is also increased or diminished in the same proportion; and conversely, when a gas is employed as a dielectric, induction across it diminishes its pressure, the gas then adjusting itself so as to become rarer and consequently less inductive.

If a given charge Q will raise a condenser in which air is the dielectric to a potential V, it will only raise a similar condenser whose dielectric has sp. ind. cap. = K to the potential V/K; and since in the special case of a conducting material used in the place of a dielectric the difference between the inner and outer coats is zero, the sp. ind. cap. of a conductor may, for some purposes, be considered infinite, for $0 = 1/\infty$.

Similarly, the greater the sp. ind. cap., the less is the mechanical Force across the dielectric between two given charges at a given distance; $F = QQ'/Kd^2$.

This is the most general formula: the formula $F = QQ'/d^2$ applies only to air as the dielectric. From the air-formula we get the following Equation of Dimensions: $[Q] = [\text{Mechanical Force} \times \text{distance}^2]^{\frac{1}{2}} = [ML/T^2 \cdot L^2]^{\frac{1}{2}} = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$; but from the general formula we get $[Q] = [K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$. In electrostatic measurements we arbitrarily assume air as a standard and, for air, K = 1, a Number merely, without Dimensions; but if we approach the subject from any other point of view, we must use the general formula, not the air-formula, in order to ascertain the true Dimensions of electric quantities on any other system of measurement.

Whatever be the medium, the quantity of induced charge remains the same: this is expressed by saying that the same number of Lines of Induction pass from a given charge whatever be the medium, and if the given charge be Q, measured in the usual air-units, the total number of these lines is $4\pi Q = \mathbf{I}$, the Total Induction; or, at distance d the lines of induction are Q/d^2 per sq. cm. $= \mathbf{i}$ the induction per sq. cm., $= 4\pi\sigma$ per sq. cm., where σ is the superficial density of the induced charge on any bounding conductor-surface. But in any medium of sp. ind. cap. = K, say higher than that of air, the quantities of electricity would have to be increased in the ratio $1 : \sqrt{K}$ in order to give the same mechanical force F: the units of quantity are thus larger: the numerical value of the potential due to a given charge thus varies inversely as $\sqrt{K}$, and ϕ, the numerical value of the potential-slope or Electric Force, in a medium of sp. ind. cap. K, varies inversely as $\sqrt{K}$ for charges of given numerical value, measured with reference to that medium, or inversely as $\sqrt{K} \times \sqrt{K} = K$ for charges of an equal number of air-units, these units being smaller. Therefore ϕ, $= 4\pi\sigma$ in air, becomes $\phi = 4\pi\sigma/K$ in any other medium, when σ is measured in the usual air-units; but $\mathbf{i}$ remains equal to $4\pi\sigma$; hence $\mathbf{i} = K\phi$, and in a medium of sp. ind. cap. K, the lines of induction are more numerous than those of force in the ratio K : 1. The units of quantity vary *pari passu* in the terms for $\mathbf{i}$ and ϕ: hence $\mathbf{i}$ is always equal to $K\phi$: but in air $\mathbf{i} = \phi$.

Given charges, Q and Q′, measured in air-units, thus produce a mutual force $F = QQ'/Kd^2$; the potential is Q/Kd: the potential-slope or electric force ϕ varies as $1/K$: the equipotential surfaces are at mutual distances K times as great as in air: the capacity of a given conductor varies as K,—*e.g.*, that of a sphere of radius r is Kr: the work done in communicating a given charge to a given conductor varies inversely as K, for $\frac{1}{2}QV' = \frac{1}{2}Q \cdot V/K$.

If a field of force be made up of layers of different dielectrics, the potential-slope in each is inversely proportional to K. There is thus a kind of refraction of the potential-slope at the bounding surfaces of the layers.

The sp. ind. cap. of a dielectric diminishes with the time, and is therefore difficult to measure directly; and when a condenser is discharged by metallic communication set up between its two coatings, its charge does not at once completely vanish, but the condition of the dielectric is apparently very similar to that of a body which, being imperfectly elastic, recovers slowly and irregularly its primitive form and condition after deformation; and it is curious that the same means—vibration, shaking, jarring, etc.—which facilitate the return of such a body to its normal condition after a strain, facilitate the prompt and complete discharge of a condenser whose two coatings are put in metallic connection. On sending alternate charges into a condenser, the residual discharge liberates them in the reverse order (Hopkinson); a result strikingly like that of Boltzmann with reference to successive torsions. Quartz employed as a dielectric has one-ninth the residual capacity of glass; Iceland spar seems to have no residual capacity at all, and permits prompt discharge.

The dielectric of a condenser may become double-refracting under the influence of Electric Stress, which tends, without altering its total volume, to dilate it at right angles to the lines of force: its optical axis is parallel to the lines of force. Glass and olive oil become like Iceland spar (negative crystals, p. 555); bisulphide of carbon, paraffin, resin, become positive (Kerr). Solids slowly, liquids instantly, acquire or lose this condition of stress: and when an air-condenser is released from its stress by discharge, there is a distinct sound.

Since electrostatic capacity or permittance varies as K, the general formula for that of a condenser, such as a Leyden jar, is $(K/d)\cdot(\text{Surface}\ /4\pi)$.

The form of condenser known as a **Leyden jar** usually consists of a glass vessel lined internally and externally with tinfoil. The inner coating communicates by wire with a smooth metallic knob projecting externally and insulated from the outer coating. By contact between the knob and a charged conductor the inner coating is charged. By induction through the glass there is produced an electrical separation in the external tinfoil. The external surface of this is temporarily connected with the earth. Thereafter there remains a Field of Force in the glass between the two tinfoil coatings. This may be discharged by establishing a metallic communication between the two coatings, the outer tinfoil being first touched, then the inner.

A Leyden jar when charged dilates somewhat, and as it expands its capacity increases; the potential, to which a given charge is competent to

raise the jar, sinks to a corresponding degree. When discharged, the jar makes a dull sound, and the glass glows at the edges of the tinfoil, while the internal air becomes warm.

A submarine telegraph cable is, in effect, a very long Leyden jar. The copper core is the inner coating; the guttapercha or other insulator represents the glass; the outer coating of tinfoil is represented by the protecting iron wire or by the bounding surface of the sea-water. When a charge of electricity is passed into a deep-sea cable, the cable takes some time to become fully charged: it then bears, for a considerable time, an electrostatic charge upon the surface of its copper core.

An ordinary aërial telegraph-wire is again, but to a less marked degree, a Leyden jar. The inner coating is the surface of the wire itself; the dielectric is the air; the outer coating is the surface of the earth. The electrostatic capacity of an aërial wire is small in comparison with that of a submarine cable; but it is not insignificant.

If the two coatings of a Leyden jar be slid past one another so as to diminish the opposed surfaces, the capacity diminishes and the potential due to the actual charge of the jar increases: the potential may thus be adjusted (Sliding Condenser).

Batteries of Leyden Jars. — When a Leyden jar has its inner coating placed in simultaneous metallic communication with the inner coats of a series of uninsulated jars, the whole becomes in effect one great Leyden jar of increased surface, and the jars are said to form a battery connected in Surface. The charge of the first jar, being then distributed throughout an enlarged conductor, brings it to a reduced potential; and energy is lost in the production of sparks when the battery is charged by the first jar.

A series or battery of Leyden jars is said to be charged in Cascade when the outer coat of one jar is connected by metal with the inner coat of the next, and so forth, while a charge is imparted to the inner coating of the first. The difference of potential between the inner coating of the first jar and the outer coating of the last is distributed between the jars of the battery, and thus the risk is diminished of any of the jars being destroyed through an excessive difference of potential in any one jar causing a spark to pass and perforate the glass. The charge of the whole system is only equal to that of a single jar, and the difference of potential in each of n jars is $(V_{,} - V_0)/n$, where $V_{,}$ and V_0 are the potentials of the first and the last coatings; whence the energy of the whole (= half the whole charge × the whole potential-difference) is the same as the energy of a single jar loaded with the same charge as the battery.

If the conductors surrounding an inducing charge do not completely enclose it, the charges induced upon them are each numerically less than the inducing charge, and the sum of those of each kind is also numerically less than that charge. In no case can the induced charges exceed the inducing charge.

Coefficient of Mutual Induction. — The Coefficient of Induction of a conductor A on a conductor B is the ratio of the Charge (or change of charge) developed in B to the Potential (or change of potential) of A. It can be proved that the coefficient of induction of A on B is always equal to the converse coefficient of B on A; and this reciprocally valid coefficient is

called the Coefficient of Mutual Induction. It depends upon the relative positions of A and B.

Inverting the statement, a unit charge on either body will, by induction, alter the potential of the other by an amount equal in both cases.

The effect of induction is seen when an electrified body—such as a glass rod rubbed with a dry silk-handkerchief — is brought into the neighbourhood of light bodies suspended or floating in the air. Over each of these bodies there is a separation of electricities; the aspect nearer to the inducing body is charged with electricity of the opposite kind, and is attracted; the farther aspect is charged with electricity of the same kind, and is repelled, but to a less extent, because it is more distant; its charge not being "bound" is, besides, more readily dispersed into the surrounding air. On the whole, these light bodies are attracted. If they come in contact with the inducing body, they acquire a part of its charge, and are thereupon repelled.

As another effect of induction we find that while two similarly-charged bodies at the same potential within the same field will always repel one another, yet if they be not at precisely the same potential, the one of higher potential will, by its presence, alter the distribution of electricity over the other, the weaker, in such a sense that the weaker one may even become oppositely charged over the nearer aspect, and the attraction of the more highly-charged body for this side of the weaker may prevail over its repulsion of the farther side; and on the whole, two such bodies will, if they be placed at a sufficiently small distance and if no spark pass between them, attract one another. In a certain intermediate position there will be unstable equilibrium, and at all greater distances there will be repulsion.

When a conducting body is brought into the neighbourhood of a system of insulated and charged conductors, the energy of that system falls, for the interposed body causes by its presence a redistribution of the charge of the system; and such a redistribution of the charge causes a fall of the potential and therefore of the energy of the system. If the body introduced be a dielectric, the effect produced is similar but smaller.

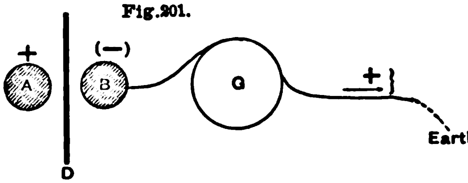
Electric Screens. — A conducting sphere surrounding an insulated electrified body and connected with the earth will, as we have seen, shelter an external particle from the inductive action of the enclosed electrified body; and conversely, it will shelter the internal electrified body from the distribution-disturbing and potential-lowering influence of the outside particle. A screen of perforated tinfoil or a cage of wire gauze has nearly an equal effect: such screens are used to protect delicate instruments from the inductive action of external electrified bodies.

The place of an enveloping sphere may be taken by a plate

of metal connected with the earth. If the diameter of this be infinite — or practically, if it be very great as compared with the distance between the electrified and the protected particle — the screening action will be perfect.

In Fig. 201, A is an insulated body positively charged by a galvanic battery or a frictional electric machine; D is a large metallic screen; B is a metallic body connected by a wire with the earth; this wire passes round the magnetic needle of a Galvanometer, G; the screen D is suddenly removed: there is a sudden separation of electricities in B: a positive charge escapes round the galvanometer and deflects its needle by an instantaneous twitch.

Fig. 201.



The phenomena of electricity in equilibrium are very similar in their mathematical aspect to those of the steady flow of Heat; equipotential surfaces represent isothermal surfaces; lines of force represent lines of flow of heat; specific inductive capacity takes the place of thermal conductivity, and potential-slope that of temperature-gradient.

Again, the calculation of the variation of the force throughout a dielectric field resembles very closely that of the distribution of the flow in a steadily-flowing mass of incompressible fluid; just as the stream lines in a field of liquid-flow may be held to exert lateral pressure upon one another, so do the lines of force in an electric field laterally repel one another, as is specially manifest at the surface of a conductor, where the "elements of charge" repel one another; within the same medium, in each of the tubes of force, or tubes of flow, the product of the force or of the flow into the cross-sectional area is constant (Law of Continuity in Hydrodynamics): and the Energy per unit of Volume in a field of force or of flow is at each point numerically equal to the electrostatic or hydrostatic Pressure per unit of Area at that point.

In Faraday and Maxwell's theory of Ether-stress, the flow of charge across an electrified surface is insisted on. This flow, which takes place whenever a Separation of Electricities occurs, is of the nature of a Displacement in the Ether permeating the Field of Force, and is directed along the Lines of Force at every point in that field. One end of a line of force is in a condition which gives rise to what we call positive, the other to what we call negative electrification or charge at the surface of the electrified body. When a thin insulated sheet of tinfoil is exposed to the inductive influence of a charged conductor, there is a separation of positive from negative charge across the conductor influenced, and on each side there is a charge induced whose density is $\pm\sigma$. This means that the lines of Induction or of Displacement, across which the tinfoil lies, are directed towards the tinfoil on one of its sides, away from it on the other. The Quantity of Electricity thus induced to flow in either direction is, over any given area A, equal to $A \cdot \sigma$: this quantity is equal to $A \cdot \phi/4\pi$, since $\phi = 4\pi\sigma$. If any other dielectric than air intervene between the inducing charge and the conductor

acted upon, $\phi = 4\pi\sigma/K$, and the amount of the induced charge, the Quantity of Flow, the so-called **Electric Displacement**, is $Q = A\sigma = AK\phi/4\pi = A/4\pi \times \mathfrak{i}$ the Induction per sq. cm. A conductor offers no permanent resistance to the displacement of an electric charge through it, and as long as there is maintained between the extremities of a conductor a permanent difference of potential, so long will the electric displacement produced be continuously relieved by an electric flow or Current; but in a non-conductor or dielectric the extremities may remain under a permanent difference of potential, a permanent stress or state of Polarisation, for the Electric Displacement, the flow set up in it during the first instant of exposure to electric stress, is arrested by a certain Electric Elasticity or Elastivity (Heaviside) of the Dielectric, which, being represented by the fraction $\frac{\text{electric stress}}{\text{electric strain}} = \frac{\text{electric force acting across each } unit \text{ of area}}{\text{quantity of flow across each } unit \text{ of area}} = \phi \div (K\phi/4\pi) = 4\pi/K$, is inversely proportional to K, the specific inductive capacity of the dielectric.

The Electrostatic Energy of the dielectric is the product of the average displacing electric force, $\frac{1}{2}\phi$ per sq. cm., into the electric displacement, $K\phi A/4\pi$ over area A, effected along a distance d: the energy of volume Ad is thus $\frac{1}{2}\phi(K\phi A/4\pi)d$, and that of unit volume is $K\phi^2/8\pi$; while the dynamical Energy-Slope f, or Electric Tension p_0 or Traction $\mathbf{t}$, in the direction of the Lines of Force, is also $K\phi^2/8\pi$ dynes across each unit of area of the bounding surface of the dielectric.

Dimensions of Electrostatic Measures, in air.—The Absolute unit of Quantity of electricity in *electrostatic measure* is a quantity which, placed at a certain distance, in air, from a similar and equal charge, repels it with a certain mechanical Force. The Force between two quantities at a given distance is therefore equal to (Product of Quantities) ÷ (Distance²). The Dimensions of this expression are $[Q] \times [Q] \div [L^2]$; but the dimensions of a Force are otherwise known to be $[ML/T^2]$; whence $[Q^2/L^2] = [ML/T^2]$ and $[Q] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$.

Surface-Density σ; quantity of electricity per unit of area: its dimensions are those of (Quantity) ÷ (Area), or $[\sigma] = [Q] \div [L^2] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

Difference of Potential, E: quantity of Work required to move a unit quantity of electricity from one point to another: its dimensions are those of (Work done) ÷ (Quantity moved); whence $[E] = [ML^2/T^2] \div [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T]$.

Electric Force, or "Electromotive Intensity," ϕ: the Electric Force at any point in a field is the mechanical force acting upon a unit quantity of electricity placed there: Mechanical Force ÷ Electric Quantity $= [ML/T^2] \div [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$: or, Force per sq. cm. $\times$ 2 ÷ Surface-Density, $= [M/LT^2] \div [M^{\frac{1}{2}}/L^{\frac{1}{2}}T] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$. Otherwise, [Potential-Slope] = [E ÷ Distance] $= [M^{\frac{1}{2}}L^{\frac{1}{2}}/T] \div [L] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$. Also, Number of Lines of Force per sq. cm.; $[4\pi Q] \div [4\pi r^2] = [Q/L^2] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [L^2] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$. Lastly, $[4\pi\sigma] = [\sigma] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

Induction per sq. cm., $\mathfrak{i}$: same as $[\phi]$, in air.

Capacity, C: the Quantity necessary to produce a certain rise of Potential: its dimensions are those of (Quantity) ÷ (Potential-difference); $[C] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [M^{\frac{1}{2}}L^{\frac{1}{2}}/T] = [L]$, a Length simply. The relative capacities of conductors of similar form are simply proportional to their diameters.

Specific Inductive Capacity, K: $K = \text{Quantity displaced} \times 4\pi / \overline{\text{area} \times \phi}$, see equation, p. 603: $[K] = [Q] \div [\phi \times \text{area}] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div \{[M^{\frac{1}{2}}/L^{\frac{1}{2}}T] \times [L^2]\}$; it is therefore simply a Number, when air is taken as the standard medium.

Coefficient of Induction; the ratio of a Charge developed to a Potential inducing; Quantity $\div$ Potential; $[M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [M^{\frac{1}{2}}L^{\frac{1}{2}}/T] = [L]$.

Electrostatic Dimensions in any medium. — Let K be the sp. ind. cap. of the medium; what the Dimensions of K may turn out to be we do not know; but $F' = QQ'/Kd^2$, whence $[Q] = [M^{\frac{1}{2}}L^{\frac{3}{2}}K^{\frac{1}{2}}/T]$. Similarly, $[\sigma] = [M^{\frac{1}{2}}K^{\frac{1}{2}} / L^{\frac{1}{2}}T]$; $[E] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/K^{\frac{1}{2}}T]$; $[\phi] = [4\pi\sigma / K] = [\text{Potential-Slope}] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}K^{\frac{1}{2}}T]$; $[\mathbf{i}] = [M^{\frac{1}{2}}K^{\frac{1}{2}}/L^{\frac{1}{2}}T]$; $[C] = [KL]$; $[K] = [K]$; and [Coefficient of Induction] $= [KL]$.

Relations of Electrostatic Quantities, in any medium. — Potential-Difference; $V_{\prime} - V_{\prime\prime} = E = d\sqrt{8\pi \cdot F/AK} = d\sqrt{8\pi f/K} = d \cdot \phi = 4\pi \cdot d \cdot \sigma/K = 4\pi \cdot d \cdot Q/AK = d \cdot \mathbf{i}/K$. Electric Force, $\phi = E/d = \sqrt{8\pi \cdot F/AK} = \sqrt{8\pi f/K} = 4\pi\sigma / K = 4\pi \cdot Q / AK = \mathbf{i} / K = 2f / \sigma$. Surface-Density, $\sigma = K\phi / 4\pi = \sqrt{KF/2\pi A} = \sqrt{Kf/2\pi} = KE/4\pi d = Q/A = \mathbf{i}/4\pi$. Induction per sq. cm., $\mathbf{i} = 4\pi\sigma = K\phi = \sqrt{8\pi KF/A} = \sqrt{8\pi Kf} = KE/d = 4\pi Q/A$. Quantity, $Q = A\sigma = A\sqrt{KF / 2\pi A} = A\sqrt{Kf / 2\pi} = AK\phi / 4\pi = AKE / 4\pi d = A\mathbf{i} / 4\pi = \mathbf{I} / 4\pi$. Force F across area A; $F = A \cdot KE^2/8\pi d^2 = KA\phi^2/8\pi = 2\pi \cdot A\sigma^2/K = 2\pi Q^2/AK = A\phi\sigma/2 = A\mathbf{i}\sigma/2K = \mathbf{I}\sigma/2K = A \cdot \phi\mathbf{i}/8\pi = A\mathbf{i}^2/8\pi K$. Force f per sq. cm.; $f = KE^2/8\pi d^2 = K\phi^2/8\pi = 2\pi\sigma^2/K = 2\pi Q^2/A^2K = \phi\sigma/2 = \mathbf{i}\sigma/2K = \phi\mathbf{i}/8\pi = \mathbf{i}^2/8\pi K$. Energy of Field $= \frac{1}{2}QV = \frac{1}{2}\{A\sqrt{KF/2\pi A} \times d \cdot \sqrt{8\pi F/AK}\} = d \cdot F = Ad \cdot f$; $= \frac{1}{2}(AK\phi / 4\pi) \cdot d \cdot \phi = Ad \cdot K\phi^2 / 8\pi$; $= Ad \cdot 2\pi\sigma^2 / K = Ad \cdot \mathbf{i}^2 / 8\pi K = AKE^2 / 8\pi d = Ad \cdot 2\pi Q^2 / A^2K = Ad \cdot \phi\mathbf{i} / 8\pi = Ad \cdot \phi\sigma / 2 = Ad \cdot \sigma\mathbf{i}/2K$. Energy of Field per cm. cube $= f = F / A = K\phi^2/8\pi = 2\pi\sigma^2/K = KE^2/8\pi d^2 = 2\pi Q^2/A^2K = \phi\mathbf{i}/8\pi = \phi\sigma/2 = \sigma\mathbf{i}/2K = \mathbf{i}^2/8\pi K$. Capacity or Permittance $C = Q/V = KA/4\pi d$. Dielectric Elasticity $= \phi/\sigma = 4\pi/K$.

The fundamental equations for the above relations are: $\phi = E/d$; $f = 2\pi\sigma^2/K$; $\mathbf{i} = K\phi = 4\pi\sigma$; where σ is measured in the usual air-units. If in the above expressions we make $K = 1$, we obtain the ordinary **air-equations.**

Observation of Differences of Potential.

Observation of Differences of Potential is effected by means of instruments called Electroscopes and Electrometers; the former indicate the nature, the latter measure the amount, of differences of potential.

Gold-Leaf Electroscope. — A glass flask with a vulcanite stopper: through the stopper passes a metal rod, surmounted by a metallic sphere or plate, and terminated below by a pair of freely-suspended strips of gold leaf. If the metallic part of the electroscope be charged by contact with an electrified body, the gold leaves, becoming similarly charged, repel one another, and diverge, slightly if the charge be feeble, widely if it be great. The electroscope may also be temporarily charged by induction: a + electrified body brought into the neighbourhood of the sphere or plate causes that sphere or plate to become negatively, while the more distant gold leaves within the flask are positively charged. If, while the electroscope is electrified by induction, its upper extremity be momentarily touched by the experimenter, the gold leaves collapse, for their charge escapes to the earth: the plate or sphere, however, retains its charge, and when the induc-

ing body is removed, the opposite charge borne by the sphere or plate becomes free to distribute itself over all the metal of the electroscope, and the leaves again diverge, for the instrument is now permanently charged.

If the electroscope be permanently charged, the approach of a body similarly charged will cause a farther divergence of the leaves: the approach of a body oppositely charged will cause the leaves to repel each other with less force: whence the nature of the electrification of a given charged-body can be ascertained.

The deficiencies of the electroscope are: that its indications are qualitative, not accurately quantitative; and that the glass does not thoroughly screen the gold leaf from the direct inductive action of external charged-bodies. In order to obviate the latter defect, the inner surface of the flask is sometimes lined with perforated tinfoil, or the whole is surrounded by a cage of wire gauze.

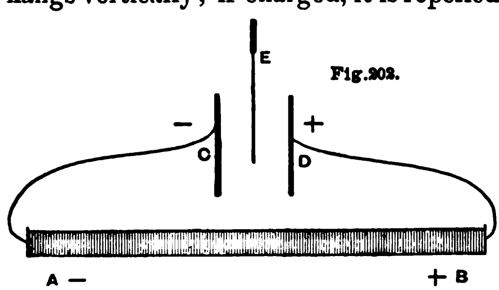
The gold-leaf electroscope is a development of earlier instruments, in which straws, plain balls of elder-pith, or gilt pith-balls, were employed.

In the discharging electroscope the gold leaves, when they diverge, come in contact with two metallic uprights which communicate with the earth: they are thus discharged and collapse, again to be charged: the number of oscillations of the gold leaves affords a rough measure of the quantity of electricity borne by a conductor which is discharged to earth through such an electroscope.

Peltier's Electroscope. — A vertical brass ring, insulated; attached to its inner circumference, at the lowest point, a vertical pointed rod; on the pointed rod is poised either a magnetic needle or else a metallic rod whose direction is determined by a small magnetic needle attached to it. The whole is turned round a vertical axis until the ring and the poised metallic rod lie in the same plane. If the ring be charged, the charge is shared with the poised metallic mass, and the ring and the poised mass repel one another; the latter swings round until the force of electrical repulsion is balanced by the tendency of the magnet to point to the magnetic north and south. This instrument may, by imparting to it a series of successive known charges, be so graduated as to act as an electrometer.

Bohnenberger's Electroscope. — Two vertical dry piles (p. 622), the one with its + pole, the other with its − pole uppermost; between these oppositely-charged uppermost poles there is a field of force, within which a strip of gold leaf is suspended. If uncharged, this strip hangs vertically; if charged, it is repelled by one pole and attracted towards the other.

Fig. 202.



Instead of two piles, the two extremities of one and the same dry pile may be used to make such an electroscope. In Fig. 202, AB is a dry pile whose poles are connected with the metallic plates

C and D, between which there is thus formed a field of force, in which the gold leaf E is suspended.

On the same principle the **Quadrant Electrometer** of Lord Kelvin is based. In Fig. 203 the two opposite quadrants A and D are connected with one another by wire, but are insulated from B and C. A and D are thus at the same potential, while B and C are also at the same potential, — a potential which may differ from that of A and D. A and D may be brought to the potential of the earth by means of a wire connected with gas or water pipes; B and C may be brought to the potential of any given object by connecting them with it by means of a wire. The quadrants A, D, and B, C, are thus at different potentials, and a metallic needle—an aluminium needle of a flat dumb-bell

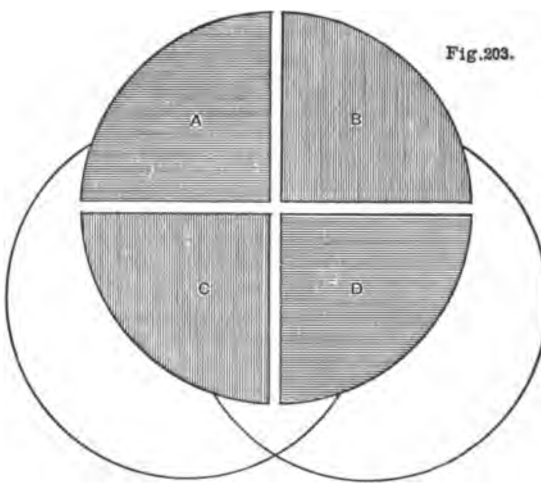


Fig. 203.

shape — will, if it be suspended symmetrically over the quadrants by means of two threads arranged parallel to one another, and if it be kept charged by constant connection with one coating of a Leyden jar (which may be replenished when necessary), impose a certain amount of torsion upon those two suspending parallel threads; the amount of this torsion will indicate the nature and — approximately — the amount of the difference of potential between the two pairs of quadrants, and therefore between the earth and the object whose Potential is to be measured.

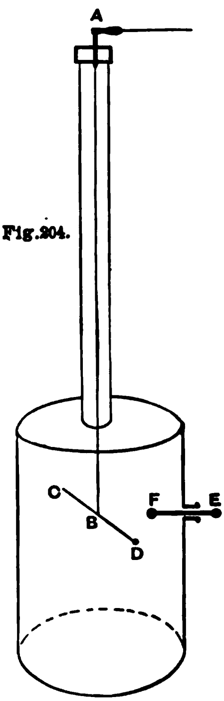
If the quadrants be made hollow, and the needle suspended within them, the arrangement is better adapted for electrometric purposes.

The whole arrangement is well adapted for testing the adjustment to equality of the potentials of two bodies.

It would come to the same thing if the potentials really measured were those of the air in the neighbourhood of the quadrants, provided that the quadrants be all of the same metal, or that the potential of the air in the neighbourhood of one uncharged metal be the same as that in the neighbourhood of another.

The amount of deflection of the suspended needle may be observed by connecting with it a very light mirror, upon which a very narrow beam of light shines; as the needle is deflected, the beam of light reflected from the mirror is deflected through an angle twice as great as that of the deflection of the mirror; and the beam of light, if received upon a distant scale, thus acts as a weightless pointer.

Fig. 204.



Upon the scale the deflection of the spot of light may be read off; that deflection is, on a straight scale, proportional to the tangent of twice the angle of deflection of the mirror: for small angles it is nearly proportional directly to twice the angle (Fig. 161).

Coulomb's Torsion Balance. — A long, vertical, slender, hard-wire or silk-fibre AB, Fig. 204, by which there is suspended in a horizontal position a thin counterpoised rod of glass or shellac, CD, which bears at one of its extremities a little gilt sphere D. In one position of the suspending wire the gilt sphere D comes into contact with a sphere-ended metal rod EF: this rod projects through the walls of the glass case in which the whole is encaged, and is therefore insulated. This metal rod terminates externally in a sphere E, which may be charged by contact with an electrified body, such as a proof-plane. A proof-plane is a small metallic disc provided with an insulating glass or ebonite handle. It is used by laying the disc upon the surface of an electrified body: when the disc is withdrawn, it bears with it a charge proportional to the charge previously borne by that part of the surface of the electrified body with which it had been placed in contact: it is then made to touch the sphere E of the torsion balance. EF being charged, the two spheres F and D, when they come in contact, become charged with electricity of the same kind, and repel one another: they do this until there is equilibrium between the electric repulsion and the torsion of the suspending wire AB. The proof-plane may be used directly in the place of EF; and instead of a proof-plane a proof-sphere may be used when the curvature of the body, whose charge is to be examined, is but small.

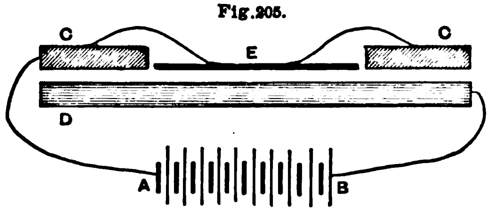
Different charges may be compared by comparing the amounts of torsion necessary to bring the two mutually-repellent bodies, D and F, to equal distances. A preliminary charge is given to the ball D; a charge Q of the same kind is imparted to F, or brought in by a proof-plane or a proof-sphere. Let the repulsion, between Q and the charge on D, be such that the suspended

horizontal fibre makes an angle FBD of 10° with that position in which D is in contact with F, while the upper end A is twisted in the contrary direction — so as, as it were, to tend to force F and D together — through an angle of 410°; the total torsion of the wire AB is 420°. Now remove the charge Q and substitute a charge Q′; the index at A indicates 95° of rotation there when D is in its former position: the total torsion of the wire is now 105°. The charges Q and Q′ are proportional to the torsions which their repulsions balance; and Q : Q′ : : 4 : 1.

Coulomb also made use of the method of oscillations (p. 39): he swung an electrified needle in presence of an electrified ball; the periods of the oscillations varied as the distance; but the period varies inversely as the square root of the force acting: therefore the force acting varies inversely as the square of the distance. When the distance is kept fixed, the charges of the needle or ball being varied, the periods of the oscillations vary inversely as the square root of the varied charge.

The **Absolute Difference of Potential** between two bodies may be ascertained by measuring the attraction between two metallic plates which are respectively connected by metallic wires with the two bodies in question. In Fig. 205 AB is a galvanic battery, the extremities of which are permanently at different potentials: it is desired to find the difference between these potentials. Connect A and B with the plates C and D. The field of force between C and D is uniform at its centre. D is fixed; but E, the central part of C, is movable. The attraction between E and D may be measured by observing the distortion of a spring which tends to pull E upwards while the electrical attraction tends to pull E downwards, this observation being made when the distance of D is so adjusted that the lower surface of E is flush with that of C. It is sometimes found advantageous in the use of instruments of this kind to connect D alternately with B and with the earth: the spring tends to become differently distorted in the two cases, and in order to render its distortion equal in both cases the distance of D must be varied. The amount of approximation or retraction of D may be measured by a micrometer-screw.

Fig. 205.



The spring which keeps up E against the attraction of D may be replaced by transforming E into one pan of a delicate balance, of which the other pan may be loaded with known weights.

The potential of E is V_E; that of D is V_D: the difference of potential to be measured is $(V_E - V_D)$. The traction across the field, along the lines of force, or the pull of the field upon the plate E, is $\frac{1}{2}\phi\sigma$ per sq. cm., or $\frac{1}{2}\phi \cdot A\sigma$ upon its whole area A. This is equal to $2\pi\sigma^2 \cdot A$: and it is balanced by F, the stress upon the spring; $F = 2\pi\sigma^2 \cdot A$. Also, $4\pi\sigma = \phi = (V_E - V_D)/d$. Hence the Difference of Potential in absolute measurement is $(V_E - V_D) = d\sqrt{8\pi F/A}$, in which expression d, A, and F can be directly measured; d being the distance between the plates E and D.

Since $\sigma = (V_E - V_D)/4\pi d$ per unit area, the charge on the attracted circular disc of radius r is $(V_E - V_D)r^2/4d$: the capacity of the system is therefore $r^2/4d$, and can thus be measured absolutely. Standards of Electrostatic Capacity can thus be constructed.

When E and D are connected with A and B, the respective potentials of E and D are V_A and V_B; and $V_A - V_B$, the difference of potential between the ends of the pile, $= d\sqrt{8\pi F/A}$. When D, instead of being connected with B, is connected with the earth, its potential becomes zero; and when D (movable in this case) is brought by its micrometer-screw into such a position that the plate E again assumes a position flush with the fixed guard-ring C, the stress F upon the spring is the same as before, and $V_A - V_{Earth} = V_A - 0 = d_{,}\sqrt{8\pi F/A}$, where $d_{,}$ is the new distance between the plates E and D. Hence $V_B = (d_{,} - d)\sqrt{8\pi F/A}$; and the Potential of B is easily measurable, for $(d_{,} - d)$, the change of distance between E and D, is much more easily measurable than d, the absolute distance between them.

Lord Kelvin, to whom the above method is due, has also devised instruments by which the difference of potential (or voltage) of electric lighting currents may be electrostatically measured. This is effected by observing the extent to which an aluminium strip, charged, succeeds in rising up from its position of gravitational equilibrium, in order to place itself immediately between two fixed aluminium plates, oppositely charged (Kelvin's Electrostatic Volt-Meters). If the electrifications be reversed, the attraction is the same; and if they be rapidly alternated, the general action of the instrument remains the same. These instruments are graduated so as to measure the potential-difference in Volts, not in electrostatic C.G.S. units.

Production of Difference of Potential.

The principal source of Difference of Potential is commonly stated to be the **Contact of dissimilar surfaces** — that is, either of different substances or of two pieces of the same substance whose surfaces are in different conditions. A piece of resin and a piece of glass will, upon contact, be more difficult to pull asunder than two pieces of resin or two pieces of glass: and if they be rubbed together, so as to multiply the points of contact, the effect is multiplied. When pulled asunder, two such bodies are found to be charged equally and oppositely: across the surface of contact there has been a Separation of positive from negative electricity. The development of electrical condition is thus necessarily a phenomenon of continual recurrence: and

it greatly influences the adhesion of one body to another. In all probability, wherever there is friction, the energy ultimately converted into heat is, in the first place, converted into the energy of electrical separation.

When two substances have different molecular velocities at their common surface of mutual contact, the molecules hamper one another and energy is lost: this energy, formerly that of molecular motion, now takes the form of the energy of electrical displacement. Within the interior of a homogeneous body the same thing must happen between colliding molecules whose velocities are different; but, all being alike, and the average molecular velocity being the same throughout the mass, there is on the whole no effect.

Non-conductors in contact become electrified; but only on their surfaces of actual contact. When they are separated their final discharge is incomplete, and the residual charges — their superficial distribution being restricted to those parts of the surfaces which have been most nearly in actual contact — are small in quantity but of great density, and therefore of high potential; and as these charges are not diffused by conduction over the whole surface, their potentials remain high after separation.

When sulphur is melted in a glass test-tube, after cooling the sulphur is found to bear permanently a negative, the glass a positive charge.

In the following series, due to Faraday, each member becomes positively charged when rubbed on one following it, negatively when rubbed on one preceding it: Cat and Bearskin — Flannel — Ivory — Feathers — Rock Crystal — Flint Glass — Cotton — Linen — Canvas — White Silk — the Hand — Wood — Shellac — the Metals (+ Fe, Cu, Brass, Sn, Ag, Pt) — Sulphur — (Soapstone). There are certain irregularities here to be observed: for example, a feather lightly drawn over a piece of canvas becomes negatively electrified, whereas if it be drawn through a pressed fold of canvas it becomes positively charged.

The separation of electricities by contact and friction is utilised in the various forms of electric **frictional machines**, which range in complexity from a simple piece of sealing-wax or a glass rod rubbed with a catskin or a silk handkerchief, or a stout glass tube rubbed with a piece of dry flannel, to a machine in which a glass or vulcanite disc or cylinder, set in rotation, rubs against silk rubbers: these rubbers, whose conductivity is improved by anointing them with a mixture of fat and mercury, communicate with the ground, and their negative electricity is thus carried off to the earth; the positive charge, borne by the rotating glass or vulcanite, blends with a negative charge developed by induction in the tips of a comb-like series of sharp metallic points which come almost in contact with the rotating glass; while the complementary induced positive-charge is conveyed either to a large insulated Conductor connected with these points by a metallic chain or wire, or to the surface of a large insulated hollow conductor which surrounds the rubbing parts of the machine, or to the inner coat of a Leyden

jar, or to the inner coat of one of the constituent members of a battery of Leyden jars. A charge of positive electricity may be thus accumulated. If, on the other hand, the positive charge of the glass be conveyed to the earth, while the insulated conductor is metallically connected with the rubbers, a charge of negative electricity may be accumulated in the conductor. If the conductor, in which positive electricity is being accumulated, be connected by wire with the negatively-charged rubbers, a current of electricity will pass along the connecting wire so long as the machine is worked, and that wire will be heated. If this current be sent through a second electric machine it will tend to cause in it a reversed rotation. It is possible (Gaugain) thus to produce continuous currents by the friction even of dissimilar metals.

When two **metals** come in contact, **in air** or other gas, they at once become electrified, positively and negatively respectively. The amount and kind of charge on each metal depends upon (1) the nature of the metals, (2) the condition of their surfaces, (3) their temperatures, and (4) the nature of the surrounding or intervening gas, if there be any. In the case of copper and zinc in air, the **copper** becomes **negatively** and the **zinc positively charged.**

The older view as to this was, that there was electrical separation at the surface of contact between the metals, and that each metal was at an equal potential throughout: then the potential of the air in the immediate neighbourhood of the metals was the potential of the metals themselves. The newer view is that the metals are, before contact, by reason of a tendency to chemical action (oxidation, etc.) each at a potential different from that of the surrounding air or gas; that their respective potentials are more or less different from one another; that when the two metals are brought into contact, the potential throughout the whole conjoint metal becomes uniform, and a momentary current runs in the conjoint metal mass, so as to charge the one metal (zinc) positively and the other (copper) negatively; that while this is taking place, the surrounding dielectric is being electrically displaced so as to produce a Field of Force; that the Energy necessary for this is derived from a trifling amount of chemical combination (oxidation, etc.) at the metal-gas surface; that in the case of a zinc-copper couple, before contact the potential of the air is say V_0 C.G.S. electrostatic units, that of the zinc is $V_0 - (x + 0{\cdot}0025)$, and that of the copper is $V_0 - x$; that after contact the potential of the copper-zinc couple is uniformly $V_0 - (x + 0{\cdot}00125)$, that of the air near the zinc is $(V_0 + 0{\cdot}00125)$, and that of the air near the copper is $(V_0 - 0{\cdot}00125)$; and that the Field of Force

is maintained in the air, through its being a dielectric, the Ether in which offers elastic resistance to further displacement.

But there is also another effect. We have spoken of the two metals coming to the same potential when brought into contact. It appears, however, that they cannot perfectly do this, on any view of the facts, even independently of the air, except at a particular temperature. There is almost always a *slight* difference of potential, a **true contact-effect**; and this varies so remarkably with the temperature as to give rise to the phenomena of *Thermo-electricity*, of which later (p. 624).

If a metallic disc be composed of four quadrants, soldered together and consisting alternately of zinc and copper respectively, and if the disc be arranged horizontally, a needle suspended horizontally over the centre of the disc will, if that needle be charged with positive electricity, be repelled by the zincs and attracted by the coppers, and it will therefore swing round so as to lie over the copper quadrants; while if it be charged negatively it will come to lie over the zinc quadrants. The needle may be so suspended by two threads that, when uncharged, it lies along a diameter of the disc, a diameter which coincides with a line of junction between quadrants.

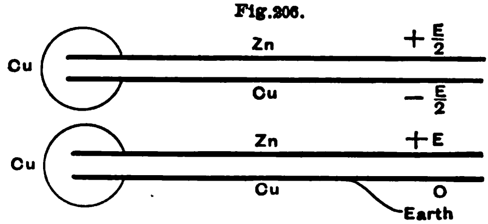
Take an electroscope surmounted by a copper plate, varnished on its upper side; upon this plate lay a zinc plate varnished on its lower side: these plates, separated by the varnish, act as a condenser. Bring a copper and a zinc plate, both of which are unvarnished and insulated, into contact: separate them; with the zinc touch the zinc, with the copper the copper of the condenser. Repeat this operation several times: then remove the zinc plate of the condenser: the copper is found to be strongly charged with negative electricity, while the zinc plate removed is positively charged.

Copper filings falling through an insulated zinc funnel, as they leave that funnel carry with them a negative charge.

These experiments may be interpreted in accordance with either of the above views.

In Fig. 206 a zinc plate Zn and a copper plate Cu, both in air, are connected by a copper wire. Then, either the zinc is at potential $\frac{1}{2}E = 0{\cdot}00125$ electrostatic units, and the copper at potential $-\frac{1}{2}E = -0{\cdot}00125$; or, according to the newer view, they are at equal potentials, while between them there is a Field of Force, the potentials within which present a potential-difference of 0·0025 units. If one of the plates be connected to earth, the potentials of the copper and zinc are altered: but the Field of Force, though its terminal potentials are altered, remains constant in its potential-fall.

Fig. 206.



Within the field of force between such plates arranged with an intervening dielectric, $\phi = 4\pi\sigma = \text{const.} = (V_{Zn} - V_{Cu})/Kd$, where d is the thickness of the dielectric, and V_{Cu}, V_{Zn}, the potentials at the copper and the zinc respectively. Hence the superficial density $\sigma = (V_{Zn} - V_{Cu})/4\pi Kd$; but if the numerator of this fraction, the difference of potential, be constant, as it is between two metals, $\sigma = \text{const.} \times (1/d)$, or $\sigma \propto (1/d)$. As the thickness d diminishes, the electrostatic capacity or Permittance of the field of force increases, and since the D.P. remains constant, σ increases. If we suppose the plates, nominally in contact, to be at a mean molecular distance of about $\frac{1}{20,000,000}$ cm., the density is so great that if the copper and the zinc could be separated from one another before their charges are allowed to recombine, they would then spark across 20 feet of air. But they could not be so removed; in air, at any rate, the striking distance falls off more rapidly than the potential-difference does; the opposed charges almost wholly discharge themselves when, after being placed in contact, the plates are pulled asunder, and there then remain in these only residual charges of small density, which vary very slightly in amount, according to the mode in which the plates are pulled asunder. The moving molecules must therefore, even though the masses in contact seem to be at rest, be constantly discharging and renewing the separation of electricities.

When the plates of copper and zinc, of Fig. 206, are connected not by a copper but by an iron wire, we have three metals and two contacts; but the difference of potential between the terminal copper and zinc is the same as when the copper and zinc were directly in contact; and, more generally, if any number of metals be arranged in linear series, in Open Circuit (say n metals or pieces of metal, with $n - 1$ contacts), there is a difference of potential developed (in the air or in the metals themselves, according to the view adopted) between the terminal metals; and this difference of potential is equal to that which would have been developed between these two terminal metals if they had touched each other directly.

If we make both terminals of the same metal, there will thus be no difference of potential between them; and if we connect these similar terminals with one another, we have now a Closed Circuit consisting of various metals. The potential will vary from part to part of the circuit: according to the one view it will vary in the metal, each metal being equipotential throughout; according to the other, it will vary only in the surrounding air: but on either view, taking it all round the circuit, there is no preponderance of potential-difference in the one direction or in the other, and there is no current round the circuit. The closed circuit remains in electrostatic equilibrium. No continuous current can, therefore, be obtained from a closed metallic circuit, or indeed from any closed circuit of conductors in which

the material of the circuit suffers no alteration, unless Energy be supplied, either from without or else at the expense of the energy (*e.g.*, the Heat) of the circuit itself.

If, on the other hand, one of the conductors of the circuit suffer a chemical change, Energy may be liberated, which may take the form of the Energy of a Continuous Current. Let us consider a circuit consisting of copper — hydrochloric acid — zinc — connecting wire — copper. Since it does not matter what the material of the connecting wire may be, we may use copper; the circuit is then Cu — HCl — Zn — Cu. If all the members of this series were mere conductors, the circuit as a whole would attain a condition of electrostatic equilibrium, and there would be no current. But they are not all mere conductors; in the circuit we have **Chemical Action,** which results in the liberation of Energy; and this energy is transformed into that of a continuous current round the circuit.

If a piece of zinc and a piece of copper be placed in hydrochloric acid, but **not in contact** with one another, the **zinc** becomes charged, **negatively** with respect to the copper, and the **copper positively** with respect to the zinc.

It is not said, observe, that either of the metals is absolutely positive or negative in its potential. The fact appears to be that the potential of the zinc is negative to that of the acid by about 0·0060 electrostatic units of potential, while that of the copper is also negative, but to a less extent, namely, about 0·0035 units.

This phenomenon has been thus explained. In the aqueous solution of hydrochloric acid, the HCl molecules are already broken up into H and Cl atoms or ions; these atoms are permanently charged, the hydrogen positively and the chlorine negatively, each with definite quantities of electricity. Then, whatever may turn out to be the cause of chemical affinity, there is no reason to doubt that chlorine atoms are more attracted by zinc than hydrogen atoms are. On the whole, there is a certain amount of combination between the chlorine and the zinc, with production of zinc chloride; but the negative charge of the chlorine ions is communicated to the remainder of the metallic zinc, which thus acquires a negative electrification. There thus arrives an ultimate condition of equilibrium, in which the electrified zinc repels the similarly-charged chlorine atoms as much as it attracts them chemically. Whether this be a correct explanation or not, it seems clear that all chemical action ceases (if the zinc be perfectly pure and homogeneous) when the zinc is at a potential lower than that of the acid by 0·0060 electrostatic units. Similarly, the copper is at a negative potential, and for the same reasons; but the chemical affinity is less, and equilibrium is reached when the potential of the copper is 0·0035 units below that of the hydrochloric acid.

It has been considered possible that a similar charging of metals by dissociated atoms of oxygen may account for the phenomena observed in air.

The acid itself is at zero potential. The separation of electricities takes place across a thin film of liquid between the mass of the metal and the bulk of the liquid; and electrostatic equilibrium is attained.

Now connect the zinc and the copper by a thick wire which offers little or no resistance or obstruction to the equalisation of potential between the copper and the zinc, and which passes out into the surrounding air, and back; the potential in the whole connected metal becomes approximately uniform. But when this happens, the circumstances are precisely analogous to those presented when copper and zinc are brought into **contact** in air; the acid or electrolyte here tends to become a Field of Force, the extremities of which are (in the instance supposed) at potentials about $+0{\cdot}00125$ and $-0{\cdot}00125$ respectively. The whole potential-slope now tends to exist within the electrolytic Field of Force, that is, within the acid.

If, now, the electrolyte had been a dielectric or insulator, the whole arrangement would have attained a condition of electrostatic equilibrium, and have maintained that; but this is not the case. The insulation of the electrolyte breaks down; but its Field of Force tends to be continually restored at the expense of the energy of chemical combination. This tendency to continual breaking-down and re-formation of the Field of Force results in the continuous passage of an electric current through the acid in the direction zinc to copper, and **along the connecting wire** in the direction **copper to zinc**; this current is kept up until there is either no more hydrochloric acid in solution or no more zinc to be dissolved; and the total Energy of the current is equal to the Heat which would have been evolved if the zinc had been directly dissolved in the hydrochloric acid.

If the copper had been associated with platinum instead of zinc, the copper surface would have been more negative than the platinum surface, and chemical action would have been manifest at the copper surface: but when copper is used in conjunction with zinc, the tendency to chemical action at its surface is reversed by the actual current, passing in a direction opposed to that of the current which chemical action on the copper would itself tend to produce. So far is this the case that Energy, instead of being given out at the copper surface, is absorbed there, and appears at that surface in the form of Heat; and this is, like the rest of the energy of the current, ultimately derived from the energy of chemical combination of the zinc.

The galvanic cell and circuit is thus a kind of engine, in which Energy is absorbed from a Source, partly returned to a condenser or Sink, and partly converted into the Energy of Electric Current.

If there be any obstruction or Resistance, or any absorption of energy, between the copper and the zinc, so that these fail to come readily to the same potential, then the fall of potential in the electrolyte is diminished to a corresponding extent; but the difference of potential between the zinc and the acid, and that between the copper and the acid, remain unaffected by this; and the effective potential-fall in the electrolyte, *plus* that along the connecting metal, is equal to the difference between the potentials at the copper and the zinc, when these are in the acid but not in contact or metallic communication with one another.

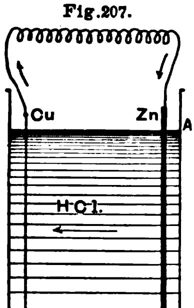
The potential-falls and differences referred to are slightly modified by the minute rise and fall of potential which occur at the junction of dissimilar metals, due to the true contact-effect.

Different metals have different chemical affinities for different chemical fluids; and consequently the amount and even the direction of the electromotive difference of potential within a circuit of the kind described depends not only upon the nature of the metals, but also upon the nature of the fluid or electrolyte employed. Copper and iron in dilute sulphuric acid give a current running along the conducting wire from copper to iron, and the iron is attacked, not the copper: in a solution of sulphide of potassium the copper is attacked, and the current runs along the wire from iron to copper. In the presence of facts of this order the theory must as yet be considered incomplete, for chemical affinity remains unexplained.

For each liquid it is possible to make up a table of relative potentials. In dilute sulphuric acid the series is, commencing with the most negative:—Amalgamated zinc—ordinary zinc—cadmium—iron—tin—lead—aluminium—nickel—antimony—bismuth—copper—silver—platinum.

A circuit of the kind just described is a **Galvanic Circuit.** In Fig. 207, A is a glass vessel containing hydrochloric acid in solution; Cu is a plate of copper, Zn a plate of zinc. The two metals are connected by a wire of any conducting material: the current runs in the direction copper — conducting wire — zinc — acid — copper. Excluding the connecting wire, such an arrangement is called a Galvanic Element or Cell: while a number of such cells may be arranged so as to form a Galvanic Battery or Pile.

Fig. 207.

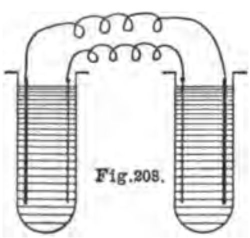


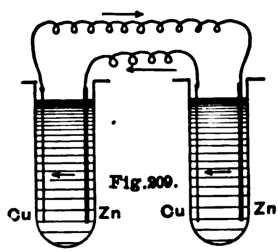
The total E.M.D.P. within a galvanic circuit or battery is measured by the electro-

static difference of potential between the free extremities of an open circuit, with terminals of the same metal at the same temperature; such an open circuit might be obtained by cutting through the conducting wire of Fig. 207 (compare p. 643).

Sometimes the **copper** end of a cell or battery is said to be negative, perhaps because copper itself is "electronegative" to zinc in contact with it in air;* sometimes it is said to be **positive,** because if there be any resistance at all between the copper and the zinc — and there must always be some — it is positively charged relatively to the zinc end, and **because the current flows from it** along the wire to the zinc. The reader will please clearly understand that in this volume the **latter** of these expressions is employed.

If two equal galvanic cells be set against one another, as in Fig. 208, no current is produced: the aggregate E.M.D.P. within the entire circuit is equal to zero.





If there be the slightest difference between the chemical constitution of the two liquids in the two cells, a feeble current will pass. (Gore's Voltaic Balance.)

If a cell containing copper and zinc in dilute sulphuric acid be set in this way against one containing platinum and zinc in the same liquid, the E.M.D.P. is the same as that of a single cell containing platinum and copper in dilute sulphuric acid.

If two cells be coupled side by side, copper to copper, zinc to zinc (*i.e.* "in Surface" or "in Parallel"), and if a conducting wire be led from any one of the coppers to any one of the zincs, the whole acts like one cell of double surface, and the E.M.D.P. within the circuit is not increased. So also for *n* cells so arranged.

If two cells be set behind one another, as in Fig. 209, copper being connected with zinc, the difference of potential between the first copper and the last zinc is twice as great as that

* Also because in the earliest forms of Volta's pile there was a superfluous zinc at the copper end, and *vice versâ*. The current then flowed from the apparent zinc end to the apparent copper end.

between the copper and the zinc in a single cell: or if n cells be arranged one behind the other tandem-fashion, in Indian file or "in Series," the copper of each being connected with the zinc of the next in regular succession, the effective difference of potential is n times that of a single cell.

If the battery be immersed in a conducting medium, the electricity escapes by its sides and ends, and establishes return-paths through the medium surrounding it.

Principal Forms of Galvanic Cells and Batteries.—These may be divided into two principal classes: (1) those which have in each cell one fluid; (2) those which have in each cell two fluids.

One-fluid cells and batteries.—Copper, sulphuric acid (diluted), and zinc, form the most commonly used triad of materials. Volta's pile; a number of repetitions of the sequence:—Copper plate, cloth dipped in water or acid, zinc plate: the terminal copper is positive, the terminal zinc negative. Volta's corona di tazze: a number of cups containing dilute sulphuric acid, in each of which are placed a plate of copper and a plate of zinc, not in contact with one another: each copper is connected with the zinc of the preceding cup. For practical purposes this is made in guttapercha-lined boxes divided into cells by partitions which are themselves made of copper on one side, zinc on the other; and to avoid spilling, the whole may be filled up with sand or stuffed with asbestos. The form of a single cell may vary; a cylinder of zinc placed within an open-ended hollow cylinder of copper, but not in contact with it, the whole being immersed in acid: a copper cylinder within a similar hollow-cylinder of zinc (Oersted); a sheet of copper and a sheet of zinc separated by flannel, rolled up and immersed in acid (Hare's Deflagrator); a larger piece of copper, bent so as to face both sides of a smaller sheet of zinc, and thereby to diminish the "resistance" within the cell (Wollaston). In all these cases the difference of potentials between the free extremities of an *open circuit* with similar terminations, but containing a battery of n cells, is, when we employ pure copper, pure zinc, and a 2% solution of pure sulphuric acid in water, about $\frac{921}{300000}n$ C.G.S. electrostatic units, or $\cdot 921n$ "Volts;" if dilute hydrochloric acid of the same strength be used, about $\frac{753}{300000}n$ C.G.S. units or $753n$ Volts.*

Instead of ordinary zinc, zinc whose surface is amalgamated may be employed: it is not corroded by the acid except while the current is passing, and the difference of potential within the circuit is raised about ·13 Volts for each cell of the battery. Ordinary zinc wastes away when left in acid, because it is not homogeneous; local differences of potential are set up in it, and local circuits are formed. Zinc may be amalgamated by setting it to stand in contact partly with mercury, partly with dilute hydrochloric acid, or by rubbing mercury into it with a rag dipped in acid, or by dipping it in a liquid prepared by dissolving 200 grms. Hg in a mixture of 250 grms. of nitric and 750 grms. of hydrochloric acid, and, when the solution is clear, adding 1000 grms. of hydrochloric acid; or by mixing Hg, 4%, with the melted zinc on casting. Iron or platinum can be wetted by mercury containing sodium.

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* 1 Volt = $\frac{1}{300}$ C.G.S. Electrostatic Unit of Difference of Potential, nearly.

The difference of potential set up by single-fluid batteries diminishes seriously when their action is prolonged, in consequence of their so-called **Polarisation.** Hydrogen is liberated at the copper or positive plate, and remains there as a film; this hydrogen is positively charged, and tends to repel all other atoms of hydrogen, and to attract the negative components of the fluid. In consequence of this there is a certain tendency towards the production of a current opposed to the main galvanic current: and if a copper-zinc couple, which has been for some time in action, be taken out of sulphuric acid and immersed in water, a reverse current, comparatively feeble, will run for some time from the zinc to the copper through the conducting wire. In order to minimise this polarisation various devices have been resorted to: the hydrogen has been swept off the positive plate by air from a bellows (Grenet), or by shaking the cell, or by rapidly rotating the positive plate in the fluid (Mocenigo); or it has been removed by covering the positive plate with a film of oxide of copper, which is reduced by the hydrogen (Becquerel), or by covering the positive plate with a film of clay (Pulvermacher), or by otherwise roughening its surface so that bubbles of hydrogen may readily form and rise; this was done by Poggendorff, who electrolytically deposited a rough film of copper on the positive copper plate, and by Smee, who used a similarly-platinised platinum or silver or lead plate as the positive plate. Platinised iron (Paterson), amalgamated iron (Münnich), and platinised charcoal (Walker), have been recommended as positive plates. Bunsen used gas coke with dilute sulphuric acid and amalgamated zinc.

For the negative plate zinc is used, because it is very readily oxidisable, convenient, and moderately cheap. Magnesium would give a higher effective difference of potential, but is too expensive: iron gives with copper too feeble a current, but may be in some cases advantageous as compared with the more expensive zinc, although to obtain a given current by its aid a greater number of cells is required.

For the intervening fluid or electrolyte, instead of sulphuric or hydrochloric acid other liquids may be employed, which oxidise the hydrogen liberated at the positive plate. Nitric acid oxidises hydrogen, being itself reduced to nitrous acid: a solution of iodine with iodide of potassium in water (Laurie) forms with it hydriodic acid: chromic acid is reduced by the hydrogen to chromic oxide: instead of chromic acid a mixture of bichromate of potash and sulphuric acid may be employed, and the reaction then is

$$3Zn + K_2Cr_2O_7 + 7H_2SO_4 = 3ZnSO_4 + \underset{\text{Chrome-alum.}}{2KCr(SO_4)_2 + 7H_2O}$$

A common bichromate-cell, in which gas-carbon, bichromate-mixture — 1000 water, 100 $K_2Cr_2O_7$, 300 pts. by wt. H_2SO_4 (Grenet); 6·182 grms. $K_2Cr_2O_7$, 6·282 cub. cm. strongest H_2SO_4, 60·47 cub. cm. water (Bunsen); with the addition (Ducretet) to each litre of about 2½ grms. of $HgSO_4$ in order to keep the zinc well amalgamated — and zinc are employed, gives a difference of potential of about 2 Volts, which remains fairly constant when the circuit is closed for about three-quarters of an hour, but which in an hour and a half sinks to about 1 Volt.

Chloride of ammonium, chloride of zinc, used as exciting fluids, also tend to check polarisation. In Leclanché's cell the materials are zinc, a solution of chloride of ammonium, and a positive plate: this plate consists in the older Leclanché cells of a mixture of moistened binoxide of manganese and

crushed gas-coke surrounding a central rod of carbon, all in a porous pot; and in the newer, of a mixture of 55 gas-coke powder, 40 binoxide of manganese, and 5 shellac, pressed upon a carbon core. The action is $Zn + 2NH_4Cl + 2MnO_2 = ZnCl_2 + 2NH_3 + H_2O + Mn_2O_3$. The zincs of Leclanché batteries are very little corroded when they are not in use; hence these cells are much used for occasional telegraphy or bell-ringing; and for therapeutic purposes a large number of such elements, each the size of a small test-tube, can (Beetz) be packed within a very small space; but elements of this kind cannot be used for continuous work, unless the resistance of the circuit be extremely high, because they polarise very rapidly. In Gassner's dry cell, hydrated ferric oxide is used as a depolariser; in presence of the NH_4Cl it gives up oxygen.

In some other cells the electrolyte is somewhat complex, and there are differences of potential, due to chemical action, set up even within it. A cell whose metals are silver and zinc, separated by an intervening mass of chloride of silver moistened with or lying within a solution of alkaline chloride (Warren de la Rue), chloride of zinc (Gaiffe), or alkali (Scrivanoff), gives a very constant and, relatively to the bulk of the cell, a powerful current. The difference of potential in a silver — salt-water-and-chloride-of-silver — zinc cell is (Warren de la Rue) 1·065 Volts. Becquerel used salt water and sulphate of lead between zinc and lead.

Two-fluid cells. Daniell's cell:—A hollow cylinder or cell of porous ware, as thin as practicable, and containing dilute sulphuric acid and a rod of zinc, is surrounded by a saturated solution of sulphate of copper and a larger cylinder of copper. The current runs through the fluid in the direction $Zn - H_2SO_4 \parallel CuSO_4 - Cu$ (where the symbol $\parallel$ is used to indicate the porous cell), and through the conducting wire as usual from copper to zinc. The chemical action, which may be expressed by the diagram

$$- \mid \overline{Zn \quad SO_4} \; \overline{H_2 \parallel SO_4} \; \overline{Cu \quad Cu} \mid +$$

results in the formation of sulphate of zinc within the porous cell, sulphuric acid near or within the walls of the porous cell, and the deposition of copper upon the inner surface of the copper cylinder. There is thus no evolution of hydrogen, and no polarisation. The effective difference of potentials is 1·124 Volts when the liquids employed are a neutral saturated solution of sulphate of zinc and a saturated solution of copper sulphate, and when the zinc is amalgamated and the copper electrolytically deposited; and this does not vary very much with the strength of the solution of sulphate of zinc, nor does it do so to any great extent though the temperature of the cell rise from 3° to 70° C.; and it is equal to almost exactly 1 Volt when the liquids used are, the one a solution of sulphuric acid, 1 vol. to water 22 vols., and the other a saturated solution of nitrate of copper. The "internal resistance" sinks (Preece) to one-third when the cell is heated to 100° C. Batteries of this construction were originally due to Becquerel; and they are very constant, lasting even for months if the resistance in the circuit be kept very great; but if the external resistance be small, as where the copper and the zinc are connected by a short piece of wire, the current produced rapidly falls off.

The solution of sulphate of copper is kept saturated by crystals placed in it. Any metal, if "electronegative" to zinc, can be used as a positive plate, for it soon becomes covered with copper.

In some forms of Daniell's cell the porous cell, which is fragile, and

which tends to have its pores blocked up, is dispensed with: in gravity batteries — *e.g.*, Callaud's — a stratum of acidulated water or of a solution of sulphate of zinc floats upon a denser solution of sulphate of copper: in the former stratum the zinc is suspended; in the latter the copper lies. Sometimes, as in Minotto's battery, the copper is protected by sand or sawdust, beneath which a layer of copper-sulphate-crystals rests upon the copper.

In Maidinger's cell the crystals lie in a special inverted flask filled with zinc-sulphate solution; the heavy solution in this flask sinks down whenever the density of the lowest layer, the solution of sulphate of copper, diminishes in consequence of the deposition of its copper upon the positive plate.

These cells without porous diaphragms are liable to diffusion of the copper-sulphate-solution upwards into the upper layer, the solution of sulphate of zinc: the zinc suspended in this is attacked, and a film of copper is deposited on it, which interferes with the efficiency of the cell. Cells of this kind are therefore good only for frequent use, such as tends to exhaust the layer of sulphate of copper solution.

The copper plates submerged in the lower layer of liquid are connected with the external circuit by wires passing down through the whole liquid, and protected by an insulating covering of guttapercha.

In Remak's portable form of Daniell's battery, discs are arranged in the following sequence: — Copper plate, cloth dipped in solution of copper sulphate, porous earthenware disc, cloth dipped in dilute sulphuric acid, zinc plate, copper plate, etc.

In Beetz's dry Daniell-cell, which is exceedingly constant even when the circuit is kept closed, a U-tube has one limb filled with plaster-of-Paris or gelatine made up with sat. $ZnSO_4$ soln. and containing a Zn wire; the other limb similarly with $CuSO_4$ and Cu wire. The E.M.D.P. is 1·04 Volts.

In Grove's cell the current passes through $Zn - H_2SO_4 \parallel HNO_3 - Pt$. The nitric acid dissolves the hydrogen liberated by the sulphuric acid, and is itself reduced to nitric peroxide or to nitrous acid; these, if not too abundant, are dissolved by the remaining nitric acid. The difference of potential maintained by a Grove's cell is equal to about 1·92 Volts. This is 1·708 × that of a Daniell, and the internal resistance of a Grove is much less; for a short time, and against a small resistance, a Grove can produce a much stronger current than a Daniell of the same size; but its fumes are unwholesome, noxious in a laboratory, and destructive to the binding-screws of the Grove cell itself.

Grove's cell, like Daniell's, may be made either cylindrical or flat-plated: the former is preferable, because cylindrical porous-cells are not so liable to break as flat ones.

The difference of potential maintained by a Grove mounts from 170·8 to 240 (Daniell = 100), when the dilute sulphuric acid surrounding the zinc is replaced by a concentrated solution of caustic potash.

The nitric acid surrounding the platinum is often mixed with strong sulphuric acid, which exercises a dehydrating action, takes water to itself, and keeps the nitric acid concentrated.

Instead of platinum, carbon may be used, as in that modification of Grove's cell known as Bunsen's cell, originally due to Grove; or iron, which becomes what the chemists call "positive," and is not dissolved by strong nitric acid; or, as in Callan's cell, platinised lead.

The nitric acid of Grove's cell may be replaced by bichromate-of-potash-

and-sulphuric-acid mixture. In the place of nitric acid a saturated solution of ferric chloride, to which 4 per cent of nitric acid has been added, forms an excellent liquid: when it is used, the total difference of potential kept up by the cell is about midway between that of a Daniell and that of an ordinary Grove: this liquid is readily renovated by boiling it with a little nitric and hydrochloric acid. In Fuller's cell the components are Zn – H_2SO_4 dil. ‖ bichromate solution – carbon; the zinc rod stands in mercury, which creeps up the zinc and keeps it amalgamated.

In Marié-Davy's cell the current runs through Zn – pure water ‖ paste of Hg_2SO_4 with water – carbon. Any mercury passing through the porous cell merely amalgamates the zinc and does no harm. Polarisation is great in this cell, but it is very convenient because very portable.

In Latimer Clark's Standard Cell the current runs through Zn – pure concent. $ZnSO_4$ soln. ‖ Hg_2SO_4-and-water-paste – Hg. This cell is very constant, and its difference of potential = 1·434 Volts at 15° C. It may be used as a standard for comparative electrostatic measurements of difference of potential, but it is greatly lacking in constancy if it be allowed to send a sensible current. Such cells differ among themselves by about ± 0·2 per cent, mainly because some cells take longer than others to reach their condition of equilibrium. If the paste be made of subsulphate of mercury and a concentrated solution of sulphate of zinc, the constancy is even greater, and the difference of potential is 1·4455 Volts.

The greatest difference of potential yet observed as having been produced and maintained by a single cell is that of a combination devised by Goodman in 1847. The current in this runs successively through potassium-amalgam — inner porous cell — solution of caustic potash — outer porous cell — solution of permanganate of potash — and lastly, as a positive plate, stick-sulphur. The difference of potential is (Beetz) 302·3 (Daniell = 100).

Two fluids or melted substances separated by a porous diaphragm will give a current even though plates of the same metal be immersed in both. Iron in nitric acid and iron in sulphuric acid (Grove), or copper in dilute sulphuric acid and copper in dilute nitric acid (Napoléon III.), aluminium in dilute caustic soda and aluminium in dilute hydrochloric acid (Wöhler), or platinum in caustic-potash-solution and platinum in nitric acid (Becquerel), will give a current, and as the one metallic plate is dissolved away the other is thickened. In a flask set aside and containing a lower layer of solution of sulphate of copper and an upper layer of acidulated water, together with a copper wire set to stand in the liquid, it will be found that the part of the copper wire which is within the acidulated water becomes thinned away, while that part which is within the solution of sulphate of copper becomes thickened. Further, two plates of the same metal, immersed in acids or alkalies of different degrees of concentration, will give a current which, in the case of sulphuric and hydrochloric acids, flows from the stronger through the porous diaphragm into the weaker acid, but which, in the case of caustic alkalies, flows towards the stronger solution.

In Shelford Bidwell's cell the materials are sulphurised silver — compressed Ag_2S — compressed CuS — copper. The sulphides of silver and copper, though solid, conduct electrolytically. The E.M.D.P. is 0·053 Volt, and the resistance about 7 Ohms.

Dry piles may be constructed as follows: — Pieces of "gold paper" and of "silver paper" may be pasted back to back and cut into small discs:

these discs are then piled up and pressed into a glass tube, or, better, strung upon a silk thread, their similar faces all looking in the same direction. Such a pile developes a considerable difference between the potentials of its extremities, and it remains thus charged for apparently indefinite periods. In principle a dry pile resembles a Volta's pile, the discs of wet cloth in which have almost dried up. Paper is never perfectly dry: the paper between the metallic faces of each disc takes the place of the moist discs of cloth; and besides this, the air acts more on the one metallic face of each disc than on the other. In consequence the chemical action is not *nil;* and a definite difference of potential is set up, by which chemical change, otherwise too feeble to be detected within any reasonably short period of time, is rendered strikingly manifest. The quantity of energy liberated by a dry pile is very small, and little work can be done by it; but one extremity of a dry pile can keep a charged gold-leaf steadily repelled for a long time. If the two ends or poles of a dry pile be brought near one another, an insulated strip of gold leaf suspended between them and alternately attracted by, coming in contact with, and repelled from, each pole, may oscillate between the poles for a very long time, but only so long as the chemical decompositions going on within the pile can furnish the energy requisite to overcome the friction of the air and the small rigidity of the gold leaf.

Difference of potentials is the most delicate test that we possess for chemical action.

The chemical action set up under the influence of actinic rays also produces difference of potential, which may serve (Becquerel) to measure the chemical energy of sunlight.

Difference of potential is also produced by friction of water against steam or air, as where a jet of partly-condensed steam or of suddenly-expanding undried air is driven through a conical nozzle of metal or glass or wood: the steam or air becomes positively, the vessel from which it is driven becomes negatively charged. If the nozzle be of ivory there is no charge. If the vessel contain some turpentine-oil the charges are reversed.

When a liquid is brought into the spheroidal state it assumes an electrical condition, which varies with the nature of the liquid and with that of the hot surface on which it lies.

When saline solutions are evaporated, the vapour and the liquid assume different electrical conditions if there be either friction of the crystals on the vessel, as when the crystals crackle, or friction of the heated water upon the salt. In the evaporation of water there is no difference of potential set up unless there be friction between the water and the vapour: if there be friction, the steam becomes positively charged.

Pressure or traction applied to tourmaline crystals, if the force applied have a component parallel to the crystallographic

axis, causes a separation of electricities; opposite extremities of the axis become oppositely electrified, and the amount of difference of potential produced depends only on the amount of force applied. The same result follows, whether the alteration of form of the crystal affected be the result of force or of the application of heat or cold.

Such crystals behave as if they were always in a state of electric stress interiorly, with a neutralising surface-charge; then on altering the interior stress by any means, the surface-charge no longer neutralises the stress, and the electrification becomes apparent.

Electro-capillarity. — Mercury standing under water has a convex surface and a definite surface-tension. The water and the mercury are at different potentials. The surface of the water and the surface of the mercury, though nominally in contact, are at a mean distance of about one twenty-millionth of a centimetre. The two surfaces therefore act as an accumulator which has a definite capacity. The surface of contact between mercury and water has thus three properties — Surface-Tension, Difference of Potential, and Electrostatic Capacity; and these depend upon one another, so that if one be varied the other two will vary (Lippmann). Thus if we vary the surface-curvature of mercury, as by setting it in vibration in a conical tube, and thus altering the area and the amount of tension of the surface; or if we heat the mercury or the water and thus again alter the surface-tension, the capacity and the difference of potential will also vary. Part of the work done upon the mercury in setting it in vibration, or of the heat supplied, is spent in setting up a difference of potential, the very existence of which causes a tendency to restitution of the original surface-tension; for if we vary the difference between the potentials of the water and the mercury by charging either the one or the other, the surface-tension, and consequently the surface-form, of the mercury varies also.

Thermo-electricity. — The difference of potential set up between two metals by their mere contact — that is, the **true contact-effect** — depends upon their temperature as well as upon their chemical nature and state of purity or their physical state — their hardness, their tension, and so forth. If bismuth and antimony (in the form of commercial pressed wire) develope, on contact at 19° C., a difference of potential of V Volts, the same materials develope, on contact at 20° C., a difference of potential of $(V - \cdot 000103)$ Volts. If a semicircle of bismuth wire and one of antimony wire be joined so as to form a circle, and if one of the two junctions be maintained at 19° C., while the other is kept at 20° C., then, since the colder junction presents a greater difference of potential than the hotter, the aggregate difference of potential within the circuit is not zero, but is equal to $\{V - (V - \cdot 000103)\}$ Volts $= \cdot 000103$ Volts, or 103 microvolts (millionths of a Volt). This difference of potential

within the circuit is maintained as an electromotive difference of potential, and there is therefore a constant current round the circuit, so long as the junctions are kept at these fixed temperatures; and the energy of this current is partly (but in part only) derived from the heat supplied at the hotter thermo-electric junction, and is due to transformation of the energy of molecular motion. Across the colder junction the current runs from the antimony to the bismuth: but across the **hotter** junction it runs from **bismuth to antimony.**

The bismuth is accordingly said to be "thermo-electrically positive" to the antimony, though it is really negative to it at both junctions.

The D.P. at the colder junction is only partly neutralised by the counter D.P. at the hotter; a differential effect is produced. On the whole, the cold junction is analogous to a galvanic cell, and the remainder of the circuit, including the hot junction, to the wire connecting the terminals of that cell; but, as will be seen later, the current is kept up by means of energy absorbed, as Heat, both at the hot junction and in the circuit itself.

Antimony and bismuth are the extremes of a thermo-electric series, which is at ordinary temperatures, according to Becquerel, the following:— Bi, Pt, Pb, Sn, Cu, Au, Ag, Zn, Fe, Sb.

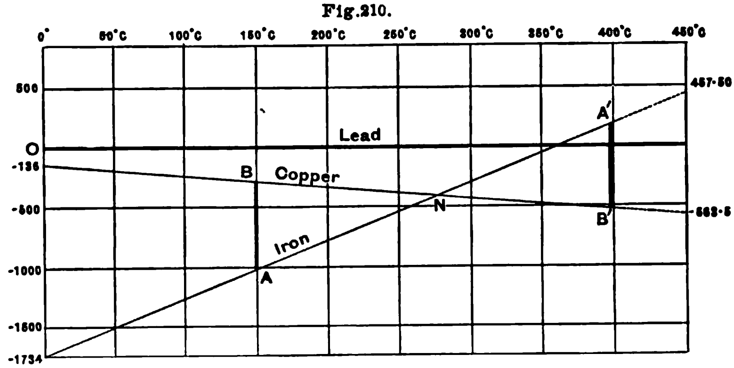
The electromotive difference of potential produced and maintained within the closed circuit is approximately proportional to the difference between the temperatures of the two junctions, if this difference be very small; and it is therefore, when measured in microvolts, equal to the product of the difference of temperatures into a Number. When the E.M.D.P. is measured in microvolts (one microvolt being $\frac{1}{1,000,000}$ Volt or $\frac{1}{300,000,000}$ electrostatic unit of D.P.), this number is called the **thermo-electric power** between the two given metals at the given mean temperature. For Bismuth and Antimony, at a mean temperature of 19½° C., it is 103; for E.M.D.P. = 103 microvolts = 103 × (20° C. − 19° C.). If the E.M.D.P. be measured in C.G.S. electromagnetic units, of which 100 make a microvolt, the thermo-electric power in this case is 10,300.

The thermo-electric power between any two metals is not a constant number, but varies with the temperature. In Fig. 210 it may be seen that near the freezing-point of water a difference of one degree between the temperatures of two junctions of a lead-iron circuit makes between the two junctions a potential-difference of 17·34 microvolts, or 1734 electromagnetic units, while at higher mean-temperatures the thermo-electric power is progressively less, becomes *nil*, and ultimately changes its sense. The thermo-electric power between copper and lead, on the other hand, increases.

A diagram of this kind is called a **Thermo-electric Diagram,** and indicates the Thermo-electric Power between its metals at any mean temperature within its range.

The lines of iron and copper cross one another at 274°·5 C. An iron-copper couple, one of whose junctions is at a temperature slightly over, the other at a temperature equally under 274°·5, will develope within its circuit no current. That mean temperature, 274°·5 C., is for iron and copper the so-called **neutral point.**

Fig. 210.



If one copper-iron junction be at 150° C., at what temperature must the other be in order that there may be no current? The temperature required is 399° C., which lies as far beyond the neutral point, 274°·5 C., as 274°·5 does beyond 150° C. The triangle ABN (Fig. 210) represents the total E.M.D.P. when the junctions are respectively at 150° C. and 274°·5 C.; * the triangle NA′B′ represents the total and opposite E.M.D.P. which would be developed if the junctions were at 274°·5 and 399° respectively: these triangles are equal: their sum is *nil:* the total electromotive potential-difference between 150° and 399° is *nil:* there is consequently no current.

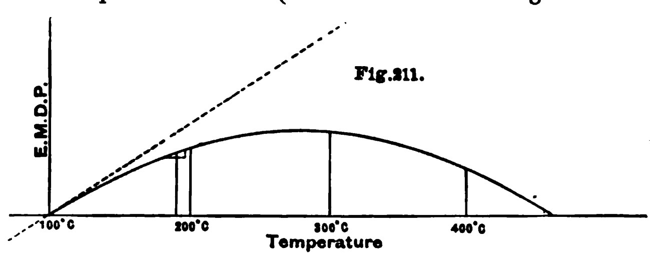
If one copper-iron junction be maintained at the constant temperature of 100° C., and the other be successively exposed to temperatures 101°, 102°, 103°, and so forth, each step in the temperature of the hotter junction produces an increment of the effective E.M.D.P. within the circuit; but each successive increment is smaller than its predecessor: as the temperature of the hotter junction nears 274°·5, the successive increments of

* Since, for a small difference of temperature, E.M.D.P. = Therm.-elect. power × diffce. of temp. measured in °C., each step in temperature multiplied by its corresponding thermo-electric power forms in the thermo-electric diagram a small rectangle, which represents the E.M.D.P. developed by each difference of temperature: the sum of all these rectangles between 150° and 274°·5 represents the total E.M. difference of potential set up when these are the temperatures of the two junctions: this sum is equal to the triangle ABN.

E.M.D.P. become less and less: when the hotter junction is at 274°·5, the **neutral point**, the increment is *nil*, and the electromotive difference of potential and the current which it causes to run round the circuit are at their **maximum**. Thereafter, as the hotter junction is still more strongly heated, the E.M.D.P. at first gradually and then more rapidly sinks. When at length the hotter junction is at 449° (the colder one still remaining at 100°) there is no E.M.D.P., and no current round the circuit: and when the temperature of the hotter junction exceeds 449°, the direction of the current is **reversed**, being now from iron to copper across the hotter junction; and thereafter, successive increasing differences of temperature develope successive numerically greater negative E.M.D.P.'s. The Neutral Point is thus a fixed temperature for each pair of metals; at that temperature there is **no** (true) **contact-effect**, and the temperature of the colder junction on the one hand (whatever that temperature may be) and the corresponding **Temperature of Reversal** on the other, are equidistant on either side of it, so long as the lines in the diagram are straight, which they are in most cases within pretty wide limits.

Curves indicating the relation between the differences of Temperature between two junctions and the electromotive differences of Potential developed in consequence of them (sometimes called Gaugain's curves),

Fig. 311.



have a form which, for most pairs of metals, is that of a parabola: and the numerical value of the tangent of the angle made by this curve with a line parallel to the axis of x, and cutting the curve at that point of it which corresponds to any given temperature, x° C., is a numerical measure of the thermo-electric power at that mean-temperature: for both the tangent and the thermo-electric power are numerically equal to the fraction

$$\frac{\text{Increment of E.M.D.P.}}{\text{Increment of mean temperature}} = \frac{\text{Change of ordinate}}{\text{Change of abscissa}}.$$

Even within one and the same bar, except, apparently, in lead, differences of potential are set up when a bar is unequally heated, and some of the heat supplied is expended in setting up this electrically-stressed condition; but in a homogeneous metallic ring, however irregularly heated it may be, there is no current. The metal on either side of a hot or cold junc-

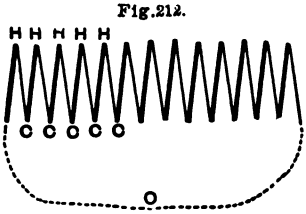
tion of two metals is, on the other hand, like a single bar, and differences of potential are set up within it; these may modify the amount of the effective difference of potential within the whole circuit, and are found to supply an explanation of the phenomena of inversion (p. 651).

Metals interpolated in the circuit produce no effect on the amount of the effective difference of potential within the circuit, unless, indeed, their junctions be at different temperatures. If that be the case, their thermo-electric effects form a part of the general thermo-electric effect of the circuit.

When a number of pieces of bismuth and antimony are arranged end to end, their alternate junctions being hotter and colder respectively, the E.M.D.P. maintained between the extremities of a pile of n pairs of elements is n times that found to be due to one such pair.

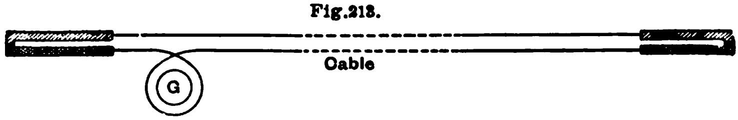
This principle is utilised in the Thermo-electric Pile, which consists of a number of pieces of bismuth and antimony (or, better, of an alloy of 10 parts by wt. of bismuth and 1 of antimony, and an alloy of 15 parts of antimony and 7 of cadmium), arranged after the fashion of Fig. 212. When the face of this pile marked HHH is exposed to heat, the junctions H, H, H become warmer than the junctions C, C, C, and a current passes through the circuit O from the antimony to the bismuth terminal. If it be placed opposite a piece of ice, the face HHH will cool itself by radiation, and the current is now in the reverse direction.

Fig. 212.



When a junction of two metals is connected, by a pair of copper wires, with a similar junction at the same temperature, no current passes, whatever be the length of the intervening cable or its local variations of temperature; but if one of the junctions assume a different temperature from the other, then a current passes, and the temperature of the distant junction may be inferred from the strength of the current which passes, this being measured by a galvanometer, or directly determined by heating or cooling the similar junction situated under the observer's control until its temperature becomes the same as that of the distant one: this is known to have occurred when the current through the galvanometer ceases. A couple of junctions of this

Fig. 213.



kind, with an intervening double wire and galvanometer, form a differential thermometer, the indications of which must be interpreted with reference to the thermo-electric diagram of the two metals used.

As sources of electricity, thermo-electric piles are not much in use. Becquerel's thermo-electric piles, made of thirty pairs of blocks or rods of artificial sulphide of copper (which fuses only at about 1000° C.) and of

German-silver, can decompose water when the differences of temperature employed are from 250° to 300°. In Řebiček's form of Noë's thermo-electric pile, twenty-five pairs of plates of German-silver and of an alloy of zinc and antimony are ranged round a Bunsen gas-burner: each such pile maintains an effective difference of potential of from 2 to 2·75 Volts, so long as the Bunsen burner is kept lighted, while the internal resistance is 0·75 Ohms. In Clamond's pile, about 6000 couples of iron and of bismuth-antimony alloy are ranged round a coke fire, and the E.M.D.P. produced is, if the couples be arranged in file, about 218 Volts. The disadvantages of thermic piles as sources of electricity are that, in general, the E.M.D.P. produced is so extremely small that moderately-slight external resistances make the current extremely weak, and that it is difficult to keep the cold junction cool; and even in Clamond's pile, which is able to keep a pair of electric arc-lamps in action, about 95 or 96 per cent of the heat of the fire is not converted into the energy of a current, and is thereby practically wasted. For many purposes, such as electroplating on the small scale, Noë's batteries, three of which produce an E.M.D.P. nearly equal to that produced by seven Daniell's cells, are very useful, for when they are once built up their current can be produced or arrested at will. An arrangement like that of Fig. 213 has been used as a self-acting source of electrical currents, and therefore of energy, sufficient to maintain in action a self-winding clock.

The most important source of electricity is the transformation of the energy of work into that of electrical separation by means of magneto-electric and dynamo-electric machines, the action of which will be explained in the sequel.

Atmospheric Electricity.—The atmosphere in different regions is often found to be at different local potentials, which differ from that of the earth sometimes even by as much as 3000 Volts within 100 feet. This is possibly (Tait) due to a contact-effect between air and aqueous vapour; and it is possibly necessary that there should be at least traces of dust present, as well as water-vapour. A conductor insulated from the earth may be brought to the same potential as any point in the air, by leading to that point a metallic wire, and by furnishing this exploring wire with an extremely fine point, or, better, by fixing at its extremity a sponge dipped in spirit and set on fire, or a little cistern from which a quantity of water is allowed to drop. In the former case the flame continuously conveys masses of gas away from the end of the exploring wire; and so long as there is any difference of potential between the region of the air explored and the conducting system of which the exploring wire forms a part, there will be a current along the wire, and finally the whole conducting system will come to the same potential as the air around the flame. Similarly, waterdrops, on falling from an insulated cistern, bring the cistern to the same potential as the air around it: each drop, just before falling off, becomes electrified with a charge opposite, while the nozzle, the cistern, and the main mass of water are electrified with a charge similar to that of the air in the neighbourhood of the falling drop. As the drop is in the act of falling off, it is attracted by the cistern: it is held back as it falls: it falls down with less speed than it would have assumed if it had fallen from an uninsulated cistern; and when it reaches the ground it produces less heat. The energy of the electrification acquired by the cistern is equal to the missing kinetic energy of the falling drops.

In an analogous way the air within a room may be strongly electrified; connect a flame with the conductor of an electrical machine, and work the machine: in one minute a Holtz machine will raise the potential of the air of a room by 2000 Volts. Combustion alone will effect electrification of the air, which is negatively charged by burning coal-gas, positively by burning charcoal.

When difference of potential has once been produced, it may be turned to account for the conversion of Work into Electrical Energy.

Charge a plate, whose free capacity is C, to potential V; its charge will be $Q = CV$. Bring up to it a second plate, parallel and at a distance d; the two plates now form a condenser. The capacity of this condenser is $C' = K \times \text{surface}/4\pi d$ (p. 599). Connect the second plate to earth. Its potential becomes zero, and on its outer surface it has no charge. The capacity of each plate for "free charge" may be reckoned as having been reduced to $\frac{1}{2}C$ by the mutual approach, for lines of force can now only pass from one face of each plate towards surrounding objects. The potential of the inducing plate will now be $V' = CV/\overline{C' + \frac{1}{2}C}$, which is less than V. The original charge CV on the inducing plate is now divided into two parts. Of these, one is "free charge" $= \frac{1}{2}C^2V/\overline{C' + \frac{1}{2}C}$; the other is "bound charge," $C'CV/\overline{C' + \frac{1}{2}C}$, which faces an equal and opposite charge on the second plate of the condenser. Now insulate the second plate and remove it to a distance d': the capacity of the condenser decreases in the ratio d/d'; then as the capacity of the condenser decreases, the potential of the second plate tends to fall from zero to $-VC'/\overline{C' + \frac{1}{2}C}$, while that of the first tends to return to V.

In the case of conducting plates in free air, the negative potential imparted to the second plate by this method cannot, therefore, become numerically quite equal to the original potential of the first plate; but in the Electrophorus, next to be considered, the charge cannot travel along the surface of the original charged plate. In that case, the whole charge remains "bound"; and since $E = 4\pi Qd/A$, and Q is constant, then on increasing d, the potential-difference will rise.

Difference of potential may also be increased by the expenditure of Work, with the assistance of induction (Holtz, Voss, Wimshurst).

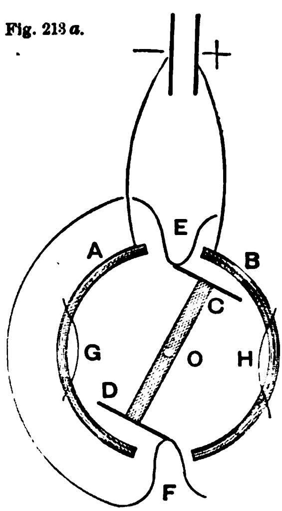
The work is done in stretching the Field of Force against the mechanical traction $\mathbf{t}$ across the field; $\mathbf{t} = E^2/8\pi d^2$, in dynes per sq. cm., where E is the rise in potential occasioned by pulling the plates apart through a distance d; whence $E = d\sqrt{8\pi \mathbf{t}}$.

The **Electrophorus** consists of a cake of resin or vulcanite and an insulated metallic plate. The former is slightly charged by being rubbed with a catskin or a dry silk-handkerchief: the metallic plate is then laid upon it. The contact between the two can never be perfect at all points; practically there is an intervening film of air between the resin and the metallic plate, and the latter is charged by induction with an attracted and a repelled charge. The latter charge may be withdrawn by touching the metallic plate with the finger, or by making metallic communication between the metallic plate and the earth; the former remains, facing and

attracted by the original charge on the resin. Work is now done from without in pulling the metallic plate away from the resin; as the distance between the metal and the resin increases, the electrostatic capacity of the electrophorus, considered as a condenser, diminishes; the potential therefore increases, both on the metal and over the resin: the knuckle applied to the edge of the insulated metallic plate may now receive a spark. When the metallic plate is next laid on the resin a new charge is induced in it, which may again be withdrawn in the same way when the plate is removed. Small original charges may thus induce successive charges of high potential.

Sir William Thomson's (Lord Kelvin's) Replenisher. — This instrument, which is used as a means of keeping the Leyden jar connected with the suspended needle of Kelvin's electrometer at a constant potential, is sketched in the accompanying diagram (Fig. 213*a*). A, B, two metal half-cylinders, insulated from one another; C, D, two metallic plates insulated from one another and capable of rotation round the axis O; E, F, an insulated spring capable of touching both C and D when they are in the position shown in the figure; G, H, two springs connected with A and B and capable of being pressed upon by C and D as they rotate. Start from the position shown in the figure. B is positively charged by contact with one of the plates of the Leyden jar; C becomes negatively, and D positively, charged. Rotate C to the left, D to the right. Their metallic connection with one another is broken and they remain oppositely charged. As they pass G and H, C's − charge escapes to A, and D's + charge to B; and thereafter D and C respectively acquire − and + charges, and stand in the former positions of C and D. The + charge of the + plate of the Leyden jar may thus by continuous rotation of CD be continuously increased. If, on the other hand, C be rotated to the right, D to the left, C's negative charge is conveyed to B, and the positive charge of the + plate of the condenser may, by continuous rotation in this sense, be reduced to any desired extent, or even reversed. The potential of the Leyden jar may thus be adjusted to any desired amount, which may be determined by a subsidiary pair of plates, connected with the inner and outer coatings respectively and separated by springs, coming to assume a position at any pre-arranged fixed distance from one another.

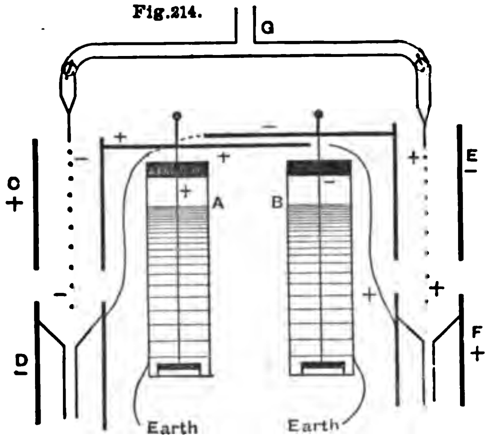
Fig. 213 *a*.



Sir William Thomson's (Lord Kelvin's) Water-Gravity Electric Machine. — In Fig. 214, A and B are two Leyden jars, whose inner coatings consist of sulphuric acid and are connected with the metal tubes C, F and E, D respectively. C and D are co-axial: so are E and F. Water falls in drops from the bifurcated metal-tube G, which, being connected with the ordinary water supply, is in communication with the earth, and is therefore at zero potential. A small initial charge, consisting (say) of positive electricity, is imparted to one of the Leyden jars, say A. Water is made to flow from G in streams so thin as to break up into drops within the tubes C and E. Just before these drops break off from the stream, they are by induction

within C charged with negative electricity, while the complementary positive charge is conveyed along G to the earth. When the drops have become separate, they fall down charged negatively. They then fall upon a metallic funnel placed in the tube D, and charge the exterior of that tube negatively: this charge is shared with the Leyden jar B. This Leyden jar, thus negatively charged, by a corresponding inductive action causes the drops which fall through E to become positively charged. When these drops fall upon F they increase the positive charge of the Leyden jar A. Thus the Leyden jars A and B become more and more highly charged, the one with positive, the other with negative electricity on its inner coat. The energy of their electrification is derived from the work done by gravity upon the falling water; and thus this contrivance is an electrical machine worked by gravity.

Fig. 214.



Steady Electrical Currents.

If the plates of a charged condenser be connected by a wire, the condenser will be discharged; the quantity Q of electricity disappears in a time t, and during that time t certain phenomena occur, with waning vigour, which are spoken of as those of a **Current of Electricity.** It is as if electricity ran out of a place where it was stored up under a potential-difference E, down to the ordinary potential-level $V = 0$; and as if its path were along the wire. In truth, the phenomenon is one of the release of the Field of Force from constraint, and of transmission of energy through that field, not through the connecting wire. Still, with this reservation, it is convenient to adhere to the terminology according to which such a discharge is spoken of as a Current in the wire.

If a very large condenser be discharged through a very long and thin wire, the phenomena of Current may remain fairly steady for any short interval of time; and if the E.M.D.P. be that between the terminals of a galvanic cell, these phenomena are, apart from Polarisation, etc., continuously uniform or steady, as if there were a practically inexhaustible reservoir of electric quantity to draw upon, so long as the cell holds out.

The **Intensity** or **Strength** of a current — *i.e.*, the Quantity of electricity which passes any cross-section of the conductor during one second of time — depends, on the one hand, upon the effective difference between the potentials at different parts of the conductor, and, on the other, upon the nature of the conductor — that is to say, upon its size and its substance. A long or thin wire is a worse conductor — has less Conductance and offers more **Resistance** — than a thick or short one: a silver wire conducts better than a copper one of the same size.

The **Density** of a Current, Δ, is the quantity of electricity passing per sq. cm. of cross-section of the conductor. It is therefore equal to Intensity $\div$ Cross-Section $= I/o$.

The relation between E the electromotive difference of potential, I the Intensity of the current, and R the Resistance of a uniform conductor, is, when the flow is steady, expressed by the equation, $I = E/R$. When there are several sources of difference of potential within the circuit, or several successive conductors, each of which offers its own resistance to the onward flow of the current, the law assumes the generalised form that $I = \frac{\Sigma E}{\Sigma R} = \frac{\text{effective sum of all the differences of potential}}{\text{sum of all the successive resistances}}$. This is **Ohm's Law.**

The Resistance, as defined by Ohm's Law, is E/I or $\Sigma E/I$, and it must be specially noted, as an experimental fact, that for any given conductor or set of conductors, this fraction, once found, remains almost absolutely constant, whatever may be the value of E or ΣE, provided that the temperature and the structure of the conductor remain unchanged. The Measurement of the Resistance of a conductor is the experimental determination of this fraction.

Ohm's Law may also be written $I = E/l \div R/l$; E/l is the "electromotive force" or Potential-Slope ϕ; R/l is the Resistance per linear centimetre. Also, if Δ stand for current-density, $\Delta = E/l \div \text{R} = \phi/\text{R} = \phi\text{D}$, where R and D are the Resistivity and Conductivity, as defined below.

The C.G.S. Electrostatic Unit of Intensity is the intensity of a current in which one C.G.S. electrostatic unit of quantity passes a given section of the conductor during one second. It is the current which passes when the difference of potential $E = 1$ C.G.S. electrostatic unit, and the total resistance is also $R = 1$ C.G.S. electrostatic unit of resistance.

The C.G.S. Electrostatic Unit of Resistance is the resistance offered by a conductor which, when it is interposed between two bodies whose potentials are maintained at a constant difference of one C.G.S. electrostatic unit, allows one C.G.S. Electrostatic unit of Quantity to pass along it, per second.

These units are inconvenient for practical purposes, and electricians use as their practical units certain fractional or integral multiples of these.

The **Resistance** of a uniformly-cylindrical conductor, such as a wire, depends upon three things: (1) its length l, directly; (2) its cross-section o, inversely; (3) its Conductivity D, inversely. It is therefore equal to $l/o\text{D} = \text{R}$.

The **Conductance** of a conductor is the reciprocal of its Resistance. In a wire it is therefore equal to $o\text{D}/l$. Ohm's Law may therefore be written $\text{I} = \text{DE}$, $\text{E} = \text{I}/\text{D}$, or $\text{D} = \text{I}/\text{E}$, where D is the Conductance.

The C.G.S. Unit of Conductance is that of a conductor of unit resistance.

The specific **Conductivity**, D, of any substance is a constant, special to each substance, and even found to differ from sample to sample of that which is nominally the same substance. It represents the number of units of electricity which can pass, per second, between two bodies kept at a constant potential-difference of one unit, when the conductor interposed between these bodies has a length of 1 cm. and a cross-section of 1 sq. cm. It varies very greatly from one substance to another.

The reciprocal of D, $(1/\text{D}) = \text{R}$, the **Resistivity** of a substance. The Resistance of a conductor of length l and cross-section o is therefore equal to $l\text{R}/o = \text{R}$.

In the following table the first column of figures gives the Resistivities, the next column the Conductivities of a certain number of substances, in electrostatic measure; while the third column gives the numbers which denote their Relative Conductivities when the conductivity of mercury is taken as a standard and called unity. It is very usual to take the conductivity of silver as a standard = 100.

The numbers in the following table have (with the exception of those for the last four substances) been calculated from the data of the authorities named, on the assumption that 14·4521 grammes of mercury (sp. gr. 13·5955), in the form of a column of uniform cross-section (1 sq. mm.) and 106·3 cm. in length, has a resistance equal to the 900,000,000000th part of an electrostatic unit of resistance, that is, equal to one Ohm or practical unit (see p. 711) of Resistance.

An Ohm-coil is a coil of wire whose resistance is one Ohm.

The conductivity of an Ohm-Coil is called a Mho.

If the following table be read without the multiplier or divisor, V^2, it then expresses the specific resistivities and conductivities in another system — the Magnetic or Electromagnetic system of C.G.S. units, from which the Ohm and the Volt are primarily derived, the Ohm being 10^9 electromagnetic units of resistance, and the Volt 10^8 electromagnetic units of potential-difference. This system depends upon the laws of Magnetism, afterwards to be explained.

Substances.	Resistivities in C.G.S. Electrostatic Units.	Conductivities in C.G.S. Electrostatic Units.	Relative Conductivities; Hg = 1.
Mercury	94073 ÷ V^2	·000010,63 × V^2*	1 (Matthiessen)
Soft Silver	1433 ,,	·000697,74 ,,	65·64 ,,
Soft Copper	1525 ,,	·000655,87 ,,	61·70 ,,
Soft Gold	1968 ,,	·000508,22 ,,	47·92 ,,
Zinc	5370 ,,	·000186,24 ,,	17·52 ,,
Platinum	14562 ,,	·000068,671 ,,	6·46 ,,
Antimony	33718 ,,	·000029,657 ,,	2·79 ,,
Bismuth	125430 ,,	·000007,97255 ,,	0·75 ,,
Nitric Acid, 29·7 % (Kohlrausch)	1283,400000 ,,	·000000,000779 ,,	0·000073,3
Sulphuric Acid, 45·84 % (Wiedemann)	1158,600000 ,,	·000000,000863 ,,	0·000081,2
Do. 11·42 % do.	2143,500000 ,,	·000000,000466 ,,	·000043,89
Do. 3·37 % do.	7266,500000 ,,	·000000,000137,62 ,,	·000012,95
Do. concent. do.	7397,800000 ,,	·000000,000135,18 ,,	·000012,72
$ZnSO_4$ sol. saturated (Kohlrausch)	20812,600000 ,,	·000000,000048,05 ,,	·000004,52
$CuSO_4$; sat. sol. (Kohlrausch)	21382,800000 ,,	·000000,000046,77 ,,	·000004,40
Pure water (Kohlrausch)	3762,936000,000000 ,,	·000000,000000,000266 ,,	·000000,000025
Glass (Beetz); the mean of several experiments—			
At about 345° C.	...	·000000,000000,002657 ,,	·000000,00025
At about 190° C.	...	·000000,000000,000059,1 ,,	·000000,000000,55
At ordinary temperatures	Immeasurably great.	Immeasurably small.	Immeasurably small.
Guttapercha (Ayrton and Perry)—			
At 83° C.	500,800000,000000,000000 ,,		
At 24° C.	83000,000000,000000,000000 ,,		
Paraffin (Ayrton and Perry)—			
At 46° C.	34,000000,000000,000000,000000 ,,		
At 77°·8 C. (melted)	1,354000,000000,000000,000000 ,,		
Vulcanised rubber (Ayrton and Perry)—			
At 67° C.	5,391000,000000,000000,000000 ,,		
At 90°·7 C.	1,015000,000000,000000,000000 ,,		
Ebonite (Ayrton and Perry)—			
At 36° C.	61,030000,000000,000000,000000 ,,		
At 96°·8 C.	9,696000,000000,000000,000000 ,,		

* $V^2 = 9 \times 10^{20}$; or rather, V = the Velocity of Ether-Wave Propagation, assumed $= (3 \times 10^{10})$ cm. per sec.

The conductivity of metals decreases, that of most bad conductors (including carbon) increases, with their temperatures: a heated wire or dynamo-electric machine increases the resistance in the circuit of which it forms a part; while at the temperature of the electric arc, carbon appears to offer no resistance. Very roughly, and with well-marked exceptions in the cases of iron and mercury, the resistivity of a pure metal is proportional to its absolute temperature: but pure metals appear (Dewar) to have no resistivity when exceedingly cold, whereas in alloys there is no such result.

When metals melt, their conductivities fall suddenly. Alloys are in general worse conductors than the arithmetical consideration of their percentage composition and the conductivities of their component metals would lead us to expect; but with changes of temperature, their conductivity varies less than that of pure metals does.

There is a broad resemblance between the conductivities of metals for electricity and for heat: the best conductors of the one are in general the best conductors of the other; and in both cases alloys offer a relatively high resistance. The series are, however, not identical. The velocities of light through films of the metals are also (Kundt) closely related to their conductivities.

Variable Conductivity. — Conductivity varies not only with varying temperature, but also with varying magnetisation, tension, torsion, or pressure. It increases with longitudinal stretching, diminishes with longitudinal compression of a wire, and diminishes in iron, but increases in tin and zinc, when the stress, being transverse, tends to widen the wire (Tomlinson). In powders or porous material, such as metal filings, platinum sponge, charcoal, it increases with the pressure; and if the pressure vary, within small limits, the variations of conductivity follow and are proportional to the variations of pressure. This is the principle of the Microphone. In such materials Heat raises the internal pressure and therefore the contact, and this modifies the amount of resistance and the heat produced within the conductor: this last itself affects the conductivity, as in the Tasimeter, which detects changes in temperature by the variation of a current passing through a rod of carbon fixed between metallic supports, and exposed to varying temperatures. Selenium, which in the amorphous form is a non-conductor, but in the crystalline form is a conductor, varies in conductivity with its state of aggregation, its temperature, the length of time during which a current has been passing through it; and crystalline selenium, when acted upon by light (especially the yellow and the red), and to a less extent when acted upon by dark rays, increases in conductivity: in the case of very bright sunlight this increase being sometimes even tenfold. Light of variable intensity produces corresponding and rapidly-responding variations in the conductivity of the crystalline selenium

upon which it may fall—a fact utilised in the construction of the Photophone. Metals, unlike selenium, become worse conductors as the temperature rises; but (Siemens) at 210° C. selenium changes its character and comes to act like a metal.

Reduced Resistance and Reduced Length of a Conductor.—This may be explained by a few numerical examples. We suppose the unit of resistance to be the Ohm, as above defined, the resistance of freshly-distilled mercury in a column of 1 sq. mm. section and 1·063 metres in length.

1. What length of soft-copper wire of 1 sq. mm. sectional area will give a resistance equal to one Ohm? $1·063 \times 61·70 = 65·5871$ metres. The figure 61·70 is taken from the table of conductivities above. The Resistance of 65·5871 metres of copper is thus equal to that of 1·063 metres of mercury: the Reduced Length of 65·5871 metres of copper is 1·063 metres of mercury.

2. What will be the resistance of a column of mercury 100 metres long and 1 sq. cm. in section? It will be equal to that of a column of mercury 1 sq. mm. in section × 1 metre in length multiplied by $l = 100$, and divided by $o = 10^2$. It is therefore 0·940734 Ohm. Its reduced length is 1 metre of standard mercury-column, 1 sq. mm. in cross-section.

3. What will be the absolute resistance, and what the resistance in Ohms, of 1000 metres of platinum wire whose diameter is $\frac{1}{4}$ millimetre? Its sectional area $o = \pi r^2 = \pi\ (\frac{1}{80})^2$ sq. cm. $= \frac{\pi}{6400}$ sq. cm.; its length $l = 100{,}000$ cm.; its resistivity R is $14562·13 \div V^2$; the Resistance of the wire is

$$R = \frac{l\text{R}}{o} = 100{,}000 \times \frac{14562·13}{V^2} \times \frac{6400}{\pi} = \frac{2{,}966560{,}000000}{V^2}$$

C.G.S. electrostatic units, or 2966·6 Ohms.

The **strength** or **intensity** of a steady current is measured by a Galvanometer (p. 713), round the magnetic needle of which the current is passed: in the Tangent Galvanometer the tangent of the angle of deflection of the needle is proportional to the intensity of the current.

The strength of a current is equal throughout all parts of a circuit in which there is a steady flow. A magnetic needle is equally deflected when brought into the neighbourhood of any part of the circuit, whether the circuit be locally composed of solid, of liquid, or of heated or rarefied gas.

The Practical Unit of Intensity is the intensity of that current which is produced in a conductor whose total resistance is 1 Ohm (=1/900,000,000000

C.G.S. electrostatic unit), when there is kept up between its extremities a potential-difference which constantly amounts to one Volt, or 1/300 C.G.S. electrostatic unit.

$$\text{Since } I = \frac{E}{R} = \frac{1 \text{ Volt}}{1 \text{ Ohm}} = \frac{1/300 \quad \text{C.G.S.E.S. unit}}{1/900,000,000000 \text{ C.G.S.E.S. unit}} = 3000,000000,$$

the practical unit of intensity, the **Ampère**, is equal to 3000,000000 C.G.S. electrostatic units of intensity.

In a current whose Intensity is one Ampère, the practical unit of quantity, one **Coulomb**, passes any given section during each second: the Coulomb is thus equal to 3000,000000 C.G.S. electrostatic units of quantity.

Electrical engineers have adopted the Ohm, the Volt, etc., as means of practical measurement. The Ohm and the Volt in electrical workshops are not abstract calculations, but standard wires and standard batteries (or multiples or fractions of these), by comparison with which the resistance or the E.M.D.P., the so-called electromotive force, of any given combination of materials may be relatively measured; *e.g.*, an average pint Daniell cell will deliver a maximum current of about $\frac{1}{4}$ Ampère, its D.P. being about 1 Volt, and its internal resistance about 4 Ohms; an average pint Grove cell will deliver a maximum current of ten Ampères, its D.P. being about 2 Volts, and its internal resistance about $\frac{1}{5}$ Ohm.

Dimensions of Electrostatic Measure in Air. — Current-Intensity — a quantity passing per second: $[I] = [Q/T] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [T] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T^2]$.

Resistance: $[R] = [E/I] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T] \div [M^{\frac{1}{2}}L^{\frac{3}{2}}/T^2] = [T/L]$; Conductance $= [1/R] = [L/T]$, a Velocity.*

Resistivity: $[\text{R}] = [R] \times [o/l] = [T/L] \times [L^2/L] = [T]$.

Conductivity: $[\text{D}] = [1/\text{R}] = [1/T]$.

In **any medium** of sp. ind. cap. K, the Dimensions in E.-S. measure are: — Current-strength, $[M^{\frac{1}{2}}L^{\frac{3}{2}}K^{\frac{1}{2}}/T]$; Resistance, $[T/LK]$; Conductance, $[LK/T]$; Resistivity, $[T/K]$; Conductivity, $[K/T]$.

The above dimensions are based on the assumption that the Quantity of electricity in a current is the same thing as the Quantity of an electrostatic charge: they are therefore called Dimensions in Electrostatic Measure.

Fall of Potential in a Homogeneous Conductor of uniform thickness. — During the maintenance of a steady current one end of a homogeneous conductor is at a higher, the other at a lower potential, and between these points the fall is gradual, so that intermediate points are at intermediate potentials. Fig. 215 shows that if the length of the uniform conductor be repre-

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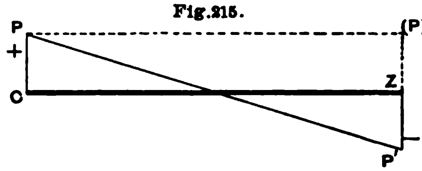
* Suppose a sphere of radius r and therefore of capacity $C = r$ to be charged, in air, with quantity Q; the potential will be $V = Q/r$; and $Q = Vr$. If this sphere be connected with the earth by a wire, whose resistance is R, for a short time t, along that wire a current will run, whose mean intensity is I; the quantity conveyed by that current in time t is It; and this is lost by the sphere, whose charge sinks to Q'. Hence $Q - Q' = It$. If the potential of the sphere is not to sink, the radius must diminish. If the radius shrink to r' in time t, the velocity of its contraction is $(r - r')/t$: and $Q = rV$ as before; and also, $Q' = r'V$, V being unchanged. From these we find that $(r - r')/t = I/V = 1/R = D$, the conductance of the wire. But $(r - r')/t$ is a Velocity; whence, in electrostatic measure, the Conductance of a wire is a Velocity.

sented by CZ, the end C connected with the positive terminal of the battery is at a potential which differs by (P)P′ from the potential of the end Z, connected with the negative terminal. The Fall is steady, and depends (1) upon the difference of potential between the ends of the conductor, and (2) upon the length of the conductor; it is measured by the Slope of the line PP′, the amount of fall of potential per unit of length.

Fig. 215.



If the battery be connected at its midpoint with the earth, the conductor CZ is near C at a positive potential; towards the midpoint of CZ this diminishes; the midpoint of the conductor is a point of zero potential (the potential of the earth); and as we approach Z we find the potential increasingly negative.

Necessarily, the Potential-Fall or -Slope along the conductor depends upon the *difference* between the potentials of its extremities, not upon the values or signs of these in relation to the arbitrary earth-zero of potential.

Resistance in a Heterogeneous Conductor. — When a conductor is made up of a succession of conductors which, on account of their differing materials or conditions or thicknesses, present different resistances to the current, it may become necessary to consider each conductor as reduced to an equivalent length of a standard conductor, such as a column of mercury 1 sq. mm. in cross-section. For example: a current passes successively along (1) a metre of mercury 1 sq. mm. in section; (2) 10 metres of mercury 1 sq. cm. in section; (3) 1 mm. of pure water 1 cm. in section; (4) 61·70 metres of soft copper wire 4 sq. mm. in cross-section: what is the total resistance of this combination? We must reduce all to a common term, to Reduced Lengths of our standard mercury column. Then, above, (1) is equivalent to a metre of such a column, (2) is equivalent to $\frac{1}{10}$ metre, (3) to 400,000 metres, and (4) to $\frac{1}{4}$ metre of such a mercury column; and the whole resistance is that of 400,001·35 metres of the standard conductor.

Resistance in a Galvanic Circuit. — In a galvanic circuit we have to consider two sets of resistances: those internal to the cells, the internal resistance, R_i; those in the conducting media, the external resistance, R_e. Then Ohm's Law is $I = E/\overline{R_i + R_e}$.

Let *n* cells be arranged side by side, copper to copper, zinc to zinc; the E.M.D.P. of the combination is the same as that of one cell, and = E Volts;

the internal resistance (the combination being virtually one cell of n-fold surface) is R_i/n Ohms; the external resistance is unaltered. The intensity or current-strength is therefore

$$I = \{E/\overline{(R_i/n) + R_e}\} = \{nE/\overline{R_i + nR_e}\} \text{ Ampères.}$$

If the internal resistance be extremely small in comparison with the external, R_i may vanish from this expression; then $I = \{nE/nR_e\} = E/R_e$ Ampères, and the current-strength is little increased by the use of many cells; but if the external resistance be extremely small, the current-intensity becomes $\{nE/R_i\}$, and the side-by-side arrangement in Surface* secures the highest strength of current.

If n cells be arranged in file, copper to zinc, the E.M.D.P. is nE Volts; the internal resistance is nR_i Ohms; and the external, as before, R_e Ohms. The current-intensity is now $I = \{nE/\overline{nR_i + R_e}\}$ Ampères. This arrangement of cells behind one another in Indian file or in Series† is the best for securing the highest attainable strength of current when the internal resistance is extremely small in comparison with the external; for then, R_i vanishing, the current-intensity is $\{nE/R_e\}$ Ampères; while if the external resistance, on the other hand, be exceedingly small in comparison with the internal, the intensity is $\{nE/nR_i\} = E/R_i$, which differs but little from $\{E/\overline{R_i + R_e}\}$, the strength of the current produced by one cell.

For extremely great external resistances, then, arrange in Series, if the highest attainable strength of current be aimed at; for extremely small external resistances, arrange in Surface.

When neither the internal resistance nor the external can be considered as vanishingly small the one in comparison with the other, the best arrangement, for high intensity, is to unite cells, ab in number, into a series of b each: in each series of b, the b cells are placed side by side, copper to copper, zinc to zinc; then a such series are arranged in file, the copper terminal of each series being connected with the zinc of the next. In this way we virtually make up a large cells, each of b-fold surface, and we arrange these in file or series.

In each of these virtual large cells the E.M.D.P. is E Volts; the resistance is 1-bth of r Ohms, the resistance of a single cell. Now couple a such large cells in Series; the E.M.D.P. of the combination is aE Volts; the internal resistance of the whole, R_i, is equal to $a \times (r/b)$ Ohms; the intensity or strength of the current produced is

$$I = \frac{aE}{a\frac{r}{b} + R_e} = \frac{aE}{\frac{a^2r}{n} + R_e} = \frac{E}{\frac{ar}{n} + \frac{R_e}{a}} \text{ Ampères,}$$

where $n = ab$. The denominator of the last fraction is the least possible, and the strength of the current consequently the greatest, when $R_e/r = a/b$. When the current is strongest, R_e is thus equal to ar/b, or the external resistance is equal to R_i, the internal. If the external resistance be equal to nr, and still more if it be greater than nr, the problem of the most advantageous arrangement of the cells in rank and file becomes an insoluble one, and the cells must be arranged in series.

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* Obsolete synonym — "*in Quantity.*"

† Obsolete synonym — "*in Tension.*"

Problem. — Sixty Grove cells, in each of which the resistance is ·6 Ohm, are at disposal: a resistance of 10 kilometres of soft copper wire of 4 mm. diameter is to be encountered; what is the best arrangement of the cells? The external resistance, R_e, is that of 1,000,000 cm. of copper wire of cross-section ·125664 sq. cm. and relative conductivity 61·70: this is equal to 12·13308 Ohms. Now in the equation $R_e = ar/b = a^2r/n$, $R_e = 12{\cdot}13308$, $r = 0{\cdot}6$, $n = 60$; whence $a = 34{\cdot}83$. The nearest feasible number corresponding to this value of a is 30; and the best arrangement is the division of the 60 cells into 30 virtual double-surface cells, arranged in Series.

If the external resistance be that of one kilometre of such wire, a being found equal to 11·015, the best arrangement is 12 sets of cells, each containing 5 cells joined in surface, and these sets joined to one another in Series.

To obtain maximum current-strength is not, however, the most economical way of using a battery; half the energy is wasted in overcoming internal resistance: this internal resistance must be proportionally reduced in order to reduce this waste; and if this be done, then, though the current is not the maximum obtainable, the amount of zinc consumed is reduced in a still greater ratio. For economical working, therefore, keep the resistance of the cell or battery as low as possible compared with that of the general circuit; that is, work with high external resistances.

The Fall of Potential in a Heterogeneous Conductor. — If we draw a diagram, setting out on a base-line and using as abscissæ the Reduced Lengths of the several successive conductors which make up a heterogeneous conductor, and if for a moment we let drop from view any local differences of potential set up by contact of different materials, then the line of potentials slopes uniformly down from one end of the heterogeneous conductor to the other end, and from such a diagram we may find the total fall of potential along each component

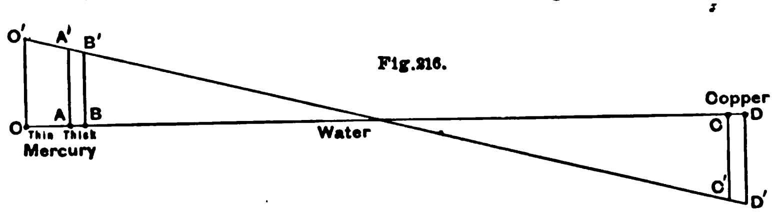


Fig. 216.

conductor. Fig. 216 very diagrammatically represents the fall of potential in the composite conductor specified in the preceding large-type paragraph, p. 639. AA' is the potential at the junction of the slender and the thicker column of mercury, BB' that at the one surface, CC' that at the other surface of the water, OO' and DD' the terminal potentials.

If now we follow this up with another diagram in which the real lengths of the conductors are supposed to be represented, we find a remarkable appearance presented by it. The poten-

tial-line, which indicates the successive falls of potential, is represented by the line O'A'B'C'D' in Fig. 217. The fall of potential is exceedingly rapid along the bad conductors, for bad conductors keep up a great difference of potential; and the whole fall

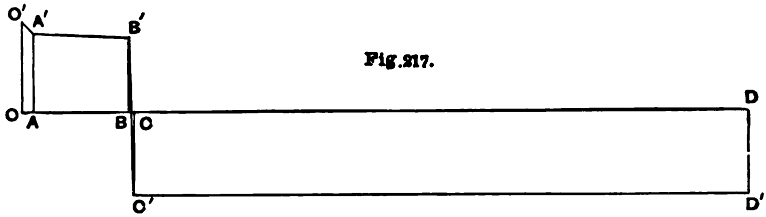


Fig. 217.

of potential is distributed among the component conductors, to each according to its Reduced Resistance.

If there be local differences of potential, these must be added to or subtracted from the total fall of potential for which the conductor has to provide. By way of illustration, let AB be a conductor, of which one-half consists of copper wire, the other half of zinc wire, of an equal thickness, and let its extremities be kept at potentials which differ by 2 AX. In Fig. 218, AC is the reduced length of the copper wire, and CB $(= \frac{61 \cdot 70}{17 \cdot 52}$ AC) the reduced length of the zinc wire. Between the copper and the zinc there is (on the older view discussed on p. 611) a rise of potential represented by DE, which would make the slope of the line of potentials steeper throughout the conductor. To BX' add X'F, which is equal to DE: connect X and F by a

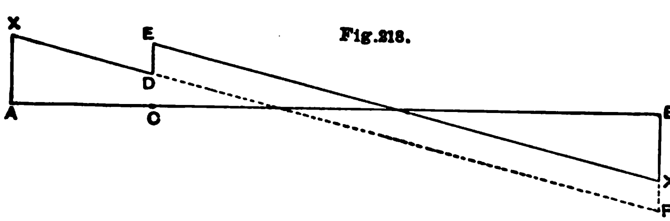
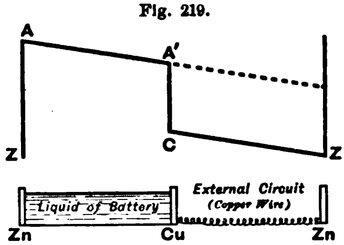


Fig. 218.

dotted line. Of this the portion XD would represent the fall of potential along the copper: the sudden rise of potential at D would bring the line of potentials up to E, whence it would be continued parallel to XF, along the course EX', arriving at the terminal potential X'. From this diagram another might be constructed in which, instead of the reduced lengths AC and CB, the corresponding true lengths would be represented and

the corresponding true slope of the potential-line found for each.

Fig. 219.



In Fig. 219, the line ZAA'CZ shows the slope of potentials in a galvanic circuit, the various parts being supposed to be brought to their reduced lengths. We might again reduce this diagram to another, in which the reduced lengths of the different parts of the circuit would be replaced by their true lengths, and the true slope of the potential-line found for each.

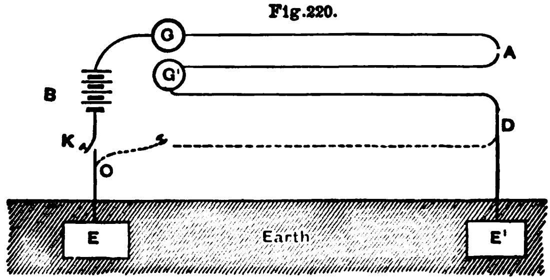
From this we find that the actual difference of potentials between the plates of a battery in a *closed circuit* depends upon the relation between the internal and the external resistance.

Thus if R_i be the internal and R_e the external resistance, the difference of potentials between the plates, available for the service of the part of the circuit external to the plates, is to the whole E.M.D.P. of the battery (measured between terminals on open circuit) as $R_e : R_i + R_e$. It is therefore equal to $ER_e/(R_i + R_e)$ and we cannot assume E, the whole E.M.D.P. of the battery, to be available unless R_e be so great in comparison with R_i that the latter may be neglected.

Flow along large Conductors.—If a conductor be very wide in comparison with the wires leading to and from it, the current widens out, and no part of the conductor is free from equipotential surfaces and lines of flow. If it be practically infinite, the resistance offered by it depends on the radius of the wires or plates connecting it with the battery, and on the resistivity of the conductor itself: not on the distance traversed by the current in the wide conductor.

In Fig. 220 the battery B is connected with two galvanometers, G and G′, by a long telegraphic wire interrupted at A: at K there is a key by

Fig. 220.



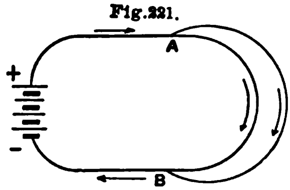
which contact may be made or broken. CD is a wire, the continuity of which may be broken by another key. When C and D are connected

through the wire CD, and connection made at K, both galvanometers are deflected. If, however, the connection CD be broken, and connection be suddenly made at K, the galvanometer G is alone deflected: the earth between E and E′ does not simply replace the wire CD between C and D. On the other hand, it is beyond doubt that currents do run in the earth's crust. A telephone, part of whose circuit runs in the straight line joining two telegraph stations, will pick up signals from the earth-currents: and the use of the earth in place of a return wire is familiar in telegraphy.

Simultaneous Currents. — Any number of currents may co-exist on the same wire, and the resultant at any point is the algebraic sum of the separate currents, positive or negative respectively. Thus two currents in opposite directions and of equal strengths may produce no effect in a single wire which is made a part common to two circuits; and if this wire be led round a magnetic needle, no deflection will be produced. If the one of these two currents be stronger than the other, the effect will be that corresponding to their difference.

Derived Currents. — When a steady current finds the conductor to divide and then to reunite, it divides into portions which run along the several paths open to it. In Fig. 221 the current arriving at A divides into two moieties; if the two paths be equal in their resistance, these moieties will be equal. If the resistances be not equal, the current passing along each branch will be inversely proportional to its resistance, for the difference of potential between the extremities is the same for every branch, and in each branch the product of the current-strength into the resistance is equal to the difference of potential.

Fig. 221.



The double path acts like a single conductor whose resistance is equal to $1/\{1/R' + 1/R''\}$, where R' and R'' are the resistances of the two branches. The conducting power or Conductance of the double path is the sum of the conductances of the two branches; these are respectively $1/R'$ and $1/R''$; their sum is $\{1/R' + 1/R''\}$, and the Resistance of the double path is the reciprocal of this sum.

Kirchhoff's Laws. — I. Where a steady current branches, the quantity of electricity arriving by the single wire is equal to the quantity leaving the junction by the branches. The algebraical sum of the currents passing towards (or passing from) the junction is equal to zero; $\Sigma I = 0$.

II. In a metallic circuit comprising within it a source of permanent difference of potential E, the products of the intensity

of the current, within each part of the circuit, into the corresponding resistance are, if the elements of current be all taken in cyclical order, together equal to E; Σ (IR) = E. In a metallic circuit in which there is no source of permanent difference of potential, E = 0, and Σ (IR) = 0.

This law applies to each several mesh of a wire network as well as to a single metallic loop, and it holds good even when an extraneous current is passed through the loop.

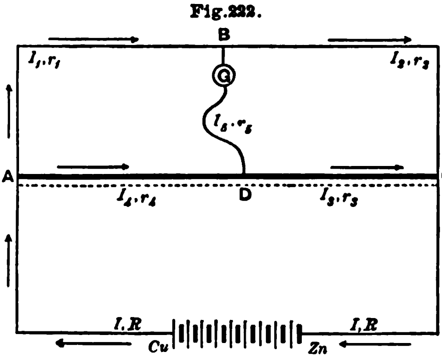
Shunts.—If between A and B (Fig. 221) a single wire run whose resistance is R, a certain current I will pass; if a lateral path or Shunt be made available, the resistance in which is $\frac{1}{99}$R, the current in the shunt is $I_{,,}$, and the current in the original wire will sink to $I_{,}$, 1–100th of its former intensity. This result may be found from the equations $I = I_{,} + I_{,,}$ and $I_{,}R - (I_{,,} \times \frac{1}{99}R) = 0$. If the original wire contain a galvanometer which would suffer risk of damage if the whole original current were sent through it, the current-intensity I can thus, by the use of shunts, be moderated to any desired degree.

If the shunt have a very high resistance the current running in it is proportionately very small, and the distribution of potential in the circuit, as well as the intensity of the current in the original path, is very little interfered with by the interposition of this new path. If in this new path, with its known resistance, there be arranged a galvanometer or a sensitive current-measurer of any other kind, the indications of this instrument will measure the intensity of the current, and therefore the difference of potential between A and B. This is realised in Lord Kelvin's Voltmeter.

The Resistance of two conductors may be compared, by means of a Voltmeter, by observing the relative differences of potentials between pairs of equidistant points in the two conductors, when these conductors are successively, in the same circuit, traversed by one and the same current.

Wheatstone's Bridge. — In Fig. 222 there is represented an arrangement of conductors known by this name. The respective resistances, intensities, and directions of the current are indicated in that figure. Kirchhoff's Laws give us the relations between these. Law I. shows that at A, I (the intensity in the wire CuA) = $I_1 + I_4$ (1); that at B, $I_1 = \pm I_5 + I_2$ (2); and that at C, $I_2 + I_3 = I$ (3). Law II. shows that within the loop CZnCuABC, in which there is a source of difference of potential E, the E.M.D.P. of the battery, $E = IR + I_1R_1 + I_2R_2$ (4); while within the loop ABD there is no galvanic cell, E = 0, and $I_1R_1 \pm I_5R_5 - I_4R_4 = 0$ (5); and similarly within the loop BCD, $I_2R_2 - I_3R_3 \pm I_5R_5 = 0$ (6). From these equations the value of I_5 may be shown to be equal to zero, when $R_1 : R_2 :: R_4 : R_3$, or as regards resistances, AB : BC :: AD : DC.

Fig. 222.



To compare the resistances of two conductors, one is placed between A and B, the other between B and C; then the relative resistances of AD and DC are altered by moving the end of the wire BD along AC (or otherwise by means of resistance-coils) until the galvanometer G gives no deflection. At that moment the resistance of the conductor in AB is to the resistance of the conductor in BC as the length of AD is to the length of DC. A scale under the wire AC enables this last ratio to be read off. If one of the conductors compared — say that in AB — be a wire of known resistance, a Standard Resistance-Coil, the resistance of the other may be absolutely measured. If the resistance in AB be exchanged for a tenfold resistance, the value of the resistance in BC will seem to be numerically diminished to a tenth, and thus resistances ten times as great as before can be measured by being placed between B and C. In this way the range of the instrument can be increased.

When the above ratio obtains between the several resistances, the currents will remain unchanged whether BD remain open or closed, or be closed intermittently by a key.

Sometimes the battery is not kept continuously in action, and a key is interposed in BD: the resistances are adjusted until closing both the battery-circuit and the galvanometer-circuit produces no deflection in G; but to avoid complications due to Self-Induction (p. 704), the battery-circuit must be closed first and then the galvanometer-key pressed down. One key is so arranged as to perform these operations successively.

The Resistance of an Electrolyte may be measured (Kohlrausch's Method) by making AB (Fig. 222) the adjustable resistance, BC the electrolyte in question; instead of a battery insert in AGC the secondary coil of an Induction-Coil, which delivers alternating currents: in BD interpose a Telephone. When nothing is heard in the telephone, $R_{AB} : R_{BC} :: AD : DC$, as before.

Measurement of the Constants of a Galvanic Cell or Battery. — The E.M.D.P. — 1. Wiedemann's Method. Two batteries or cells, the one a standard; connect in series: the joint E.M.D.P. is $E_{,} + E_{,,}$; the total resistance is $R_i' + R_i'' + R_e$; the intensity of the current produced is $I = (E_{,} + E_{,,}) / \overline{R_i' + R_i'' + R_e}$. Now turn one of the batteries round, and connect so that the two now oppose one another; the joint E.M.D.P. is $\overline{E_{,} - E_{,,}}$: the total resistance is as before: the intensity is $I' = (E_{,} - E_{,,}) / \overline{R_i' + R_i'' + R_e}$. Hence $E_{,} : E_{,,} :: (I - I') : (I + I')$. $E_{,}$, the E.M.D.P. of the standard cell, is known; I and I′, the intensities, can be observed: whence $E_{,,}$ can be calculated.

2. Potentiometer-Method. — Connect a low-resistance battery of fair constancy with the extremities of a stretched wire AB, one metre long, divided into millimetres. To one end A of this conducting wire connect, in series, a resistance-coil, a galvanometer, and a Clark's standard cell: the free terminal of the Clark's cell is connected with a wire or "slider" which may be made to touch any point of the stretched wire AB. Adjust the position of the slider on AB, gradually diminishing the resistance, until there is no deflection in the galvanometer. The D.P. between A and the point touched, due to the main current along the wire, is then equal and opposite to the E.M.D.P. of the Clark's cell. Do the same thing with the cell or battery to be tested. For convenience the Clark's cell and the battery to be tested should both be connected with the galvanometer, and the adjustments made with each in succession: and then the chance of variation in

the main battery may be eliminated by touching the stretched wire with the two sliders alternately at the points determined, and thus checking the results. This method eliminates polarisation and the effect of the internal resistance of the battery to be tested.

The Internal Resistance of a Battery may be measured (by Mance's method) by making it one of the four resistances within a Wheatstone's bridge (Fig. 223); one of the other resistances, say AB, being rendered adjustable either by making AB consist of standard resistance-coils or by the use of a Rheostat or Rheochord, by which variable quantities of wire or mercury, or fluids of various kinds equivalent to so many Ohms resistance, may be introduced into AB. A galvanometer is placed in AGC; a key in BD. The adjustable resistance in AB is varied in amount until the deflection of the galvanometer becomes unaffected by making or breaking contact in BD. The relation $R_{AB} : R_{BC} :: AD : DC$ again holds good.

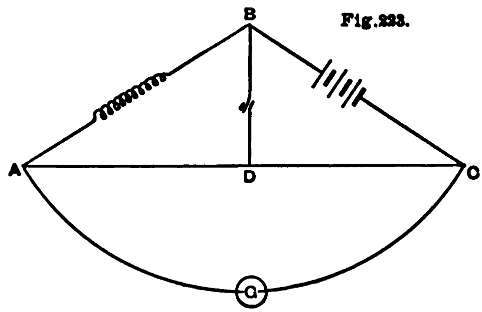


Fig. 223.

If we make the galvanometer G and the battery between B and C exchange places, we have (Lord Kelvin) a very easy method of finding the resistance of a galvanometer coil.

The Energy of a Steady Current. — In a steady current of intensity I, a quantity I of electricity passes during each second from a place where the potential is $V_{\prime}$ to a place where it is V_0; but $V_{\prime} - V_0$, the fall in its potential, is E, the electromotive difference of potential within the circuit. This fall is constant, for the electromotive difference of potential is kept up: the Energy transmitted by the current is therefore $I \times (V_{\prime} - V_0) = IE$ per second; or, in the course of that period of time during which a quantity Q passes, the Total Energy transmitted is equal to QE; all in C.G.S. units or ergs.

Since by Ohm's law $I = E/R$, we find that the Energy per second, $IE, = E^2/R$; and that it is also equal to I^2R, per second.

The energy transmitted by a steady current of one Ampère-intensity under an electromotive potential-difference of one Volt is equal, since Energy per second = EI, to $(\frac{1}{300} \times 3000,000000) = 10,000000$ C.G.S. units or ergs per second. This Rate of Transference of Energy, an Activity of 10 megergs (one Joule) per second, is called an Ampère-Volt or a Watt; and it is equal to 1/746 Horse-power nearly, or to 1/735·75 Cheval-vapeur.

A steady current of V Volts and A Ampères thus represents $AV/746$ horse-power.

This applies to any selected part of the circuit. Let this part be the battery or cell itself. If the resistance of this be R_i and if the actual current-strength be I, then the energy used up in the battery is I^2R_i: and E_i, the potential-fall in the battery, is $E_i = IR_i$. But the whole potential-fall in the whole circuit is $E = I(R_i + R_e)$. Therefore the fraction $R_i \div (R_i + R_e)$ of the total energy is absorbed in the battery or cell itself; and the remainder is at the disposal of the external circuit, whose resistance is R_e.

Energy in charging a condenser. — A condenser may be charged by having its opposite plates put in metallic connection with the opposite terminals of a battery. The opposed plates acquire a potential-difference equal to the E.M.D.P. of the battery as measured on open circuit. When this is done, the energy stored up in the condenser is equal to the total energy of the current during the charging; but the latter has been converted into Heat; therefore the battery has had to evolve a double amount of energy during the accumulation of electrostatic energy in the condenser.

Energy stored in the Ether. — If there be anything of the nature of a Displacement-Current in the dielectric, as there is during the brief period in which the steady current is first being set up, the energy of that displacement-current is stored up in the dielectric, and only appears as Heat when it is restored from the Ether to the wire, on the cessation of the conduction-current.

Transmission of Energy by a Steady Current. — During the passage of a steady current, the Lines of Force in the Field are directed, not at right angles to the wire, but approximately parallel to it. They travel inwards, moving broadside-on, towards the wire; and when they reach it, they give up their momentum (J. J. Thomson). Energy is thus conveyed, not through the wire, but through the Field, the dielectric.

The direction of transmission of energy, being at right angles to the lines of force, is always somewhere along the electric equipotential surfaces in the field. These surfaces radiate out sideways from a cell (Poynting), from the margin of the zinc plate; and those which do not cross over to the copper, and thus provide a path for the absorption of energy by the copper, each cut the wire once, returning to the opposite margin of the zinc plate. Where the equipotential surfaces are most crowded together in the field, as where there is the steepest potential-gradient along the circuit, the transmission of energy through the dielectric is most concentrated; but the energy supplied from any part of the field to the circuit is never any more than is necessary to make up for the local transformations of energy into Heat, Work, etc. The energy is gradually absorbed or transformed as it passes through the wire from circumference to axis, and at the axis it has become wholly transformed. Prof. O. J. Lodge has worked out a scheme of illustrative models to indicate how the Ether may thus transmit Energy; see his *Modern Views of Electricity*. The quantity of energy flowing, at any point in the field, per sq. cm. per second, is (Oliver Heaviside) $\boldsymbol{\phi} \cdot \mathbf{h}/4\pi$ ergs, where $\boldsymbol{\phi}$ and $\mathbf{h}$ are respectively the local electric and magnetic forces per sq. cm. in the field, both at right angles to the direction of transmission of energy.

EFFECTS OF A STEADY CURRENT.

Production of Heat. — If a galvanic circuit be completed and allowed, as it were, steadily to run to waste, no external work being done by it, heat is developed within the cell and in the conducting wire. The Heat produced represents the total Energy of the steady current, and is equal, like that energy, to I^2R ergs per second (Joule's Law), or to E^2/R or EI ergs per second, where R is the total resistance of the circuit, and I the intensity of the current actually passing.

The heat per cub. cm. of the wire is $EI/\text{vol.} = E/l \times I/o = \phi\Delta$.

Problem. — A uniform copper-wire whose cross-section is 4 sq. mm., and whose length is 106·3 metres, connects the poles of a cell whose effective difference of potential is one Volt, and whose internal resistance is 4 Ohms. How much heat will be developed during one minute?

E is one Volt = $\frac{1}{300}$ C.G.S. electrostatic unit of E.M.D.P. The total resistance, R, is 4 Ohms internal + $\left(\frac{1063}{1\cdot063} \times \frac{1}{4} \times \frac{1}{61\cdot70}\right)$ Ohms external = 4·405 Ohms = $\frac{4\cdot405}{900000,000000}$ C.G.S. electrostatic units. The Heat = Energy = $\frac{E^2}{R}$ per second = $\frac{1}{300^2} + \frac{4\cdot405}{900000,000000}$ = 2,270148 ergs per second = 136,208880 ergs per minute = about 3·3 *ca* per minute.

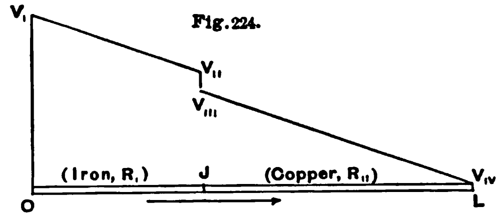
If a current be maintained in a wire, the temperature rises until a point is reached at which the loss by radiation and convection, on the one hand, and the heat supplied at the expense of the energy of the current, on the other, exactly balance one another. Thereafter the temperature remains constant. The wire, being warm, expands. This expansion may be measured. It varies as the intensity varies. If the current-intensity do not materially vary, the amount of expansion may be utilised as a means of ascertaining E, the difference of potential between the ends of the wire. This principle is applied in Major Cardew's and Ayrton and Perry's *Voltmeters*.

When a current is made to pass through a *heterogeneous* conductor composed of different metals, between which there is developed a difference of electric potential (*true contact-effect*, p. 624), the energy, which is wholly converted into heat when no work is done by the current, is divided into two parts. Of these one part obeys Joule's Law, and is equal, per second, to I^2R, the product of the total resistance into the square of the actual intensity: the other, which may be positive or negative, goes to produce **Peltier's effect**, which is the following: — Consider a junction of metals, A and B, such that when this junction is made the *hot* junction of a thermo-electric circuit a current passes through it from A to B: let a current be

made to run *ab externo* through that junction in the same direction, A to B; that junction will under such circumstances be cooled, while if, on the other hand, the current be made to flow from B to A, the junction will be heated.

In Fig. 224, OL is a conductor composed partly of iron, partly of copper; a steady current is made to flow through the conductor from O to L,

Fig. 224.



between the potentials $V_{\prime}$ and V_{IV}. At the junction J there is a sudden fall of potential $(V_{\prime\prime} - V_{\prime\prime\prime})$. In OJ the intensity $= \dfrac{\text{Fall of potential}}{\text{Resistance of OJ}} = \dfrac{V_{\prime} - V_{\prime\prime}}{R_{\prime}}$; in JL the intensity is $\dfrac{V_{\prime\prime\prime} - V_{IV}}{R_{\prime\prime}}$. In both it is equal: hence

$$I = \frac{V_{\prime} - V_{\prime\prime}}{R_{\prime}} = \frac{V_{\prime\prime\prime} - V_{IV}}{R_{\prime\prime}} = \frac{(V_{\prime} - V_{IV}) - (V_{\prime\prime} - V_{\prime\prime\prime})}{R_{\prime} + R_{\prime\prime}} = \frac{E_{OL} - E_J}{R},$$

where E_{OL} is the total fall of potential, E_J the fall at J, and R the total resistance. Hence $E_{OL} = RI + E_J$. The Energy of the current is $E_{OL} \times I = RI^2 + E_J I$. The first part of this expression, RI^2, represents heat distributed over the whole conductor; the second part, $E_J I$, represents heat locally developed at J, and proportional to the *fall* of potential there. If the current be made to pass from copper to iron there will be a rise, a negative fall: the heat developed at the junction will be a negative quantity, and the junction will be cooled.*

In a thermo-electric circuit of copper and iron, the current flows from the copper to the iron across the hot junction. At the hot junction the current passes through a rise of potential (copper-iron); the current therefore tends to cool the hot junction. At the cold junction the current passes through a fall of potential (iron-copper); it therefore tends also to heat the cool junction. This cooling of the hot and heating of the cool junction is **Peltier's Effect.**

When a current is passed *ab externo* through iron, copper, iron successively, it again heats the iron-copper and cools the copper-iron junction. The main current is weakened by the reverse thermo-electric current secondarily produced, and when the main current is cut off, the latter acts alone until the junctions come to the same temperature.

Thomson's Effect (Lord Kelvin's). — The same thing may occur even within a single metal. Hot iron is negative to colder iron; a current, made to pass within a mass of iron from a hotter region to a colder, travels against

* Prof. O. J. Lodge points out that there is no such phenomenon at a junction of copper and zinc: whence he concludes that there is at such a junction no real fall of potential, and that the apparent D.P. of copper-zinc is really the sum of a copper-air and a zinc-air contact-difference.

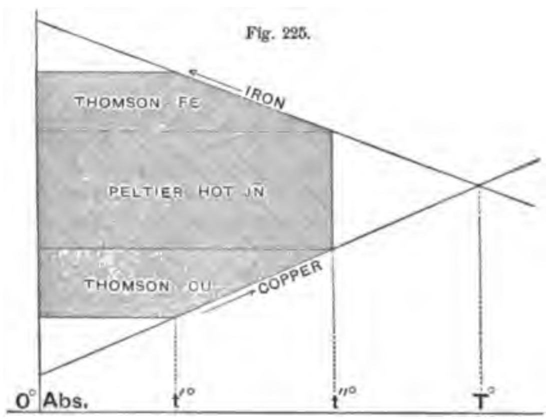
progressively rising potentials and cools the iron in the cooler region; made to pass from cold to hot iron, it heats the iron in the hotter region. It thus tends to exaggerate the existing differences of temperature. These effects are reversed in copper or brass.

The convection of heat by a current of electricity in unequally heated iron is "negative," that is, it is opposed to that convection of heat which would be brought about by the flow of water through an unequally heated tube. In copper, on the other hand, the electric convection of heat is "positive."

In a thermo-electric circuit, therefore, the current, as it travels in the iron from hot to cold, absorbs heat and gains energy; in the copper, travelling from cold to hot it again absorbs heat.

The Thermo-electric Diagram may be made to represent the Thomson and Peltier effects. Let Fig. 225 be a diagram for iron and copper between the temperatures $t_{,}$ and $t_{,,}$. The area marked "Peltier hot junction" represents the amount of energy absorbed at the hot junction when a unit-current passes; the area marked "Thomson-Fe" represents the energy

Fig. 225.



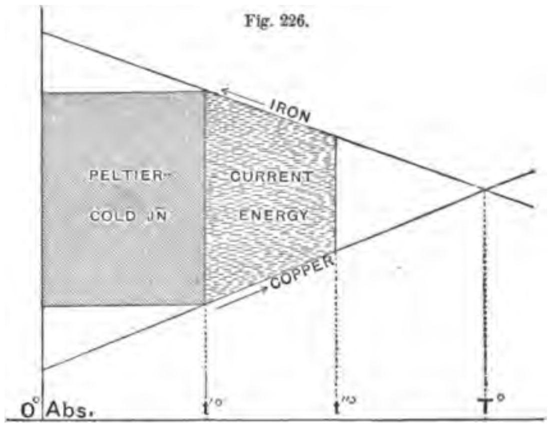
absorbed from the iron when a unit-current passes in it, from hot to cold; the area marked "Thomson-Cu" in the same way represents the Thomson effect in the copper, the amount of energy absorbed from the copper when a unit-current passes in it from cold to hot. The whole shaded area thus represents the energy absorbed, by the cooling of the hot junction and of the unequally heated iron and copper, when the unit-current runs in the direction indicated by the arrows. Plainly, if the hotter junction be heated to T°, the Neutral Point, we shall have the two Thomson effects, and, at the hot junction, no Peltier effect. There is at that temperature no D.P. between the two metals.

Now turn to the energy evolved. This takes two forms: (1) Heat liberated at the colder junction (Peltier effect); and (2) the Energy of Electric Current. The latter, when the current-intensity is unity, is equal to the E.M.D.P.; and we have already seen that this E.M.D.P. is represented by the area between two metal-lines and the ordinates corresponding to the two temperatures. Hence the accompanying diagram (Fig. 226) needs little explanation. If the colder junction be at a temperature of T°,

the Neutral Point, there will, at that junction, be no Peltier effect, no liberation of energy as Heat.

The student may now exercise himself in showing that when the colder junction is at temperature T° the effect is the reverse of that obtained when the hotter junction is at T°; that when one junction is as far below T° as

Fig. 226.



the other is above T°, the area representing the Current-Energy vanishes; and that when the hotter junction is at a temperature farther above T° than that of the colder is below it, the current is reversed.

In these figures the energy supplied is equal to the energy accounted for. The whole arrangement is a kind of thermic engine, in which Heat is absorbed from a Source, partly restored to a Condenser or Sink, and partly converted into the Energy of an Electric Current.

The Thomson effects are themselves reversed in iron at a low red heat, and probably again at a higher temperature, so as to make one if not two new neutral points. The same phenomena occur in nickel at comparatively low temperatures.

When a circuit is composed of various conductors which successively offer different resistances to the current, the Heat produced is distributed among them, **to each according to its resistance,** or its total potential-fall.

Numerical Example:—A circuit consisting of one cell whose E.M.D.P. is 1·8 Volts, and whose internal resistance is 0·7313 Ohm, and of an external conductor composed of 6·170 metres of soft copper-wire 4 sq. mm. in cross-section, in which is interpolated a piece of platinum wire $\frac{1}{10}$ mm. in diar. and 4 cm. in length, will have a total resistance amounting to—Battery 0·7333 Ohm, copper wire $\frac{1}{40}$ Ohm, and platinum wire (equivalent to a mercury column $\left(\frac{4}{100} \times \frac{1}{6{\cdot}46}\right)$ metres long and ·007854 sq. mm. in section) ·7417 Ohm; or on the whole 1·500 Ohms, or $\frac{1{\cdot}5}{900000{,}000000}$ C.G.S. electrostatic units. We assume that a steady current can be set up and maintained for a second within such a circuit, and further, that radiation and convec-

tion of heat may be set aside. Of the total heat produced, $\frac{0 \cdot 7333}{1 \cdot 500}$ is developed in the battery, $\frac{\cdot 025}{1 \cdot 500}$ in the copper wire, and $\frac{\cdot 7417}{1 \cdot 500}$ in the small piece of platinum wire. The total heat produced in a second is

$$\frac{E^2}{R} = \left\{ \left(\frac{1 \cdot 8}{300}\right)^2 \div \frac{1 \cdot 5}{900000,000000} \right\}$$

C.G.S. units or ergs; this is 21,600000 ergs or (21,600000 ÷ 41,593000) *ca.* The heat evolved in the battery — ·4889 of the whole — would be, if we suppose the battery to contain 1 kilogramme of material of a mean specific heat of 0·8, sufficient to raise its temperature by about ·000314° C. in a second; that evolved in the copper wire (whose weight is about 217 grammes and sp. heat = 0·095) by about ·0004° C.; while that liberated in the platinum wire (whose weight is about 0·0276 grammes, and whose sp. heat = 0·0325) would be competent to raise it in a second to the temperature of 289° C.

In electric welding, the two pieces of metal to be welded together are made terminals for a powerful current. They are brought into contact: the current runs: the point of contact offers resistance and becomes very hot: and the hotter it is the worse is its conductivity, and therefore all the greater is its resistance. Heat is thus locally developed: and the metal pieces may, by this means, even be fused together. In electric blasting and the electric cautery the current is made to flow through a very thin piece of platinum wire, which locally becomes red-hot or white-hot; and in electric fuses an excessive current heats a specially fusible part of the circuit so far that it melts, and thus breaks the circuit or "cuts-off" the current.

When a steady current is divided into derived currents (Fig. 221, *et seq.*), the division is such as to correspond to the least possible value of $\Sigma(I^2R)$ in the branches; that is, to the minimum aggregate production of Heat.

Production of Light.—When one part of a circuit presents a relatively-great resistance, the greater part of the heat developed within the circuit is concentrated within that part. When the local resistance is due to a thin platinum wire or a thin filament of carbon or of carbonised paper or vegetable fibre or paste, that bad conductor is so far heated as to emit a considerable amount of light. This is illustrated by the various forms of incandescent lamps or electric "glow-lamps."

Those in which the carbon filament is arranged within a vacuum give out, according to the type of lamp, the number in circuit, and the intensity of the current employed, a light equal to that of from 1 to 1000 candles each (usually 8 or 16). The ordinary data are:—2½ candle-power, 5 Volts × 1·9 Ampères to 25 Volts × 0·4 Ampères; 8 c.-p., 15 *V* × 1·9 *A* to 55 *V* × 0·6 *A*; 16 c.-p., 30 *V* × 1·85 *A* to 105 *V* × 0·58 *A*; 32 c.-p., 55 *V* × 2·0 *A* to 105 *V* × 1·05 *A*; 100 c.-p., 80 *V* × 4·4 *A* to 105 *V* × 3·3 *A*; 200 c.-p., 80 *V* × 8·5 *A* to 105 *V* × 6·5 *A*; 1000 c.-p., 80 *V* × 43·5 *A* to 105 *V* × 33 *A*. The consumpt of energy is, per candle-power, from 3½ to 4 Ampère-Volts per second, or from 0·0047 to 0·0054 horse-power, absorbed from the energy of the current.

If the carbon filament be so constructed as to present the form of a hollow tube, of relatively-great surface and small actual cross-section, the luminous efficiency of the lamp is greatly increased (Bernstein, Cruto).

When too strong a current is driven through such a lamp, the superficial particles of the heated carbon are scattered throughout the vacuum; the carbon is volatilised, and condenses on the wall of the lamp; so with platinum and iridium heated above 1700° C.

When a strong electric current is driven through a carbon rod to a thicker piece of carbon, the thin rod becomes heated; when this takes place in air the carbon burns away rapidly; but if the rod rest loosely by one end upon the thicker mass, the contact is always maintained, and the light is fairly steady so long as any carbon remains.

When the interposed resistance is that of a certain thickness of air, the current will not pass unless the interval be so small or the difference of potential on its two sides be so great that a spark can fly across it. When this is the case the current is established across the interval. If the poles be of carbon, their extremities become intensely hot and wear away by oxidation in the air. The intervening air is so good a conductor when intensely heated that, when the arc has once been established, the poles may be separated to a distance greater than the striking distance in cold air; still, the resistance of the hot air will not alone explain the resistance offered by the voltaic arc to the transmission of the current. This is due to a kind of thermo-electric effect. The positive pole is hotter (4000° C.) than the negative (3000° – 3500° C.); and the greater part of the fall of potential is at the positive carbon, while there is another potential-fall at the negative carbon. These sudden falls of potential are equivalent to a reverse E.M.D.P.; and the resistance of the arc presents two terms, the one constant, the other increasing with the length of the arc. The air in the arc is dark, not bright.

The temperature attained seems to be that of the volatilisation of carbon. In small arc-lamps, this temperature is attained over a small area; in large, with more powerful currents, a larger area attains it; but the brightness of the incandescent carbon per unit of luminous area is the same in both large and small.

The positive pole, being the hotter, wears away about twice as fast as the negative, and becomes hollowed. The problem of electric lighting is to keep the arc in the same place, the carbons

being allowed to approach one another as far as, and only as far as, is necessary in order to make up for their wear.

In Jablochkoff's and Jamin's candles the two carbons were rods, parallel to one another and of equal length; the arc passed between their apices. If the current passed in one direction only, one carbon would wear away faster than the other; the carbons would thus cease to be of the same length. The currents used must rapidly alternate in their direction; both carbons are thus equally worn away, and the length of the arc is constant. The usual fall of potential in Jablochkoff's lamps was from 42 to 43 Volts; the intensity of the current producing the light from 8 to 9 Ampères, and the candles per horse-power about 400.

In arc-lamps two carbon points are placed opposite to one another, and it is the part of a special regulatory mechanism to keep the carbons at a constant distance (3 to 4½ mm.) apart. Such regulating mechanisms depend for their action (1) upon variations of intensity of the current traversing the lamp, or (2) upon variations in the differences of potential between the two ends of the arc, or (3) upon departures from a predetermined relation between this difference of potential and the intensity of the current, or (4) upon variations in the amount of heat developed in the arc. The light given generally varies very much according to the angle from which the lamp is viewed.

The resistance of the voltaic arc is 4 to 10 Ohms; the fall of potential is from 32 to 58 Volts; a 12,000-candle lamp consumes about 7 engine horse-power, an 875-candle lamp about 1 engine h.-p., or ⅔ h.-p. electrical (say 10 Ampères × 50 Volts).

The heat developed in the arc has been utilised by Messrs. Siemens and Huntington, who produced the electric arc within the interior of a crucible, and by its means fused very refractory metals with considerable expedition. M. Moissan has recently succeeded in effecting many extraordinary chemical reductions by the temperature of an arc (450 A and 70 V) enclosed between two blocks of burned lime. At the temperatures obtained, he distilled platinum and evaporated silicium and carbon.

When the electric arc is produced between carbons *in vacuo* a beautiful glow is obtained, the negative pole being surrounded by a blue aureole, and the positive by a stratified pale-blue light. The carbon evaporates, the vessel becomes filled with a blue vapour which darkens to indigo, and this condenses and renders the whole opaque.

If a very little vapour of bisulphide of carbon be introduced into the vacuum, the light becomes insupportably bright, and of an extremely brilliant green. Its spectrum presents channelled regions in the red, yellow, green, and violet, which look like duplicates of one another, reproduced in different colours (Jamin).

Geissler's Vacuum Tubes. — When a discharge of high D.P., as from an electric frictional-machine, or an "induction coil," or a battery of 400 Groves, is sent through a mass of rarefied gas (about $\frac{3}{1000}$ atmos.) contained within a so-called vacuum tube, that gas glows with a bright light, characteristic, as regards its spectrum, of the gas exposed to this operation. The positive pole is surrounded by a bright glow, the negative by a dark space and a set of striæ, and in this case the negative pole is the hotter.

If the vacuum be very good and the tube containing the rarefied gas be somewhat narrow at its middle, the glow breaks up into striæ, which flow and flicker if the current which produces them be in the slightest degree variable. (See p. 662.)

The discharge through a vacuum is shown to be disruptive by the fact that the fall of potential in the vacuum tube is independent of the E.M.D.P. of the circuit, provided that this be sufficient to produce such a discharge at all; and there is a large fall of potential at the negative pole.

The approach of external conductors repels the internal glow and causes its deflection; and the glow is deflected by a magnet in the same way as a wire bearing a current would be.

In very high vacua the discharges from the two poles of a vacuum tube appear to be independent of one another: each pole discharges itself without, as it were, feeling the condition of the opposite pole of the tube. Even where both are positive they may discharge towards one another into the same space.

Electrification of Radiant Matter. — When the rarefaction of a gas is extreme (one-millionth) its matter becomes radiant. The movement of its molecules may be guided and rendered manifest by electrification. In a Geissler's tube, the matter filling which is radiant, the molecules which come in contact with the negative pole are at once repelled from it in lines at right angles to its surface. Energy is imparted to these molecules by the electrified negative-pole, and where these molecules strike each other or other molecules they produce an internal glow; where they strike glass (or diamond or ruby) they produce light and cause phosphorescence; they also produce heat, so that when they are directed from a concave negative-pole upon a piece of platinum, the energy conveyed by them brings that piece of metal to its melting point; and when they strike a movable body they produce obvious mechanical effect (Crookes).

Two streams of molecules proceeding from a forked negative-pole repel one another like two similarly-electrified gold leaves, and the negatively-electrified particles which constitute such a stream are attracted and deflected from their course by a magnet.

Chemical Effects of a Steady Current—Electrolysis.—In most cases, if a liquid permit the passage of a steady current through it, different chemical elements or groups of elements, contained within it, travel in opposite directions along the lines of force, the result being apparent decomposition; in other words, most liquids which possess conductivity are Electrolytes. A few liquids, such as alcohol and ether, though not absolutely non-conductive, are not decomposed by the passage of a current. As a rule, substances which conduct when melted, but insulate when solid and cold, are electrolytes, *e.g.*, glass.

Let us take as an example the effect of a current upon a solution of hydrochloric acid in water. In such a solution insert two platinum plates,—the one, the **positive-electrode,** connected by wire with the copper terminal of a sufficient battery; the other, the **negative-electrode,** with the zinc or negative terminal. Hydrogen is liberated on the surface of the negative-electrode; chlorine is liberated upon the positive-electrode, which by a secondary reaction it attacks and dissolves, with the formation of $PtCl_4$.

According to recent researches, the mechanism of Electrolysis seems to be somewhat the following:—If the liquid lying between the positive and negative electrodes were pure water, these electrodes would be brought by the outside battery to a definite difference of potentials, and there would thereafter be no current, but a condition of electrostatic equilibrium would result, in which the water constituted a field of force; for pure water is a non-conductor. If, however, the water contain, say, HCl in solution, then some hydrogen and some chlorine are already dissociated from one another, and exist equally disseminated throughout the solvent as separate atoms or ions; the hydrogen-atoms are positively and the chlorine-atoms negatively charged with definite and equal quantities of electricity. The positive hydrogen-atoms are attracted by the negative electrode, the negative chlorine by the positive. Each atom, as it comes up to its electrode, discharges its electricity into the general circuit; it is then free to combine with another similar atom, similarly discharged, to form a molecule of ordinary free non-electrified chlorine, or of hydrogen. The phenomena of Elec-

trolysis are, therefore, not phenomena of decomposition, but of discharge of already-dissociated ions upon the electrodes; and the number of free charged ions which can come up to the electrodes in a given time limits the quantity of electricity which they can bring to the electrodes in that time, and thus determines the Conductance (or, inversely, the Resistance) of the electrolyte. Any molecules which are not decomposed by dissociation appear to play no part in the electrolytic conduction: and the electrolytic conductivity of a solution, after making due allowance for dilution, thus measures the extent to which dissociation has taken place in it.

The ions cannot travel with indefinite velocity through the electrolyte; each kind of ion has its own specific velocity under the electromotive action of a given slope of potential.

This causes differences in the number of free ions which reach the electrodes, and corresponding differences in the quantities of electricity discharged upon the electrodes in a given time; hence the total Conductivity depends upon the number of free ions of each kind and the specific velocity of each: and it increases with temperature so long as dissociation remains incomplete, but no further.

In the electrolysis of a solution of hydrochloric acid in water, since the hydrogen travels five times as fast as the chlorine, the solution becomes more weakened towards the positive than towards the negative electrode: while in the electrolysis of a solution of potassium chloride, since the velocities of the ions happen to be nearly equal, the solution is weakened almost equally at the two electrodes.

In copper sulphate, $CuSO_4$, the copper plays the part of the hydrogen of the previous example, while the part of the atom of chlorine is taken by the atom-group or salt-radicle SO_4. The copper liberated at the negative-electrode forms a film upon that electrode; the SO_4 liberated at the positive-electrode reacts secondarily upon the water present; $SO_4 + H_2O = H_2SO_4 + O$; the positive-electrode is surrounded by sulphuric acid, and oxygen is liberated on its surface.

If a solution of sulphate of potash be electrolysed, the ions liberated are K, K, at the negative and SO_4 at the positive electrode; the SO_4 causes the evolution of oxygen as a secondary product at the positive-electrode; the potassium, by the reaction $K + K + 2H_2O = H_2 + 2KHO$, causes the evolution of hydrogen at the negative-electrode. Here the water **seems** to have been decomposed; the apparent decomposition of the water is, however, a secondary result of the liberation of the potassium and SO_4 ions.

Alcohol, oil, and bisulphide of carbon are practically like water, non-conductors when in a pure state; but if metallic salts be dissolved in them, the solutions are electrolytes.

The secondary reactions met with in electrolysis depend upon the time which is allowed for them, and are therefore favoured by currents of small intensity.

If copper chloride be electrolysed between copper electrodes, the one, the negative-electrode, is thickened by a deposit of copper, while the other is worn away, being attacked by the chlorine; and the intervening solution of copper chloride is, if the electrolysing current be feeble, maintained in its state of saturation; but if the electrolysis be very rapid, the solution of the copper electrode does not keep pace with the evolution of chlorine upon it, and the solution becomes weaker in copper and acid in its reaction.

The secondary reactions observed are sometimes very peculiar. When hydrochloric acid is electrolysed, the chlorine evolved at the positive-electrode attacks the water present and liberates oxygen, which in its turn attacks some of the hydrochloric acid present and produces chloric and perchloric acids. When Na_2CO_3 is electrolysed, Na appears at the negative pole (producing a secondary evolution of hydrogen) and $CNaO_3$ at the positive pole; this last group reacts upon water and forms oxygen and $NaHCO_3$; $2CNaO_3 + H_2O = O + 2HNaCO_3$. When $NaHCO_3$ is electrolysed, it produces Na and CHO_3, and then $2CHO_3 = 2CO_2 + H_2O + O$. When formic acid (H.COOH) is electrolysed it breaks up into H and COOH; then $2COOH + H_2O = 2H.COOH + O$, or formic acid and oxygen; but the oxygen reacts upon some of the formic acid present, and then $H.COOH + O = H_2O + CO_2$. When fused caustic-potash is electrolysed, K appears at the negative, HO at the positive; this coalesces into H_2O_2, and breaks up into H_2O and O; but if the action be slow the K acts upon some of the water, and hydrogen is evolved. The nascent hydrogen evolved at the negative pole will attack aldehydes, forming alcohols, and thus certain ill-tasted rough alcohols may be greatly improved.

Faraday's Laws of Electrolysis.— First Law.— The quantity of material liberated at the electrodes from a given electrolyte in a given time is directly p r o p o r t i o n a l t o t h e i n t e n s i t y or **strength** of the actual current; or, in other words, the quantity of electricity which passes in a given time, in a circuit of which a given electrolyte forms part, is proportional to the quantity of the ions actually liberated at the electrodes in that time.

Faraday's Second Law. — This Law of Electrochemical Equivalence may be divided into the following propositions, of which the fourth may be regarded as a paraphrase of the law itself: —

1. The gramme-equivalent of a metal is that quantity which will chemically replace one gramme of hydrogen. For example: in comparing HCl ($H = 1$, $Cl = 35{\cdot}5$) with AgCl ($Ag = 108$, $Cl = 35{\cdot}5$), we find Ag ($= 108$) to be equivalent to H ($= 1$); the gramme-equivalent of silver is 108 grammes. In comparing $CuSO_4$ with H_2SO_4 we find Cu ($= 63{\cdot}46$) equivalent to H_2 ($= 2$); the gramme-equivalent of copper is 31·73 grammes.

2. The gramme-equivalent of a salt-radicle or halogen is that quantity which will combine with one gramme of hydrogen. In HCl, 35·5 grammes of chlorine unite with 1 gramme of hydrogen; the gramme-equivalent of chlorine is 35·5 grammes. In HNO_3 ($H = 1$, $NO_3 = 62$), 62 grammes of NO_3 unite with 1 gramme of H; the gramme-equivalent of NO_3 is 62. In H_2SO_4 ($H_2 = 2$, $SO_4 = 96$), the gramme-equivalent of SO_4 is 48 grammes. In H_3PO_4 ($H_3 = 3$, $PO_4 = 95$) the gramme-equivalent of PO_4 is $31\frac{2}{3}$.

3. The gramme-equivalent of a salt or acid is that quantity which contains 1 gramme-equivalent of the halogen or salt-radicle.

4. When a current whose intensity, after the current has become steady, is equal to A Ampères passes in a solution of a salt, $0{\cdot}000{,}010352A$ gramme-equivalents of the salt are electrolysed during each second; $0{\cdot}000{,}010352A$ gramme-equivalents of the salt-radicle or halogen being liberated at the positive electrode and a corresponding quantity of the metal at the negative.

If both ions be monovalent, as in $Ag \mid NO_3$, the electrolysis of the solution causes the liberation or deposition of $0{\cdot}000{,}010352A$ gramme-equivalents of the metal as well as of the negative ion, where A is the number of Ampères. A current of 1 Ampère deposits from a solution of pure nitrate of silver 0·001,118 gramme of silver per second, or 4·025 grammes per hour; this corresponds to 0·000,010352 gramme of H per second.

If the ions differ in chemical valency, the rule holds good that the amount of the negative ion, the halogen or the salt-radicle, liberated at the positive-electrode, is equal to $0{\cdot}000{,}010352A$ gramme-equivalents.

Thus let a current pass simultaneously, in "series," through a solution of cupric chloride and a solution of cuprous chloride, and continue to pass through both solutions for five minutes; its intensity is 18 Ampères; compare the amounts of copper deposited on the negative-electrodes in the two solutions. In each the amount of the halogen — the chlorine — liberated is ($\cdot000010352 \times 18$ Amp. $\times 300$ sec.) gramme-equivalents, or ($\cdot000010352 \times 18 \times 300 \times 35{\cdot}5$) grammes. In $CuCl_2$ every 71 parts of chlorine are combined with 63·46 of copper; the copper deposited from the $CuCl_2$ is therefore

$$\left\{(\cdot000010352 \times 18 \times 300 \times 35{\cdot}5) \times \frac{63{\cdot}46}{71}\right\} \text{ grammes.}$$

In Cu_2Cl_2 every 71 parts of chlorine are combined with 126·92 of copper; the copper deposited from the Cu_2Cl_2 solution is therefore $\left\{ (\cdot 000010352 \times 18 \times 300 \times 35{\cdot}5) \times \frac{126{\cdot}92}{71} \right\}$ grammes, double the quantity deposited by the same current from a solution of cupric chloride.

When acidulated water is electrolysed, $0{\cdot}000010352\,A$ grm.-equivts. of salt-radicle are set free: these take up the hydrogen of $0{\cdot}000010352\,A$ grm.-equivts. of water, and set free $0{\cdot}000010352\,A$ grm.-equivts. of oxygen, that is, $(0{\cdot}000010352\,A \times 8)$ grammes. Correspondingly, $0{\cdot}000010352\,A$ grammes of hydrogen are set free at the negative-electrode. Thus it is said that a current whose intensity is A Ampères will decompose $0{\cdot}000{,}092{,}961\,A$ grammes of water per second: though a current will not decompose water at all, except by secondary reactions such as the above. A Voltameter is an instrument in which a current is made to pass through acidulated water between platinum electrodes; the hydrogen and oxygen liberated at the electrodes are collected, either together or separately, in a graduated tube or tubes, and measured; a simple calculation gives the strength of the current actually passing through the voltameter. Instead of sending the whole current through the voltameter, we may send a known fractional part of it by arranging the instrument in a Shunt.

Since each Ampère will liberate 0·000010352 grm.-equivt. of salt-radicle or halogen per second, each Coulomb of electricity will liberate that quantity independently of the time; and each C.G.S. electrostatic unit will liberate one 3000,000000th part of this quantity, *i.e.*, 0·000000,000000,0034506 gramme-equivalents. This last quantity is otherwise known as the electrostatic **Electrochemical Equivalent** of the salt-radicle or halogen liberated or salt electrolysed.

The Energy of a current passing a quantity Q down a constant potential-fall E is equal to EQ; when $Q = 1$ this energy is, numerically, E ergs; but it is also, if no energy be lost or gained collaterally, equal to the work done by the unit quantity in electrolysing one electrochemical equivalent of a salt. Hence, conversely, the chemical energy liberated by the production of one electrochemical equivalent of a salt may be measured in terms of a potential-fall or an electromotive difference of potential.

The E.M.D.P. in a galvanic cell may thus be computed in terms of the chemical energy set free in it. Let us enquire what is the value of the E.M.D.P. of a Daniell's cell. Here we have a chemical action going on which liberates electric energy: this energy, if not utilised as the energy of a current of electricity, may wholly appear as heat; this heat has been measured in various ways, and the mean result of several observations is that (between extreme values 714 and 805) the amount of heat liberated when one gramme of zinc is dissolved in the cell amounts to 760 *ca* or 31,610,680000 ergs. The gramme-equivalent of zinc is 32·645 grammes (Marignac); the electrostatic electrochemical equivalent of zinc is $(0{\cdot}000000{,}000000{,}0034506 \times 32{\cdot}645) = 0{\cdot}000000{,}000000{,}112645$ grammes. The amount of energy liberated on the solution of one electrostatic electrochemical equivalent of zinc is therefore $(0{\cdot}000000{,}000000{,}112645 \times 31{,}610{,}680000)$ ergs or 0·00356 ergs. This being equal to the amount of energy liberated during the solution of one electrostatic electrochemical equivalent of zinc in a Daniell's cell and the production of a current of one electrostatic unit-intensity for one second, is necessarily equal to the energy which would be spent by a current of the same intensity, and enduring for the same time,

on reversing the whole chemical process which results in the production of an electrochemical equivalent of zinc sulphate, that is, on electrolysing one electrochemical equivalent of that salt. This we have seen to be numerically equal to E, the difference of potential under which the unit current passes. The E.M.D.P. of a Daniell's cell is therefore $E = 0{\cdot}00356$ electrostatic units of difference of potential. This is equal to 1·068 Volts; a theoretical result which does not depart widely from the experimental values, which range between 1 Volt and 1·124 Volt. This mode of computation is due to Lord Kelvin.

Electrolysis in Gases.—Cold gas will not conduct, if dustless: hot gas will: it seems that some amount of dissociation is necessary for conduction. Those gases which are most readily dissociated are the best conductors. Mass for mass, highly rarefied gases have high conductivities: and dissociated molecules appear to be concerned in conduction. These molecules appear to be disposed in chains here and there along the path of the current, those of each kind acting together, and the average length of the chains being the same as the distance between the striæ. When a current goes through rarefied steam, oxygen and hydrogen are liberated, in approximately the same amount as in a voltameter traversed by the same current (Perrot and J. J. Thomson). A current once started in a gas can be kept up with comparative ease; the molecules have been sufficiently dissociated to act as carriers. When oxygen is sent through a space across which a silent discharge is passing, it becomes in part converted into ozone, especially if there be rapid variations in the potential-difference. Prof. Schuster has shown reason for believing that in highly rarefied gases there is considerable dissociation of molecules into atoms around the negative electrode. In mercury-vapour, which is monatomic, the phenomena of glow are the same round both terminals.

In Electrolysis, Work is done by the current; its Energy is spent in separation of the charged ions of the electrolyte. The energy absorbed in the electrolytic production of, say, 9 grammes of oxy-hydrogen gas from acidulated water, is approximately equal to the Heat liberated by chemical action and change of physical state when 1 gramme of hydrogen and 8 of oxygen are exploded together and condensed into water.

Work done on electrolysis, if such work be other than chemical, causes divergences from Faraday's second law: Such work is done when the products of electrolysis of a liquid become gaseous, or when the metal of an electrolyte is lifted up towards the negative-electrode; while the energy of the current is increased when these circumstances are reversed.

The relation of the energy liberated by the chemical action of the battery to the heat produced in the battery and the energy spent in doing electrolytic work is represented by the equation —

Battery-energy = Heat evolved in battery + Electrolytic work done.

The last term cannot be greater than the first; if it were, the heat developed in the battery would be a negative quan-

tity, and the battery would cool itself as well as all surrounding objects.

That this should go on indefinitely is impossible: and in such cases the E.M.D.P. of the battery falls as the temperature falls: and, further, heating the battery causes a rise of E.M.D.P. Similarly, if the battery become heated in action, heating it causes a fall in its E.M.D.P. If the battery become neither heated nor cooled, as in the case of Daniell's cell, heating or cooling it from without causes no change in its E.M.D.P.; and it is only in such cases that the E.M.D.P. can be directly calculated from the chemical energy alone.

In a Daniell cell the replacement of one grm.-equivt. of copper by one grm.-equivt. of zinc is attended with the evolution of 24,200 *ca* of heat; in a Grove the energy evolved is 47,000 *ca* for every grm.-equivt. of Zn dissolved. When 1 grm. H and 8 grm. O unite to form one grm.-equivt. of water, 34,462 *ca* of heat are evolved. A single Grove cell can therefore electrolyse acidulated water; a cell with a potential-difference equal to $\frac{34462}{24200}$ times that of a Daniell would just be able to do so; a single Daniell cannot. Two such cells are, however, able to effect electrolysis of water; the energy supplied by two cells arranged in series is double that supplied by one, even though the resistances be so adjusted that the current produced is of the same intensity: for Energy per second = EI, and if E be doubled while I remains the same, the energy is doubled.

A single cell will, however, electrolyse water if the positive-electrode be of a substance such as copper, which will combine with oxygen, and thereby liberate energy sufficient to make up the deficiency in that supplied by the cell.

The process of electrolysis is turned to practical account in the arts of electroplating, etc.; the article to be covered with a metallic film is made the negative-electrode in a suitable solution of the metal. The positive-electrode is often itself made of the metal to be deposited from the solution; as the metal is deposited from the solution upon the negative-electrode, the positive-electrode is attacked and its substance dissolved in the solution, which is thus kept saturated if the action be not too rapid.

In the electrolysis of mixtures, the reaction which requires the least energy is the first to be completed; and thus, by regulating the potential-difference, the different metals may be successively deposited from a mixed solution.

When a mixed solution of acetates of lead and copper (Nobili), or a solution of litharge in caustic potash (Becquerel), is electrolysed between two electrodes, of which the negative is a sharp-pointed platinum wire or steel needle, while the positive is a large plate of silver, german-silver, silvered copper (spangle metal), or even thin sheet-iron, the current being one of relatively-great E.M.D.P. (15 to 20 very small Bunsen cells mounted in file), there is formed on the positive-electrode a series of rings concentric with the point of the negative-electrode, — **Nobili's rings**, produced by deposition of PbO_2. If the positive-electrode be complex and consist of two or more points, or if currents be made to run through some of these points towards the plate, while in others the direction is reversed, the rings are modified into representations, in iridescent hues, of complete systems of equipotential surfaces (Guébhard).

Polarisation of Electrodes. — When platinum electrodes have been used in the electrolysis of water, the negative one is found to contain a certain quantity of hydrogen not only adherent to its surface, but also occluded within its substance; while the positive one carries similarly a certain quantity of oxygen. These oxygen and hydrogen films tend to produce a reverse current, which weakens the main current. When the main current ceases, the reverse current continues for some time and dies away gradually.

The smallest potential-difference between the electrodes is sufficient to direct the ions so as to set up this state of polarisation; and the reverse current tending to be produced by this is equal and opposite to the main current, so that between the two there is no effect, until the latter reaches a certain limit. When it exceeds this limit, a current passes; but it is only the excess of the main current above the virtual reverse current which can pass through the electrolyte.

The occluded hydrogen is very gradually reduced by the oxygen brought to it by a reverse current, and a corresponding quantity of hydrogen is liberated on the positive pole, where it is occluded by the electrode or oxidised by the occluded oxygen or dissolved by the water. Thus, though a reverse current passes when the main current ceases, no products visibly appear.

In Grove's **gas-battery** a number of such electrodes are arranged so as to give a current, or — which amounts to the same thing — a circuit is arranged in which a current passes through: (1) water; (2) a plate of platinum standing partly in water, partly in hydrogen gas; (3) conducting wire; (4) a plate of platinum standing partly in an atmosphere of ozoniferous or electrolytic oxygen, partly in the water.

If electrodes of palladium be used in the decomposition of water, the negative one absorbs as much as 936 vols. of hydrogen, with which it forms an alloy, greater in size than the original electrode.

The electrodes in such a case are said to be **polarised**, and their polarisation within a given substance is (if the resistance of that substance be not so great as to prevent any perceptible current from passing under any potential-difference) the best test as to whether that substance is really an electrolyte. Warm glass is thus found to be actually an electrolyte.

The capacity of electrodes so polarised is very great. Two square inches of platinum electrode immersed in dilute sulphuric acid have (Varley) when the E.M.D.P. is one-fiftieth that of a Daniell's cell, a capacity equal to that of an electrostatic condenser whose plates have an area of 80,000000 square inches separated by $\frac{1}{8}$ inch of air; *i.e.*, a capacity of 175 microfarads: while as the E.M.D.P. increases, the capacity increases also.

Electrodes of amalgamated zinc are not at all polarised when used to transmit a current through a perfectly neutral solution of $ZnSO_4$ (Beetz).

When a powerful current is sent through a metal immersed in dilute sulphuric acid, hydrogen evolved on its surface offers local resistance, and

heat is there evolved; the metal may rapidly become even white-hot (Lagrange and Hoho's process for welding).

Secondary Cells and Batteries. — When acidulated water is electrolysed between electrodes of lead, the negative electrode remains bright, but the positive becomes covered with a film of PbO_2. When the current ceases, if the positive and negative electrodes be connected by a conducting wire, a reverse current — a polarisation-current, or Secondary Current — passes; the film of PbO_2 is electro-negative like the copper in an ordinary cell, the lead electro-positive, like the zinc; and the current passes in the conducting wire from PbO_2 to Pb; for which reason the PbO_2 pole of the secondary cell or battery is called the positive pole.

The reverse action appears to be (Gladstone and Tribe) in the main the following: —

$$PbO_2 \mid H_2SO_4 \mid H_2SO_4 \mid Pb = PbO \mid H_2O \mid H_2SO_4 \mid PbSO_4$$

which becomes

$$PbSO_4 \mid H_2O \mid H_2O \mid PbSO_4.$$

On passing in a charging current, the reaction is, for thin layers,

$$PbSO_4, H_2O, O; \; H_2, SO_4Pb = PbO_2, SO_4H_2; \; H_2SO_4, Pb.$$

An arrangement of this kind, into which a current of electricity can be passed and a reverse or secondary current obtained at will, is a Secondary Cell; and secondary cells may be grouped into Secondary Batteries. Planté's original form of secondary cell consisted of two large lead electrodes, separated by a sheet of felt, rolled up into a spiral and immersed in 10% dilute sulphuric acid. Faure improved this by covering both electrodes with a layer of red-lead of about 10 kilogrs. per sq. metre, held on by layers of felt and parchment between the opposed plates. When a current has been passed into a so-called 'Faure-accumulator' for some time, the red-lead on the surface of the negative-electrode is converted into spongy lead, while that on the positive-electrode is oxidised to PbO_2. This improvement greatly abridged the tedious process previously necessary for charging Planté's 'accumulators.'

In the Faure-Sellon-Volckmar 'accumulators' there is no felt; the plates of lead are pierced or cast with holes, into which there is compressed a quantity of red-lead, of reduced lead, or of a salt of lead, or a mixture of PbO, Pb_3O_4, and $PbSO_4$. One of these, weighing 140 kgr., and composed of 43 plates, gave (Hospitalier) a current of 120 Amp. for 6 hours.

The efficiency of these batteries is in part due to the fact that their porous metal or oxide is in close contact with the lead plate, and is, on

account of its porosity, able to retain large quantities of the oxygen or hydrogen which is evolved when an external current is passed through the accumulator.

When the secondary current has passed for some time, both films become mainly converted into sulphate of lead, and the apparatus is ready for a renewed charge.

When a secondary battery is charged by two or three Grove cells and disconnected from them, it will be rapidly discharged if connection be established between its poles by means of a thick wire. From sixty to seventy per cent, or, in the newer forms, a still greater proportion of the energy actually sunk in it can be recovered in the form of the energy of the secondary current, especially if the battery be not allowed to run down too far. When the cells of a secondary battery, charged side by side, are disconnected from the source, and then connected in series and discharged, the electric current produced is one of "high tension" or great fall of potential. The D.P. of a single cell is about 2·25 Volts, and hence a hundred such cells, first arranged in surface and charged by prolonged connection with a few cells, can be made, if arranged in series, to pass for a short time a current of E.M.D.P. = 225 Volts; such a current produces a high temperature, together with vibration and crumpling of the conducting wire. The internal resistance of these cells is very small, being only 0·006 Ohm in a single cell whose surface is about 300 sq. cm.

A secondary cell of about 18 lbs. weight, the plates in which are each about 2 sq. feet in area, will, when charged by three Leclanchés, keep at a white heat during 5 to 10 minutes a platinum wire of $\frac{1}{40}$ inch diar. and 8 inches long. A pile of this weight, kept constantly connected with three or four Leclanché elements (which require little attention beyond that of keeping them moist), is a very convenient means of heating such a thing as a galvano-cautery wire, which must be raised to a high temperature for a short time. The rate of discharge should not exceed 1 Ampère per sq. decimetre of plates.

Electrical Storage of Energy.— When energy is stored up in bent steel springs, about 3924 megergs per kilogramme can be stored up — *i.e.*, 40 kilogrs. can be lifted through 1 metre by the elasticity of a spring weighing one kilogramme.

When it is stored up in compressed air, 1 kilogr. of air compressed to one-sixth contains 2,250,000 megergs, of which about 450,000 can be recovered in the form of work.

When it is stored in secondary batteries, about 500,000 megergs are stored up per kilogr. of secondary battery. Of these,

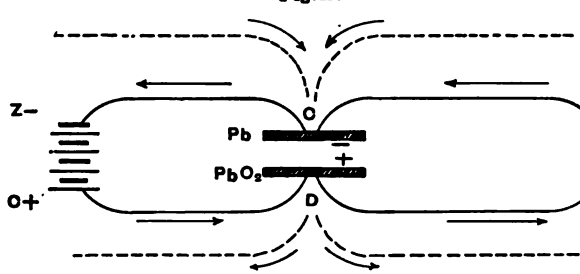
from 250,000 to 330,000 may be recovered if the batteries be used within a day or two after charging.

The great fault of these "accumulators" in their present form is their want of durability.

Equalisation of a Current. — A current passed through one plate of an Electrostatic Condenser will be apparently absorbed when the current is increased, and will be given out equably when the current falls off; such a condenser is therefore competent to play the part taken by a flywheel in the mechanical transmission of power.

Similarly, Secondary Cells may be made to serve as regulators of a current if they be fitted up in the course of the conductor in the manner indicated in Fig. 227. The direction of

Fig. 227.



the secondary current is indicated by the dotted lines and arrows connected with the secondary cell; it opposes the main current in CZnCuD, but aids it in DEC.

The Dynamical Properties of a Steady Current.

If a straight conducting wire, forming part of a wide circuit, bearing a steady current, be passed vertically through a hole in a piece of card or of silver-paper adjusted to a horizontal position, and if iron filings be then sprinkled upon the card, and if the card be gently tapped downwards so that the filings may leap into positions spontaneously assumed by them, they will be found to range themselves in concentric circles round the current, while each filing becomes, for the time being, a little magnet.

The space round the current is therefore an **Electromagnetic Field of Force,** permeated by concentric circular Lines of Magnetic Force or of Magnetic Induction and by Magnetic Equipotential Surfaces, which are at right angles to these. The magnetic equipotential surfaces all have

the line of the current for their common edge or boundary. If the current be straight, these equipotential surfaces are planes; and if they were visible, and if the current could be looked at end-on, so as to appear a mere point, these surfaces would seem to radiate from it like equidistant radii from a centre.

The lines of Magnetic Force mark the direction in which an ordinary magnet, such as a small compass-needle, when placed within the field, tends to place itself. The one end of the magnet is driven in one direction, the other end equally in the opposite direction, along these lines of force: the magnet is acted upon by a couple, which acts upon the two extremities or Poles like the hands on the handles of a copying press — one pole being pushed or repelled, the other being pulled or attracted, until the magnet lies along a line of force. The moment of the couple gradually diminishes as this position is being assumed, and the couple ultimately ceases to produce farther rotation; and further, since one pole is attracted as much as the other is repelled, the magnet as a whole undergoes in such a field no movement of translation.

The direction in which a current tends to throw the positive or north-seeking pole of a magnet placed in its neighbourhood is shown by Fig. 228. This direction is called the **Positive Direction** of the lines of magnetic force. The Current in the figure passes vertically Upwards; the Positive pole is thrown to the Left Hand of the Current. This expression, left hand of the current, is obtained by supposing the current to be replaced by a person whose head is at B and feet at A, and who turns so as constantly to keep the magnet-pole in full view. The relation between the direction of the current and the positive direction of the lines of magnetic force is always the same as that between the propulsion of the point and the twist of the hand in the ordinary use of a European corkscrew. Conversely, action and reaction being opposite, stationary positive magnet-poles tend to throw movable conductors, bearing upward currents, to their left. For negative magnet-poles the directions given are reversed.

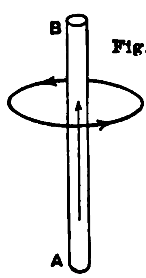


Fig. 228.

Proposed Mnemonic Rule. — If a pen be held in the right hand in the usual way, the penholder may represent the wire, and the direction of flow of ink (towards the point) the direction of the current; if then the thumb be stretched across the

penholder it will represent the magnet, and the thumb-nail its marked or north-seeking pole. The same relation may be still more simply brought to mind by laying the thumb across the forefinger of the right hand; either of these will then represent the current (flowing towards the finger-tip or the thumb-tip, as the case may be), the other the magnet.

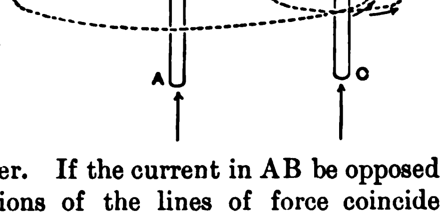
A magnet-pole may be made to rotate round a current by keeping the other pole in the axis of rotation. In general, magnet-poles tend to rotate round currents so long as these are maintained; and, conversely, currents tend to rotate round magnet-poles: and the deflection of a magnet by a current is only a particular case of this. The action between a current and a magnet-pole is thus at right angles to the line joining them.

There is a magnetic field even within the wire; if a magnet be made itself a conductor, the steady current being led from its midpoint to one end, it will spin.

Further, Linear Currents act upon other Linear Currents; but they do not throw them to the right or left; they attract or repel them directly.

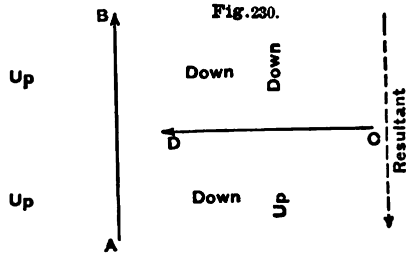
In Fig. 229, AB is a steady current; round it there are lines of magnetic force, the number of which per sq. cm., at any point, varies inversely as the mean distance from the axis of the wire. Let another current, parallel and in the same direction, be brought to CD: then, between AB and CD the lines of force of the two currents are opposed, but beyond CD they concur in their direction. The result is that the medium beyond CD is in a state of greater constraint than that between AB and CD; so is that to the left side of AB; and the two conductors are impelled towards one another. If the current in AB be opposed to that in CD, the directions of the lines of force coincide between AB and CD, but are opposed beyond AB or CD, and the stress is such as to drive AB and CD asunder. If the currents in AB and CD be at right angles, approaching one another (Fig. 230), the current in AB has lines of force whose direction is upward on the left side of AB, downward on its right. The current CD has lines which would depress a positive pole placed in the upper, and raise a positive pole placed

Fig. 229.



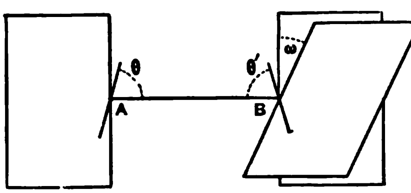
in the lower part of the figure. The concurrence of lines of force is in the upper part of the diagram (Fig. 230): if the current AB be fixed, the conductor CD is repelled in a direction contrary to that of the course of the current in AB. If the current in CD pass from D towards C, the conductor CD tends to move in the same direction as the current in AB.

Fig. 230.



These results were first summarised in **Ampère's formula.** The mechanical force of Attraction between very small elements of two linear circuits is equal, in dynes, to $ii' \cdot ll'$ $(2 \sin \theta \sin \theta' \cos \omega - \cos \theta \cos \theta')/d^2$, where i and i' are, in magnetic measurement (p. 707), the intensities of the currents passing in the two wires; l and l' the lengths of the elements; d the distance between their midpoints; and θ, θ', ω are the angles which determine their relative direction as follows:— Each element makes an angle θ or θ' with the line AB; but further, these elements are situated in planes which make an angle ω with one another.

Fig. 231.



We may take some particular cases.

Let both currents lie in the plane of the paper; the angle $\omega = 0$; $\cos \omega = 1$; $F = ii' \cdot ll' \cdot (2 \sin \theta \sin \theta' - \cos \theta \cos \theta')/d^2$.

1. Let both currents be parallel to AB; $\theta = 0$, $\theta' = 0$; $\sin \theta = \sin \theta' = 0$; $\cos \theta = \cos \theta' = 1$; $F = - ii' \cdot ll'/d^2$. Two elements of current, end-on to one another, and running in the same direction, repel one another.

2. Let both be at right angles to AB; $\theta = \theta' = 90°$; $\cos \theta = 0$; $\sin \theta = 1$; $F = {}^*2\, ii' \cdot ll'/d^2$. Two elements of current, parallel and abreast of one another, running in the same direction, attract one another. If they be opposed in direction, the product ii' is negative; then F is negative, and the mutual action is one of repulsion.

3. Let one be at right angles to AB, the other parallel to it; $\theta = 90°$; $\theta' = 0$. $\cos \theta = 0 = \sin \theta'$; $\cos \theta' = 1 = \sin \theta$. $F = 0$. There is no appreciable action between two extremely small elements one of which points end-on and at right angles to the centre of the other.

4. Let one be at right angles to AB, the other at any other angle, say 45°. $\theta = 90°$, $\theta' = 45°$. $F = \sqrt{2} \cdot ii' \cdot ll'/d^2$. Two currents diverging from

* If in this case $l = l' = d = 1$ cm., and if $l = l' = 1$ electromagnetic unit, $F = 2$ dynes; but by using smaller intensities, each equal to (1 E.-M. unit $\div \sqrt{2}$), F becomes equal to one dyne. The reduced intensity so found is the electrodynamic unit of current-intensity, the basis of a system of measurement now disused, but still occasionally referred to, especially in French works.

a point attract one another; if one of the currents be reversed, they repel one another; if both be reversed, so that both converge upon an angle, they attract one another.

5. Two elements, B and A, parallel in their direction; the resultant force is one of attraction between B and A. A similar element at A′ attracts B towards A′. Of these two attractions the resultant is towards O. Pairs of such elements, symmetrically ranged to an infinite distance on either side

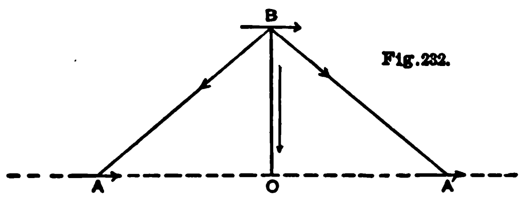


Fig. 232.

of O, make up an infinite conductor whose attraction for B is, in the aggregate, $F = 2\ (ii'/OB) \times$ length of element B. The attraction of an infinite current for an element of current running parallel to it is directed along a line at right angles to both currents, and is inversely proportional to the distance between them.

6. The element B flows towards O; A and A′ are equal elements, symmetrically arranged, of an infinite current along AA′: A attracts B: A′

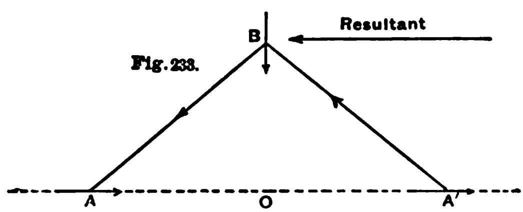


Fig. 233.

equally repels B; the resultant is in a line parallel to AA′, and tends to drive the element B in the direction A′A.

The formulæ of Grassmann, Clausius, von Helmholtz, and others, we here pass over.

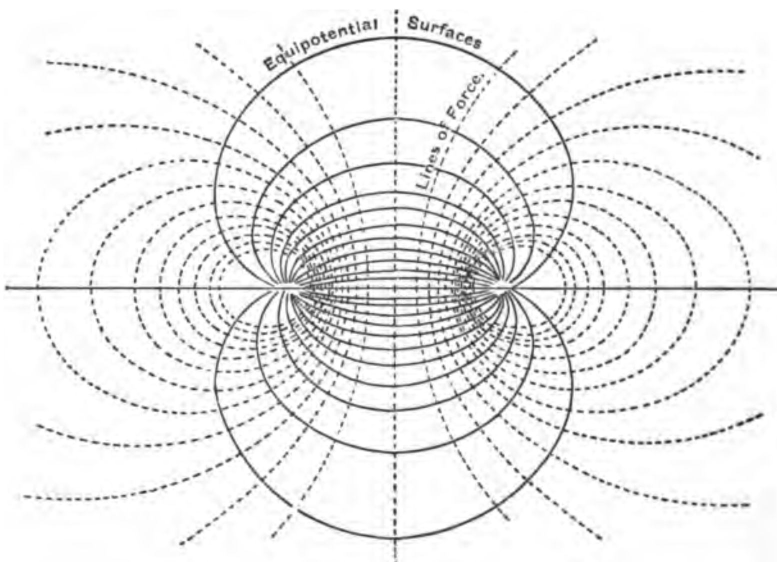
In all the above examples, it is assumed that the medium between the two wires is air; if it be not air, all the above expressions for the value of F will have to be multiplied by a factor μ, special to each medium, and known as the Magnetic Permeability, p. 685.

The difference between an Electrostatic and a Current Field is, that in the former the Lines of Electric Force are steady, while in the latter they are in motion. In the former case there is no magnetic induction; in the latter there is. The relation between the lines of Electric Force and those of Magnetic Induction may be expressed by a simple illustration. Conceive a guide-post, which of course points upwards, bearing two pointers, pointing respectively N. and E.; let the guide-post itself represent a Line of Electric Force, directed upwards; let the one pointer, the one pointing Northwards, be marked "This way the Direction of Motion of the Line of Electric Force, of Momentum, of Transmission of Energy, and of Repulsion of a Current in the Electromagnetic Field;" and the other, the one pointing Eastwards, be marked "This way the Magnetic Force and Induction:"

and further, conceive the pointers to shrink to nothing when the guide-post is at rest, but to grow simultaneously, proportional to one another, when the post is moved. Then if a model of this be made, and turned into any position, and moved by translation, but always so as to follow the direction of the Motion-pointer, it will denote the respective directions of the lines of Electric Force, of the lines of Magnetic Force and Induction, and of their Motion in the Field. A surface joining the two rectangular pointers would be at right angles to the post and would represent an Electric Equipotential Surface in the field; one joining the post with the Motion-pointer would be at right angles to the magnetic force, and would represent a Magnetic Equipotential Surface in the field.

If a wire bearing a straight steady current be bent into a closed circuit or loop, its equipotential surfaces are modified into a series of bowl-shaped surfaces which still have the wire, the contour of the circuit, for their common edge or boundary. A circular current would have equipotential surfaces whose general form, looked at in section, is indicated by the undotted curves of Fig. 234, in which the lines of force are shown at right

Fig. 234.

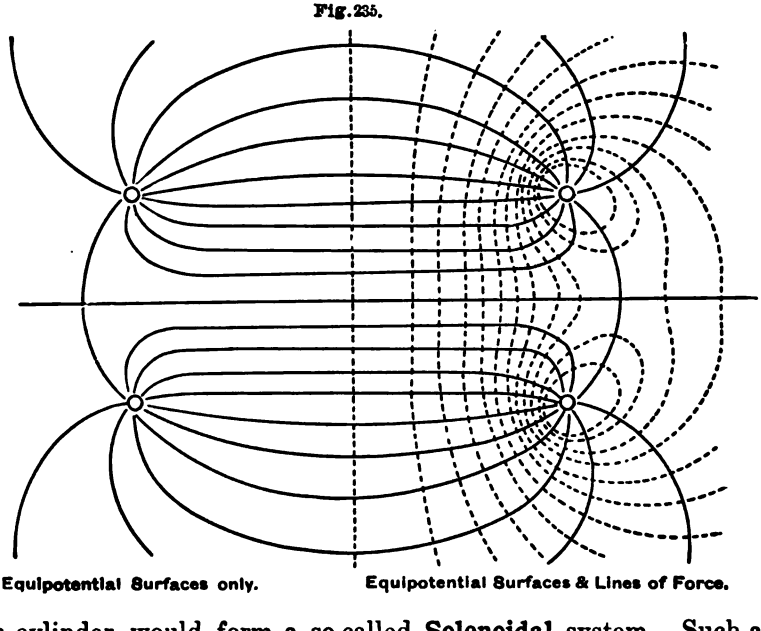


angles to the equipotential surfaces. To one side of the circuit the magnetic potential is positive, to the other it is negative. The **positive side of the circuit** is such that an observer standing girdled by the current, with his head in the positive region, would see the current pass round him from his right towards his left hand.

If another circular circuit be brought near, and if the direction of the current within it be the same as that within the former, the lines of force or of induction of the two circuits coalesce, and the two circuits attract one another. The resultant system of surfaces and lines takes the form indicated in Fig. 235. The field of force between the two circuits is approximately uniform. The lines of force are all closed curves, but some of them, those which pass up the centre of the region between the circuits, take a relatively-wide sweep into space, and seem to radiate from or converge upon the external face of either circuit.

A large number of such circular circuits arranged so as to have a common axis, and thus to form, as it were, the outline of

Fig. 235.

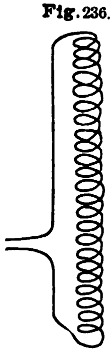


Equipotential Surfaces only. Equipotential Surfaces & Lines of Force.

a cylinder, would form a so-called **Solenoidal** system. Such a system would have lines of force or of induction radiating from each extremity, taking a more or less ample sweep into space, returning into the opposite extremity and passing up the axial region of the cylinder from the negative to the positive region; each line of force or induction being thus a closed curve. The external electro-magnetic field of such a solenoid system would be identical with that produced by a system consisting of an attracting disc at the one and a repelling disc at the other

extremity of the solenoid; and such a solenoid would by one of its extremities attract the north and repel the south pole of a compass-needle; while by the other it would attract the south and repel the north pole. Such a solenoid would, so far as its external action is concerned, act like a bar-magnet; and Ampère's theory of Magnetism is, that magnets and solenoid systems of currents are fundamentally identical.

Fig. 236.



A solenoid may be roughly realised by winding a wire into a narrow spiral and bringing the two extremities back to the same point. The error introduced in each turn of the spiral by its departure from a perfect ring-form is roughly compensated by the return of the wire (Fig. 236).

This identity of action of Magnets and of Solenoidal Steady Current-systems being premised, we now proceed to give a rapid summary of the main phenomena of Magnetism.

Magnetism.

Some bodies — a piece of loadstone, a compass-needle, a wire spiral through which a current is passing — tend, when suspended by their centre of gravity, to lay themselves in a definite direction, and to place a definite line within them, their Magnetic Axis, in a definite direction, which, roughly speaking, lies north and south. The same bodies have the power of attracting iron. Such bodies are called Magnets.

Curiously enough, this directive power is, according to Gore, shared by crystals of Cyanite, an anhydrous monosilicate of alumina.

Magnets may be divided into Permanent (loadstone, hard steel magnets) or Temporary (a solenoid current, or an Electro-magnet, *i.e.*, a bar of soft iron, whose magnetic properties are induced by the presence of an electric current circulating round it, but endure, in soft iron, no longer than the persistence of that current); or again, into Natural (loadstone) and Artificial.

The constituent particles of a magnet are themselves magnets. A permanent magnet may be cut into a very large number of minute fragments, each of which will be a little magnet, the original magnetic axis in which will continue to point to the magnetic north and south. When a steel bar is converted into a permanent, or a soft-iron bar into a temporary magnet, some operation must be effected, not upon the mass as a whole,

but upon its constituent molecules, or groups of molecules. The magnetic axis of a bar-magnet or compass-needle coincides more or less closely, but hardly ever with perfect accuracy, with its geometrical axis of figure. The magnetic axis joins the two **Poles** of the magnet.

One mode of expressing the mechanical action of magnets is to feign a distribution of imaginary magnetic matter at the Poles; positive at the one pole, equal and negative at the other; the attractions and repulsions observed are exercised mainly to and from these poles.

Another method is to feign a distribution of magnetic matter partly over the surface, partly within the substance of the magnet (Poisson) or over the surface only (Gauss and Green).

Positive and negative magnetic distribution may be feigned to be either heaping-up of positive matter towards or at positive poles, and of negative matter towards or at negative; or else to be distribution in excess and defect respectively (or inversely) of one and the same all-pervading imaginary "magnetic fluid."

These modes of representation are convenient for calculation and exposition merely; and indeed the only case in which it can be said that there is in a magnet a real Pole, a point at which the imaginary mass may be considered as concentrated, is that of an ideally-thin, infinitely-long, uniformly-magnetised wire. In every other case the distribution of forces in the field surrounding the magnet is more complex; but it is convenient to assume, to begin with, that every Magnet has two Poles, each of which is a Point.

A long thin bar so magnetised that all its molecules would, considered as magnets, be absolutely equal, would have its poles at its ends. Such a theoretical bar-magnet is called a **Solenoidal Magnet.** In practice the action of bar-magnets is the same as that of a theoretical Solenoid whose Poles are at a somewhat less distance from another than the extremities of the bar: for which reason the Poles of a bar-magnet are often said to be within its substance, at a short distance from its ends.

The North-seeking pole of a magnet is *called* its Positive pole; the other, its south pole, is *called* its negative pole.

In different magnets, unlike poles attract one another; like poles repel one another.

The **force** F of repulsion or attraction between the poles varies inversely as the square of the distance between them. It also varies directly with, and furnishes the basis for the measurement of, the **strength** m of each pole, or the quantity m of imaginary magnetic matter conceived to be concentrated at each. It is therefore, **in air,** $F = mm'/d^2$, and is repulsive when the poles are similar, both positive or both negative; attractive when they are dissimilar.

If any other medium than air intervene between the mutually repelling or attracting magnet-poles, $F = mm'/\mu d^2$, where μ is the Magnetic Permeability of the medium.

Two poles are said to be of Equal Strength when they can replace one another in their action upon external magnetic poles.

Two poles are said to be each of **Unit Strength** if they be equal and have between them a repulsion-force equal to one dyne when their mutual distance is one centimetre through **air.**

A unit pole placed at 1 cm. in air from a similar pole of m units will be repelled with a force of m dynes.

The opposite poles of a magnet are of strengths opposite but numerically equal; these are $+m$ and $-m$; their sum is zero; the sum of the magnetisms of every magnet is always zero.

It is not possible to isolate a single magnetic pole or to produce any numerical difference of strength between the two poles of a magnet.

Round a magnet there is a **Magnetic Field of Force**, permeated by Magnetic Lines of Force or Induction and Magnetic Equipotential Surfaces. An isolated positive-pole (if such a thing were possible) placed in the neighbourhood of the positive extremity of a bar-magnet would be repelled and would travel to the negative extremity, not by the shortest path, but by following the wide sweep of any line of force on which it might happen to lie.

The Direction of a magnetic Line of Force is the direction in which a positive pole is driven, or, conversely, a negative pole pulled upon.

A magnet within a magnetic field is acted upon by a Couple: its positive pole is driven, its negative pole drawn, in the direction of the Lines of Force passing through them: the magnetic axis of the magnet tends to coincide as nearly as possible with a line of force passing through its centre. If this line of force be curved, the axis of the magnet is set tangentially to it.

If there be magnetic attraction between any two magnetised surfaces, the Lines of Magnetic Force run across from the one surface to the other. If Magnetic Lines of Force, proceeding from the respective surfaces, present their sides to one another, the medium between the two surfaces tends to expand so that these lines of force come to lie farther apart; and the two surfaces accordingly appear to repel one another.

The Condition of a Magnetic Field at a point is determined (1) by the Direction of the Line of Force passing through the point, and (2) by the local Intensity or **Strength h of the**

field—*i.e.*, by the amount of mechanical force with which a unit-pole there situated is repelled or attracted.

The unit intensity of field would be that produced by a unit of magnetic quantity at 1 cm. distance through air. This unit of intensity is called a Gauss.

When the local intensity is $\mathbf{h}$, a magnet whose length is l, and whose poles have the respective values $\mathfrak{m}$ and $-\mathfrak{m}$, is acted on by a couple: the force acting on the positive pole is $F = \mathfrak{m}\mathbf{h}$, and its moment round the midpoint of the magnet is $(\mathfrak{m}\mathbf{h} \times \frac{1}{2}l)$; the moment of that acting on the negative pole is $(-\mathfrak{m} \times -\mathbf{h} \times \frac{1}{2}l) = \frac{1}{2}\mathfrak{m}\mathbf{h}l$: the moment of the couple is thus $\mathfrak{m}\mathbf{h} \cdot l$. At a spot where the intensity $\mathbf{h} = 1$—that is, within a unit magnetic field—the moment is equal to $\mathfrak{m}l$, the numerical Strength of either pole multiplied by the Length of the magnet. This moment $\mathfrak{m}l$ is called the **Magnetic Moment, $\mathfrak{M}$,** of the magnet.

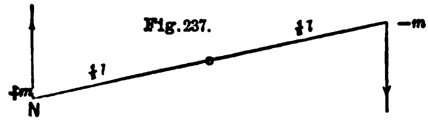


Fig. 237.

If a magnet of length l be broken into fragments, each of length l/n, the magnetic moment of each fragment is $\mathfrak{m} \times l/n$; the sum of the magnetic moments of all the n fragments is $n \times \mathfrak{m} \cdot l/n = \mathfrak{m}l = \mathfrak{M}$, the magnetic moment of the original magnet, and every fragment possesses poles of strength $\mathfrak{m}$ and $-\mathfrak{m}$, equal to those of the original magnet.

When two equal magnets are arranged thus—N–S, N–S, the extreme poles are effective, the intermediate ones mask one another; when work is done upon them in separating them, the original condition is restored and all the poles are again manifest. When n such magnets are connected in this way, all the poles except the extremes mask one another. A uniform bar-magnet l cm. long and o sq. cm. in cross-section, and therefore having a volume of lo cub. cm., may be considered as a collection of lo magnets, each 1 cm. in length and 1 sq. cm. in section, and therefore each of unit volume. The magnetic moment of the whole is equal to that of lo such magnets: the magnetic moment of each of these unit-volume magnets is that of the entire magnet $\mathfrak{M} = \mathfrak{m}l$, divided by lo, the volume, and is therefore equal to $\mathfrak{m}/o$. This is $\mathfrak{I}$, the so-called **Intensity of Magnetisation** of the bar-magnet.

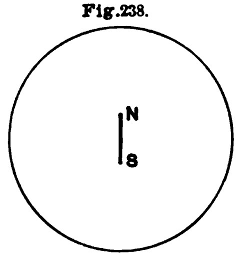
If the intensity of magnetisation of a bar-magnet were equal throughout, its poles would be situated exactly at its extremities. We generally find, however, that magnets present abnormalities in this respect, and that they may even have secondary poles,

produced by local inequalities in this intensity and by consequent deficient compensation of the internal poles.

Such complex distributions can, within a bar, generally be represented by the superposition of a number of solenoids of different lengths.

A steel sphere will be magnetised uniformly if it be placed for some time within a uniform magnetic field. It then has a moment equal to that of a small axial magnet NS, Fig. 238; and it tends to lay its axis NS along that line of force which passes through its centre.

Fig. 238.



The Earth considered as a magnet is not uniformly magnetised; its intensity of magnetisation is not equal throughout; it does not act upon bar-magnets placed near its surface exactly as a distant bar-magnet would do, for the law of its action is not even approximately expressible by any formula less complicated than one which contains at least twenty-four coefficients (Gauss).

Terrestrial Magnetism. — The neighbourhood of the surface of the Earth is a great Magnetic Field, nearly uniform within such small spaces as the interiors of rooms. The lines of force point in the northern hemisphere downwards and northwards: in the southern hemisphere upwards and northwards. A compass-needle thus tends to place itself, in the northern hemisphere, so that its magnetic axis points downwards and to the Magnetic North, which is inclined to the west of the true or Geographical North by a so-called Declination or, as sailors call it, a 'variation' of 17° 6′ (at Greenwich, 1894); 20° 13′ at Edinburgh, 22° 11′54″ at Valentia; 4° 5′ 48″ W. at Washington, D.C., 1893; 14° 22′ 54″ E. at Los Angeles, 1893. This declination towards the west is at present decreasing at Greenwich by 6′ per annum; at Edinburgh by 7′·8. If the needle cannot move except round a vertical axis, its axis cannot point downwards or upwards: it therefore tends to point to the Magnetic North, lying, as it does so, with its magnetic axis in a line situated in the same vertical plane with the true line of force. This plane is the Magnetic Meridian; and the magnetic axis of a given magnet may be found if the magnetic meridian passing through the place of observation be known.

To find the magnetic meridian, a single observation is not sufficient. The axis of figure of a needle may not coincide with its magnetic axis, and the needle (which is provided with an agate cup on each of its flat faces) is therefore observed when it lies poised on one side, and again when it lies on the other. The mean of the two positions gives the position of the magnetic axis of the needle, and therefore indicates the magnetic meridian. The downward or upward direction of the lines of force, their departure from the horizontal line, is the Inclination or Dip of the needle. This is downwards in the northern hemisphere, upwards in the southern. A needle suspended on a horizontal axle will, by the mean of two readings, give the inclination of its magnetic axis. Friction prevents the attainment of very

great accuracy in this measurement; but it can be greatly diminished by slinging the axle of the needle upon silk threads. The inclination is at Greenwich, 1894, 67° 16′ 30″, diminishing by 1½′ per annum; at Edinburgh, 70° 30′; at Valentia, 68° 45′·7, 1893; in Washington, 71° 4′ 30″, 1893; at Los Angeles, 59° 29′, 1893.

If the inclination be found, and if the horizontal component $\mathfrak{h}$ of the attractive force of the earth's magnetism, acting upon a unit pole, be known, we have the data required for determining the whole intensity of the earth's magnetic field in the direction of the lines of force at any point. The horizontal component $\mathfrak{h}$ at Greenwich in 1894 is 0·1832 dynes, increasing by ·0002 per annum; Washington, 0·19860, 1893; Los Angeles, 0·2725, 1893. The vertical component at Greenwich is 0·4374 dynes, scarcely changing, since 1889, from year to year; Washington, 0·57928, 1893; Los Angeles, 0·4630, 1893.

The line along which the lines of force are horizontal, and at which the Inclination or Dip is equal to zero, is the Magnetic Equator, which does not coincide with the geographical equator, and is not a great circle of the earth. The lines, roughly parallel to the magnetic equator, along which the Dip is equal, are the Magnetic Parallels: these are lines along which equipotential surfaces cut the surface of the earth. The intensity of the earth's magnetic force may be indicated by the distance between these parallels, just as those maps, which give contour-lines indicating equal levels, may show by the crowding together or separation of these lines the tendency of water to rapid or to slow flow over the face of a country. The magnetic parallels are not great circles of the earth; they are not even parallel to one another; in circumpolar regions they are irregularly elliptical, and the needle points to their centres of curvature. A Magnetic Pole is a spot where the equipotential surfaces of the magnetic field graze the earth's surface; the needle there stands vertical, the dip being 90°. There are two true poles, one Arctic (negative), the other Antarctic (positive), together with other points towards which surrounding magnetic needles seem to converge, but which are only the centres of curvature of the irregularly-shaped magnetic parallels. The line joining the Magnetic Poles does not coincide with anything which may be termed the Magnetic Axis of the earth.

The terrestrial magnetic field undergoes remarkable Variations. The direction of the lines of force, and therefore the dip, the declination and the position of the magnetic north, as also the intensity, undergo secular changes; and there are other changes, some of which depend, like the period of sunspots, upon a cyclical period of about eleven years, others upon the rotation of the sun, upon the position of the moon, upon the time of the year and the hour of the day; while other disturbances, productive of electrical currents in the crust of the earth, so powerful and so irregular as sometimes to render telegraphic signalling perfectly unintelligible — disturbances known as Magnetic Storms, and possibly due to long waves in the Ether — are observed to occur with special frequency in sympathy with outbreaks of sunspots and of solar storms and appearances of the Aurora Borealis. The nature of the undoubted connection between the Sun and the magnetism of the earth is in the highest degree obscure; it is clear, however, that the sun and moon cannot exercise any important direct effect as magnets, although when one side of the sun is turned towards us the terrestrial magnetic intensity is greater than when that side is turned away.

The diurnal variations have been traced by Schuster to causes above

the earth's surface; probably those electric currents in the upper regions of the atmosphere which are (Balfour Stewart) produced by the action of the Sun, and which give rise to the Aurora Borealis, and are best marked during periods of maximum sunspots.

To find the Magnetic Moment of a Magnet.—We must combine the magnetic moment $\mathfrak{M}$ of the magnet with $\mathfrak{h}$, the horizontal component of the intensity of the Earth's Magnetic Field at the place of observation. By one process we can find the value of $\mathfrak{M}\mathfrak{h}$; by another we can find that of $\mathfrak{M}/\mathfrak{h}$; from these data we can find not only $\mathfrak{M}$, the magnetic moment (for $\mathfrak{M}\mathfrak{h} \times \mathfrak{M}/\mathfrak{h} = \mathfrak{M}^2$), but also $\mathfrak{h}$, for $\mathfrak{M}\mathfrak{h} \div (\mathfrak{M}/\mathfrak{h}) = \mathfrak{h}^2$.

1. To find $\mathfrak{M}\mathfrak{h}$ ("Method of Vibrations"):—Suspend a very long magnet by one thread attached to its centre; load it so that it may swing horizontally round a vertical axis: observe the time of its oscillation under the earth's magnetic attraction of one pole and repulsion of the other. The time of a *complete* oscillation is $T = 2\pi\sqrt{N/\mathfrak{h}\mathfrak{M}}$, where N is the moment of inertia (page 162). In the simple-pendulum formula (page 212) $T = 2\pi\sqrt{N/G\mathfrak{l}}$. Here G, the weight of the pendulum, is replaced by $\mathfrak{h}\mathfrak{m}$, the attraction of the one, and $-\mathfrak{h}\mathfrak{m}$ the repulsion of the other pole, or together by $2\mathfrak{h}\mathfrak{m}$. Also, the magnet of length l swings suspended on its midpoint like a couple of simultaneously-oscillating pendulums, each of length $\mathfrak{l} = \frac{1}{2}l$. Whence T, the period of a complete oscillation to-and-fro, $T = 2\pi\sqrt{N/\mathfrak{h}\mathfrak{m}l} = 2\pi\sqrt{N/\mathfrak{M}\mathfrak{h}}$. T can be found; N can be ascertained for any given needle; whence $\mathfrak{M}\mathfrak{h}$ may be calculated.

This operation is difficult; for $\mathfrak{M}$, the magnetic moment, and therefore the rate of oscillation, varies with every slight vibration or change of state or of the temperature of the suspended magnet.

If we take the torsion of the suspending thread into account we find that a restitution-pressure has been developed, proportional to the displacing force $F = \mathfrak{h}\mathfrak{m}$ and also to l; it is therefore proportional to $\mathfrak{h}\mathfrak{m}\cdot l$ and consequently to $\mathfrak{h}\mathfrak{M}$; call it $p\cdot\mathfrak{h}\mathfrak{M}$. The couple tending to restore the needle to its mean position is not $\mathfrak{h}\mathfrak{M}$ but $\mathfrak{h}\mathfrak{M} + p\cdot\mathfrak{h}\mathfrak{M}$; and the divisor in the value of T is not $\sqrt{\mathfrak{h}\mathfrak{M}}$, but $\sqrt{\mathfrak{h}\mathfrak{M} + p\mathfrak{h}\mathfrak{M}}$.

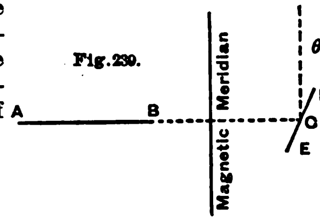
To find the value of N, the Moment of Inertia, we must attach to the oscillating needle a mass whose moment of inertia $N_{,}$ is known from geometrical considerations. Let this be, for instance, a ring of rectangular cross-section whose mass is m and whose radii are $r_{,}$ and $r_{,,}$; the moment of inertia is (No. 6, p. 163) $\frac{1}{2}m\,(r_{,,}^2 + r_{,}^2)$. When this ring is fixed to the needle in such a way that the horizontal oscillations of the needle cause the heavy mass to rotate horizontally round its own centre, the time of a complete oscillation is increased to $T_{,}$, which is equal to $2\pi\sqrt{(N + N_{,})/\mathfrak{h}\mathfrak{M}}$. These data are sufficient to give the value of N, the moment of inertia of the needle.

If the needle be a straight wire of length l, $N = ml^2/12$; whence $\mathfrak{h}\mathfrak{M} = \pi^2\cdot ml^2/3T^2$, where m is the mass of the needle in grammes (Equation **a**).

To find $\mathfrak{M}/\mathfrak{h}$ ("Deflection-Methods"):—We may use the same needle to produce a deflection in a compass-needle free to swing round its midpoint; by observing the deflection from the Magnetic Meridian (p. 679) when the compass-needle has come to rest under the influence of the two couples, we find the ratio of these couples and thus learn the value of $\mathfrak{M}/\mathfrak{h}$. There are two main methods; the End-on deflection-method and the Broadside deflection-method.

In the former, the **End-on**, Fig. 239, DE is the deflected magnet or compass-needle; AB the magnet whose moment we are investigating; θ the deflection; d the distance between C and the midpoint of AB, a distance very considerable as compared with the dimensions of DE; l the length of AB; then

Fig. 239.

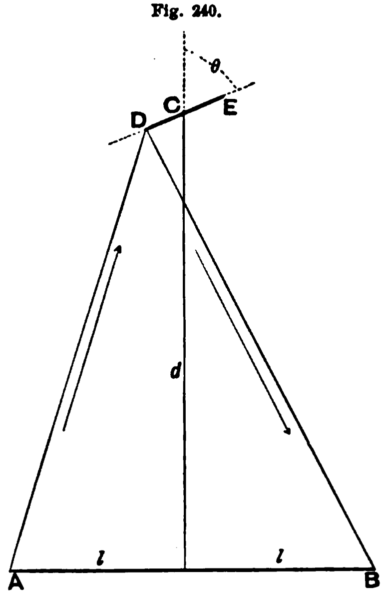


$$\tan\theta = \frac{\mathfrak{M}}{\mathfrak{h}} \cdot \frac{2d}{(d^2 - \frac{1}{4}l^2)^2},\ \textbf{(b)}:*$$

a formula which, when the length l of the magnet AB is small in comparison with the distance d, becomes $\tan\theta = 2\mathfrak{M}/\mathfrak{h}d^3$.

In the latter, the **Broadside** method, Fig. 240, AB is fixed so that its midpoint is in a line with the magnetic meridian passing through C, and d being, as before, the distance between the centres of the magnets, the deflection θ is such that

Fig. 240.



$$\tan\theta = \frac{\mathfrak{M}}{\mathfrak{h}} \cdot \frac{1}{(d^2 + \frac{1}{4}l^2)^{\frac{3}{2}}},\ \textbf{(c)}:\dagger$$

which, if l be relatively insignificant, becomes $\tan\theta = \mathfrak{M}/\mathfrak{h}d^3$; the twisting couple in this case is therefore half that due to AB when the end-on method is applied.

By blending equations **(a)** and **(b)** we find that the data of the end-on measurement give

$$\mathfrak{M} = (\pi l/2\mathrm{T})(d^2 - \tfrac{1}{4}l^2)\sqrt{2\tan\theta \cdot m/3d},$$

and

$$\mathfrak{h} = (\pi l/\mathrm{T}(d^2 - \tfrac{1}{4}l^2))\sqrt{2md/3\tan\theta},$$

expressions which involve only measurable terms, and which give numerical values for $\mathfrak{h}$ and $\mathfrak{M}$ in proper C.G.S. units. Similarly by blend-

* DE is supposed so small that all forces acting on it act along the line BC, and that deflections do not modify the forces upon it. This distance BC is $(d - \frac{1}{2}l)$; the strength of B is $\mathfrak{M}/l$; the strength of D is $\mathfrak{m}$; the force between B and D is $\mathfrak{m}\mathfrak{M}/l(d - \frac{1}{2}l)^2$. Similarly the force between A and D is opposite in sign, and equal to $\mathfrak{m}\mathfrak{M}/l(d + \frac{1}{2}l)^2$. The force upon D is thus

$$(\mathfrak{m}\mathfrak{M}/l)\left\{\frac{1}{(d - \frac{1}{2}l)^2} - \frac{1}{(d + \frac{1}{2}l)^2}\right\} = \mathfrak{m}\mathfrak{M} \cdot 2d/(d^2 - \tfrac{1}{4}l^2)^2.$$

An equal force acts upon E. The couple acting on DE is thus $\mathrm{DE} \times \mathfrak{m}\mathfrak{M} \cdot 2d/(d^2 - \frac{1}{4}l^2)^2 = \mathfrak{M}_{,}\mathfrak{M} \cdot 2d/(d^2 - \frac{1}{4}l^2)^2$, where $\mathfrak{M}_{,}$ is the magnetic moment of DE. When DE is deflected through an angle θ this couple becomes $2\mathfrak{M}\mathfrak{M}_{,}d\cos\theta./(d^2 - \frac{1}{4}l^2)^2$. When this couple is in equilibrium with the terrestrial horizontal couple $(\mathfrak{h} \cdot \mathfrak{m} \cdot \mathrm{DE} \cdot \sin\theta)$ or $(\mathfrak{h} \cdot \mathfrak{M}_{,}\sin\theta)$,

$$2'\mathfrak{M}\mathfrak{M} \cdot d \cdot \cos\theta./(d^2 - \tfrac{1}{4}l^2)^2 = \mathfrak{h} \cdot \mathfrak{M}_{,} \cdot \sin\theta, \text{ whence}$$

$$\tan\theta = \frac{\mathfrak{M}}{\mathfrak{h}} \cdot \frac{2d}{(d^2 - \frac{1}{4}l^2)^2}.$$

† It is supposed that all parts of DE are appreciably at the same distances from

ing equations (a) and (c) we may interpret the data of the broadside method.

If we have any doubt as to the true value of l, the distance between the "poles" of AB, we can find it by repeating at a different distance d the observations which lead to, say, equation (b). We now have a different θ, a different d, but still the same l. From two such equations we can obtain the numerical value of l.

Magnetic Potential. — A magnetic pole, if isolated, would be surrounded by concentrically-spherical equipotential surfaces traversed by radial lines of force. But a magnet has two poles of opposite kind, and the field of force around it presents a character approximately represented by Fig. 234, if the lines there marked Equipotential Surfaces be held to represent Lines of Magnetic Force, and *vice versâ*.

The potential at any point due to the positive pole $\mathfrak{m}$ at distance d, through air, is $\mathfrak{m}/d$; that due to negative pole, $-\mathfrak{m}$, at distance d', is $-\mathfrak{m}/d'$; due to both together the potential is $\Omega = \{\mathfrak{m}/d - \mathfrak{m}/d'\} = \mathfrak{m}(1/d - 1/d')$, which has the same value for every point on one and the same equipotential surface.

In any other medium than air, the permeability being μ, the force between two magnetic poles $\mathfrak{m}$ and $\mathfrak{m}'$ is $F = \mathfrak{m} \cdot \mathfrak{m}'/\mu d^2$; then, just as in the corresponding electrostatic case where $F = Q \cdot Q'/Kd^2$, the Magnetic Potential Ω at a point varies inversely as μ; the field-intensity $\mathbf{h}$ also varies inversely as μ; and the magnetic induction per sq. cm., $\mathbf{b}$, which is always equal to $\mu\mathbf{h}$, is independent of μ.

Magnetic Shell. — We may arrange a number of extremely short magnetised bars side by side, so that their similar poles all point in the same direction; a metal sheet is thus built up, of which the one face is negatively, the other positively magnetised. Such a sheet is called a Magnetic Shell. The Magnetic Moment of such a shell is the sum of the magnetic moments of all its portions. Let it be supposed to be first a continuous shell, and then to be divided into portions each 1 sq. cm. in area. Each such portion will have a magnetic moment $\mathfrak{M}$. The magnetic moment of each such unit-area portion, if this be invariable over the whole shell, is called the **Strength, φ, of the Shell**; it is equal to the magnetic quantity per unit of area $\times$ the thickness of the shell.

AB as the central point C is; *i.e.*, at distance d from the midpoint AB, at distance $\sqrt{d^2 + \frac{1}{4}l^2}$ from either A or B. The pole D is attracted by the one end of AB *and* repelled by the other; in each case with a force $\mathfrak{m} \cdot \mathfrak{M}/l \cdot \overline{AD}^2$, or $\mathfrak{m}\mathfrak{M}/l \cdot (d^2 + \frac{1}{4}l)^2$. The resultant force on D is parallel to AB, and is therefore equal to the whole force acting $\times \frac{1}{2}l/\sqrt{d^2 + \frac{1}{4}l^2}$; *i.e.*, it is equal to $\mathfrak{m}\mathfrak{M}/(d^2 + \frac{1}{4}l^2)^{\frac{3}{2}}$. The resultant on E is equal and opposed in direction. The couple on DE is therefore $\mathfrak{m}\mathfrak{M}/(d^2 + \frac{1}{4}l^2)^{\frac{3}{2}} \times \mathrm{DE} \times \cos\theta$ $= \mathfrak{M}_{,} \cdot \mathfrak{M} \cdot \cos\theta./(d^2 + \frac{1}{4}l^2)^{\frac{3}{2}}$. This is equal to the terrestrial couple $\mathfrak{h} \cdot \mathfrak{M}_{,} \cdot \sin\theta$; whence $\tan\theta = \mathfrak{M}/\mathfrak{h}(d^2 + \frac{1}{4}l^2)^{\frac{3}{2}}$.

The quantity of magnetism per unit of area is the Magnetic Superficial Density, ς, $=\varphi/d$, where d is the thickness of the shell.

The **Potential** of a Magnetic Shell upon a Unit Positive-Pole placed at any point facing the Positive Aspect of the Shell will, in air, be the product (see p. 200) of the Strength of the Shell into the apparent Surface of the shell, as seen from the unit-pole — this apparent surface being measured by the projection of the shell upon an ideal sphere whose centre is occupied by the unit-pole, and whose radius is 1 cm., or, in other words, by the value of the Solid Angle ω subtended by the Shell at the point. [Magnetic Potential $\Omega=\omega\varphi/\mu$.]

If the shell be fixed, a positive pole would tend to move away to regions of less potential, and thus to travel round to the negative side of the shell. If the unit pole be fixed, a shell would tend to move in such a way as to diminish its apparent area, and even to present what is equivalent to a negative area, namely, its negative side, to the positive pole; it would therefore tend to rotate.

In the immediate neighbourhood of a magnetic shell the angle subtended by it is 2π; the potential near the positive surface is therefore $2\pi\varphi$, where φ is the strength of the shell; near the negative surface it is $-2\pi\varphi$; hence, when a unit positive-pole moved, in air, from the + to the − surface, $4\pi\varphi$ units of work would be done by it.

Equivalence of Magnetic Shell and Electric Circuit. — The Equipotential Surfaces in the neighbourhood of a magnetic shell are such that from every point on any one of them the area of the shell will for that surface appear invariable. But equipotential surfaces as determined by this criterion are identical in form with those bowl-shaped equipotential surfaces which surround a closed circuit bearing a steady current of electricity (Fig. 234); provided that the contour of the shell and that of the circuit be the same. A magnetic Shell and a Closed Current of electricity may therefore have in their vicinity an identical Magnetic or Electromagnetic Field; and an electromagnetic field is a magnetic field produced by a current.

Magnetic Shells and equivalent Currents of the same contour can thus replace one another, the difference being that the shell is impervious, while the circuit is not. Hence, when a closed current-bearing circuit is placed with its positive face facing a positive magnet-pole, there is mutual repulsion. The positive pole is repelled along the lines of magnetic force, which

trend positively from the positive face of the circuit; such a pole would tend to travel repeatedly through the circuit along the closed lines of force. The potential energy of the system, which always tends to become a minimum, is thus found to have no fixed value, but to depend on the number of times the pole has passed through the circuit. This anomalous result would be due to the continuous supply of energy by the current itself.

As regards the Lines of Force and of Induction in the electromagnetic field surrounding a current-bearing Circuit, these are, for one particular intensity or strength of current, exactly the same as those in the Magnetic Field surrounding a magnetic Shell of the same contour and of a given strength; and by adopting a system in which that particular Intensity of the Current is said to be the same numerically, in air, as the Strength of the equivalent magnetic Shell, we are able to state all electrical and magnetic quantities in terms of **Magnetic** or **Electromagnetic Measurement**.

In other words, $i = \varphi$, where φ is the strength of the equivalent shell; and if the area of shell or circuit be A, $Ai = A\varphi =$ the Magnetic Moment of the equivalent shell; but if the circuit be coiled, so as not to be a single loop, but to surround its own axis n times, the magnetic moment of the circuit $= Ani$; and if the medium constituting the field be any other than air, the magnetic moment $A\varphi$ of the equivalent shell $= \mu \cdot An \cdot i$, or for a single loop, $\varphi = \mu \cdot i$.

Magnetic Induction. — Soft iron filings are attracted by a magnet, and themselves become temporary magnets. This they do even though they be not in contact with a magnet, but merely exposed to such forces as can act upon them within a magnetic field. Soft iron completely loses its magnetic properties when removed from the neighbourhood of a magnet; but a steel or hard iron bar, which is with greater difficulty induced to become a magnet, will not, when removed from the field, entirely lose its magnetic state, but preserves a certain Residual Magnetisation. The property of steel or hard iron, in virtue of which it slowly takes up and slowly parts with a magnetic condition, is traditionally named its Coercitive Force. Any vibration or jar which facilitates relative movement of particles of the iron will enable its molecules to yield to the inducing forces, and will facilitate the magnetisation of the iron: and after its removal from the field, such a jar will facilitate its loss of magnetic condition.

A poker suspended near the earth's surface and repeatedly struck will become feebly magnetic; so does an iron ship which is exposed to much hammering during construction; and all working machinery is magnetic.

The effects of the inducing forces within a magnetic field differ from those within an electric field of force in the following respects: — (1) The action is one which affects the state of each molecule; (2) There is no repulsion of a mass of iron or steel which comes in contact with a magnet; and (3) The power of taking up a magnetic condition in any marked degree is limited to a very small number of bodies, though to a slight extent it is possessed by all.

The strength of the poles of an induced magnet depends on the nature of the magnetic field, and therefore on the strength, the distance, the direction, the form, of the inducing magnet; and also upon the nature of the body acted upon, its form, its direction, its temperature, and its size.

In some substances the magnetisation induced is such that the north pole of the induced magnet lies as far as possible along the lines of force, — as far as possible away from the north pole of the inducing magnet. Such substances — iron, nickel, cobalt, manganese, chromium, oxygen, etc. — are **Paramagnetic** or **Ferromagnetic.**

In other substances the direction of the induced magnetisation is the reverse of this. Such substances — bismuth, antimony, silver, copper, hydrogen, nitrogen, etc. — are **Diamagnetic.**

Intermediate between these are such substances as air, which do not become magnetic, or but very slightly so.

In still other substances the induced magnetisation is not parallel to the lines of force, but is along certain **Lines of Induction** within the body, whose direction depends upon the molecular agglomeration or the crystalline constitution of the body.

The Lines of Induction within an induced magnet must therefore be distinguished from the Lines of Force, with which they do not in all cases coincide; within a magnetic field in air, on the other hand, they are coincident in all respects.

The magnetisation induced in a magnetisable body exposed to induction is proportional to the local strength $\mathbf{h}$ of the field; and the number of lines of magnetic induction per sq. cm., or the Magnetic Induction or Flux per sq. cm., within that body, is $\mathbf{b} = \mu\mathbf{h}$, where μ is a coefficient, the Coefficient of Magnetic Induction, or the **Permeability,** or Inductivity, of the substance. The Total Magnetic Induction or Flux, or the Total Number of Lines of Magnetic Induction across a given area A is $\mathbf{B}, = A\mathbf{b}$ if $\mathbf{b}$ be uniform over that area.

The number of Lines of Magnetic Induction round a pole $\mathfrak{m}$ is always $\mathbf{B} = 4\pi\mathfrak{m}$; $\therefore \mathbf{b} = \mathbf{B}/4\pi r^2 = \mathfrak{m}/r^2$ per sq. cm. at distance r: the number of

Lines of Force, within a medium of permeability μ, is $4\pi\mathfrak{m}/\mu$ all round the pole: $\therefore \mathbf{h} = \mathfrak{m}/r^2\mu$ per sq. cm., at distance r.

The number of lines of induction from 1 sq. cm. of either face of a flat magnetic shell, of superficial density $\varsigma = (\mathbf{b}/4\pi)$, is $4\pi\varsigma = \mathbf{b}$ lines per sq. cm.; for the lines do not diverge, but all pass in one direction, 4π lines from each unit of magnetic quantity. Conversely, if there be $\mathbf{b}$ lines of induction crossing each sq. cm. of a given area, these will be associated with a displacement or separation of magnetism, or Magnetic Flux, along the line of their direction, to the amount of $\mathfrak{m}' = \varsigma = \mathbf{b}/4\pi$ units of magnetic quantity per sq. cm. This magnetic displacement, $\mathbf{b}/4\pi = \mu\mathbf{h}/4\pi$ per sq. cm., multiplied by half the inducing force $\mathbf{h}$ per sq. cm., gives the local magnetic **energy of the magnetic field**, *i.e.*, $\mu\mathbf{h}^2/8\pi$ ergs per cub. cm.; provided that that energy be wholly induced and be stored up conservatively in the field, so that there is no tendency to retention of magnetisation when the inducing cause ceases to act.

If we consider the air in the neighbourhood of a magnet, we find a certain number of lines of force or of induction passing through a given bulk of it. If we replace the given bulk of air by an equal bulk of iron, we find the lines of induction passing through the iron to be more numerous than those previously within the undisturbed magnetic field. The equipotential surfaces are also farther apart within the iron, so that iron may be said, on the analogy of Electric Conduction, to transmit inductive effect better than air or a vacuum.

Faraday called the permeability μ the Conductivity for Lines of Magnetic Force.

The number of lines developed in a body of permeability μ, exposed to the influence of induction within a field of magnetic force $\mathbf{h}$, is such as to meet two requirements; those lines shall remain which would have been present had there been no magnetisable body brought into the field; that is, there shall be $\mathbf{h}$ lines per sq. cm.; and secondly, there shall be lines developed in consequence of the presence of the magnetisable body, the number of which lines is proportional to the magnetic separation or displacement induced, per sq. cm. cross-section of the magnetised body. This displacement, being proportional to the strength $\mathbf{h}$ of the field, is equal to, say, $\kappa\mathbf{h}$ units of magnetic quantity per sq. cm.; κ is the Coefficient of Induced Magnetisation for the substance acted upon, or its **Magnetic Susceptibility**: and the number of induced lines running through the magnetised substance, in the direction along which the displacement has occurred, is $4\pi\cdot\kappa\mathbf{h}$ per sq. cm. Together, the lines per sq. cm. $= \mathbf{b} = \mathbf{h}(1 + 4\pi\kappa)$; whence $\mu = (1 + 4\pi\kappa)$.

For air, earth, and almost all unmagnetisable substances, κ is approximately $= 0$, and $\mu = 1$; for bismuth $\kappa = -0{\cdot}000{,}0025$, and $\mu = 0{\cdot}999{,}968{,}584$. Bismuth is the most strongly diamagnetic substance known, and has the least known permeability. For iron, κ varies according to the inducing force $\mathbf{h}$ applied, and tends to fall to 0 for successive increments of $\mathbf{h}$; the value of $(\mathbf{b} - \mathbf{h})$ therefore tends to a limit. When $\mathbf{h}$ is about 50 in good soft iron, κ is about 25·39 and μ about 320; but κ falls to nearly 0 and μ to 1, for increments of $\mathbf{h}$, when $\mathbf{h}$ is about 24,000, and $\mu\mathbf{h}$ about 45,400.

The value of μ also varies to some extent with vibration, the mechanical condition, and the temperature.

The permeability μ of a substance can be measured by comparing the deflection produced in a distant magnet by a magnetising coil, first alone, and then provided with a core, consisting of the substance to be examined; or by arranging an exploring coil of wire so as to embrace all the lines of induction developed through a given cross-section of that substance in a field of known magnetic intensity, and connecting this exploring coil with a Ballistic Galvanometer (p. 713); then, when the substance is magnetised or acted upon by the inducing current, a secondary current (p. 700) is induced in the galvanometer circuit, and the throw of the needle affords the means of comparison with an earth-inductor (p. 718) of known power.

In medical magneto-electric machines, an adjustable piece of soft iron is used, in order to weaken the field of the permanent magnet when it is brought near the magnet-poles, by drawing off, through its substance, some of the lines of that field.

A diamagnetic substance has the reverse property; fewer lines of induction pass through it.

The result of this distortion of the lines in the magnetic field is to set up stresses, which tend to cause an iron bar to assume a position parallel with the lines of force; while a bar of bismuth tends, in a non-uniform field, to move into a position of least negative magnetisation, and to lie across these lines, at right angles to them if possible. Substances which in the form of bars take up this cross-position in a non-uniform field—Diamagnets—comprise the great majority of the substances found in nature.

In a uniform field, a magnetically **isotropic** substance (*i.e.*, one in which all directions are magnetically similar), if its form be **spherical**, becomes simply magnetised and remains at rest. If its form be ellipsoidal or otherwise **elongated**, it tends to rotate until its greatest length lies parallel to the lines of force, and this whether it be paramagnetic or diamagnetic.

If the substance be **aelotropic** (*i.e.*, having different susceptibilities, κ, in different directions) and **spherical**, the sphere tends to rotate until it brings its direction of greatest susceptibility parallel to the lines of force. If it be **elongated** we have two cases: (1) in the case of very small susceptibility the form has no effect, and the axis of greatest susceptibility comes to lie along the lines of force; (2) in the case of great susceptibility the longest axis lies parallel to the lines of force.

In a non-uniform field, an **isotropic sphere** tends to move along the Lines of Slope (p. 200) into regions of greater force if the substance be paramagnetic, into regions of less force if it be diamagnetic. An **elongated** body, if it be paramagnetic, lies along the lines of force, a position in which it lies as far as possible in the strongest part of the field; if it be diamagnetic it rotates so as to lie across the lines of force, a position in which it lies on the whole in the weakest possible part of the field.

If the substance be **aelotropic**, the force tending to produce motion along the Lines of Slope is greatest when the axis of greatest paramagnetic susceptibility is parallel, or that of least diamagnetic is at right angles to the lines of force: and in the case of crystals, the susceptibility being small, there will be rotation into this position whatever the form of the crystal.

A **diamagnetic** substance has therefore two obvious characteristics: (1) a bar of it places itself across the lines of force in a non-uniform field, and (2) a sphere of it in a non-uniform field, such as the neighbourhood of either pole of a magnet or of an electric current, is repelled into regions of weaker force.

A **paramagnetic** substance in bar-form places itself along the lines of force, and a sphere of it is attracted by either pole of a magnet.

With these we may compare a sphere of an **already magnetised** substance, which is attracted by the one pole of a magnet and may be repelled by the other, if the disturbance of its magnetic condition due to induction do not overpower the already existing magnetic distribution.

Any substance in which μ has a less value than it has in the surrounding medium will behave as if it were diamagnetic: for example, a tube filled with a weaker solution of ferric chloride and immersed in a stronger solution of the same salt will take up a cross-position in a strong magnetic field, though both solutions are ferromagnetic.

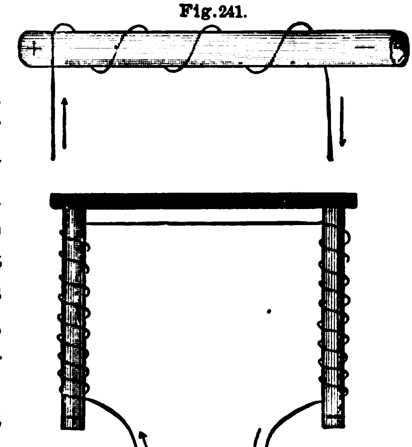
Limit of Magnetisation.—The intensity of induced magnetism increases with the intensity of the magnetic field, and varies with that intensity; but, in the case of iron, within certain limits only. As the intensity of the magnetic field rises, the induced magnetism of soft iron verges towards a limit. This would appear to favour that theory of induction which regards the magnetisation of induced iron not as created, but as directed by the forces within the field. According to this theory (Weber's), the molecules of iron are already little magnets, but their directions are promiscuously discrepant. When they come into a magnetic field they are directed so as to lie with their axes parallel to lines of force, and the whole mass of iron thereupon becomes obviously magnetic. The lines of force of the directed magnetic molecules are added to those of the electromagnetic field; and this operation has obviously a limit, beyond which any increase in the number of lines can be due to the electromagnetic field only. The value of μ therefore falls off as **h** increases. In diamagnets there is some reason for believing that the particles are magnetised *de novo* in the magnetic field; but the particle-magnets thus produced are feeble, and their strength does not tend to a limit.

When an iron bar is magnetised there seems to be an actual twist set up in it.

Magnets used to be made by exposing steel for some time to the influence of an existing magnetic field; as by rubbing bent or straight bars from centre to ends with the opposite poles of bar-magnets, or by leaving them in contact by their extremities with the opposite poles of a strong horse-shoe magnet or electromagnet.

Magnets are now produced by electric currents. A simple bar-magnet, as we have seen, tends to lie across a current, its positive pole to the left, its negative to the right of the current. If a bar of soft iron be placed along the lines of force within an Electromagnetic Field, it becomes a temporary magnet, or **Electromagnet**; there is a kind of separation of magnetisms; the left-hand end of the bar becomes magnetically positive, the right-hand end negative. If the current be wound round the bar, so that every part of the current exerts a similar action upon the bar, the bar becomes strongly magnetic (Fig. 241).

Fig. 241.



If it be of very soft iron it loses this property the instant the current ceases; but if it be of **steel** (or nickel, or cobalt), and if the current be powerful and continued for some time, it becomes a **permanent magnet.**

Cobalt, iron, and steel become more susceptible to magnetic induction when they are slightly warmed. At about 785° C., a soft iron or steel magnet suddenly loses all its magnetism, with evolution of energy as Heat; nickel does so at a lower (635° C.) and cobalt at a higher temperature (that of melting copper). If exposed to magnetic induction while above 785° C., iron manifests no susceptivity; then shut off the inducing current and allow the bar to cool; the bar becomes magnetic when it reaches that temperature. Steel presents similar phenomena at 690°–880°, according to its composition. An alloy, Fe 75, Ni 25, cannot be magnetised unless below – 20° C.; if then magnetised, it remains magnetic up to 580° C.; it then loses its magnetism and does not recover it unless and until cooled to – 20° C. There is some molecular change at these critical temperatures; at about the same temperatures, the electrical resistivity and the thermo-electric properties also change, and there is a sudden evolution of latent heat on cooling (J. Hopkinson).

Thermo-magnetic Motors. — A soft-iron disc in a strong field may become magnetised along a particular diameter; but if one end of that

diameter be heated, the susceptibility of the iron is diminished along that diameter, and a cooler diameter is pulled round to take its place. Continuous rotation may thus be set up, with transformation of Heat into Work by magnetic means (Thomson and Houston). In Edison's pyro-magnetic motor, a drum is made up of soft-iron tubes: some of these are heated by hot air, others cooled by cold air blown down them: the cool tubes are continuously drawn into the strongest part of the field, but no sooner do they reach it than they are heated, and have to make way for their cooler successors. A motor of this kind gave 3 horse-power, at 120 revolutions per minute.

When an iron or cobalt bar is magnetised it becomes longer and somewhat more slender, but does not appreciably alter in volume; it also emits a slight sound, a "magnetic tick." A nickel or a steel bar shortens and thickens.

Magnetisation, induced or residual, in an iron or steel wire is diminished on stretching, provided that the magnetisation correspond to an inducing force above a certain critical value known as Villari's Critical Value; this being (Lord Kelvin) about 24 times the terrestrial intensity. Below that critical value, tension increases the magnetisation of the magnetised wire. In nickel the magnetisation is always diminished. The effects of transverse expansive stress are opposed to those of longitudinal stretching.

Energy is absorbed during magnetisation, and if an electromagnet be made and unmade in frequent quick succession, it becomes hot; the energy is derived from that of the intermittent inducing current.

Hysteresis. — When a soft iron core of permeability μ is put into a solenoid field, of intensity $\mathbf{h}$, the number of lines which traverse the core is, per sq. cm., $\mathbf{b} = \mu\mathbf{h}$, where μ itself depends upon the value of $\mathbf{h}$. When $\mathbf{h} = 0$, $\mathbf{b} = 0$, to begin with; but if we induce magnetism in ordinary iron, it does not lose all its magnetism when we withdraw the inducing force $\mathbf{h}$. To take an example, of Prof. Ewing's: $\mathbf{h} = 0$, $\mathbf{b} = 0$, to begin with: $\mathbf{h}$ was raised to 90, and $\mathbf{b}$ was then 14,000; when $\mathbf{h}$ came back to 0, the value of $\mathbf{b}$ in the iron had only sunk to 10,500: to make $\mathbf{b}$ sink to 0, it was necessary to make $\mathbf{h} = -24$: when $\mathbf{h}$ was -90, $\mathbf{b}$ was $-14{,}000$; when $\mathbf{h}$ again became 0, $\mathbf{b}$ was $-10{,}500$; $\mathbf{h}$ had to be increased to $+24$ before $\mathbf{b}$ became 0; and when $\mathbf{h}$ was 90, $\mathbf{b}$ was again 14,000. On successive oscillations of the value of $\mathbf{h}$ between $+90$ and -90, the same cycle was repeated; and on plotting out these results in a diagram it will be seen that an area is described, after the fashion of the Indicator Diagram, showing products of $\mathbf{b}$ into $\mathbf{h}$, and such that the area of the figure is equal to $8\pi \times$ the number of ergs per cub. cm. wasted at each half-alternation by being transformed into Heat; for the Energy of Field, within the magnet in this case, is $\mathbf{h}\cdot\mathbf{b}/8\pi$ ergs per cub. cm., in any medium. Vibration or a high temperature reduces this effect. The form of the area described is somewhat like a $\int$, varying in central thickness from one substance or condition to another.

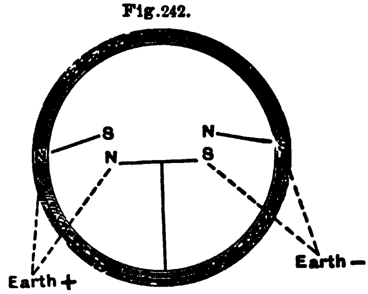
If the induced magnetism of iron be due to the directive action of the magnetic field, the residual magnetism of steel may perhaps be due to a sort of imperfect elasticity of the medium surrounding the particles; the particles are wrenched into definite directions, and retain these as a permanent set, or very slowly reassume their discrepancy of direction (Maxwell).

If we dissolve away the outer skin of a steel magnet by means of acid, we find (Jamin) that the remainder has a very small intensity of magnetisation. Perhaps the outer shell is the hardest part of the magnet and has the greatest amount of the so-called coercitive force, the least amount of elasticity of the medium.

Astatic Arrangements. — A needle tends to point to the magnetic north; but it is often desirable to mask the action of the earth's magnetism, in order to increase the ratio of the torque due to any deflecting magnet to that due to the terrestrial intensity, and thereby to increase the sensitiveness of galvanometers. This may, roughly, be done by bringing another magnet of opposite effect into the neighbourhood, so as nearly to neutralise the earth's directive force; or again by coupling together on the same suspending thread two equal magnets with their poles opposed. In the latter case the earth tends to direct the two magnets in opposite senses, and if the two magnets were equal and their axes parallel, the joint system would be practically unaffected by the earth's directive action.

It is better to enclose a single needle in a shell of very soft iron, as in Lord Kelvin's marine galvanometer. This shell, within the earth's magnetic field, becomes magnetic. The needle is now under the inducing action of two magnets, the earth and the induced shell. The actions of these are opposed, and if the shell be thick enough are approximately equal: the earth's magnetic field is thus nearly destroyed within the shell, and the magnet is free to obey the directive impulse of any current which may be sent round it. Such a shell acts as a Magnetic Screen; and such a screen, efficient as a protection from the influence of an external magnet, may be a sphere, an infinite or a very large plane, or an equipotential surface of any form.

Fig. 242.



Magnetic Circuit. — The actual phenomena of a Magnet are in some respects better correlated on considering the Lines of Induction in an Electromagnet than they are on considering the Poles of a permanent Magnet. Suppose a solenoid, of n turns and of length l cm., and bearing a current of intensity i; the magnetic force $\mathbf{h}$ at any point inside the solenoid, in any medium, is $4\pi i \cdot n/l$. The induction per sq. cm., the number of lines of induction per sq. cm., is $\mathbf{b} = \mu\mathbf{h} = 4\pi ni \cdot \mu/l$. The Total Induction along the electromagnet is $\mathbf{B}$ = cross-sectional area $\times$ $\mathbf{b} = 4\pi ni \cdot \mu A/l$. Write this $\mathbf{B} = (4\pi ni) \div (l/\mu A)$; then the Magnetic Induction or Flux $\mathbf{B}$ is said to be equal to the "**Magnetomotive Force**" $4\pi ni$ $(= \mathbf{h} \cdot l)$, divided by the "**Reluctance**" $(l/\mu A)$. The lines of induction, each of which is closed upon itself, form together a "closed circuit," like an electric current; and the above expression is analogous to Ohm's Law, $I = E/R$. The ideal Magnetic Circuit is a magnetised ring, magnetised so that the lines of induction pass continuously and equably round the ring, each of them being circular. Such a ring has no "poles," for there is no place at which the lines of force escape into the outer air. But if we cut such a ring, so as to produce an air-gap in it, we then have two poles, or rather polar regions; the lines are, as it were, unwilling to bridge the air-gap, and thus,

in order to produce a given **b**, it is then necessary to apply a greater **h**; the lines tend to become fewer in number, while of those which remain, some find their way back by shorter return-paths through the surrounding air, there then being "leakage" of these lines. The air-gap has acted as if a considerable length of iron, μ times, or say 400 times its own length, had been inserted in the magnetic circuit, and the Reluctance of that circuit had been correspondingly increased. But the formula, in the form $\mathbf{B} = (4\pi ni) \div \Sigma(l/\mu A)$, is general; there is always some kind of a return-path, the reluctance of which is ascertainable. In the ordinary horse-shoe magnet, the circuit for the lines is up and down the U, and through the cross-bar or armature; many lines escape, however, at the air-joints, and do not travel through the armature. In an ordinary bar-magnet, a large proportion of the lines of induction escape laterally; and at the terminal faces they are crowded towards the edge of that face, so that the field is stronger towards that edge. In any dynamo, the air-gaps should be reduced as far as possible, so as to diminish the reluctance of the circuit: and in every magnetic circuit in which electromagnets are employed, the soft iron parts must be so designed that, with the desired value of **B**, **b** shall not exceed a limit which suits the iron made use of, say 16,000 per sq. cm., *e.g.*, $\mathbf{h} = 50$, μ then $= 320$; for if the iron be too slender, and if the current be forced up so as to produce the required value of **B**, **b** tends to grow to values at which μ falls off.

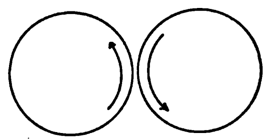
The lines which are enabled, by the presence of air-gaps, to return through the air tend to shorten and shrink; hence a permanent magnet tends to become demagnetised. Keeping its armature upon a permanent magnet tends to favour the retention of the energy of magnetisation, for a good path is thus provided for the lines.

The **general problem of magnetic induction** is a problem of potential and lines of force, in which the body acted upon consists of perfectly-conducting molecules scattered through an absolutely non-conducting medium. This kind of problem involves difficulties of calculation, but is of the same nature as that of electrostatic induction through a heterogeneous dielectric, or that of conduction of heat or of electricity through a heterogeneous conductor, or that of the flow of a frictionless incompressible fluid through a heterogeneous porous material. A given amount of force within the magnetic field produces a certain amount of separation of magnetisms and a corresponding density of magnetic distribution; this may be regarded as an arrested flow, an accumulation, which is proportional to the continuous flow which is dealt with in problems of conduction: and the nature of the substance acted upon brings into the calculation a term, the Coefficient of Magnetic Induction or the Permeability, μ, which resembles the permeability of bodies to fluids, or the specific inductive capacity of electrostatic dielectrics.

As to the **nature of magnetism**, Ampère's theory is that every molecule of a magnetic substance is the seat of a separate current, circulating round it in a plane at right angles to the magnetic axis. This explanation meets most of the facts with great readiness; but in view of the doctrine of the Conservation of Energy we must postulate the entire absence of resistance to these molecular currents—a circumstance of which it is somewhat difficult to form a clear conception.

When all the molecules of a substance have their currents running in the same direction, and when all these currents are equal, the substance is uniformly magnetised, and in the interior any two contiguous molecules (Fig. 243) have currents in opposed directions whose effect on exterior particles is *nil*. The result of the whole is equivalent to a superficial sheet of electric current, the action of which may be approximately reduced by a kind of centre-of-gravity problem to the action of two Poles.

Fig. 243.



As to the **direction of the currents** within a magnet: a person standing on the Arctic Pole of the earth would, if those currents to which the earth's magnetism is supposed to be due were visible to him, see them, or rather their resultant, the current-sheet, travelling over the surface, circulating round him from east to west; those in front of him would therefore travel towards his right hand. The observer there situated would be at the negative end of the earth; the Positive pole is its Southern pole — that pole, namely, from which the positive or north-seeking end of the compass-needle is driven. An observer stationed at the positive or southern pole of the earth, the Antarctic Pole, would therefore see these currents pass round him, still from east to west, but apparently towards his **left**. These currents within a magnet are known as Ampère-currents.

Dimensions of Magnetic Measures, in Air. — Quantity of magnetism, $\mathfrak{m}$: force $= \mathfrak{m}\mathfrak{m}' \div$ distance^2; whence, just as in the case of electric quantity, p. 603, the dimensions of magnetic quantity are $[\mathfrak{m}] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$.

Magnetic Force, or Strength or Intensity of Field, $\mathfrak{h}$: mechanical force acting on unit quantity of magnetism; its Dimensions are Mechanical Force $[ML/T^2] \div$ Magnetic Quantity $[M^{\frac{1}{2}}L^{\frac{3}{2}}/T] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

Magnetic Moment, $\mathfrak{M} = \mathfrak{m}l$: a magnetic quantity $\times$ a length; $[\mathfrak{M}] = [M^{\frac{1}{2}}L^{\frac{5}{2}}/T]$.

Intensity of Magnetisation, $\mathfrak{I}$: magnetic moment per unit of volume; $[\mathfrak{I}] = [M^{\frac{1}{2}}L^{\frac{5}{2}}/T] \div [L^3] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

Magnetic Potential, Ω: work done in moving unit quantity of magnetism; its dimensions are those of (Work done) $\div$ (Magnetic Quantity $\mathfrak{m}$ moved); $[\Omega] = [\text{Work}/\mathfrak{m}] = [ML^2/T^2] \div [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T]$; the same dimensions as those of electric potential, electrostatically measured.

Magnetic Surface-density, $\mathfrak{s}$: quantity of magnetism per unit of area; $[\mathfrak{s}] = [\mathfrak{m}/\text{Area}] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [L^2] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

Strength of Shell, φ: Surface-density $\times$ thickness; $[\varphi] = [\mathfrak{s} \times \text{thickness}] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T] \times [L] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T]$.

Magnetic Induction, $\mathfrak{b}$ per sq. cm.; Number of Lines of Induction from a given Pole across a given Area; $[4\pi\mathfrak{m}/\text{area}] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [L^2] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

Coefficients of Induced Magnetisation, κ(=Intensity of Magnetisation ÷ Magnetic Force), and of Magnetic Induction, μ(=Magnetic Induction ÷ Magnetic Force): Numbers simply; no Dimensions.

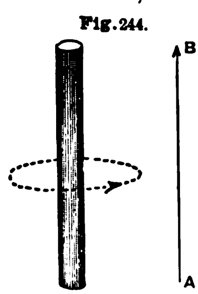
Magnetic Dimensions in medium of permeability μ.—Magnetic Quantity, $[\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$: Magnetic Field-Intensity, $[M^{\frac{1}{2}}/L^{\frac{1}{2}}T\mu^{\frac{1}{2}}]$: Magnetic Moment, $[\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{5}{2}}/T]$: Intensity of Magnetisation, $[\mu^{\frac{1}{2}}M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$: Magnetic Potential, $[M^{\frac{1}{2}}L^{\frac{1}{2}}/T\mu^{\frac{1}{2}}]$: Magnetic Surface-Density, $[\mu^{\frac{1}{2}}M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$: Strength of Shell, $[\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$: Magnetic Induction per sq. cm., **b**, $[\mu^{\frac{1}{2}}M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$; κ and μ, $[\mu]$.

Magnetic Rotatory Polarisation of Light.—If a plane-polarised beam of light or of radiant heat be sent through a magnetic field occupied by a transparent medium, its plane will, by the retardation or acceleration in phase of one of its circular components, be rotated. The sense of this rotation depends upon the direction of the lines of force and upon the nature and chemical constitution of the medium; its amount upon the thickness and the nature and physical state of the medium and upon the intensity of the magnetic field, resolved in the direction of the ray; but it does not occur in free Ether.

This has, in the hands of Becquerel and Lord Rayleigh, been made the basis of a method of measurement of the intensity of a current; the current, when passed through a solenoid, produces within this an electromagnetic field, the intensity of which is proportional to the strength of the current; a plane-polarised beam sent through glass along the axis of the solenoid is rotated to an extent proportionate to the current-strength.

The direction of rotation of the plane of polarisation is in most cases, including flint glass and thin films of iron, cobalt, and nickel, positive, being that shown by Fig. 244, in which AB represents a line of magnetic force, and the arrowed circle represents the direction of rotation of the plane. Whether the ray travel in the direction AB or in the direction BA, the absolute rotation imparted to it on its transmission through the magnetic field remains the same; whence, if it be reflected from a mirror and sent back through the field, the rotation of its plane will be doubled.

Fig. 244.

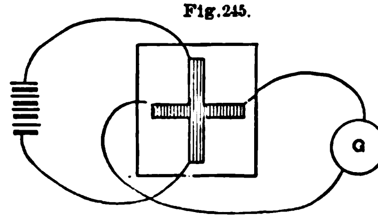


In uniaxial crystals, the rotation is most marked along the axis. In a concentrated solution of perchloride of iron it is negative.

Hall's Experiment.—A film of metal, in the form of a cross, laid upon glass. A current from a battery passes through two opposite arms of the cross, and does not affect a galvanometer connected with the other two arms. When the cross is made to face the lines of force of a strong magnetic field, a small constant current is indicated by the galvanometer (Fig.

245). The strength of this current depends upon the intensity of the magnetic field and the strength of the primary current; and also upon the kind of metal of which the film consists. Its direction depends upon the direction of the field and that of the primary current, and on the metal of the film.

Fig. 245.



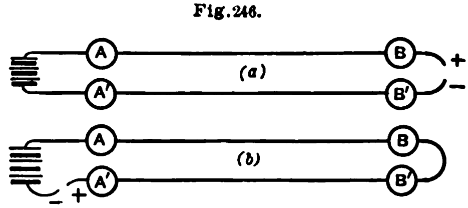
Kerr's Experiment. — Polarised light reflected from the polished face of a magnet undergoes rotation of the plane of its polarisation; when reflected from the north-seeking pole, the rotation is negative. When reflected from the sides of a magnet, it also undergoes rotation, the sense of which varies with the plane of polarisation and the angle of incidence. Kundt has also found a variety of phenomena of rotation of the plane of polarisation of light transmitted through thin magnetised metallic films.

Thus far we have dealt with Steady Currents and Steady Fields, electrostatic, magnetic, or electromagnetic. We have now to consider the properties of Varying Currents and Fields.

The Variable Period.

When an open circuit is abruptly closed for an instant, and an instantaneous current is produced in the wire, this current is not felt simultaneously over the whole circuit. In the case (*a*) of Fig. 246 the galvanometers B and B′ twitch first when the interrupted circuit is momentarily completed; in case (*b*) of that figure, under similar circumstances, the galvanometers A and A′ twitch first. The distance between A and B must be great in order to show this effect.

Fig. 246.

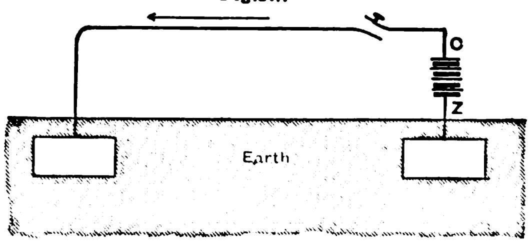


By the time a state has been arrived at, in which a steady current passes, a certain amount of energy has been elastically accumulated in the dielectric; but between the instant at which the current begins to flow and that at which it has assumed its steady state, there is a period of adjustment, the **variable period**.

During this period the field has Kinetic Energy, and there is a Displacement- or Polarisation-Current.

When, as in Fig. 247, a battery of which one pole is connected to earth has its other pole suddenly brought into communication with a long wire whose other extremity is connected with the earth, the time which elapses before the current through the wire becomes steady is found to vary as the square of the length of that wire.

Fig. 247.

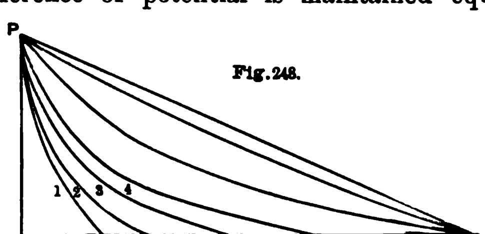


In long lines, the time spent in acquiring at any point of the wire a certain definite intensity of current is approximately proportional to RCl^2/E, where E is the D.P. employed, R the resistance, and C the electrostatic capacity of the wire per cm., and l its length. The time which elapses before a certain proportion of the ultimate intensity is attained varies approximately as RCl^2; it is also, in practice, not independent of the effective D.P. set up by the galvanic cell employed. The product RCl^2, which measures the Electrostatic Retardation, is thus of great importance in long lines; but in short lines, the effects of electrostatic retardation are masked by those of self-induction and the induction of other circuits.

A lightning discharge through a lightning conductor is so brief that the laws of steady flow do not hold good: it is of advantage, in order to diminish the risk of lateral divergence, to render the current more uniform, or, in other words, to retard it; for this purpose the capacity of the conductor should be increased, and therefore its surface; and lightning conductors should be broad flat plates of metal rather than compact rods. The effect of self-induction of the current also aids in bringing about this result; currents running parallel and in the same direction retard one another. The circumstance that the flow is too brief to affect the interior of the wire to any considerable extent also aids in making it more important to increase the relative surface of the conductor than to increase its cross-section; for the phenomena of so abrupt a discharge are practically restricted to the field of force, the dielectric, surrounding the wire.

In a uniform wire, OL, between whose extremities a difference of potential is maintained equal to OP (Fig. 248), the ultimate Line of Potentials is PL; and when such a distribution of potentials has once been produced along the conductor, Ohm's law is obeyed; but at various instants during the preliminary variable period, the distribution of poten-

Fig. 248.



tials along the wire is such as is indicated by the curved lines, 1, 2, 3, etc., sketched in Fig. 248.

The momentary and local intensity is always the momentary and local E/R (= Potential-Slope ÷ Resistance per linear cm.), but during the variable period it varies from point to point and from instant to instant.

When the extremity of a long wire is momentarily charged by contact with a charged conductor or with one pole of a battery, its home end suddenly acquires a high potential, which is immediately thereupon reduced by communication of the charge acquired by the extremity of the wire to the rest of the wire. In Fig. 249 the end O of the conductor OL is suddenly raised to the potential OP. A point such as A is found, as it were, to leap up to a high potential and then to descend. A wave of sudden increase of potential thus travels along the conductor, but falls off progressively, both in abruptness and in height, the farther it travels.

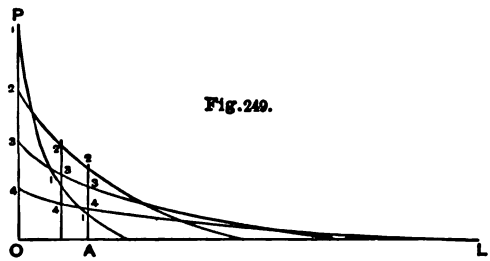
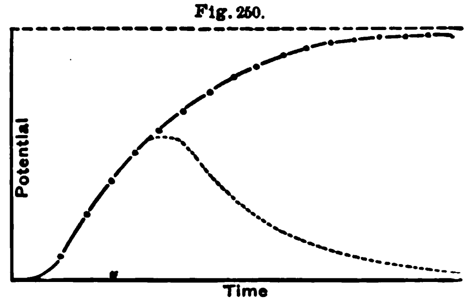


Fig. 249.

At the distant end, for a short interval after the circuit has been actually completed, no effect is perceived; the current then begins to become sensible: and it then, if the contact be kept up at the home end, appears to increase in intensity after the manner indicated by the Arrival-Curve represented in Fig. 250. A current, even though it be constantly maintained at the home end, would take an infinite time to acquire its maximum value at the distant end of such a conductor as an Atlantic cable, if that conductor had, when the current commenced to traverse it, been uncharged; it would, however, require only about 108 seconds to attain $\frac{9}{10}$ of its maximum value, and about the fifth part of a second to attain $\frac{1}{100}$ of its maximum value. The apparent velocity of transmission of signals in a given conductor is thus seen to be mainly an affair of the delicacy of the instruments which detect the current on its arrival at the distant

Fig. 250.


end, and is perfectly distinct from the velocity of propagation of an electromagnetic disturbance; and it depends on the capacity of the conductor, for the transmission is greatly delayed in conductors whose capacity is great, such as submarine cables, appreciably so in long air-lines, inappreciably so in short air-lines.

The attainment of the steady state is greatly facilitated, though the currents produced are weakened, by leakage. The signals produced are thus rendered clear (Oliver Heaviside).

When the current suddenly stops after having acquired a steady flow, its cessation at the distant end presents a similar deliberation.

When a wire is momentarily connected with a charged body and then connected with the earth, or "put to earth," the arrival-curve at its distant end is a curve due to the superposition of two arrival-curves; the first of these is the arrival-curve, resembling that of Fig. 250, due to the contact with the charged body; the second is curved in the opposite sense, and is due to the sudden discharge of the conductor. The dotted curve of Fig. 250 is the result of the superposition of two such opposed arrival-curves. This curve indicates that there is an abrupt and brief variation of potential at the end of the wire distant from the galvanic cell.

More effective still than this in producing an abrupt and brief current is the process of following up each positive charge, immediately after putting to earth, with a negative one, after which the wire is again put to earth. The disadvantage of this is that the potential, while it abruptly ceases to be positive, sinks at once to a negative condition, as in Fig. 251; for which reason it is customary so to arrange the mechanism at the signalling station that each apparently simple making of contact is in reality a complex operation, in which an odd number of currents of opposite kinds are sent in rapid succession into the wire, the wire being, after each, put to earth; each of these currents being briefer than its predecessor, and correcting it. The arrival-curve for such a combination indicates an abrupt rise of potential, an abrupt fall, and then a slightly-wavy line, which at no point diverges to any material extent from the base-line.

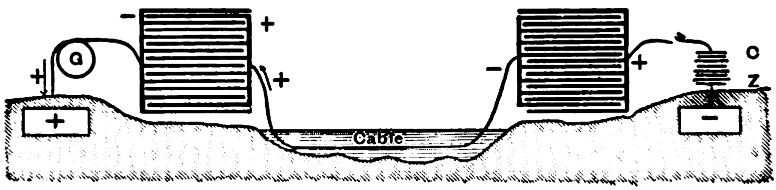
Fig. 251.

Even these methods are increased in effect, the arrival-curve being rendered still more abrupt, by the use of Condensers. Each condenser is composed of a large number of plates of tinfoil separated by waxed paper and paraffin: the alternate plates are in metallic communication with one another. One series of alternate plates in each condenser is in communication with the cable; the other set is in communication with the galvanic battery or with the galvanometer G (Fig. 252).

Any sudden variation in the potential of the landward plates of the home condenser is immediately followed by an equally-sudden flow of electricity

into or from the cableward plates of that condenser: this flow takes place either from or into the cable itself; this disturbance is propagated along the cable; the potential of the cableward plates of the condenser at the receiving station is affected; by induction the distribution of electricity in the landward plates of that condenser is affected, and a current passes through the galvanometer, either from the condenser to the earth or in the

Fig. 252.



reverse direction. On connection of the home condenser with the positive pole of the battery employed, a positive current runs through the distant galvanometer G to the earth; and on putting the home condenser to earth a reverse current passes through G, which may be corrected as before.

Even if the key be kept permanently pressed down at the transmitting station the current passing through G is but momentary, for both condensers quickly assume a condition of electrostatic equilibrium.

During the Variable Period, the Lines of Force are slipping along the wires with the velocity of Light. They travel with their ends on the wires, and approximately lie at right angles to these, until the condition approaches that of the steady state. During the variable state, they accumulate (or thin away) in the field; when the steady state has been attained, there is no accumulation of them in the field, but only transit, while their direction becomes approximately parallel to that of the wires. At the same time, during the variable period, the Lines of Transmission of Energy through the field are themselves in motion; and the axis of the wire is the last thing to be affected. During this period there may be production of induced currents in neighbouring conductors.

Electromagnetic Current-induction.

If there be a closed current-bearing circuit, with its positive face facing a positive magnetic pole, there will be mutual repulsion between that circuit and pole; and if the current and the magnetic pole be brought nearer one another, then, since work must be done in order to bring about this approach in the face of mutual repulsion, the potential energy of the system is increased by a fixed amount: a portion of this energy takes the form of a temporary increase of the current in the closed circuit; while the remainder may, by induction, produce an increased magnetic condition in the magnet.

Now replace the magnet by an equivalent closed circuit

("circuit B"), the positive aspect of which faces the positive aspect of the original closed circuit ("circuit A"). These two circuits, again, repel one another: and if work be done in forcing them together, the energy appears in the form of a temporary increase in the intensities of both currents, and is presently converted into heat in the circuits.

Conversely, when the magnet or the equivalent circuit is withdrawn, there is a corresponding temporary diminution in the corresponding current-intensities, and possibly in the corresponding magnet-strength.

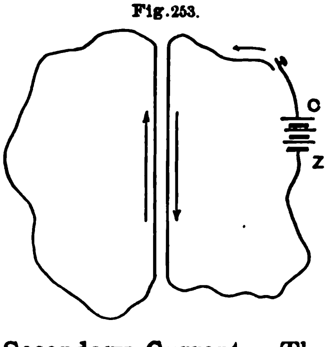
These increases and diminutions in current-intensities are equivalent to the Induction of New Currents. The duration of these new induced currents is limited to the Variable Period, the time spent in changing the relative positions of the mutually-inducing magnets or currents.

The lines of electric force accumulate, or else fall off, upon and parallel to the circuit-wire. The steady state is thus interfered with, and an electrical effect is produced, analogous to mechanical acceleration.

If a circuit bearing a current be brought towards a circuit capable of bearing a current, and if the former, the inducing current, have its positive face turned towards the circuit approached, there will be two effects produced: (1) an increase in the intensity of the inducing current, and (2) a new current developed by induction in the circuit approached, which had previously appeared to bear no current. This current has its positive face turned towards the approaching positive face of the inducing current, and is therefore opposed to it in its direction.

If two wires be laid alongside one another (Fig. 253), and if one of these wires be connected with the two poles of a battery, and thus form part of a Primary or Battery Circuit; while the other wire is merely a part of a complete metallic circuit, a so-called Secondary Circuit; then, when contact is suddenly made in the primary circuit, a current of brief duration — a duration not exceeding in time the variable state of the primary current — is produced in the secondary circuit and is known as the **Secondary Current.** The primary current and the secondary current are, in the wires laid alongside one another, opposed in their direction.

Fig. 253.



So long as the intensity of the primary current remains constant, the secondary circuit has in it no current; but any increase is accompanied by a brief opposed secondary current.

When the primary current is diminished, the primary circuit again presents a variable state; and so long as that variable state lasts, there is again a current in the secondary circuit, which is on this occasion in the same direction as the waning primary current. When the primary current stops abruptly, there is a very abrupt secondary current, parallel to the ceasing current.

These secondary currents represent a definite amount of energy subtracted from the energy of the primary current,—an amount which depends only on the initial and final states or intensities of that current. Being of extremely short duration, they are of correspondingly great intensity and high potential. The secondary current produced on breaking the primary current is briefer, and therefore more intense than that produced on making it.

When a magnet is thrust into the axis of a bobbin which forms part of a closed circuit, there is a current produced in that circuit. The current is opposed in direction to the magnetic molecular-currents, the Ampère-currents, of the pole which is introduced first. If a long magnet be drawn wholly through such a coil, there is at first a current in one direction as the one pole approaches; then, as its midpoint passes the midpoint of the coil, the current is *nil*, but is reversed as the opposite pole passes out. The current is at first opposed to the Ampère-currents of the approaching pole; and as all parts of a bar-magnet, looked at end-on, have their currents in the same direction in space, the induced current changes in its direction as the magnet passes through.

These statements may be generalised by saying that wherever a closed circuit, capable of bearing an electric current, lies wholly or in part in a Magnetic or Electromagnetic Field of Force, any disturbance in the Intensity of the Field of Force will induce a Current in the circuit; and the direction of the induced current is determined by the rule (**Lenz's Law**) that the new current will increase the already-existing resistances, or develope new resistance to that disturbance of the field which is the cause of induction.

A telephone circuit passing through a disturbed field of force will pick up signals: for example, at every lightning flash the instrument is heard to roar; and in order to prevent such effects of induction, no part of the current

is entrusted to earth, but the double wire necessary is coiled round itself so as to form a strand composed of two insulated wires. The effect of induction on one wire is then equal to the opposite effect of induction on the other wire. During thunderstorms military mine-fuses have been known to explode through induction in the wires controlling them.

We have seen that a closed current, A, whose positive aspect faces a positive magnetic pole or face of a magnetic shell or equivalent electric current, B, sends towards the latter, from within its own contour, a number of positive lines of force or of induction, which radiate from its positive face. If we change our standpoint, and regard the current first mentioned — a current borne by circuit A — as placed within the magnetic field of the shell B or the electromagnetic field of the equivalent circuit, then the positive lines proceeding from the latter are, as regards the circuit A, negative, for they trend not from but towards its positive aspect.

Circuit A, as we have seen, tends to move by translation to a greater distance from circuit B. It will also tend to rotate until its negative aspect faces the positive side of B; it is then attracted towards B.

In the former case, as A moves away, the number of *negative* lines which pass towards its area diminishes. In the latter case — that of rotation — A tends, as it turns, first to set itself edge-on to B's lines of induction, and then so to place itself (its negative face opposite to B's positive face) that the lines of induction which emerge from B positively also emerge from A's positive face positively, and are positive with respect to A.

In either of these cases, translation or rotation, the number of negative lines met by the area of A is diminished as far as possible, or the number of positive lines embraced by its contour attains a maximum.

A little movable circuit may be made — De la Rive's floating battery — by thrusting a strip of copper and a strip of zinc through a cork, and connecting them by an arch of copper wire: when the whole is floated in water, the arch tends to lay itself at right angles to the magnetic meridian, copper to the west, zinc to the east; in this position the positive face of the arch is to the north, and the magnetic lines of force or induction which trend towards the north are embraced by the arch in the greatest possible number.

The general statement of the phenomenon is: — A movable circuit tends so to place itself as to meet as few negative or to have as many positive lines of induction passing through it as possible; a line passing through a circuit being held positive when, after passing through, it emerges from the positive face in a positive direction; a line being held to be negative when its direction is towards the positive face. The position thus assumed is the position of least potential energy, that into which the whole system tends, as it were, to sink. A circuit in this position of least potential energy embraces as great a number as possible of *positive* lines of induction.

Mutual Attraction and Repulsion of Currents. — Suppose two currents in the plane of the paper, similar in their directions and having in consequence their nearer portions opposed in direction, as in Fig. 243. Let their directions be the same as those of the two currents in that figure. The left-hand current in that figure has lines of induction which ascend from the plane of the paper and tend to descend through the contour of the right-hand circuit, meeting its ascending lines. These descending lines are therefore negative to the right-hand circuit, and that circuit tends to move away so as to meet as few of them as possible. The portions of the currents which are nearest one another, running in opposite directions, thus seem to repel one

another. The area in which downward lines meet upward lines is thus diminished as far as possible, and this enables us now to understand the propositions illustrated by Figs. 229, 230.

If a circuit embracing the greatest possible number of positive lines of induction, and therefore occupying the position of least potential energy, be pulled or turned into any other position, work must be done upon it; and this work is done against mutual attractions. This doing of work is associated with diminution of the number of positive lines of induction embraced by the movable circuit. As the circuit moves in the field, lines of induction must be cut through by it. All cutting through lines of induction, when the number of lines enclosed by the circuit is diminished by the operation, is effected by the expenditure of work. The process attains its maximum when the movable circuit has been swung round through 180°. If it be still farther rotated, it comes to meet fewer negative lines, then to enclose an increasing number of positive lines, until it regains its original position.

The work done takes the form of the energy of induced currents, which always increase the resistance to the actual movement; if A and B repel one another, their intensities are increased when they are urged together, diminished when they are drawn asunder; if they attract one another these actions are reversed. This may be otherwise expressed by saying that when a circuit is made to meet a greater number of negative lines of induction, or to enclose a smaller number of positive lines, its current is increased in intensity, or a new induced current set up in it; while if it be made to meet a smaller number of negative or to enclose a greater number of positive lines, the intensity of its current is diminished, or a new reverse current is set up in it. The result is the same whether it move so as to enclose more or fewer lines in an existing magnetic field, or whether the magnetic field itself vary so that its lines either open out and become fewer, or become more numerous and approach one another — a smaller or a greater number of them consequently passing through the given circuit.

If a part of a circuit of total resistance r be movable in a magnetic field which presents $\mathbf{b}$ lines of magnetic induction per sq. cm., it will cut through all the magnetic lines in a certain area A in the course of time t; it will therefore cut through $\mathrm{A}\mathbf{b}/t = \mathbf{B}/t$ lines of induction per second. The current set up is, firstly, such that if the movable part of the circuit uniformly *diminish* the area of the circuit as it moves in the terrestrial magnetic field, the current will run in the circuit (which is supposed to be set in a plane at right angles to the magnetic meridian) in a direction which seems from the standpoint of an observer stationed to the south to be the same as that of the hands of a watch: and, secondly, its intensity is proportional to $\mathbf{B}/rt$. In Electromagnetic measure, the units are so adjusted that the intensity of the induced current is **equal** to $\mathbf{B}/rt = \mathrm{A}\mathbf{b}/rt$. If $\mathbf{B}$ be the *additional* number of lines which the circuit comes to enclose, the intensity of the induced current is $i = -\mathbf{B}/rt = -\mathrm{A}\mathbf{b}/rt$. This relation is the same, whatever be the permeability μ.

When a block of copper is whirled within a magnetic field, currents are set up in it, which produce resistance to the motion; the motion of the block very rapidly ceases, as if the magnetic field were highly viscous, and the block becomes hot. When a magnet-needle is suspended immediately above a copper plate, any oscillation in the magnet developes retarding currents in the copper, and the magnet almost immediately comes to rest.

Self-Induction. — A current suddenly formed in a spiral wire is retarded by the mutual action of the different turns; it does not flow on, and its intensity is, at first, less than it would have been in a straight wire: when suddenly broken it is prolonged and is as it were piled up, so that the so-called Extra-Current can force its way through greater resistance than the steady current can. In fact, a single Daniell cell can be made to electrolyse water by delivering a part of the energy of its current, at high potential, in the form of the so-called Extra-current.

These phenomena closely remind us of the phenomena of momentum in a water-pipe, already discussed under the Hydraulic Ram; and they can be explained as phenomena of momentum of the Ether in the electromagnetic field.

Two wires bearing currents in opposite directions, and twisted round one another, present no phenomena of self-induction; for which reason the wires leading to and from a galvanometer should be twisted together for some distance from the needle.

Coefficient of Mutual Induction of two Currents. — The Ether surrounding a pair of current-loops of intensities $i_{,}$ and $i_{,,}$ must possess Energy, which Clerk Maxwell showed to be proportional to squares and products of the intensities, and which may be written thus: $\{{}^1\mathrm{L}i_{,}^2 + \mathrm{M}i_{,}i_{,,} + \frac{1}{2}\mathrm{L}'i_{,,}^2\}$.

The second term vanishes when the currents are at an infinite distance from one another; it is at its greatest practical when the two circuits almost touch one another, its greatest theoretical when they absolutely coincide: at intermediate points it has intermediate values. It can be shown that $\mathrm{M}i_{,}i_{,,}$ is, in any given position of A and B, numerically equal (1) to the Mutual Potential Energy of the two circuits and (2) to the Number of Lines of Induction which, being due to A, pass from A through B or, equally, being due to B, pass from B through A; and M is styled the Mutual Inductance or the Coefficient of Mutual Induction. M varies with the relative position of the two circuits.

The maximum value of M is its value when the two currents are made to run in the same circuit; let this be called M_0. The number of lines $(= \mathrm{M}_0 i_{,}i_{,,})$ due to $i_{,}$ and the number due to $i_{,,}$ are to be added together for the conjoined current $(i_{,} + i_{,,})$; for they all pass through the same circuit; hence the number actually threading the circuit will be $2\mathrm{M}_0 i_{,}i_{,,}$. In this case $\mathrm{M}_0 i_{,}i_{,,}$ is equal to the part of the energy of the field which is due to the approximation of the currents $i_{,}$ and $i_{,,}$ from an infinite distance and their coincidence; and M_0 is equal, numerically, to half the number of lines of induction which pass through the circuit itself when $i_{,}$ and $i_{,,}$ are both unity, that is, when the conjoined current has an intensity $= 2$. It is therefore equal to the number of lines of induction passing through when $i = 1$.

Coefficient of Self-Induction, or Inductance. — We next see that $\mathrm{M}_0 = \mathrm{L}$. If the intensity of the second current $i_{,,}$ be 0, the energy of the field is $\frac{1}{2}\mathrm{L}i_{,}^2$ only. A second current of the same intensity in a circuit of the same size, etc., at an infinite distance will have energy also equal to

$\frac{1}{2}Li_{,}^{2}$. Together the energy will be $Li_{,}^{2}$. Now bring the two currents together and blend them; the energy is $\frac{1}{2}L(2i_{,})^{2} = 2Li_{,}^{2}$. The system possesses energy equal to $Li_{,}^{2}$, due to the approximation; but this is also $M_{O}i_{,}^{2}$ if both currents be equal to $i_{,}$; whence $L = M_{O}$. L is the Coefficient of Self-Induction; and the coefficient of self-induction of a circuit is equal, numerically, to the number of Lines of Magnetic Induction which thread that circuit when it bears a current whose intensity is unity in electromagnetic measure. Within the contour of a circuit, $\mathbf{B} = Li$.

In a solenoid of n turns, and length l cm., the number of lines of induction for unit current is $4\pi n \cdot \mu/l$ per sq. cm., or $4\pi n \cdot A \cdot \mu/l$ across area A. If a second solenoid, of n' turns, surround the first, each turn of it embraces $4\pi n \cdot A \cdot \mu/l$ lines once; and its n' turns embrace $4\pi n \cdot n' \cdot A\mu/l$ lines. Hence for two such solenoids, $M = 4\pi \cdot nn' \cdot A\mu/l$; and for a single solenoid, $L = 4\pi n^{2}A\mu/l$. Hence the self-induction of a coil of many turns is very great.

Extra-Currents.—If the energy of a current traversing a single circuit be derived from any external source, such as a battery, which is independent of induction, the energy supplied from that source during a very short time δt will be equal to $ei \cdot \delta t$, where e is the E.M.D.P. and i the current-intensity. (All our measurements in these paragraphs are supposed to be made in electromagnetic measure.) This energy is divided into three parts.

(1.) Heat in the circuit. This is equal to $ri^{2} \cdot \delta t$, where r is the resistance.

(2.) External work, mechanical, chemical or other. This we shall suppose $= 0$.

(3.) Work spent in imparting energy to the electromagnetic field. This is equal to $\frac{1}{2}L\{(i + \delta i)^{2} - i^{2}\}$, where δi is the small change in the intensity produced during the time δt.

We thus have the equation

$$ei \cdot \delta t = ri^{2} \cdot \delta t + \tfrac{1}{2}L\{(i + \delta i)^{2} - i^{2}\}; \qquad \text{(i.)}$$

an equation which can be dealt with by integration, the effect being that we find the intensity, at any time t after the introduction of a new E.M.D.P. $= e$ into the circuit, to be

$$i_{t} = e/r - e/r\,(2{\cdot}718281^{-rt/L}). \qquad \text{(ii.)}$$

The intensity never comes fully up to the value e/r; but it approaches it indefinitely nearly as the time t increases. If, however, the coefficient L be large, as it is in a coil of wire, the second term on the right-hand side is not immeasurably small, and it represents what is equivalent to a reverse current lasting for an appreciable time, and delaying the development of a current of full intensity e/r. This reverse current is called the Reverse Extra-Current or the Extra-Current of Closure or of Making.

When a circuit is suddenly broken, the intensity at a time t after the current has been stopped is $+ (e/r)\ (2{\cdot}718281^{-rt/L})$. This indicates that there is still an onflow, a Direct Extra-Current or Extra-Current of Opening or Breaking; an onflow which results in a high potential at the broken extremities of the wire, and, since the capacities of these extremities are small, in a high value of σ at these extremities.

These extra-currents are thus associated with absorption of energy by the electromagnetic field while currents, the energy of which is derived from extraneous sources, are being produced or increased, and with liberation of energy by that field when such currents are broken or while they are being diminished.

They may also be looked at as phenomena of Induction. When the intensity of a current is increased, the circuit is made to embrace more lines of induction: if it embrace **B** more lines of force in time δt, an E.M.D.P. is set up, equal in electromagnetic measure, to $-\mathbf{B}/\delta t$. Where L is the self-induction of a circuit, the establishment of current i in it causes the development of $\mathrm{L}i$ lines of induction embraced by it; and this causes an E.M.D.P. $= e_{,} = -\mathrm{L}i/\delta t$; whence $i_{,}$, the mean intensity, $= -\mathrm{L}i/\overline{r\cdot\delta t}$; this being the mean intensity of an induced current opposed in its direction to the originating current. When the main-current ceases, the induced current now produced on the disappearance of lines of induction is direct, the Direct Extra-Current.

The steady intensity $i = e/r$; whence the new "electromotive force" $e_{,} = -\mathrm{L}e/\overline{r\cdot\delta t}$. Since δt is very small this may greatly exceed e; and, for a given length of wire, it is greatest when the current passes through conductors of a spiral form, in which the value of L is great. The extra-current may thus be able to spark across striking distances beyond the power of the main-current.

The quantity of electricity in the extra-current is in each case $q = i_{,}\cdot\delta t = e\mathrm{L}/r^2$. These relations are the same, whatever be the permeability μ.

Measurement of Inductance L.—Take a Wheatstone Bridge (Fig. 222), with arms AB, BC, AD, DC, with a battery as in the figure, and with a galvanometer between B and D; and then lengthen the wire of the battery circuit AC, and of the galvanometer branch BD, so as to bring them both up to a rotating commutator or contrivance which shall, in repeated succession, perform the following cycle of operations, viz.:—(1) disconnect the battery, (2) disconnect the galvanometer, (3) connect up the battery, and (4) bring in the galvanometer. Then, if the resistances in the arms AB, BC, CD, and DA be respectively r', r'', r''', and a resistance r'''' which comprises that of the loop or coil which is to be tested; and if these be so adjusted that there is no current in the galvanometer; then, on setting the commutator in action, the balance of resistance will appear to be disturbed, for the "impedance" of the coil (p. 722) is not the same as its Resistance to steady currents; and the galvanometer-needle will be deflected. The resistance r'''' will have to be reduced by a certain amount δr to restore the balance; and Ayrton and Perry have shown that if n be the number of complete cycles per second, the inductance L is equal to $\delta r/n$. (Ayrton and Perry's Secohm-meter.)

Induction Coils.—The effect of induction is multiplied when the two wires, that of the primary and that of the secondary circuit, though kept insulated from one another, are wound together round the same axis. The secondary current is then proportional in its intensity to the product of the number of turns in the two wires, provided that the resistances introduced by multiplying the coils, or the differences between the mutual distances of the different turns, be not too considerable. In Induction Coils the wires of the two circuits are wound round separate bobbins, which are then slipped the one over the other to a greater or less extent. On this extent depends the inten-

sity of the secondary current. The primary current is made and broken with great frequency by means of a Contact-breaker.

This may be a mere mechanical contrivance, or it may be automatic.

In the latter case there lies a bar of soft iron in the axis of the inner, the primary, bobbin. When the primary current passes, this bar or core becomes an electromagnet. This electromagnet pulls towards itself an armature, a mass of soft iron, which is arranged near one of its extremities; this mass of soft iron is an integral part of the circuit of the primary current, and by its movement the primary current is suddenly broken. The electromagnet now loses its magnetic condition; it ceases to attract the armature; the latter, under the pressure of a spring, returns to its former position, and again completes the primary circuit; the electromagnet is again made, and the armature again displaced. The soft iron armature is thus caused to oscillate and to impart to the primary current an intermittence, whose frequency depends upon the intensity of the current and upon the pressure of the spring.

Magnetic or Electromagnetic Measure.

A current of given intensity, I in electrostatic units, must be represented by smaller numbers when magnetic units are used; a current of $I = 60000,000000$ is a current of $i = 2$; for the C.G.S. magnetic unit of current-intensity or -strength is 30000,000000 times as great as the C.G.S. electrostatic unit.

The basis of the Magnetic or Electromagnetic system of measurement is the identity of effect, **in air,** between a magnetic shell of strength φ and a closed current of the same contour and of a particular intensity i; the units of current-intensity are so adjusted that i becomes, in air, numerically equal to φ.

If $i = 1$, the current is equivalent in magnetic effect to a magnetic shell whose strength φ is unity and whose area and outline are the same as that of the circuit; and this is the **Magnetic C.G.S. Unit of Current-Intensity (Definition i.).** [$\varphi = \mu i.$]

If we suppose a wire bearing a current, and one cm. in length, to be bent into a circular arc whose radius is one cm., and if we suppose a unit magnetic pole to be placed at the centre of the circle of which the circular arc forms a part; and if we further suppose that the mechanical force exerted by the current upon the unit magnet-pole is equal to one dyne; — then the current is one whose intensity is, in magnetic measure, equal to unity. In such a case $i = 1$; and this may be taken as **Definition ii.** of the Magnetic C.G.S. Unit of Current-Intensity. The general formula is $F = \mathfrak{m} \cdot il/r^2$, where F is the force exerted upon a magnet-pole placed at the centre of such an arc, $\mathfrak{m}$ the strength of that pole, i the intensity of the current, l the length and r the radius of the circular arc into which the wire is bent. [F does not depend upon μ.]

If the wire be bent into a complete single circular loop of radius r, $l = 2\pi r$, and $F = \mathfrak{m} \cdot i \cdot 2\pi/r$.

If the pole $\mathfrak{m}$ be in the axis of the single loop, but not necessarily in its plane, $F = / \cdot i \cdot 2\pi \cdot r^2/u^3$, where u is the distance, more or less oblique to the axis, between the pole and the wire. When u is reduced to $u = r$; that is, when the pole comes into the plane of the loop, $F = \mathfrak{m} \cdot i \cdot 2\pi \cdot r^2/r^3 = \mathfrak{m} \cdot i \cdot 2\pi/r$; which agrees with the expression above.

If a current, $= i$ in magnetic units, pass along a wire across a uniform field of magnetic force or strength $\mathbf{h}$, in air, the wire is acted upon, transversely, by a force $F = \mathbf{h}il$ dynes, if the length of the wire be l cm.; and this gives us another definition (**Definition iii.**) of the Magnetic C.G.S. Unit of Current-Intensity. This also enables us to measure the intensity $\mathbf{h}$ of an intense magnetic or electromagnetic field; a current is led through it; the wire is forced in one or other direction; this force can be balanced by a known weight. If the current be sent through a column of mercury in a known magnetic field, there is a difference between the manometric pressures at the two sides of the column (Lippmann). [$F = \mu \cdot \mathbf{h}i \cdot l$.]

(**Definition iv.**): a current of intensity i, traversing a straight wire of indefinite length, acts upon a magnetic pole $\mathfrak{m}$, at a distance d from the wire, with a force $F = \mathfrak{m}\mathbf{h} = \mathfrak{m} \cdot 2i/d$. (Compare p. 189, prop. 7.) A unit current would therefore act upon a pole $\mathfrak{m}$ with a force $F = 2\mathfrak{m}/d$. [Does not depend upon μ.]

When a current i passes into a long solenoid, of n turns and length l cm., a magnetic pole $\mathfrak{m}$ anywhere within the solenoid is acted upon with a force $F = \mathbf{h} \cdot \mathfrak{m} = 4\pi \cdot \mathfrak{m} \cdot i \cdot n/l$. Whence a unit current will act with a force $F = \mathfrak{m} \cdot 4\pi \cdot n/l$, or $(4\pi \cdot \mathfrak{m}) \times$ the number of turns in the solenoid per unit of length. (**Definition v.**) [Does not depend upon μ.]

The Intensity of a Current, magnetically measured, and the Magnetic Strength of a Shell must accordingly have the same **dimensions.** The Magnetic Strength of a magnetic shell is (p. 682) numerically equal to the product of the Magnetic Quantity per unit of area into the Thickness of the shell; its dimensions must hence be those of Magnetic Quantity, divided by an Area, and multiplied by a linear Thickness; but the dimensions of Magnetic Quantity must (since the imaginary magnetic matter obeys laws resembling those of repelling or attracting electric matter, similarly imaginary) be like those of electric quantity, $[M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$; the dimensions of Magnet-Strength are accordingly $[M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [L^2] \times [L] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T]$; the Magnetic Measure of Current-Intensity has the same dimensions, and therefore differs from the electrostatic measure, whose dimensions are $[M^{\frac{1}{2}}L^{\frac{3}{2}}/T^2]$, by the term $[L/T]$, which represents a Velocity: the numerical value of this ratio must be found by experiment, which shows it to be 30,000,000000 nearly, $= \mathbf{V}$.

Measurement of V. — The ratio between the magnetic and electrostatic measures may be determined by several methods, of which two may be taken as examples.

Weber's method. — Charge a Leyden jar with a known quantity of electricity, Q; discharge the jar through a wire, which passes round a gal-

vanometer-needle. The quantity of Electricity passing through the galvanometer may be measured in terms of the deviation undergone by the needle in consequence of being thrown by the instantaneous current. This gives the quantity, q, in magnetic measure. These separate measurements of the numerical values, Q and q, of the one quantity of electricity give the ratio between the electrostatic and the magnetic unit.

Lord Kelvin's method.—The two ends of a wire of great resistance, R, are kept at a constant potential-difference, E; a constant current runs through the wire; this current is found to have an intensity i C.G.S. magnetic units; the difference of potential is (by Ohm's law) $E = IR$ or $e = ir$. E and e, or i and I, bear to one another the relation of $1 : \mathbf{V}$; whence $\mathbf{V}$ may be found numerically.

The meaning of this ratio between the Electrostatic and the Magnetic or electromagnetic units is frequently found to be puzzling. Its real basis is the following. Both these systems of units are based on independent arbitrary conventions; and neither of them can absolutely represent physical truth, though all calculations will work out accurately if we adhere to either system. In the Electrostatic system the units are so adjusted that equal charges at unit distance apart, repelling or attracting one another through air with unit force, are called "unit charges:" but more generally, the mutual action of two charges depends upon K, the sp. ind. cap. of the medium between them; $F = QQ'/Kd^2$; whence $[Q] = [K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$; and it is only by assuming the sp. ind. cap. of air to be unity, and the sp. ind. caps. of other media to be Numbers merely, that we arrive at the air-equation $F = QQ'/d^2$, and the corresponding Equation of Dimensions $[Q] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$. Similarly, the Force F between two currents i and i' varies as $\mu \cdot ii'$, where μ is the magnetic permeability of the medium: that is, [Force] $= [\mu \cdot Q^2/T^2]$, and $[Q] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/\mu^{\frac{1}{2}}]$; but by similarly assuming the magnetic permeability of air to be unity, and the magn. perms. of other media to be numbers merely, we arrive at that air-equation of Dimensions, $[Q] = [M^{\frac{1}{2}}L^{\frac{1}{2}}]$, which lies at the basis of the Magnetic system of measurement. The conventional units of electric Quantity thus bear to one another the ratio of $[M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$ to $[M^{\frac{1}{2}}L^{\frac{1}{2}}]$, or $[L/T]$ to 1; but this apparent want of equality arises from these conventions themselves. On a natural system, the same quantity ought to have the same Dimensions, whether looked at from an electrostatic or a magnetic point of view; and therefore, throwing aside these conventions, we must have $[M^{\frac{1}{2}}L^{\frac{3}{2}}K^{\frac{1}{2}}/T] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/\mu^{\frac{1}{2}}]$, or $[L/T \cdot \sqrt{K\mu}]$ = a Number merely, and that number = 1. When we assume, in accordance with our conventions, that K and μ are both numerically = 1, assumptions which cannot possibly both be true, we find that L/T, the term of difference between the conventional e.-s. and m. units of electric Quantity, is experimentally determinable as numerically equal to 3×10^{10}; and as from its Dimensions it is a Velocity, it is said to be a Velocity of 3×10^{10} cm. per second. What the last equation of dimensions more truly shows, however, is that $[K\mu] = [T^2/L^2]$, and that $K\mu = 9 \times 10^{20}$. We do not know the absolute numerical values of either K or μ separately; neither do we know their Dimensions separately. See, however, p. 746.

Dimensions in the Conventional Magnetic Air-system.—Current Intensity or Strength, i. Page 670, no. 1; attraction or repulsion (= mechanical Force) $\propto ii' \times ll'/d^2$; *i.e.*, $[ML/T^2]$ = [intensity²]; $\therefore [i] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T]$.

Quantity, q, = Intensity × Time; $[q] = \{[M^{\frac{1}{2}}L^{\frac{1}{2}}/T] \times [T]\} = [M^{\frac{1}{2}}L^{\frac{1}{2}}]$.

Potential, or Difference of Potential, e, = work done ÷ quantity of electricity upon which work is done; $[e] = \{[ML^2/T^2] \div [M^{\frac12}L^{\frac12}]\} = [M^{\frac12}L^{\frac32}/T^2]$.

Electrical Force, the mechanical force acting on magnetic unit of electric quantity. Its dimensions are those of mechanical force ÷ electric quantity; $[f] = [F/q] = \{[ML/T^2] \div [M^{\frac12}L^{\frac12}]\} = [M^{\frac12}L^{\frac12}/T^2]$. Otherwise, this is Potential-Slope, $[M^{\frac12}L^{\frac32}/T^2] \div [L] = [M^{\frac12}L^{\frac12}/T^2]$.

Resistance = difference of potential ÷ current-intensity; $\{[M^{\frac12}L^{\frac32}/T^2] \div [M^{\frac12}L^{\frac12}/T]\} = [L/T]$, a Velocity;* whence the resistance of an Ohm wire (10^9 magnetic C.G.S. units of resistance) is said to be 10^9 cm. per sec.; and so on in proportion.

Capacity is quantity of electricity stored up per unit potential-difference produced by it; its dimensions are $\{[M^{\frac12}L^{\frac12}] \div [M^{\frac12}L^{\frac32}/T^2]\} = [T^2/L]$.

Conductivity: the intensity of current passing across unit area under the action of unit electrical force. Its dimensions are those of current-intensity ÷ (electrical force × area), viz., $\{[M^{\frac12}L^{\frac12}/T] \div [(M^{\frac12}L^{\frac12}/T^2 \times L^2)]\} = [T/L^2]$.

Resistivity, the reciprocal of the Conductivity; $[L^2/T]$.

Coefficients of Self-Induction and Mutual Induction of Currents: Ratios between E.M.D.P. produced and the rate of change of current-intensity producing it: the dimensions of the former are $[M^{\frac12}L^{\frac32}/T^2]$; those of the latter are [Intensity ÷ Time] $= [M^{\frac12}L^{\frac12}/T] \div [T] = [M^{\frac12}L^{\frac12}/T^2]$; the ratio therefore has the dimensions $[M^{\frac12}L^{\frac32}/T^2] \div [M^{\frac12}L^{\frac12}/T^2] = [L]$.

Magnetic Dimensions in any medium.—Current-intensity, $[M^{\frac12}L^{\frac12}/T\mu^{\frac12}]$; Electric Quantity, $[M^{\frac12}L^{\frac12}/\mu^{\frac12}]$; Electric Potential, $[\mu^{\frac12}M^{\frac12}L^{\frac32}/T]$; Electric Force, $[\mu^{\frac12}M^{\frac12}L^{\frac12}/T^2]$; Resistance, $[\mu L/T]$; Conductance, $[T/L\mu]$; Resistivity, $[\mu L^2/T]$; Conductivity, $[T/L^2\mu]$; Capacity, $[T^2/L\mu]$; Coefficients of Induction, $[\mu L]$.

From these dimensions we find that that which is measured electrostatically as a current of intensity I e.-s. units is magnetically a current of intensity $i = (I/V)$ magnetic units. Similarly, by comparison with the electrostatic measures, we find that electrostatic quantity, Q e.-s. units, is numerically expressible as $q = (Q/V)$ magnetic units; potential-difference, E in electrostatic measure, as $e = (EV)$ in magnetic; resistance, R electrostatic units, as $r = (RV^2)$ magnetic units; capacity, C, as (C/V^2)

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* Resistance a Velocity.—In the Tangent-Galvanometer, p. 712, and by definition (ii.), p. 707, $\mathfrak{h} = \mathfrak{h} \tan\theta = il/rad.^2$; $\therefore i = \mathfrak{h}\cdot\tan\theta\cdot rad.^2/l$. By definition, p. 703, $i = A\cdot\mathfrak{b}/rt$, where r is the resistance; let $(A\mathfrak{b}/t)$ lines of induction per second be cut by a vertical slider, connecting two parallel horizontal rails which lie East and West, one vertically above the other, at a mutual distance of d cm., and which, with the aid of the slider, form part of a circuit; then, in order to cut this exact number of lines per second, the slider must travel with a particular mean velocity v. The horizontal component of intensity of the terrestrial magnetic field is $\mathfrak{h}$; the number of lines of induction cut per second is thus $A\mathfrak{b}/t = \mathfrak{h}\cdot v\cdot d$; whence i, the mean intensity of the current induced in the circuit, $= \mathfrak{h}vd/r$. Then $\mathfrak{h}\cdot v\cdot d/r = i = \mathfrak{h}\cdot\tan\theta\cdot rad.^2/l$; whence $r = v\cdot d\cdot l/(rad.^2\tan\theta)$. Now impose two conditions; first, the wire coiled in the galvanometer is to be of length $l = rad.^2/d$; and second, the velocity is to be such as to produce a deflection $\theta = 45^\circ$. Then $r = v$; the Resistance is, in this case, numerically equal to the Velocity of the slider; and it is always some merely numerical multiple of the slider-velocity.

magnetic units; conductivity and resistivity, equal to D and R electrostatic units, as respectively equal to ($\mathrm{D}V^2$) and to (R/V^2) magnetic units.

Practical Units. — Some of the units of the C.G.S. Magnetic System are inconveniently large or small. It is therefore the practice not to use the C.G.S. magnetic units of electrical quantity, intensity, resistance, etc., but to build up a magnetic system based on new units of length, l, and of mass, m. These are respectively 1000,000000 cm. (the earth's quadrant) and the 100,000,000000th part of a gramme. The unit of current-intensity is then $[m^{\frac{1}{2}}l^{\frac{1}{2}}/\mathrm{T}] = [(\mathrm{M}/100,000,000000)^{\frac{1}{2}} \cdot (\mathrm{L} \times 1000,000000)^{\frac{1}{2}}/\mathrm{T}] = \frac{1}{10}[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}/\mathrm{T}]$. The new unit of intensity, the **Ampère,** is thus equal to $\frac{1}{10}$ C.G.S. Magnetic Unit; and the new unit of quantity, the **Coulomb,** is similarly equal to $\frac{1}{10}$ C.G.S. Magnetic Unit. In the same way we find the unit of resistance, the **Ohm,** $= 10^9$ C.G.S. Magnetic Units. The Megohm = 1 million Ohms; the Microhm = one-millionth Ohm. The unit of difference of potential, the **Volt,** $= 10^8$ C.G.S. Magnetic Units; the Megavolt = 1 million Volts; the Microvolt = one-millionth Volt. The unit of capacity $[\mathrm{T}^2/l] = [\mathrm{T}^2/1000,000000\,\mathrm{L}] = \{[\mathrm{T}^2/\mathrm{L}] \div 1000,000000\} = 10^{-9}$ C.G.S. Magnetic Unit = 1 **Farad.** The Farad $= 10^{-9} \times$ one C.G.S. Magnetic Unit of Capacity; but the latter unit is equal to the electrostatic unit $\times V^2$, or to 9×10^{20} Electrostatic Units; the Farad is therefore equal to $10^{-9} \times (9 \times 10^{20}) = (9 \times 10^{11})$ Electrostatic Units of Capacity. The electrostatic capacity of a sphere is equal to its radius; a Farad is therefore the electrostatic capacity of a sphere of (9×10^{11}) cm. radius; and for convenience the standard in use is the Microfarad, the millionth of a Farad. The coefficient of self-induction, the **Henry** or Secohm or Quadrant, is $[1000,000000\mathrm{L}] = 10^9$ Magnetic C.G.S. units.

In this system the quantity $i \cdot n$, which so often occurs in our equations, is known as the Ampère-turns.

The heat developed in a wire, per second, by a steady current of A Ampères, under a potential-difference of V Volts, is $(V \times 10^8) \times (A \times 10^{-1})$ ergs $= 10^7 \cdot VA$ ergs $= 0{\cdot}24\,VA$ *ca.*

In electric lighting a certain unit is commonly made use of as a conventional basis for estimating the sum due by the consumer. This unit represents 1000 Ampère-Volt-Hours, and is equivalent to the Energy conveyed by a current of one Ampère intensity, passing down a fall of potential of one Volt, and sustained for 1000 hours. This amount of Energy =

1 Ampère-Volt or Watt × 3,600,000 sec., and is therefore equal to {10,000000 Ergs per sec. × 3,600000 sec.} = 36,000000,000000 Ergs or 2,654,340 foot-pounds, or about 865,000 *ca*, an amount of heat which would convert 2·95 lbs. of ice-cold water into steam at 100° C.; and the commercial unit of current is a current of any intensity continued until this quantity of energy has been transmitted through the consumer's apparatus.

The accompanying table, pp. 714, 715, gives a conspectus of the relations between the quantities dealt with in this chapter, and of the numerical data which are requisite in order to transform a quantity, numerically stated in terms of one of the three systems of conventional units described, into the same quantity numerically stated in terms of either of the other two systems.

Magnetic Measurement of Current-Intensity.—The magnetic units of measurement have all been derived in theory from the magnetic measurement of intensity of a steady current: the magnetic measurement of a steady current is therefore a fundamental measurement. It is effected by the use of **galvanometers** and **electrodynamometers.**

A magnet surrounded by a coil of wire will, when a current is passed through the wire, tend to place itself at right angles to the plane of the current, that is, to place its axis along the Lines of Induction. If the coil be placed vertically in the plane of the magnetic meridian, and if the needle be suspended horizontally at its centre, so that it can swing round a vertical axis of rotation, then, on the supposition that both poles of the magnet are at the centre of the coil—an ideal approximated to when the coil has an extremely large diameter, or when the needle is extremely short—the deflection of the needle from the magnetic meridian is such that its tangent is proportional to the current passing. Such an arrangement is called a **Tangent-Galvanometer.**

In a tangent-galvanometer in which the coil consists of only one turn, of radius r, the force acting upon a pole $\mathfrak{m}$ very near the centre is $F = \mathbf{h}\mathfrak{m} = \mathfrak{m}il/r^2 = \mathfrak{m}i \cdot 2\pi r/r^2 = \mathfrak{m}i \cdot 2\pi/r$, when i is measured in C.G.S. Magnetic Units, and where $\mathbf{h}$ is the force acting on a unit-pole. The mechanical force exerted by the horizontal component of the earth's magnetism is $\mathfrak{h}$ on a unit-pole, $\mathfrak{h}\mathfrak{m}$ on a pole $\mathfrak{m}$. The deflection of the needle is θ. The magnetic couple is $\mathfrak{h} \cdot \mathfrak{m}l \cdot \sin\theta$ if the moment of the magnet be $l \cdot \mathfrak{m}$. From the "Equilibrium of Couples," page 160, prop. 2, we learn that $F : \mathfrak{h}\mathfrak{m} :: \tan\theta : 1$. Therefore $i \cdot 2\pi/r = \mathfrak{h}\tan\theta$, or $i = \mathfrak{h}\tan\theta \cdot r/2\pi$. $\mathfrak{h}$ can be found as on page 681, or turned up in observational tables of local magnetic intensities; θ can be observed; r can be measured; whence i can be found numerically in magnetic C.G.S. measure: and it will be observed that it is independent of variations in the strength $\mathfrak{m}$ of either pole of the magnet. It is also the same whatever the value of μ.

If the coil consist of n turns, whose mean radius is r', the force $\mathbf{h}$ acting on unit-pole (the 'intensity of the electromagnetic field') at the centre is $i \cdot n \cdot 2\pi/r'$. If it be a coil of rectangular section with inner and outer radii $r_{,}$ and $r_{,,}$, and of length l cm., with n turns in it on the whole, the intensity of field is

$$\mathbf{h} = \left\{ i \cdot n \cdot \frac{2\pi}{r_{\prime} - r_{\prime\prime}} \cdot \log \frac{r_{\prime\prime} + \sqrt{r_{\prime\prime}^2 + (l/2)^2}}{r_{\prime} + \sqrt{r_{\prime}^2 + (l/2)^2}} \right\}.$$

The Tangent-Galvanometer is most sensitive when θ is 45°.

If the coil be placed parallel to the deflected needle, the *sine* of the deflection becomes proportional to the current-intensity: we then have the **Sine Galvanometer,** in which a long needle is used. (Compare Figs. 83 *a* and 83 *b*.)

In Galvanometers, in which a passing current produces an electromagnetic field in which a magnetic needle is deflected, the amount of this deflection indicates the strength of the current. It is well in all cases to produce as uniform a field of force as possible. This is effected by arranging a number of coils so as to surround the field, not wholly but in outline. In von Helmholtz's Galvanometer, for example, there are two parallel coils, between which the needle is placed, at a mean distance from each equal to half the mean radius of either. For sensitiveness, each winding of the wire is made to come as near the magnet as is practicable.

Galvanometer-Constant.—When a current passes through the wire of a galvanometer, the needle is in a magnetic field of a certain intensity or strength, measured, as usual, by $\mathbf{h}$, the force locally acting upon a *unit* magnetic-pole. If $i = 1$, $\mathbf{h}$ has a certain numerical value which involves only measurements derived from the construction of the galvanometer itself: this is known as the Galvanometer-Constant, and gives the numerical value of the strength of the field when the current traversing the instrument is of unit intensity. It is distinctively represented by the symbol Γ, and the force $\mathbf{h}$ acting on a unit-pole, when the intensity of the current is i, is equal to Γi; acting on a pole $\mathfrak{m}$, the mechanical force is $F = \mathbf{h}\mathfrak{m} = \Gamma i\mathfrak{m}$. For example, in a tangent-galvanometer of one turn the force acting is, as above, $F = \mathfrak{m}i \cdot 2\pi/r$; whence $\Gamma = 2\pi/r$. The dimensions of Γ are $[1/L]$.

Ballistic Galvanometer.—If a tangent-galvanometer be constructed with a short heavy needle of length l, and if a very brief current, enduring only for the exceedingly small interval δt, be passed through it, the needle will receive a twitch and, after the current has passed, will swing through an angle θ. The last equations under Ballistic Pendulum (p. 215) were $\frac{1}{2}N\omega^2 = (M + m)gh \cdot 2\sin^2 \overline{\theta/2}$ and $\omega = 2\sqrt{(M + m)gh/N} \cdot \sin \overline{\theta/2}$. The problem here corresponds closely, but instead of $(M + m)h$ we have the magnetic moment of the needle $(= l \cdot \mathfrak{m})$, which we write as $\mathfrak{M}$; instead of g the local intensity of gravity (*i.e.* the force acting upon a unit-mass), we have $\mathfrak{h}$ the effective component of the local intensity of the field within which the needle moves after the current has passed,—that is, of the terrestrial magnetic field. The equations of that paragraph therefore become for our present purposes

$$\tfrac{1}{2}N\omega^2 = \mathfrak{M}\mathfrak{h} \cdot 2\sin^2 \overline{\theta/2}$$

which represents the energy imparted to the needle, and

$$\omega = 2\sqrt{\mathfrak{M}\mathfrak{h}/N} \cdot \sin \overline{\theta/2}$$

which represents the angular velocity imparted to it.

	DIMENSIONS OF UNITS.			
	ELECTROSTATIC.		MAGNETIC.	
	Derivation.	Dimensions.	Derivation.	Dimensions.
1 Electric Quantity, Q, q	$\sqrt{\text{Force}\cdot d^2\text{K}}$	$K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T$	$i \times$ Time	$M^{\frac{1}{2}}L^{\frac{1}{2}}/\mu^{\frac{1}{2}}$
2 Electric Surface-Density, σ . . .	Q/Area	$K^{\frac{1}{2}}M^{\frac{1}{2}}/L^{\frac{1}{2}}T$		$M^{\frac{1}{2}}/L^{\frac{3}{2}}\mu^{\frac{1}{2}}$
3 Electric Force on Unit Quantity; Electromotive Intensity; Intensity of Electrostatic Field; Impressed Electromotive Force; Potential-Slope, ϕ, f	Force/Q	$M^{\frac{1}{2}}/L^{\frac{1}{2}}TK^{\frac{1}{2}}$		$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}/T^2$
4 Total Electrostatic Induction, I .	4πQ	$K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T$		$M^{\frac{1}{2}}L^{\frac{1}{2}}/\mu^{\frac{1}{2}}$
5 Induction per sq. cm., i	$4\pi\sigma$	$K^{\frac{1}{2}}M^{\frac{1}{2}}/L^{\frac{1}{2}}T$		$M^{\frac{1}{2}}/L^{\frac{3}{2}}\mu^{\frac{1}{2}}$
6 Sp. Ind. Capacity, or Permittivity, K	$4\pi\sigma/\phi$	K		$T^2/L^2\mu$
7 Electrostatic Potential V; Difference of Potential, E, e . . .	Work/Q	$M^{\frac{1}{2}}L^{\frac{1}{2}}/TK^{\frac{1}{2}}$		$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T^2$
8 Electrostatic Capacity or Permittance, C	Q/V	KL		$T^2/L\mu$
9 Current-Intensity or Strength, I, i	Q/Time	$K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T^2$	$i=\varphi/\mu$	$M^{\frac{1}{2}}L^{\frac{1}{2}}/T\mu^{\frac{1}{2}}$
10 Current-Density, Δ	I/Area	$K^{\frac{1}{2}}M^{\frac{1}{2}}/L^{\frac{1}{2}}T^2$		$M^{\frac{1}{2}}/L^{\frac{3}{2}}T\mu^{\frac{1}{2}}$
11 Conductance, D.	I/E	KL/T		$T/L\mu$
12 Conductivity, D	Δ/ϕ	K/T		$T/L^2\mu$
13 Resistance, R, r	1/D	T/LK		$\mu L/T$
14 Resistivity, R	1/D	T/K		$\mu L^2/T$
15 Magnetic Quantity, m	F=m$i$$l$/$d^2$	$M^{\frac{1}{2}}L^{\frac{1}{2}}/K^{\frac{1}{2}}$	$\sqrt{\text{Force}\cdot d^2\cdot\mu}$	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T$
16 Magnetic Force; Mechanical Force on Unit Quantity; Intensity of Magnetic Field; Potential-Slope; Number of Lines of Force per sq. cm.; h		$K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}/T^2$	Force/m	$M^{\frac{1}{2}}/L^{\frac{1}{2}}T\mu^{\frac{1}{2}}$
17 Magnetic Moment, $\mathfrak{M}$		$M^{\frac{1}{2}}L^{\frac{3}{2}}/K^{\frac{1}{2}}$	m × Length	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{5}{2}}/T$
18 Intensity of Magnetisation, $\mathfrak{I}$ . .		$M^{\frac{1}{2}}/L^{\frac{3}{2}}K^{\frac{1}{2}}$	/vol.	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}/L^{\frac{1}{2}}T$
19 Magnetic [scalar] Potential, Ω .		$K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T^2$	Work/m	$M^{\frac{1}{2}}L^{\frac{1}{2}}/T\mu^{\frac{1}{2}}$
20 Magnetic Surface-Density, s . .		$M^{\frac{1}{2}}/L^{\frac{3}{2}}K^{\frac{1}{2}}$	m/Area	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}/L^{\frac{1}{2}}T$
21 Strength of Magnetic Shell, φ . .		$M^{\frac{1}{2}}/L^{\frac{1}{2}}K^{\frac{1}{2}}$	s · thickness	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}/T$
22 Magnetic Induction or Flux per sq. cm., b		$M^{\frac{1}{2}}/L^{\frac{3}{2}}K^{\frac{1}{2}}$	4πm/Area	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}/L^{\frac{1}{2}}T$
23 Magnetic Induction or Flux within a Contour, B		$M^{\frac{1}{2}}L^{\frac{1}{2}}/K^{\frac{1}{2}}$	4πm	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T$
24 Magnetic Susceptibility, κ . . .		T^2/L^2K	s/h	μ
25 Permeability or Inductivity, μ . .		T^2/L^2K		μ
26 Coefficient of Self-Induction, or Inductance, L		T^2/LK	$e \div di/dt$	μL
27 Mutual Induction, Mutual Inductance, M		T^2/LK	$e \div di/dt$	μL
28 Magneto-motive Force across area		$K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}/T^2$	$4\pi i\cdot n$	$M^{\frac{1}{2}}L^{\frac{1}{2}}/T\mu^{\frac{1}{2}}$
29 Reluctance of Magnetic Circuit .		KL/T^2	$l/A\mu$	$1/\mu L$
30 Reluctivity (Reluctance per unit volume)		KL^2/T^2	$(l/A\mu)\div(l/A)$	$1/\mu$

Example. — The electrostatic capacity of a certain small Leyden jar is found, by the formula is $\{(300/\pi)+(9\times10^{11})\}$ practical magnetic units of electrostatic capacity, or 1/3000,000000π

Conventional.		REDUCTION-FACTORS, to transform						
		E.S.C.G.S. to		M.C.G.S. to		Practical Magnetic to		
E.S.	Magn.	M.C.G.S.	P.M.	E.S.C.G.S.	P.M.	E.S.C.G.S.	M.C.G.S.	
$M^{1/2}L^{3/2}/T$	$M^{1/2}L^{1/2}$	$\div(3\times10^{10})$	$\div(3\times10^{9})$	$\times(3\times10^{10})$	$\times10$	Coulombs, $\times(3\times10^{9})$	$\div10$	1
$M^{1/2}/L^{1/2}T$	$M^{1/2}/L^{3/2}$	$\div(3\times10^{10})$	$\div(3\times10^{-9})$	$\times(3\times10^{10})$	$\times10^{19}$	$\times(3\times10^{-9})$	$\div10^{19}$	2
$M^{1/2}/L^{1/2}T$	$M^{1/2}L^{1/2}/T^2$	$\times(3\times10^{10})$	$\times(3\times10^{11})$	$\div(3\times10^{10})$	$\times10$	$\div(3\times10^{11})$	$\div10$	3
$M^{1/2}L^{3/2}/T$	$M^{1/2}L^{1/2}$	$\div(3\times10^{10})$	$\div(3\times10^{9})$	$\times(3\times10^{10})$	$\times10$	$\times(3\times10^{9})$	$\div10$	4
$M^{1/2}/L^{1/2}T$	$M^{1/2}/L^{3/2}$	$\div(3\times10^{10})$	$\div(3\times10^{-9})$	$\times(3\times10^{10})$	$\times10^{19}$	$\times(3\times10^{-9})$	$\div10^{19}$	5
(No.)	T^2/L^2	$\div(9\times10^{20})$	$\div(9\times10^{2})$	$\times(9\times10^{20})$	$\times10^{18}$	$\times(9\times10^{2})$	$\div10^{18}$	6
$M^{1/2}L^{1/2}/T$	$M^{1/2}L^{3/2}/T^2$	$\times(3\times10^{10})$	$\times(3\times10^{2})$	$\div(3\times10^{10})$	$\div10^{8}$	Volts, $\div(3\times10^{2})$	$\times10^{8}$	7
L	T^2/L	$\div(9\times10^{20})$	$\div(9\times10^{11})$	$\times(9\times10^{20})$	$\times10^{9}$	Farads, $\times(9\times10^{11})$	$\div10^{9}$	8
$M^{1/2}L^{3/2}/T^2$	$M^{1/2}L^{1/2}/T$	$\div(3\times10^{10})$	$\div(3\times10^{9})$	$\times(3\times10^{10})$	$\times10$	Ampères, $\times(3\times10^{9})$	$\div10$	9
$M^{1/2}/L^{1/2}T^2$	$M^{1/2}/L^{3/2}T$	$\div(3\times10^{10})$	$\div(3\times10^{-9})$	$\times(3\times10^{10})$	$\times10^{19}$	$\times(3\times10^{-9})$	$\div10^{19}$	10
L/T	T/L	$\div(9\times10^{20})$	$\div(9\times10^{11})$	$\times(9\times10^{20})$	$\times10^{9}$	Mhos, $\times(9\times10^{11})$	$\div10^{9}$	11
1/T	T/L^2	$\div(9\times10^{20})$	$\div(9\times10^{2})$	$\times(9\times10^{20})$	$\times10^{18}$	$\times(9\times10^{2})$	$\div10^{18}$	12
T/L	L/T	$\times(9\times10^{20})$	$\times(9\times10^{11})$	$\div(9\times10^{20})$	$\div10^{9}$	Ohms, $\div(9\times10^{11})$	$\times10^{9}$	13
T	L^2/T	$\times(9\times10^{20})$	$\times(9\times10^{2})$	$\div(9\times10^{20})$	$\div10^{18}$	$\div(9\times10^{2})$	$\times10^{18}$	14
$M^{1/2}L^{1/2}$	$M^{1/2}L^{3/2}/T$	$\times(3\times10^{10})$	$\times(3\times10^{2})$	$\div(3\times10^{10})$	$\div10^{8}$	$\div(3\times10^{2})$	$\times10^{8}$	15
$M^{1/2}L^{1/2}/T^2$	$M^{1/2}/L^{1/2}T$	$\div(3\times10^{10})$	$\div3$	$\times(3\times10^{10})$	$\times10^{10}$	Gausses, $\times3$	$\div10^{10}$	16
$M^{1/2}L^{1/2}$	$M^{1/2}L^{3/2}/T$	$\times(3\times10^{10})$	$\times(3\times10^{-7})$	$\div(3\times10^{10})$	$\div10^{17}$	$\div(3\times10^{-7})$	$\times10^{17}$	17
$M^{1/2}/L^{3/2}$	$M^{1/2}/L^{1/2}T$	$\times(3\times10^{10})$	$\times(3\times10^{20})$	$\div(3\times10^{10})$	$\times10^{10}$	$\div(3\times10^{20})$	$\div10^{10}$	18
$M^{1/2}L^{3/2}/T^2$	$M^{1/2}L^{1/2}/T$	$\div(3\times10^{10})$	$\div(3\times10^{9})$	$\times(3\times10^{10})$	$\times10$	$\times(3\times10^{9})$	$\div10$	19
$M^{1/2}/L^{3/2}$	$M^{1/2}/L^{1/2}T$	$\times(3\times10^{10})$	$\times(3\times10^{20})$	$\div(3\times10^{10})$	$\times10^{10}$	$\div(3\times10^{20})$	$\div10^{10}$	20
$M^{1/2}/L^{1/2}$	$M^{1/2}L^{1/2}/T$	$\times(3\times10^{10})$	$\times(3\times10^{11})$	$\div(3\times10^{10})$	$\times10$	$\div(3\times10^{11})$	$\div10$	21
$M^{1/2}/L^{3/2}$	$M^{1/2}/L^{1/2}T$	$\times(3\times10^{10})$	$\times(3\times10^{20})$	$\div(3\times10^{10})$	$\times10^{10}$	$\div(3\times10^{20})$	$\div10^{10}$	22
$M^{1/2}L^{1/2}$	$M^{1/2}L^{3/2}/T$	$\times(3\times10^{10})$	$\times(3\times10^{2})$	$\div(3\times10^{10})$	$\div10^{8}$	Webers, $\div(3\times10^{2})$	$\times10^{8}$	23
T^2/L^2	(No.)	$\times(9\times10^{20})$	$\times(9\times10^{20})$	$\div(9\times10^{20})$	Same	$\div(9\times10^{20})$	Same	24
T^2/L^2	(No.)	$\times(9\times10^{20})$	$\times(9\times10^{20})$	$\div(9\times10^{20})$	Same	$\div(9\times10^{20})$	Same	25
T^2/L	L	$\times(9\times10^{20})$	$\times(9\times10^{11})$	$\div(9\times10^{20})$	$\div10^{9}$	Henries, $\div(9\times10^{11})$	$\times10^{9}$	26
T^2/L	L	$\times(9\times10^{20})$	$\times(9\times10^{11})$	$\div(9\times10^{20})$	$\div10^{9}$	$\div(9\times10^{11})$	$\times10^{9}$	27
$M^{1/2}L^{3/2}/T^2$	$M^{1/2}L^{1/2}/T$	$\div(3\times10^{10})$	$\div(3\times10^{9})$	$\times(3\times10^{10})$	$\times10$	Gilberts, $\times(3\times10^{9})$	$\div10$	28
L/T^2	1/L	$\div(9\times10^{20})$	$\div(9\times10^{11})$	$\times(9\times10^{20})$	$\times10^{9}$	Oersteds, $\times(9\times10^{11})$	$\div10^{9}$	29
L^2/T^2	(No.)	$\div(9\times10^{20})$	$\div(9\times10^{20})$	$\times(9\times10^{20})$	Same	$\times(9\times10^{20})$	Same	30

C = (K/d) · (surface/4π), to be 300/π C.G.S. electrostatic units; this, by the above table, Farad, or 1/9425 microfarad. See note, p. 635.

But again, we can in other terms express the work done on the needle during the brief period of the current; as the product, namely, of half the twisting moment into the angle of twist imparted during that period (p. 263). The twisting moment is $\Gamma i\mathfrak{m} \times l$; the angle of twist is an exceedingly small angle $\xi = (\frac{1}{2}\omega \times \delta t)$, where ω is the angular velocity imparted to the needle, and δt the time during which the current lasts. The work done is therefore $\frac{1}{2}\Gamma i\mathfrak{m} \cdot l \cdot \frac{1}{2}\omega\delta t = \frac{1}{2}\Gamma \cdot \mathfrak{m}l \cdot i\delta t \cdot \frac{1}{2}\omega = \frac{1}{2}\Gamma\mathfrak{M}Q \cdot \frac{1}{2}\omega$, where Q is the whole quantity of electricity passing in time δt. This is equal to $\frac{1}{2}N\omega^2$.

Now equating these two values of $\frac{1}{2}N\omega^2$ the energy, we have $\frac{1}{4}\Gamma\mathfrak{M}Q \cdot \omega = \mathfrak{M}\mathfrak{h} \cdot 2\sin^2(\theta/2)$; or $Q = 4\mathfrak{h}/\Gamma \cdot 2\sin^2(\theta/2)./\omega = 4\mathfrak{h}/\Gamma \cdot 2\sin^2(\theta/2) \div [2\sqrt{\mathfrak{M}\mathfrak{h}/N} \cdot \sin(\theta/2)] = 4\mathfrak{h}/\Gamma \cdot \sqrt{N/\mathfrak{M}\mathfrak{h}}.\sin\theta/2$. But T, the time of a complete to-and-fro oscillation of a needle swinging freely in the terrestrial field, is $T = 2\pi\sqrt{N/\mathfrak{M}\mathfrak{h}}$; whence $Q = 2\mathfrak{h}/\Gamma \cdot T/\pi \cdot \sin(\theta/2)$, in which all the terms are measurable.

When the current ceases, the magnet tends to oscillate for some time, like a pendulum; but if it oscillate in a strong magnetic field of force — as, for example, in the neighbourhood of a strong magnet — its oscillations will be very rapid and of small amplitude. If masses of metal be so arranged that any oscillations of the magnet tend to produce retarding induced-currents in these masses, then, especially if the needle be light, the oscillations of the magnet rapidly cease, as if it were immersed in a viscous medium, and the magnet is, without further oscillation, restored to its position of repose. A galvanometer arranged on this principle is a Dead-Beat galvanometer. The same dead-beat effect is mechanically produced by making the magnet move in a small closed chamber of air which it nearly fills; it thus moves against air-resistance.

In Differential Galvanometers, two equal and separate wires are similarly coiled round the same needle; through these wires currents may be sent in opposite directions; if the two currents be equal, the needle remains at rest; if either predominate, the needle moves.

In Electrodynamometers the current is passed through a coil which is suspended within a strong and uniform magnetic field, such as that produced by powerful electromagnets actuated by a second current, or again by fixed coils through which a second current is passing. The deflection of the suspended coil depends upon the strength of the current passing through it, and also upon the strength of the magnetic field surrounding it. If the same current traverse both the fixed and the suspended coils, the rotating couple is proportional to i^2, and therefore to the energy of the current; and it is independent of its direction.

The two coils have the respective mean radii r and $r_{,}$, r the greater, $r_{,}$ the less; the respective numbers of turns are n and $n_{,}$; when the plane of the suspended coil makes with the plane of the larger coil an angle θ, the couple, tending to bring the two coils into the same plane with their currents opposed, is $\mu i^2 \cdot 2\pi^2 \cdot r_{,}^2 \cdot nn_{,} \cdot \sin\theta./r$. If l and $l_{,}$ be the lengths of wire in the two coils respectively, this expression may be written as $\frac{1}{2}\mu i^2 \cdot ll_{,} \cdot r_{,} \cdot \sin\theta./r^2$.

If the current to be measured be a rapidly alternating one, the result is the same as if it were constant; it is reversed in both coils at the same time, and the algebraic sign of i^2 is always positive.

In the Siemens electrodynamometer, used for measuring electric-lighting currents, the same current is made to pass in succession through two thick-wire loops. These tend to place themselves in the same plane; but by the torsion of a spring, they are forced into a standard position at right angles to one another. This torsion is measured by the angle of rotation of a pointer connected with the spring; and it is proportional to the square of the intensity of the current.

If a movable coil be free to slip up and down the axis of a fixed coil in which a current is passing, the inner coil may be sucked in or repelled with a force which may be balanced and measured by known weights or elastic tensions or torsions; or if the current in the suspended coil be variable, the tension tending to draw it in to the fixed coil may be made to act against a spring, and graphically to record its own variations upon a uniformly-moving piece of paper.

If, instead of a movable coil free to slip up and down the axis of a fixed coil, we have a bar of soft iron, it will also be sucked in or repelled; but in this case the magnetisation of the bar is not strictly proportional to the intensity of the current: whence the law, that the force of suction or of repulsion is proportional to the square of the current-intensity, fails us. If, however, the bar be reduced to a very thin soft-iron tube, it rapidly becomes saturated and soon becomes practically constant in strength. When this limit has been reached, the force is directly proportional to the intensity of the current. This is the principle of Ayrton and Perry's Ammeter (= "Ampère-meter"). A slender piece of soft iron tends to move towards the centre of a current-bearing coil; it may thus be made to rotate against gravity round a fixed pivot: a pointer attached to it will indicate the amount of rotation: and this is the principle of Schuckert's Ammeter. A short thin bar of soft iron tends to be pulled so as to lie in the strongest part of the field between two electromagnet-poles (Evershed's Ammeter). The tendency towards suction of a suitable electromagnet into a coil can also be measured by balancing it against weights, as in the Electric Power Storage Co.'s Steelyard Ammeter. Ampère-meters, as their name indicates, are graduated in Ampères, not in magnetic C.G.S. units.

The principle of the differential galvanometer may be here applied, as in Prof. Langley's Thermic Balance or Bolometer. The suspended coil is composed of two separate wires wound together, but insulated from one another: a single current is divided into two equal moieties which run in opposite directions through the two wires of the coil; there is no effect. The least variation in one of these moieties, as when the conductivity of its path is affected by the local application of heat, causes imperfect compensation, and practically a small uncompensated current passes: however feeble this may be, it can be rendered manifest and measurable by increasing the strength of the magnetic field within which the double coil is suspended.

The part of the divided circuit to which heat may be locally applied may be an exceedingly thin strip of platinum. This may be moved up and down, say, in the dark region of the spectrum. In some places it is heated, in others — dark lines — it is not. By thus groping in the dark it discovers

the dark lines and the specially "bright" lines of the heat-spectrum. An instrument of this kind is sensitive to differences of temperature of $\frac{1}{10000}$ F°.

Magnetic Measurement of Resistance.—If a circle of wire, radius r, stand at right angles to the magnetic meridian it will embrace $\mu\mathfrak{h}$ lines of terrestrial magnetic induction ($\mu = 1$) per sq. cm. or $\mu\mathfrak{h}\cdot\pi r^2$ lines over its whole area; if it be turned round a vertical axis through 180°, it will come to embrace $\mu\mathfrak{h}\cdot\pi r^2$ lines oppositely directed with reference to it: the number of lines of induction passing through it has therefore been increased or decreased by $2\mu\mathfrak{h}\cdot\pi r^2$. The circle of wire thus rotated (Earth-inductor) becomes the seat of a current whose mean E.M.D.P. is numerically equal to $2\mu\mathfrak{h}\cdot\pi r^2/t$ in magnetic units, and whose mean intensity is $i = 2\mu\mathfrak{h}\cdot\pi r^2/\mathrm{R}t$, where R is the resistance (also measured in magnetic units) and t the time occupied in the rotation through 180°. If a small needle be suspended at the centre of this rotating circle, that needle will be deflected; the rotating circle acts somewhat as if it were its own Tangent-galvanometer; but instead of a deflection θ such that $\tan\theta = i\cdot 2\pi n/r\mathfrak{h}$, where n is the number of coils and r their mean radius, we have (approximately) $\tan 2\theta = i\cdot\pi^2\cdot n^2/r\mathfrak{h}$. But $i = 2\mu\mathfrak{h}\cdot\pi r^2/\mathrm{R}t$; whence $\tan 2\theta = 2\mu\cdot\pi^3 n^2 r/\mathrm{R}t$. If t be the 2Nth part of a second, the coil will make N complete turns per second, and $\tan 2\theta = 4\mu\mathrm{N}\pi^3 n^2 r/\mathrm{R}$. Therefore $\mathrm{R} = 4\mu\mathrm{N}\pi^3 n^2 r/\tan 2\theta$. Of these quantities, n the number of coils and r their mean radius are obtained by measurement; $\tan 2\theta$ is the ratio between the scale-reading (straight scale) and the distance of the scale from the mirror fixed to the centre of the deflecting needle; N can be read off on a speed-indicator; and $\mu = 1$ in air. When the resistance is so adjusted that to a speed N there corresponds a deflection θ such that the product above (with due corrections) is numerically equal to 1000,000000, the resistance employed is equal to one Ohm. This is the principle of the method by which the British Association Committee on Electrical Standards constructed the original standard Ohm.

Measurement of the Capacity of a Conductor or Condenser.—The capacity is $\mathrm{C} = \mathrm{Q}/\mathrm{V}$, and therefore we can find the value of C if we find, in terms of units of the same system, the quantity Q with which a body is charged, and the potential V to which this charge raises it. This potential V may be the difference between the potential of the body charged and that of the earth, or it may be the difference between the potentials of the opposed plates of a condenser.

This is, however, not a convenient method; and the practical method is first to construct standard condensers of known capacity, and then to compare the capacity of the body examined with that of these standards.

Standard Condensers.—Suppose a conductor of capacity C and bearing a charge Q to be discharged through a known resistance which includes a galvanometer; the resistance being so considerable that the discharge is far from instantaneous. The initial potential of the conductor is $\mathrm{V} = \mathrm{Q}/\mathrm{C}$. A current will pass through the galvanometer, but will continuously diminish in intensity. At the end of time t let the potential have sunk to $\mathrm{V}_{,}$ which is the nth part of V, and the charge to $\mathrm{Q}_{,}$; and at the end of a very small further interval δt, let these have farther sunk by the amounts $\dot{\mathrm{V}}_{,}$ and $\dot{\mathrm{Q}}_{,}$ respectively. Then the quantity which has escaped in time δt is $\dot{\mathrm{Q}}_{,}$, which is necessarily equal to $\mathrm{C}\dot{\mathrm{V}}_{,}$; it is also equal to the instantaneous intensity I multiplied by the time δt, and this product is equal, by Ohm's law, to $(\mathrm{V}_{,}/\mathrm{R})\times\delta t$. Hence $(\mathrm{V}_{,}/\mathrm{R})\cdot\delta t = \mathrm{C}\dot{\mathrm{V}}_{,}$, or $\delta t = \mathrm{CR}\cdot\dot{\mathrm{V}}_{,}/\mathrm{V}_{,}$. From this we find, by means of the Integral Calculus, that the time which

must have elapsed between the initial instant at which the potential had been V and that instant at which the potential had sunk to $V_{,}$ is equal to CR log $(V/V_{,})$. But by our supposition this time is t, and the ratio $V/V_{,}$ is equal to n. Whence $t = CR \log n$, or $C = t/\overline{R \cdot \log n}$. If we observe the successive values of the current-intensity at equal intervals of time we can find the value of n, and then, knowing the value of R in electrostatic, in magnetic, or in practical units, we can find the corresponding value of C, the capacity, measured in units of the corresponding system.

Comparison of Capacities.—1. De Sauty's Bridge-method, applicable to small capacities. Two condensers of capacities $C_{,}$ and $C_{,,}$, if charged to the same potential V, must be charged with the respective quantities $C_{,}V$ and $C_{,,}V$. If one of these be discharged through a resistance $R_{,}$ for time δt, the current which is set up is of mean intensity $I_{,}$, and the quantity passing in time δt is $I_{,} \cdot \delta t$. But this is equal to the fall in the value of Q during the time δt; that is, to $\dot{Q}$. But $\dot{Q} = C_{,}\dot{V}_{,}$, where $\dot{V}_{,}$ is the fall of potential in the condenser $C_{,}$. Whence $I_{,} = C_{,}\dot{V}_{,}/\delta t$; and $R_{,}$, which varies inversely as $I_{,}$, is proportional to $\delta t/C_{,}\dot{V}$. Similarly, if $C_{,,}$ be discharged by a current of intensity $I_{,,}$ through a resistance $R_{,,}$, that resistance must bear the same proportion to $\delta t/C_{,,}\dot{V}_{,,}$; and if $\dot{V}_{,,}$, the fall of potential in $C_{,,}$, be the same as in condenser $C_{,}$, that is, if $\dot{V}_{,,} = \dot{V}_{,}$; then the equation

$$R_{,}/R_{,,} = \frac{\delta t/C_{,}\dot{V}_{,}}{\delta t/C_{,,}\dot{V}_{,}} = C_{,,}/C_{,}$$

shows that the Resistances through which the two condensers charged to equal potentials must be discharged, in order that the potentials of the two condensers may fall concurrently and remain persistently equal to one another, must be inversely proportional to the respective Capacities of the bodies discharged through them.

This being postulated, the arrangement of the apparatus is indicated by Fig. 254. A, a battery; K, a key with which the wire B may be at will connected either with the battery A or directly with the earth, or else, as in the figure, isolated from these. When the battery is connected with B, the condensers $C_{,}$ and $C_{,,}$ are charged through the resistances $R_{,}$ and $R_{,,}$. Connect, then, A with B for a certain time; disconnect. The two condensers, if not already at equal potentials, soon become so, for an equalising current traverses EGF; when equalisation is complete, the needle of the galvanometer G returns to rest. Now put B to earth. The charges of $C_{,}$ and $C_{,,}$ escape through $R_{,}$ and $R_{,,}$ respectively. If the resistance $R_{,}$ be disproportionately great, the outflow through it is disproportionately small, and the potential at F sinks faster than that at E; a current therefore passes from E to F, and the galvanometer-needle in G is deflected. If $R_{,} : R_{,,} :: C_{,,} : C_{,}$, the galvanometer-needle remains at rest, for the potentials at E and F as they sink, sink together and are concur-

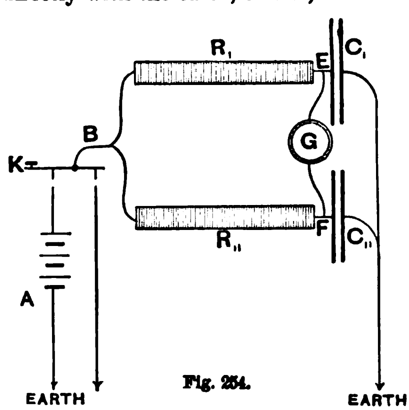
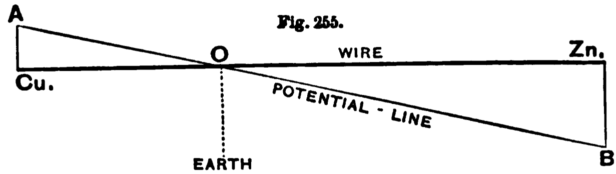


Fig. 254.

rently equal to one another. If therefore we adjust the resistances $R_{\prime}$ and $R_{\prime\prime}$ until we find that on effecting the three operations — (1) connecting B with the battery A; (2) isolating B from A until the galvanometer-needle comes to rest; (3) putting B to earth — the last of these is followed by no deflection of the galvanometer-needle, then, since we know the relative values of the resistances $R_{\prime}$ and $R_{\prime\prime}$, we know, inversely, the relative values of the capacities $C_{\prime\prime}$ and $C_{\prime}$; and as one of these capacities is a standard, we are thus enabled to state absolutely the actual value of the capacity to be measured.

2. Compensation-method. If a wire Cu Zn (Fig. 255), connecting two poles of a battery, be connected at any one point with the earth, the potential

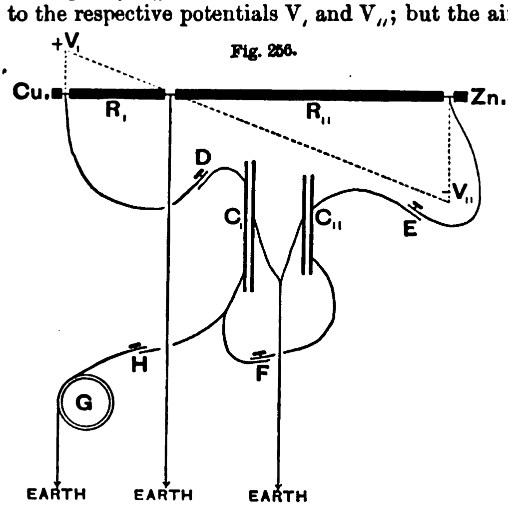
Fig. 255.



of that point must become equal to zero; but the difference of potential between the extremities of the wire remains unaffected. The positive potential at Cu (Fig. 255) bears to the negative potential at Zn a numerical ratio, the same as that between CuO and OZn; for obviously CuA : ZnB : : CuO : OZn.

Let now between the points Cu and O a resistance of reduced length $R_{\prime}$ be placed, and between the points O and Zn a resistance of reduced length $R_{\prime\prime}$. The potential at a point just between $R_{\prime}$ and Cu, and the potential at a point just between $R_{\prime\prime}$ and Zn, bear to one another the ratio of $R_{\prime} : R_{\prime\prime}$. If these potentials be $+ V_{\prime}$ and $- V_{\prime\prime}$ respectively, we have $V_{\prime} : V_{\prime\prime} : : R_{\prime} : R_{\prime\prime}$. Let these two points, at potentials $V_{\prime}$ and $- V_{\prime\prime}$ respectively, be connected with two condensers of which the one has standard capacity $C_{\prime}$, the other the capacity $C_{\prime\prime}$ to be determined. The two condensers will become charged to the respective potentials $V_{\prime}$ and $V_{\prime\prime}$; but the aim is so to adjust the potentials that these condensers shall become charged with equal but opposite quantities of electricity. Suppose this adjustment to have been effected. Then Fig. 256 illustrates the successive operations.

Fig. 256.



(i.) Connect at D and E. The condensers become charged to potentials $+ V_{\prime}$ and $- V_{\prime\prime}$ respectively. They are therefore charged with quantities $+ C_{\prime}V_{\prime}$ and $- C_{\prime\prime}V_{\prime\prime}$. Disconnect at D and E.

(ii.) Connect at F. The two charges $+ C_{\prime}V_{\prime}$ and $- C_{\prime\prime}V_{\prime\prime}$ blend, and there remains in the conjoined condensers a residual charge of $(C_{\prime}V_{\prime} - C_{\prime\prime}V_{\prime\prime})$, which, if $C_{\prime}V_{\prime} = C_{\prime\prime}V_{\prime\prime}$, is equal to zero.

(iii.) Connect at H. The residual charge, if any, runs to earth and deflects the needle of the galvanometer; if none, there is no deflection. There is no deflection when $C_{,}V_{,} = C_{,,}V_{,,}$. But $V_{,} : V_{,,} :: R_{,} : R_{,,}$. Therefore, when there is no deflection on making contact at H, $C_{,}R_{,} = C_{,,}R_{,,}$; and the capacities $C_{,}$ and $C_{,,}$ are inversely as the corresponding resistances.

Adjust therefore the resistances $R_{,}$ and $R_{,,}$ until there comes to be no deflection of the galvanometer-needle after operation (iii.), and from the known ratio of the resistances we find that of the capacities, for $R_{,}/R_{,,} = C_{,,}/C_{,}$.

OSCILLATING OR ALTERNATING CURRENTS.

If the potential, which tends to give rise to a current, itself fluctuate between positive and negative values, its variation being simple-harmonic, we have an **oscillating or alternating current** produced in the circuit. Oscillating currents differ in some important respects from the steady currents hitherto discussed. If their frequency be small, they approximate to steady currents in their character, and merely fluctuate in their intensity and direction: if it be great, they present, as it were, nothing but the Variable Period, and never arrive at the Steady State. We shall mention the main characteristics of those currents in which the frequency is great, say a million oscillations per second. How such currents are produced we shall learn later.

The phenomena of the current tend, as the frequency increases, to be entirely *confined to the dielectric*, so that only a *thin skin* of the wire is concerned in the current: but the less the magnetic permeability of the conducting wire, the thicker is that conducting skin; and the smaller the frequency of alternation, again the thicker is that skin. Even when the frequency is as low as 100 per second, the skin, in the case of iron, is practically not more than 0·3 cm. thick, while with a Leyden-jar discharge it is less than 0·001 cm. in thickness. In the dielectric there are Waves of Propagation of Lines of Force; these lines travel back and fore with the velocity of Light, with their ends on the conducting wires, and approximately at right angles to these. In the dielectric or the electromagnetic field surrounding the wire, the lines of magnetic induction have the same direction as in the case of a steady current, but the field-intensity fades away very rapidly as the wire is receded from. The Electrostatic Attraction between the two sides of the circuit tends, as the frequency increases, towards equality with the Electromagnetic Repulsion between them. The Transmission of Energy through the dielectric is approximately parallel

to the wire. The Apparent Resistance, or **Impedance,** $\mathfrak{r}$, of the wire tends towards a value inversely proportional to the circumference, instead of to the cross-section of the wire; and it is greater the greater the magnetic permeability of that wire, for the conducting skin is in that case all the thinner. As a wave of potential passes along the wire, its amplitude wanes; and the effect of this is more marked the greater the frequency; so that after passing over a long wire a complex harmonic disturbance may have its higher harmonics considerably more attenuated than the lower components. Besides this, disturbances of different wave-lengths produce their maximum effect, at a given distant point, at different times.

The conducting skin screens the interior of the wire from the inductions which would otherwise be set up in it by the alternating field; and generally, a film of metal acts as a screen, or is opaque, to the alternating condition of the field, while an insulator is not. An electrolyte is not quite opaque in this sense, but has a high absorptive coefficient, and a thickness of some millimetres acts like a film of metal.

Suppose that the E.M.D.P. is itself subject to variation which follows the S.H.M. law; that is, with a frequency n times per second, it oscillates between the extremes $+e$ and $-e$, and is at any given time t equal (assuming it to be zero at the initial instant when $t=0$) to $e\cdot\sin 2\pi nt$. Then, instead of e in equation i., p. 705, we have $(e\cdot\sin 2\pi nt)$; and that equation, when so modified, upon being integrated, gives. $i_t = (e/\sqrt{r^2+4L^2n^2\pi^2})$; $\sin(2\pi nt - \tan^{-1} 2\pi nL/r)$, a S.H. function: that is, the intensity of the current varies as time goes on, according to the simple harmonic law, and its maximum alternating value is $\pm e/\sqrt{r^2+4L^2n^2\pi^2}$. Observe from this expression that if the coefficient L be comparatively great, as in a coil of wire, the maximum values of the current may fall considerably short of the value e/r ultimately attained by a steady current, but will tend to be little affected by changes in the resistance of the circuit; and that as the frequency n of oscillation increases, the maximum intensities fall off. The apparent resistance, or Impedance, is thus $\sqrt{r^2+4L^2n^2\pi^2}=\mathfrak{r}$. Hence, when an alternating current of high frequency is sent through a coil of many turns round a soft-iron core, the impedance may have a very high value, and the current round the coil may be reduced to a minimum. The current may thus be "**throttled,**" or choked down. Observe also that in the equation arrived at above, the S.H. variation of the current-intensity does not keep step with the variations of the E.M.D.P., as it would do if the last term were simply $\sin 2\pi nt$; there is a **lag**; the variations of current fall behind those of E.M.D.P. by an amount represented by an angle whose tangent is $2\pi nL/r$. This Lag is greater the greater the frequency n of the variations of the applied E.M.D.P., or the greater the self-induction L of the circuit or coil, or the less its resistance r; but the angle cannot exceed 90°; hence the zero current is delayed after the zero E.M.D.P., by an interval of time corresponding to not more than a quarter of a revolution in the circle of reference (see S.H.M., Fig. 29); that is, it is in arrear by an interval of time not greater than a quarter period, T/4, or $1/4n$.

Since, during any S.H. variation of potential or of current-strength, etc., the average value, taken throughout the whole of a positive or a negative phase, is $2/\pi \times$ the maximum value (see p. 85), the actual mean intensity of an alternating current is $2/\pi$ or 0·6366 times the maximum intensity.

Alternating currents produce **Heat,** and therefore also **Light,** as in incandescent and arc lamps; and they can light up Geissler-tubes.

A Cardew's Ammeter, or an Electrodynamometer, produces Heat, or a Torque, proportional to the square of the current at any instant; such instruments therefore indicate the mean value of i^2, not of i; and in S.H. variations of i, the mean value of i^2 is half the square of the maximum intensity; whence the mean value of i, as given by such instruments, is the maximum intensity $\times \sqrt{\frac{1}{2}}$. This differs from the true arithmetical mean in the ratio $\sqrt{\frac{1}{2}} : 2/\pi$, or ·707/·637; and it is called the *effective* or *virtual mean intensity*. Similarly for the voltages. Hence the Heat produced by an alternating current whose maximum intensity is $i_{\prime}$ is $\frac{1}{2}i_{\prime}^2 r$.

If an object of some capacity — a small porcelain ball in a vacuum chamber — be connected with one terminal only of the secondary coil of an induction coil subjected to alternations of extreme frequency, it may itself become very hot; and if it be itself surrounded by air, it may subject the molecules of that air to collisions and shock, so that the air glows with a phosphorescent light. A Geissler-tube connected in the same way will light up. The human body may be charged in this extremely-rapidly-alternating manner by being connected in the same way; when this is done, a Geissler-tube held in the hand will light up; while the current then passing in the body, though of enormous voltage, appears to do no more harm than the impact of light-waves does. Mr. Nicola Tesla has recently devised a series of extraordinary experiments on these lines. He has constructed lamps connected only by one wire with the terminals of secondary coils; and he has obtained, at these terminals, brushes in the form of veritable flames, consisting merely of air-molecules subjected to collision and shock.

Since the current is alternating, there can be **no electrolytic effect,** except, apparently, a small residual decomposition in some cases, which is possibly due to greater ease of charging particular elements with one kind of electricity than with another.

When an alternating current is sent through a loop, as in Fig. 221, the **derived currents** are not so distributed as to produce the minimum amount of Heat, as is the case in steady currents; but they are so distributed as to keep the kinetic energy of the field down to a minimum, and to neutralise each other's pro-

duction of an electromagnetic field as far as possible. The currents in the two branches of a loop may thus be opposed, and even be each much greater than the leading current; but their mean difference will be equal to the mean intensity of that current.

The preponderating branch-current goes in the direction of the leading current along that branch which has the less self-induction; that is, along that branch the passage of a current along which would give rise to the smaller amount of total magnetic induction in the part of the field affected by that branch; but if the electrostatic capacity of the other branch be increased, this may neutralise the effect of its superior self-induction.

When an alternating current is used to excite an electromagnet, there is a strong tendency to the formation of Eddy-Currents, parallel to the wire of the inducing coils. This is combated by building up the core, of laminæ or of thin wires.

If an alternating current be used to excite a long electromagnet, the alternating magnetising effect falls off very rapidly at a distance from the exciting coil, and does so the more rapidly the greater the frequency; the lines of magnetic induction l e a k o u t laterally from the iron, and find closed return-paths through the air.

In a non-uniform magnetic or electromagnetic field, in a case where a conductor, say a **coil**, bearing a steady current, would act like a magnet, one bearing an alternating current acts like a **diamagnetic** body; it is repelled into the weakest part of the field.

Two alternating currents in the same direction and in the same phase attract one another; if in opposite phases, they repel one another; if their phases differ by $\pi/2$, they have no effect upon one another. Hence the **mutual attraction or repulsion** of two alternating currents will depend upon their relative amounts of Lag.

When an alternating current acts by induction on a coil laid parallel to the inducing coil, in the absence of self-induction the induced current would be strongest when the inducing current passed through its zero value, for the strength of that current would then vary most rapidly. But, when the secondary coil is excited by an electromagnet, itself excited by an alternating current, there is Lag in the secondary coil; there is now repulsion between the two coils; and the secondary coil tends to fly off the electromagnet. Similarly, an alternate-current electromagnet may repel copper by its action on the currents induced in that copper.

Transformers. — An alternating current of high voltage and few Ampères can be sent to a great distance, for there is comparatively little loss by transformation into heat. But for use in houses, etc., its voltage must be reduced. This is effected

by Transformers, which are, in effect, Induction Coils reversed in their action. The current of high voltage is sent through the coil of many turns (inside the other coil), and an induced alternating current of lower voltage and correspondingly greater quantity is induced in the coil of fewer turns. The soft-iron wire or laminated core may form either a closed or an open "magnetic circuit." An interrupter or contact-breaker is not necessary. When the core is large and the alternations rapid, the effective current-intensities are inversely proportional to the number of turns in the respective coils. The induced currents are opposite in phase to the inducing currents; they thus tend to demagnetise the soft-iron core when the house-circuit is closed. They thus tend to diminish the Impedance of the main circuit; and they do this the more completely, the less the resistance in the house-circuit. When the house-circuit is broken, little or no current passes through the main coil, on account of the impedance of that coil, with its core; but as the resistance in the house-circuit is reduced by increasing the number of paths along which the house-current may pass, that house-current is allowed to gain in strength in proportion to the reduction, that is, in proportion to the work to be done. From 3 to 6 per cent of the energy supplied is usually lost in eddy-currents and hysteresis.

Production of Alternating Currents.

There are two main methods of producing these; (1) by the action of dynamo-electric machinery, the frequencies produced by which range, say, from 40 to 150 per second; and (2) by means of the discharge of a Leyden-jar or other electrostatic condenser, the frequencies produced by which may amount to, say, 10,000000 per second. We shall deal with the latter first.

Oscillation in Leyden-jar Discharge.—Assume the source of electricity in a circuit,—which circuit may have an air-gap in it equivalent to an interposed resistance,—to be a Leyden jar charged with a quantity Q; the E.M.D.P. is $E = Q/C$, where C is the Capacity of the jar. The equation (i.), p. 705, is easily reduced—neglecting squares of δI, which is a proper omission in a calculation involving subsequent integration—to the form $E = RI + L \cdot \delta I$; or, since I is itself equal to $-\dot{Q}$, we have $Q/C = -(R\dot{Q} + L\ddot{Q})$; R being the total Resistance of the circuit. This equation is a Differential Equation; and, starting from a charge Q and no current at the initial instant, this equation is reduced, by appropriate mathematical treatment, to the statement that at the end of any given time-interval t, the charge Q_t left in the Leyden jar is $Q_t = Q \cdot \epsilon^{-Rt/2L} \cdot \{((2La + R)/4La) \cdot \epsilon^{at}$

$+ ((2La - R)/4La) \cdot \epsilon^{-at}\}$, in which expression the letter a is made to do duty for $\sqrt{(R/2L)^2 - 1/CL}$, and ϵ is equal to 2·718281. On interpreting this statement we find that there are two cases, in which the consequences are different; (1) when L is less than $R^2C/4$, a is a possible positive quantity, and the charge gradually diminishes as t increases, that is, as time goes on, so that the Leyden jar **steadily** discharges itself; but (2) when L is greater than $R^2C/4$, a becomes the square root of a minus quantity, and being itself therefore impossible, renders the expression an unintelligible one as it stands. But if we transform it by making $a\sqrt{-1} = a'$, we find the result to be that the expression takes the form $Q_t = Q \cdot \epsilon^{-Rt/2L} \cdot (\cos a't - (R/2La' \cdot \sin a't))$. That is to say, the charge in the jar **oscillates** in amount, being nothing when $(\sin a't/\cos a't) = 2La'/R$, and attaining a succession of rapidly-decreasing maxima (alternately positive and negative), which occur whenever $a't = n\pi$, where n is any whole number. The current-intensity is a maximum, positive or negative, when the charge in the jar is passing through a zero value; when this is the case, the intensity is equal to $-\dot{Q}_t = Q/CLa' \cdot \epsilon^{-Rt/2L} \cdot \sin a't$; and it thus oscillates in value, rapidly diminishing, with a **period** T of complete oscillation $= 2\pi/a' = 2\pi/\sqrt{1/CL - (R/2L)^2}$. This period T, when R is very small in comparison with L, is approximately $T = 2\pi\sqrt{CL}$. Whether, therefore, a steady or an oscillating discharge will be obtained depends on the relations between the capacity C of the jar, the self-induction L of the circuit, and its resistance R: diminish C or R or increase L sufficiently, and an oscillating discharge may be obtained; while by increasing L very much, as by interposing very large coils in the circuit, the rate of oscillation may be greatly reduced.

Since a maximal positive value of the current-intensity occurs once in each period, the time-interval between any two such positive maxima is $T = 2\pi\sqrt{CL}$. The successive maximal positive values of the current-intensity are thus $Q/CLa' \cdot \epsilon^{-Rt/2L} = Q/CLa' \cdot \epsilon^{-R\pi\sqrt{C/L}}$ for the first; $Q/CLa' \cdot \epsilon^{-R2\pi\sqrt{C/L}}$ for the second, and so on; the value being $Q/CLa' \cdot \epsilon^{-Rn\pi\sqrt{C/L}}$ for the nth positive maximum.

Numerical Example.—In a solenoid of n turns and of length l cm., $L = 4\pi n^2 A\mu/l$. Let $n = 100$, and $l = 20$; and let $r = 1$ cm., and, as the medium is air, $\mu = 1$. Then L = 19,739, the number of lines of induction, in magnetic measure, which thread the solenoid when unit-current passes. The Resistance of this wire may be taken as 0·0045 Ohm per metre, or 45000 magnetic units per cm.; $R = 45000 \cdot 2\pi r \cdot n =$ (nearly) 28,275,000 magnetic units. Next, suppose a Leyden jar to have opposed surfaces of 250 sq. cm. each, at a mutual distance 0·3 cm. across glass whose sp. ind. cap. is, say, $K = 2{\cdot}5$: then its Capacity C will be $K/4\pi \cdot \text{surface}/d = 165{\cdot}78$ C.G.S. electrostatic units, or $(165{\cdot}78 \div 9.10^{20})$ magnetic units. Now discharge the Leyden jar through this solenoid, it being assumed that there is no other part of the circuit to be taken into consideration. Then $a' = \sqrt{1/CL - (R/2L)^2} =$ 16,583500, in magnetic units; and the period of complete oscillation $T = 2\pi/a' = \frac{1}{2,639350}$ second.

To ascertain the progressive decrease of the successive maxima of current-intensity, put these numerical values of C, R, and L in the expressions given above for that intensity: then we find that the current-intensity is, at the first, the 10th, the 100th, the 1000th, and the 10000th maxima respectively, in the ratios of 0·99973 : 0·99742 : 0·97323 : 0·76237 : 0·06632; and the maxima of current-intensity are reduced to a millionth in 0·02428 second.

The discharge of a Leyden jar is thus **practically instantaneous**; and in order that it may keep up a continued discharge, the jar must be fed from a machine or an induction-coil.

If the apparatus of the above example be reduced to half the size, linearly, the time of oscillation will be reduced to one-half; and so on in proportion. A Leyden jar of molecular size would give an oscillating discharge whose frequency is of the same order as that of light-waves; but if the molecule be simple in its structure, the frequency, thus calculated, will lie beyond the violet.

It has, however, been assumed in the above that the current is uniform all along the wire; that is, that the wave-length is great in comparison with the length of the circuit; and that the current-density is uniform across the cross-section of the wire. If we take into account that these things are not true at high frequencies, we find that we have to replace the Resistance by the Impedance, and that the value of L is affected by the frequency; which modifies the numerical results.

If the condenser be reduced to the mere tips of the wire of a loop or coil, the to-and-fro reflexion of the disturbances in the wire will occur at a definite frequency, which depends on the size of that loop or coil. Similar phenomena of to-and-fro reflexion occur whenever abrupt signals are sent over a short circuit, but on a long one they die out; they are due to the electrostatic charge on the wire.

The second method involves the use of **Magneto-Electric and Dynamo-Electric Machines.** The former are now seldom seen, except in small apparatus such as that used for medical purposes: the latter have assumed great importance in Electric Lighting, Electrolysis, Transmission of Power and, to a smaller extent, in Heating.

Given an existing magnetic field: then, if a loop of wire be moved in this field, so as to embrace more or fewer lines of magnetic induction, a current will be set up in that wire. In the former case, Work has to be done upon the loop; in the latter, the field does work upon the loop; and both these amounts of Work appear as the Energy of Electric Current in the loop. The direction of that current depends upon whether work is being done by or against the field at the moment.

We have seen that if the lines of magnetic induction point eastwards, and the direction of motion of the lines of force in the field be northward, the corresponding lines of Electric Force will themselves point upwards. Here we have the converse case: if any small part of the wire stand vertical and move broadside-on, towards the south, across lines of magnetic induction whose trend is towards the east, there will be set up in that part of the wire a current whose direction is upward. In the former case, the lines moved up to the wire; in the latter the wire moves in the opposite direction towards them.

If a loop, or a coil, be flashed past the two poles of a permanent magnet, so as alternately to embrace the magnetic lines

radiating from these two poles, it will, as it approaches the one pole, travel into a field whose strength it finds to increase to a maximum, and then to fall away as it recedes from that pole. As the coil goes farther, it goes through a region of zero and then into one of opposite potential, which in its turn again reaches a maximum and then falls away. The currents produced in any given part of the wire are thus alternately in opposite directions.

In Pixii's machine, permanent magnets were themselves flashed past the coils, of which there were two, parallel and wound on bobbins, with a soft-iron core in each. These cores assumed, in rapid alternation, opposite magnetic characters, and exposed the wire to a more intense alternating magnetic field than they would have been exposed to had there been no cores. In other cases — Clarke's, etc. — the magnets were fixed, and the bobbins were made to pass their poles. This kind of machine, by multiplication of the magnets and of the rotating bobbins, has been made in very large sizes; and with the substitution of electromagnets for permanent magnets, the principle is still applied in some disc dynamos.

It is more common, however, instead of dragging a loop or a coil sideways past the polar face of a magnet, to provide for it an axis of rotation traversing the middle of a **nearly uniform magnetic field,** at right angles to the magnetic lines of induction of that field; and so to fit up the loop, that at two points in its rotation round that axis, it may be so placed as to embrace the greatest possible number of lines of induction. This may be done in two ways: first, by making the axis of rotation lie across the loop itself, which is spun in the field round its own diameter; or second, by making the axis of rotation lie outside the loop itself, and parallel to it, so that the loop is swung in a circle in the field. In both these cases, the loop, at one part of its rotation, will embrace a maximum number of magnetic lines, to which its own axis is then parallel; when it has turned round through 90°, the axis of the loop is at right angles to these lines, and the loop itself embraces none of them. When it is in this latter position, it is most rapidly altering the number of lines which pass through it, and the induced current in that loop is then a maximum.

The number of lines of induction embraced by it when its axis is parallel to these lines is $\mathbf{B}$ = its area $A \times \mathbf{b}$; when it is at an angle θ $(= 2\pi nt)$ to that position, the induction through it is reduced to $\mathbf{B} \cdot \cos\theta = \mathbf{B}_\theta$. But the current-strength i_θ in any position is, omitting self-induction, $-\delta\mathbf{B}_\theta / r\delta t$, r being the resistance; and this, on giving $\mathbf{B}_\theta$ its value, and differentiating, leads to the result that $i_\theta = (\mathbf{B}/r) \cdot 2\pi n \cdot \sin\theta$; and the maximum value of this is $i = (\mathbf{B}/r) \cdot 2\pi n$, when $\theta = 90°$; that is, when the axis of the coil is at right angles to the lines of induction. Hence the maximum value

of $e = 2\pi n\mathbf{B}$; and at any position θ, $e_\theta = e \cdot \sin\theta = e \cdot \sin 2\pi nt$; the condition required for finding the Lag and the Impedance (p. 722), when self-induction is taken into account. The average value of e is $2/\pi \times$ the maximum value (p. 85), and is therefore, for a single loop, equal to $4n\mathbf{B}$; n being the number of complete revolutions of the loop, through 360°.

In the latter of the two methods of rotation referred to, the whole loop, and in the former, any part of it, is carried round in the course of its rotation from a positive into a negative part of the field, and the same operation, with negative sign, is repeated there; the current now produced is, as regards the loop or the part of it referred to, now in the opposite sense, and passes through a maximum in the same way. In any given part of the loop, therefore, the current produced passes through alternating positive and negative maximal values; and its variation between these extremes is **simple-harmonic,** one complete alternation to each complete revolution of the loop.

The extremities of the loop may be connected with two separate rings, which rotate along with the loop round the axis of rotation; and if an external circuit terminate in flexible metallic "brushes," which rest upon these collecting rings, the alternating currents developed in the loop will be propagated round that circuit.

A single loop is, however, an illustrative rather than a practical apparatus. A coil would produce a greater current, for each turn in it would be acted upon, practically, as if it were a loop; and in Siemens' Inductor a coil, with a soft-iron core, was rotated round an axis passing through its own centre, and was so shaped as to lie as close to the magnetic pole-faces as possible, and at the same time to have a minimum moment of inertia. But in the course of each revolution there is a period during which such a coil or loop is very nearly idle: that is, when its own plane is at right angles to the lines of induction: and since there is only one alternation per revolution, the speed for rapidly-alternating currents would have to be excessive. It is therefore the practice in alternating-current machines, or **Alternators,** to multiply the opposed magnetic fields through which the coil has to travel; and this is done by multiplying the opposite pole-faces past which the coil is driven. Such machines are said to be multipolar: and the frequency of alternation is correspondingly increased by this device, though the alternations are now, initially, not so nearly simple-harmonic in their character as when a single loop or coil is employed in

a single uniform field. But further, instead of a single coil passing one magnet-pole at a time, a number are put into **simultaneous** action, all similarly situated with respect to their several magnetic fields; and great ingénuity has been displayed in contriving the machine so that these may act in concert. These coils may move past the magnet-poles, or the magnet-poles past them: or again, both coils and magnets may be stationary, the strength of the magnetic fields being alternately increased and diminished by masses of soft iron moving in or near those fields. For some purposes it is convenient so to arrange the coils and their metallic connections as to send along, say, three wires three equal currents differing in their phase of alternation by equal amounts; alternators of this kind are called multiphase alternators.

The congeries of coils is borne by a drum, by a ring, or on the periphery of a disc; and each coil has a laminated soft-iron core, which forms a part of the magnetic circuit, and greatly intensifies the inductive effect on the wire of the coils. The whole arrangement of coils, with their core or cores, is called the Armature.

The magnetic field is, in all modern machines of any size, that of an electromagnet; it has to be intense, while at the same time there must be plenty of iron in the magnetic circuit. The electromagnets are excited, when the machine is at work, by a part of the current from the machine itself; but as this is alternating, it would not excite an electromagnet so as to impart to it a uniformly-directed magnetic polarity, unless its alternate phases had been made to go, not in opposite, but in the same directions. This is accomplished by a **Commutator**: the two "brushes," which take current off for the electromagnets, do not each continuously touch one of the collecting rings; but each comes in contact, first with a projecting tooth of the one, and then with a projecting tooth of the other collecting ring. Thus, in step with the alternations of the current yielded by the machine, there is an alternation of the directions into which it is guided; and the result is a current not uniform in strength, but constant in direction, and useful for the electromagnet.

Two alternators put in series tend to assume opposition of phase and to deliver, jointly, no current; but they will work in parallel. They then go into step, and tend to keep step, co-phasally.

Direct-Current Dynamos.—In these the whole current of the machine is led through a Commutator. The current from a single loop or coil would vary from zero to a maximum and back to zero twice in each revolution; and as currents, merely in different states of variation of positive or of negative value on one side only of zero, do not tend to neutralise one another when sent into the same wire, but tend, by their summation, to render the aggregate current more uniform in character, the direct-current dynamo generally has its several coils or groups of coils so arranged that each sends its own current into the general circuit, in whatever phase it may happen to be, and is cut out of that circuit only during such time as there may be danger of other coils or groups of coils being short-circuited through it during its own comparatively idle period. The armature, by multiplication of loops or coils lying across the field or towards its periphery, usually takes the form of a drum or of a ring; and its soft-iron core is laminated, to prevent the formation of eddy-currents.

Both in ring and drum armatures, the armature-core tends to become magnetised transversely to the main magnetic field. The actual magnetic field is thus the resultant of the main field and a **cross-field.** If it had not been for this, the proper position for the brushes would have been at right angles to the field, so as to lead off the current through those coils which, at the moment, are least engaged in the actual production of current; but the effect of the resultant obliquity of the field is that the brushes must also lie obliquely, to an equal extent; and thus, as is said, the brushes must be given a certain lead. The amount of this lead is, further, somewhat increased by the necessity, in order to prevent sparking at the brushes, of letting each loop or coil get a little way into the opposing field before being cut-out; by which means the extra current is neutralised. This conduces, however, to demagnetisation of the magnetic circuit as a whole.

The mean current produced is proportional to the speed, less a certain number of revolutions per second, called the Dead Turns; and is also proportional to the number of coils in the field and to the strength of that field. At speeds less than the so-called Dead Turns the machine will not deliver any current at all.

As to the mode of excitation of the Field Magnets, that is, of the electromagnets which produce the magnetic field within

which the armature rotates: these are feebly excited by an ordinary magnet, or there may be sufficient residual magnetism in them to serve the purpose, or they may be feebly magnetic under the induction of the earth's magnetic field; then, when the armature is set in rotation, an extremely feeble current is generated. This feeble current is not permitted at once to pass away, but is sent, either wholly (in "Series" dynamos) or partly (in "Shunt" dynamos, by means of a shunt always kept in action), round the soft-iron magnet, and thereby increases its magnetisation. The soft-iron electromagnet, thus strengthened, induces a still stronger current in the rotating armature; and thus, the current-intensity attains in a short time a maximum, the potential of which depends upon the speed of rotation and upon the product of the intensity of the current actually passing round the field-magnets into the number of turns which it takes round them, as well as upon the number of turns within the armature, effectively connected in Series. In "Series-Shunt" machines the current is divided into two parts; one part runs in a shunt round the electromagnet: the other runs both round the electromagnet and through the external circuit.

In a Series machine, as the Ampères increase, the total voltage in the circuit also increases, rapidly at first, but then more slowly, as the permeability of the iron begins to fall off, and the cross magnetic field to become more intense; and at extreme ampereages, the total voltage even tends to droop away for these reasons. As the ampereage increases, the available voltage at the terminals differs by a steadily-increasing amount from the total voltage in the circuit, because of the increased voltage consumed in the passage of a greater current through the armature: hence, as the current increases, the available voltage at the terminals reaches a maximum and then falls off. Within this limit, however, when the resistance of the circuit is increased, the electromagnet is enfeebled, and the voltage at the same time falls.

In a Shunt machine, on the other hand, as this resistance increases, the tendency is for a larger proportion of the total current to pass through the shunt-winding round the electromagnet, and thus to strengthen it and raise the voltage. When the voltage is so increased, the Ampères rise to a maximum, and then fall off to nothing, while the voltage goes on rising to a maximum, the potential-difference on open circuit.

In Series-Shunt machines, the electromagnet is wound with shunt-coils, and the main current is also sent round it. When the resistance is increased, the opposite variations of potential, due to the shunt and to the series-winding, may partly compensate one another; when they are so adjusted that, for a particular speed of running, the machine gives a constant voltage whatever be the resistance, the machine is said to be "Compound-Wound."

In another class of machines, there is separate excitation of the electromagnet by particular coils driven on the same axis as the coils supplying the general working circuit, or by a separate subsidiary machine.

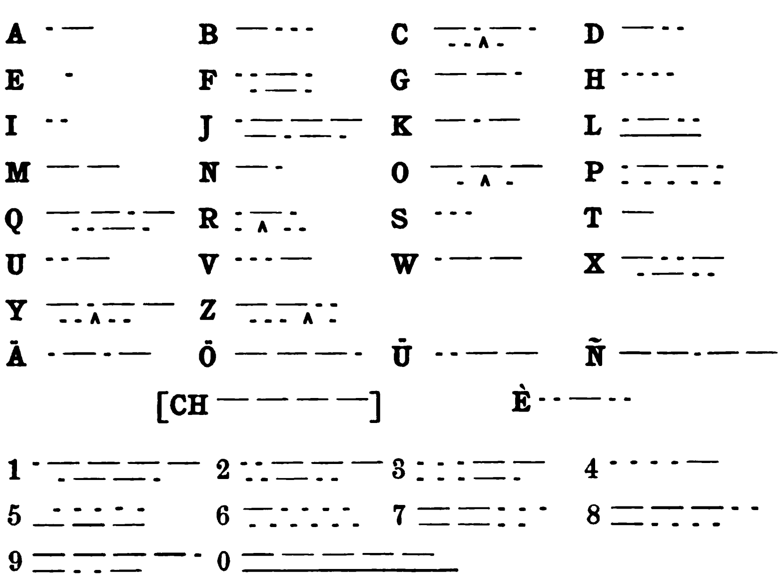
Transmission of Energy to a Distance.

All current and electromagnetic phenomena are, as we have seen, associated with the transmission of Energy to a distance, across the dielectric.

We have already considered the action of Galvanometers; and also of Ballistic Galvanometers, in which the throw of the needle renders manifest the passage of a very brief current; just as the position of equilibrium assumed by the needle, as it lies more or less completely across the current, with its axis directed along the lines of force, indicates the persistence of a steady current.

As often as a momentary-current is sent round the magnet of a galvanometer, so often will the twitch of the suspended magnet be repeated, and at intervals of time equal to those between the successive momentary-currents. This action — which is the simplest form of transmission of energy to a distance, for work is done in displacing the magnet within the field — is the basis of telegraphic signalling.

Longer and shorter currents produce longer throws and shorter twitches of the galvanometer-needle. These form the basis of a signal alphabet — the Morse code. The following is the alphabet, the upper line, where there are two, being the European or "International," the lower the American form: —



Full stop	· · — — · ·	Stroke	— — — — — —
Semicolon	· — · — · (Amer.)	Apostrophe	· — — — — —
Comma	· — · — · —	Parenthesis	— · — · — · —
Exclamation	— — · · — —	Repeat or ?	· · — — · ·
Paragraph	— — — — (Amer.)	Hyphen	— · · · · —
Italics	— · · · — (Amer.)	&	· ʌ · · · (Amer.)

In some of the American forms it will be observed that a period of time, represented by ʌ, intervenes in the midst of a set of signals representing one letter. The American form (U.S. and Canada) is the original, as devised by Prof. Morse: the European is an improved version (International Congress, 1851).

In submarine telegraphy the signals used are not long and short, but right and left deflections — that is, positive and negative momentary-currents.

By means of differential galvanometers two messages may be sent along the same telegraphic wire at the same time (Duplex Telegraphy). Station A has a single wire leading from the positive pole of his battery: the current running in this he divides into two moieties, which he sends in opposite directions round the needle of his differential galvanometer: these two moieties are then sent on, the one to the distant station B, the other to A's own earth-plate. In the course of the one or the other branch-current the operator at A interposes resistances until the intensities of the opposed currents round his galvanometer-needle are equal; then, in whatever way A may make or break circuit, his galvanometer-needle will remain steady; but the needle at B will respond. Similarly, B sends signals to A, to which his own instrument is mute. The two stations may thus signal simultaneously, two operators being employed at each end, one to transmit, the other to receive; and the variations of electric condition produced in the single connecting-wire run through one another in a manner analogous to that in which waves meeting on a cord traverse one another.

Bridge-Method in Duplex Telegraphy. — Suppose a triangle ABC; the current enters at A; B is connected with the distant station D; C is connected to earth through a resistance equal to that of the line BD; between B and C is the recording instrument of the home station. One moiety of the current which enters at A will run to earth, the other will travel to D. If the resistance in AB be so adjusted as to be equal to that in AC, B and C will be at equal potentials; no current will run through BC; the home instrument stands motionless. At the receiving station the apparatus may be precisely similar; it will then indicate the arrival of signals from A, but will be insensible to the movements of its own key.

Quadruplex Telegraphy. — A small current always runs in the circuit. There are two transmitting keys. The one reverses the direction of the current; this causes a needle within a magnetic field at the receiving station to swing to left or right; an effect which depends upon change of direction of the current within the circuit. The other key, when depressed, introduces a new battery into the circuit; the strength of the current is thereby increased, and the current is now enabled to make a certain soft-

iron electromagnet move at the receiving station; an effect which depends upon the strength, but not upon the direction of the current in the circuit. The one receiving instrument thus records reversals, the other the enhancements of current-intensity. Two sets of signals may thus be sent in the same direction at the same time; and this arrangement when duplexed, preferably by the bridge-method above described, becomes quadruplex. This is the ground-principle of Prescott and Edison's system, which is described at length in Prescott's *Telephone*. The practical details are extremely ingenious; there may, for instance, be a critical instant at which the intensity-receiver is liable to be interfered with and to fail, through the current supplied to it fading away while being reversed by the reversing-key; a condenser then acts as a reservoir, and its discharge keeps up a current which tides over the critical instant; a result which is aided by a subsidiary local battery then brought into action by means of a relay.

In Multiplex Telegraphy, each operator gets the use of the circuit several times a second; his signals are like cyclostyle writing, broken up, but practically continuous.

When at the distant end of a circuit the conducting wire is passed round a soft-iron core, that soft-iron core becomes an electromagnet just as often, and remains an electromagnet just as long, as the circuit is or remains completed by a key at the home station. This electromagnet may govern the movements of a neighbouring mass of iron, and do work upon it: and the movements of this second mass may be utilised in an endless variety of ways for the repetition of movements similar to those executed at the home station by the hand of the operator, or by any mechanical contrivance adjusted so as to make and break contact in any pre-arranged manner. The mass of iron moved at the distant station may itself, by its movement, make and break a second electric circuit, and may thus control the movement of metallic masses at still more distant stations, as in the case of telegraphic relays.

Electromagnetic Interrupter for Tuning-Forks.—A tuning-fork of known pitch is set in vibration. As it vibrates, it alternately makes and breaks a current which traverses the tuning-fork itself. This current is passed, in its course, round a little electromagnet, which is alternately made and unmade. This electromagnet is so arranged as alternately to attract and release one of the prongs of the tuning-fork, which is thus kept in continuous action. The intermittent current produced is sent round a second electromagnet, which rhythmically attracts and releases a second tuning-fork; this is thus kept vibrating in unison with the first, even although it be not precisely in tune with it.

Signalling by Alternating Currents.—The Phonophore. In this there are two wires, simply coiled together: their farther ends are both free: the nearer end of one is connected with the line-wire. When a brief current is sent, the other wire is acted upon by induction, and signals may be heard in a telephone connected with it and also to earth. This instrument is

worked by alternating currents, which do not affect the ordinary telegraphic instruments, and will not pass through their coils, being throttled by them. These signals may thus be made independently of the ordinary direct-current signals, and both systems may be duplexed.

If a current which has 300 maxima of intensity per second, and another of say 800 maxima per second, be sent along the same wire, the conjoined current will present variations of intensity such as might be represented by the curves of Fig. 45. Suppose a current presenting such variations of intensity to be passed round a soft-iron core, near the end of which is a steel reed tuned to vibrate 300 times a second, and so adjusted as to be attracted in the sense of its vibrations when the soft-iron core attains its maximum intensity. The steel reed would, among other impulses to which it would not respond, receive a set of 300 maximum attractions per second, which would set it in vibration. The same current may be also passed round a core, opposite the extremity of which is placed a reed tuned to 800 vibrations per second; that reed will pick out and will respond to the more rapid component of variation of intensity of the current, and will respond to it only. Further, suppose that the several components of variation are each not continuous, but interrupted: the corresponding vibrations of each reed will be similarly interrupted, and one telegraph clerk may be occupied with listening to each. A current whose variation of intensity is as complex as the sum of eight distinct S.H.M.'s may be practically resolved by as many distinct receiving-reeds into distinct signals; and since the duplex method of working may be applied to this plan, as many as sixteen distinct messages may travel along a single wire at the same time. This is the principle of Mr. Elisha Grey's Harmonic Telegraph.

The Telephone, in its simplest form, presents a plate of iron, P, placed in the magnetic field of a magnet, M: the plate is caused, by being spoken at, to enter into certain vibrations; the vibrating plate P, by induction, acts upon the magnetism of the magnet M; the latter is alternately strengthened and weakened in accordance with the varying position of the vibrating plate: as M varies in strength it causes variations in the strength of a current passing through a coil, C, wound round its pole, or else, if there be no appreciable current passing in that wire, it causes a current to be formed in that wire whose intensity varies continuously on either side of zero-value, being now in the one direction, now in the other. This induced current reproduces in the mode of its variation the complex-harmonic curve which might have been recorded by a delicate writing-point attached to the vibrating plate. The variable current thus produced passes at the receiving station through the coil of a similar telephone. It there causes, by induction, variations in the strength of the

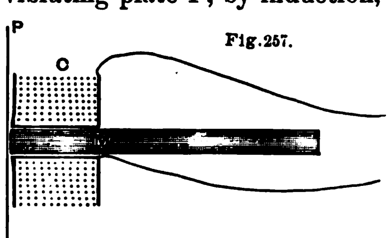


Fig. 257.

magnet, which attracts the plate with varying degrees of force. That plate is either bent as a mass towards and from the magnet, or its molecules are disturbed by the varying induction: or these actions may be combined; in any case, the plate exerts varying pressure upon the surrounding air and produces in it Sound-Waves, which approximately reproduce in their complexity the sound-waves produced by the original voice.

There are many causes of distortion of the signals sent, both in the instrument and in the line. In the latter, the higher harmonics tend to thin away more rapidly than the graver components, and they are propagated at different speeds: but the resulting distortion can be reduced to a minimum by lowering the line-resistance, and would also be reduced through increasing the inductance L by hanging the wires far apart, or through increasing the leakage (Heaviside), though this would weaken the current reaching the receiving instrument, or through reducing the electrostatic capacity of the line. Mr. Heaviside has shown that the distortion would be zero, if $R/L = D/C$, where D is the leakage-conductance, all per unit of length.

It is a matter of indifference to the receiving telephone by what means the variations of current-intensity which it reveals have been produced. These may be due to variations of electromotive D.P. (vibrations of one of the plates of an electrostatic condenser or oscillatory variations in its charge, — variations of the potential of a mass of mercury vibrating, while in contact with water, up and down a conical capillary tube), or to variations in the total resistance (length, cross-section, conductivity) of the conducting wire. The conductance of the circuit may be caused to vary by squeezing the wire, by causing a certain length of it to vibrate; or again by interposing a certain length of a conductor whose conductivity varies with varying pressure (microphone) or with varying illumination (photophone).

According to Prof. Tait, the variations of current in an ordinary telephone are equivalent to actual currents whose intensity is one-thousand millionth part of the current ordinarily used in telegraphic work. This telegraphic current may, on long lines, be stated to be about one-sixtieth Ampère.

Page-Effect. — A telephone will work feebly even without any plate P; the varying constraint of the particles of the magnet M causes them to exert varying pressure upon the air. If a plate of any substance be connected with the extremity of M, that plate will act as a sounding-board, and will enhance the sound produced.

Reversed Action. — A reverse current of high potential sent through a frictional machine may maintain rotation in it, so that a stronger machine in circuit with a weaker one may drive it backwards. If a dynamo deliver all its current in one direction, an extraneous current sent through the machine in the same sense causes a reversed rotation of its armature. In consequence of this, if we couple two direct-current dynamo-electric machines by connecting wires, so that both dynamos (so-called for the sake of brevity) are on the same metallic cir-

cuit, and if we force the armature of the one into rotation, the armature of the other rotates in a reverse sense (that is, against its brushes unless these be reversed or the armature-connections reversed) as soon as the current transmitted attains a certain intensity. The distant dynamo, which bears under such circumstances the name of **Electromotor,** may be of any size, and the simple use of a key or commutator arrests or reverses its action at will. The intensity of the current passing round the circuit is diminished by the reversed rotation of the electromotor: this is equivalent to the production of a reverse current by the electromotor. The usefulness of the arrangement, the proportion of the Energy Absorbed by the electromotor, in rotating against resistances, to the Total Energy imparted by water-wheel or steam-engine to the driving dynamo, is equal to the ratio between the intensity of the virtual reverse-current produced by the electromotor, while running, and the intensity of the current produced by the dynamo when the motor is kept from rotating. This Utility or **Efficiency** is not to be measured by the relative rapidities of rotation of the electromotor and dynamo, on account of the so-called dead turns; the rotation of the dynamo must exceed a certain speed before any current will be produced, and the current produced must exceed a certain strength before the electromotor will turn. The **Activity** of the arrangement (*i.e.* the rate at which the electromotor can do external work, the amount of energy transmitted per second) is theoretically greatest (Jacobi's Law) when the virtual reverse-current is half that produced by the dynamo when the motor is stopped — that is, when the exterior circuit, including the running motor, acts as if it were a wire-circuit whose resistance is equal to that within the dynamo itself.

In practice it is better to make the whole resistance about $\frac{10}{9}$ times the internal resistance.

Jacobi's Law is arrived at thus: — During each second the external work done is w ergs; w also represents numerically the **Activity** in question; the energy converted into heat in the whole circuit is I^2R, and the energy provided by the dynamo is EI, ergs per sec. Then $EI = I^2R + w$; a quadratic; whence $I = (E \pm \sqrt{E^2 - 4Rw})/2R$. The quantity $(E^2 - 4Rw)$ cannot have, physically, a negative sign, for its square root would then become an impossible quantity. $(E^2 - 4Rw)$ cannot be less than zero: whence w cannot be greater than $E^2/4R$: when it is equal to $E^2/4R$, $I = E/2R$, and the intensity has been diminished from E/R to $E/2R$, — that is, to one half, — while the total resistance must have been doubled.

The **Efficiency-relation** may be thus arrived at: — If the dynamo run while the motor is held fast, the E.M.D.P. and current-intensity will be

E_D and I_D. If the motor were to run at its actual speed while the dynamo was held fast, the reverse-current produced by it would be at E_M and I_M. When the two are coupled, the actual current is $(I_D - I_M)$: the energy supplied by the dynamo during each second is w_D ergs; that taken up and transmitted by the motor is w_M: the resistance of the whole circuit is R: and $(I_D - I_M)^2 R$ is the Heat developed in the whole circuit: then the Energy supplied by the dynamo is $w_D = E_D(I_D - I_M) = E_D(E_D - E_M)/R = \{w_M + (I_D - I_M)^2 R\} = \{w_M + (E_D - E_M)/R\}$; whence the Efficiency $w_M/w_D = E_M/E_D = I_M/I_D$. The ratio E_M/E_D, and therefore the Efficiency, may be raised by raising the value of E_M: and this may be done by giving the motor a small load, so that it may rotate rapidly, or by making its magnetic field a comparatively strong one; so that efficiencies of 86 per cent have been attained at such distances as 600 metres (⅜ mile); 44·8 per cent at 36 miles (Creil-Paris), with 6000 Volts.

As to the thickness of conducting wire necessary, there is no limit other than that imposed by the necessity of very good insulation. An ordinary telegraph wire could convey the whole energy of Niagara Falls, and convey it to any distance; but the wire would be at so high a potential that sparks would fly from it into the surrounding air. In the same way, if the amount of onflow of a fluid in a pipe were found to vary directly as the motive difference of pressure, any amount of energy might be transferred from one place to another by the smallest flow of water, for any water allowed to flow out of the pipe might be made to escape with any assignable velocity; provided always that the tube were strong enough at all points to sustain at all intermediate points the necessary pressure.

If a dynamo of resistance 5 Ohms, and producing a difference of potential of 1000 Volts, be the source, and a similar machine be the electromotor, while the connecting wire offers a resistance of R Ohms, the intensity of the current produced is $\left(\frac{1000}{5+5+R}\right)$ Ampères. If 500 such dynamos be coupled in file, their joint E.D.P. will be 500,000 Volts, and their resistances 2500 Ohms; if the receiving electromotors be also multiplied five-hundredfold, their resistances will be 2500 Ohms; if the connecting wire be unaltered, the intensity of the current passing will be $\frac{500,000}{2500+2500+R}$ Ampères; but if the connecting wire be also 500 times as long as at first, the intensity is $\frac{500,000}{2500+2500+500R} = \left(\frac{1000}{10+R}\right)$ Ampères, the same as in the former case. Though the intensity of the current passing is the same, the energy transmitted per second is not the same: it is 500 times as great. In the former case it is Intensity × E.D.P. $= \left(\frac{1000}{10+R}\right) \times 1000$ Ampère-Volts or Watts: in the latter it is $\frac{1000}{10+R}$ Ampères × 500,000 Volts $= \frac{500,000000}{10+R}$ Watts.

When the total resistance is the internal resistance R_i, $I = \frac{E_i}{R_i}$; when it is $(R_i + R_e)$, $I_i = \frac{E_i}{R_i + R_e}$. These two distinct sets of circumstances are linked

together by — (1) The criterion of maximum activity, $R_{i+e} = \frac{100}{58} R_i$; and (2) The energy imparted to the dynamos is a constant, $= E_{,}I_{,} = w$ ergs per second. From these equations we find $I_{,} = \sqrt{\frac{58}{100} w / R_i}$, and $E_{,} = R_{i+e}I_{,} = (\frac{100}{58}R_i)(\sqrt{\frac{58}{100} w / R_i}) = \sqrt{\frac{100}{58} R_i w} = \sqrt{R_{i+e} w}$. We must now choose numerical values for R_{i+e}, the total resistance internal and external, and w, the energy imparted to the system per second. We shall use C.G.S. Electrostatic units.

Let the resistance be that of 4000 kilometres of copper wire of 1 sq. cm. in cross-section, and that of x dynamos and x electromotors. The dynamos are each supposed to generate an E.D.P. of 1000 Volts, or $3\frac{1}{3}$ C.G.S. Electrostatic units. Their number must be $(E_{,} \div 3\frac{1}{3})$.

Let the joint resistance of each dynamo and motor be 10 Ohms, or $\frac{10}{900{,}000{,}000000}$ C.G.S. Electrostatic unit of resistance. The resistance of the $(E_{,} \div 3\frac{1}{3})$ pairs of machines will be $\{(E_{,} \div 3\frac{1}{3}) \times \frac{10}{900{,}000{,}000000}\}$, or $\frac{E_{,}}{300{,}000{,}000000}$ C.G.S. Electrostatic units.

The wire (4000 kilometres) will offer a resistance of about 648 Ohms, or $\frac{216}{800{,}000{,}000000}$ Electrostatic C.G.S. units.

The total resistance $R_{i+e} = \frac{1}{300{,}000{,}000000}\{E_{,} + 216\}$ C.G.S.E.S. units.

Let w be the energy of the Falls of Niagara, per second, also in C.G.S. units or ergs. About 100,000000,000000 grammes of water fall per hour through a height of about 4830 cm. The potential energy lost by the water is about 132,000,000000,000000 ergs per second $= w$.

The equation $E_{,} = \sqrt{R_{i+e} w}$ is now

$E_{,} = \sqrt{\frac{1}{300{,}000{,}000000}\{E_{,} + 216\} \times 132{,}000{,}000000{,}000000}$, a quadratic; whence $E_{,} = 440{,}020$ C.G.S.E.S. units or 132,006000 Volts.

If iron telegraphic wire 4 mm. in diar. were used, its resistance would be (the resistance of iron being $\frac{880}{184}$ that of copper) 31680 Ohms; the total resistance would be $\frac{1}{300{,}000{,}000000}\{E_{,} + 10560\}$; and $E_{,} = 450{,}320$ C.G.S.E.S. units, or 135,096000 Volts.

No practicable insulation could be set up, adequate to sustain permanently so great a stress; and no possible dynamo-electric machines could be ranged in file to the number necessary, for the insulation of their coils would be broken down by sparks from the wire to the outer air.

It is practicable, however, with ordinary telegraphic wires insulated in the ordinary way, and with a 16-horse-power dynamo, to drive a 6-horse power electromotor at a distance of 30 miles.

The wire must also, by possessing sufficient thickness, offer so little resistance that it is not so far heated as to deteriorate in conductivity.

Alternating Current Motors. — If two alternating current machines be coupled in circuit, and if they be once in synchronous motion, they will tend to assume and to maintain uniformity of phase; while if the mechanical load on the motor be increased, within appropriate limits, the motor will present a less complete opposition of phase, and a stronger current will run, so that the mechanical forces upon the motor-armature will be correspondingly increased. The magnetic field of such a motor must be kept constant in its direction. The practical difficulty connected with such motors is that they are not self-starting.

At pages 90–91 we learned that two S.H.M.'s, differing by $\frac{1}{4}$ period, produce an ellipse. Similarly if we have two S.-H.-varying currents, each tending to produce an alternating magnetic field in its own particular direction, but differing from one another in phase, the result will be a continuous rotation of the direction of the resultant magnetic field. In such a field any mass of metal will tend to rotate; and this is the basis of **rotary-field** alternating current motors (Tesla, Dobrowolski). In Schallenberger's alternate-current meter, a vertical coil receives the alternating current to be measured; within it is fixed another vertical coil, closed, and standing at an adjustable angle with the preceding. Inside the latter is a soft-iron disc, horizontal, pivoted on a vertical axis of rotation. The outer coil tends, alternatingly, to magnetise the disc along a certain line; the inner presents induced currents, nearly opposite in phase, which tend to magnetise the disc, alternatingly, along another line, which makes an angle with the preceding: the joint action of the two coils (or sets of coils) on the disc is to magnetise it in a direction which itself continuously rotates; the disc tends to rotate with a velocity proportional to the square of the current-strength. The tendency is for the disc to maintain a position in which the retarding eddy-currents in the iron are a minimum. In such apparatus the result is, as regards the intensity of the induced magnetisation (which tends, if the component inducing forces be not equal and at right angles to one another, and at phases differing by exactly $\frac{1}{4}$ period, to present maxima and minima), more uniform if three or more S.H. variations be simultaneously induced in directions making equal angles with each other, as in Brown's three-phase alternating-current motor. Multiple-phase motors are self-starting, and gain in speed until synchronism is attained.

The Lauffen-Frankfurt experiments of 1891 gave an efficiency of 72 per cent in the transmission of 108 horse-power over 110 miles, at 30,000 Volts.

Oscillatory Electromagnetic Disturbances in Free Ether.

Herz's Experiments.—Two metallic plates, each say 16 cm. square, are suspended in the same plane, and are connected each with one terminal of an induction-coil. They are also almost connected with one another by means of wires terminated at their free extremities by polished knobs, between which there is a small air-gap. When the induction-coil is at work, a stream of sparks runs across this air-gap, from knob to knob. The electric displacements in the region of the spark are of an oscillatory character, and are parallel to the length of the air-gap, from knob to knob. The lines of force are accordingly parallel to the length of that gap. This apparatus is called **Herz's Vibrator.** Next we have his **Resonator,** which is a single circle of wire, broken by an air-gap between two knobs. This resonator has its own fundamental period of electric oscillation. The vibrator sends out a mixture of oscillatory ether-disturbances of various

periods. Now assume that the vibrator air-gap, in its length, runs East and West, horizontally. Lay the resonator horizontally, with its air-gap also East and West, and facing the vibrator centrally. Then sparks pass in the resonator air-gap. Sparks will continue to pass in this air-gap, though with a diminished striking distance, when the resonator is turned round in its own plane into any position, so long as it is kept horizontal. Now turn the resonator into a vertical plane, which plane lies parallel to the length of the vibrator air-gap, that is, East and West; some sparks will pass should the resonator air-gap be also parallel to the vibrator air-gap, but when it is at right angles to the same no sparks will pass. Again, turn the resonator round so that its vertical plane lies North and South, or at right angles to the length of the vibrator air-gap; no sparks will pass at all, whatever be the direction of its air-gap. It is (J. J. Thomson) as if the lines of electric force in the Leyden-jar discharge through the vibrator air-gap were parallel to the length of that gap; and as if when travelling broadside-on, outwards from that gap, they produced some sparks when they struck the resonator air-gap in such a way that their length coincided with its length, and produced a maximum effect when they struck the wire of the resonator longitudinally, so that their lengths coincide with its length, while at the same time the magnetic induction was directed along the axis of the resonator. In the last case there would be reflexion, from end to end of the resonator, of these lines of force; and these lines of force would oscillate to-and-fro along that resonator, with a result analogous to Resonance in Acoustics. Those disturbances, radiated from the vibrator, which had been in tune with the resonator would be taken up and piled up by it, until sparks passed in the resonator air-gap, or until a Geissler-tube held in or near that gap would light up.

By this means the Resonator can be used to detect the existence and the direction of electric oscillatory disturbances in the Ether, such as have periods corresponding to the rate of propagation of electric disturbance along the wire of the resonator, and to the length of its wire; but the resonator will respond to disturbances of a considerable range of frequencies above and below this limit.

The waves produced by this electric method traverse brick walls with ease, but they are **reflected** by metallic mirrors. If the resonator be used to explore the path of the reflected waves,

it is found that there is **interference** between the direct and the reflected waves, exactly as in the case of Sound or of Light; the resonator gives maxima of sparking or of illumination at distances equal to half a wave-length from one another; and the wave-length, thus determined, is consistent with the velocity of Light, together with the frequency of vibration as calculated from the dimensions of the resonator. If a large prism of pitch be employed, it is found that the waves are **refracted** by the pitch; and a large lens of pitch acts in an analogous way. If a vibrator-gap be adjusted in the focus of a mirror which consists of a sheet of metal bent into a parabolic form, then a suitable resonator-gap, placed in the focus of a similar mirror opposite to the first, may give sparks. These waves, like Light **polarised** at right angles to the plane of incidence and reflexion, fail at a particular angle of incidence to be reflected from a metal mirror, provided that the vibrator air-gap be in the plane of incidence. If therefore these electromagnetic waves are like waves of Light or Radiant Heat, or Actinic Radiation, and differ from these in wave-length only, the electric oscillations, as distinguished from the magnetic inductions, are at right angles to the "Plane of Polarisation." The leading phenomena of Light—including Reflexion, Refraction, the Angle of Polarisation, and Polarisation itself, **Scattering by Haze,** and, to a large extent, **Metallic Reflexion** and the **change of phase** on transmission through thin films of metal, along with **Newton's Rings** and the black region of a thin **soap-bubble,** as well as **Diffraction**—have been imitated on the large scale by means of these electromagnetic waves; while the effect of the Magnetic Field on Light can be largely explained if it be admitted that Light consists of such Electromagnetic Waves of small wave-length, in which the electric oscillations, at right angles to the direction of propagation, are also at right angles to the plane of polarisation, while the magnetic inductions are in that plane.

The field in the immediate neighbourhood of the vibrator is in a singular condition of alternate withdrawal and emergence of Lines of Force: there are various peculiarities in the amount of the electric force and in the velocity of propagation at that place; but the upshot is, that after interference between lines passing outward and lines re-absorbed has had full swing, the ether-waves emerge as if from wave-centres about half a wave-length from the vibrator-gap, and the electric forces are thereafter exactly at right angles to the direction of propagation, and vary inversely as the square of the distance.

If an electrostatically-charged body could be whirled round a magnetic needle at the rate of 30,000,000000 cm. per second, it ought to act upon it

much in the same way as a circulating electric current. At very high speeds, such as are physically within our reach, such an effect should be observed in small degree; and Prof. Rowland of Baltimore has succeeded in making it manifest.

Maxwell's Theory of Light.—These results, obtained by Herz and others, furnish a verification of Clerk Maxwell's Theory of Light.

According to this theory, the Electric Displacements, parallel to the wave-front, and at right angles to the plane of polarisation, are the cause of Optical Phenomena. The Magnetic Inductions or Displacements, at right angles to the preceding, and parallel to the plane of polarisation, but also parallel to the wave-front, produce no effect on the eye. The electric displacements will be propagated, in an æolotropic medium, in the same way and with the same velocity as a light-wave; and the magnetic disturbance, whose energy is equal to that of the electric, is propagated with the same velocity. The intensity of Light, its average energy per cub. cm., is $2\pi\mu\sigma^2v^2$, where v is the velocity of propagation, σ the maximum electric displacement per sq. cm. at the ends of the lines of electric force, and μ the permeability of the medium. In a doubly-refracting medium, the electric displacement and the magnetic induction are propagated according to Fresnel's wave-surface. There is no dilatational wave possible: and this removes a difficulty in optical theories.

Since in non-conductors an electric Displacement produces an Electric Restitution-Force which varies as the Displacement—a criterion of vibratory movement propagated with a definite velocity; but in conductors no such force is manifest, and the energy of electric disturbance is continuously dissipated by transformation into Heat: then Light-vibrations ought not to be possible in Conductors, which should be always opaque, while non-conductors ought, if homogeneous, to be transparent. With few exceptions this is the rule.

The velocity of propagation of an electromagnetic disturbance is the same as the ratio of the electromagnetic to the electrostatic unit of current-intensity or quantity. This ratio is experimentally found to be, on the C.G.S. system, in round numbers, equal to 30,000,000000; which, in cm. per second, coincides sufficiently with the velocity of light.

On comparing the formulæ for the transverse variations of an elastic solid with those worked out to represent the stresses in an Ether concerned in electromagnetic phenomena, it is found (Clerk Maxwell, *Elect. and Magn.*, vol. ii., chap. xx.) that in the former a term V, the velocity of propagation of transverse disturbances, occupies the same place as $1/\sqrt{K}$ in the latter. By our electrostatic convention, *in vacuo* $K = 1$ and $\mu = 1/\mathbf{V}^2$; $\therefore$ $\mathbf{V} = (K\mu)^{-\frac{1}{2}}$; but electromagnetically, $K = 1/\mathbf{V}^2$ and $\mu = 1$; whence $V = K^{-\frac{1}{2}} = \mathbf{V}$; *i.e.* the velocity *in vacuo* is equal to the ratio-number $\mathbf{V}$. With other media than air, $(K\mu)^{-\frac{1}{2}}$ has other values; but μ is nearly unity in most non-conductors; roughly, $V = \sqrt{1/K}$; but in Light, $v = V$, and varies inversely as the Refractive Index; whence the Specific Inductive Capacity K of a dielectric ought (Maxwell) to be equal to β^2, where β is the refractive index

for waves of infinite length.* In some substances this is the case; it is so in sulphur (Romich, Nowak, Boltzmann), and in turpentine, petroleum, and benzol (Silow); but in vegetable and animal oils (Hopkinson) and in glass, Iceland spar, fluor spar, and quartz (Romich and Nowak), and generally in substances not simple in chemical constitution or homogeneous in structure, the sp. ind. cap. is too great. We ought not, however, to expect more than a general agreement; even the oscillations produced by a Leyden-jar discharge are millions of times less frequent than those electromagnetic alternations which we call Light; and it appears that the sp. ind. caps. for these already approach the values required by the theory.

The theory explains most optical phenomena as a part of electrical science; but it is still weak in its treatment of Dispersion, both ordinary and anomalous, of Metallic Reflexion, and of the rotation of the plane of polarisation by magnets. It does not, however, profess yet to explain the interaction of Ether and ordinary Matter.

The Ether.

The properties of the Ether which are involved in the phenomena of Electricity, Magnetism, Electromagnetism, Light, Radiant Heat, and Actinic Radiation have been referred to under these several heads, where required. We can now merely note a few remaining points. In terms of **Maxwell's Theory**, it is supposed that there is some kind of Rotation round each Line of Force, upon which rotation the Elasticity of the Ether may depend: and the facts of Electrolysis or of the Galvanic Cell, in which the charge that can be liberated upon an electrode or plate is limited to a definite quantity per free atom, seem to show that the Lines of Force are not indefinite but definite in number, so that we may perhaps have (J. J. Thomson) one line of force between each pair of free atoms. But these lines of force, on this view, need not all have free ends situated upon matter: there may be, in the Ether, closed lines, like vortex-rings: and these may, in a Magnetic Field, be so directed as to take up a position parallel to one another.

Again, if Ether be a form of Matter, it ought to have **Inertia**. In Self-Induction it appears to have inertia; but in a Steady Current it appears to have absolutely none. The only way, and mathematically an easy way, out of the difficulty seems to be to assume that the Ether is really double in its constitution, so that when we say, for example, that positive lines of

* If β, $\beta_{,}$, be the refractive indices corresponding to the respective wave-lengths, λ and $\lambda_{,}$, we know that to a rough approximation $\beta = A + B/\lambda$, where A and B are constants, found by experiment. From these, knowing the numerical values of β, $\beta_{,}$, λ, $\lambda_{,}$, we can find that of A, which is the approximate value of β when $\lambda = \infty$.

force move in a certain direction, we ought to add that an equal number of negative lines move in an opposite direction, shearing past the former: the apparent inertia would then be zero in a steady current. This would make a steady current really a double current, positive one way and negative the other; but in *Electrolysis* we actually have a double transfer of atoms, to which this would precisely correspond; and the phenomena of the electric *spark* point in the same direction. The sp. ind. cap. of a dielectric would, on this view (Lodge), correspond to a *Shearability*, while the permeability μ at the same time represented an *Inertia* or *Density* of the Ether.

Mr. W. Williams has shown (*Proc. Phys. Soc. Lond.*, xi. 357) that we are practically restricted either to this view or to the opposite, that K is a Density, or inertia per unit volume, and μ a Shearability. On the former view, the magnetic energy of the field is kinetic: and if we take care to keep in view the proper Direction of each element of Length which enters into the Dimensional Equations, these equations themselves bring out the closest analogies between magnetic and electric phenomena and those of vortex-motion and transverse stresses in an incompressible fluid. The Dimensions of μ in this view would be M/L^3, where the three L's are at right angles to one another, so that their product truly represents a volume; and that of K would be T^2L/M or more properly T^2L^2/LM; where the three directions have again to be distinguished. Such an expression as $[\mathbf{h}] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T\mu^{\frac{1}{2}}]$ then becomes $[\mathbf{h}] = [L/T]$, and $\mathbf{h}$ is a Linear Velocity along the Lines of Force; and so forth. This tends to elucidate the purely mechanical aspect of magnetic and electrical phenomena in the Ether: and the paper should be consulted.

But the subject is still more obscure when we consider the relation of Ether to ordinary *Matter*. Why the magnetic induction under a given magnetising force should be 300 times as great in a particular sample of iron as in a corresponding amount of air or of copper is still a mystery; and even if it were established that the density of the Ether was greater in iron, that would itself have to be explained.

Meantime, it will be kept in view that all our statements as to positive and negative quantities and currents are based upon the purely *arbitrary convention* that *vitreous* electricity is *positive*, and resinous electricity negative. This convention happens to harmonise with that which regards the *north-seeking* end of a magnet as its *positive* pole; and thus uniformity of language throughout the subject-matter of this chapter happens to have been readily attained.

APPENDIX.

Notation. — In adjusting the notation used in this volume, the purposes kept in view have been to provide, as far as might be, for the whole subject-matter of the book, but to depart as little as possible from the symbols ordinarily in use, while the letters employed should be distinctive and at the same time typographically suitable. One guiding principle has been to separate physical quantities in general from the same quantities per unit of area, where such a distinction seemed needful, by using, in order to represent these, capital and small letters respectively. Then Mr. Oliver Heaviside's suggestion as to the use of blackfaced type for directed quantities was found to promise to work well, and to enable useful distinctions to be made. Thus F is a force acting, in general; *f* is a force per unit of area; and **f** is a force acting per unit of area in some given direction. Again, **B** is a total magnetic induction, **b** is a magnetic induction per sq. cm.; and both these, being blackfaced, call to mind the Lines or directions in which the induction acts. Other typographical devices have had to be employed for the sake mainly of obtaining a larger number of distinctive characters; but it is hoped that none of these will offend the eye, and that the grouping is reasonably consistent. Perfect symmetry seems hardly attainable, on account of the varying demand for the different letters in different parts of the subject. Still, even with the notation as it stands, it has been interesting to the author to note in how many instances the mere necessity of ascertaining which symbol ought to be employed has enabled him to set matters forth with greater definiteness than in the former editions.

In its issue of Aug. 25, 1894, the *Electrical World* of New York has published the recommendations of the Committee on Notation of the Chamber of Delegates of the International Electrical Congress, Chicago, 1893. These recommendations as to notation are at present the subject of a good deal of discussion, and it remains to be seen to what extent they will be generally adopted. In the meantime they are recommendations only, and will have to be further considered when an International Electrical Congress next takes place; but the following conspectus shows their relations to the notation employed in this volume.

It is not explained what distinctions the respective large and small letters denote, if any; the same Dimensions apply to both the large and the small letters. The measurement in the fourth and sixth columns is in electromagnetic, not in electrostatic, units. The manuscript type in the last column is that known as the French Script of Messrs. Damon & Peets, New York.

This Volume.	Committee.	This Volume.	Committee.	This Volume.	Committee.
l	L, l	q	Q, q	$\mathfrak{m}$	m
m	M	e	E, e for so-called E.M.F.	$\mathfrak{h}$	$\mathcal{H}$
t	T, t		U,u for difference of potential	$\mathfrak{H}$	Φ
A	S, s	(C)	C, c	$\mathfrak{M}$	$\mathcal{M}$
$\mathfrak{v}$	V	i	I, i	$\mathfrak{I}$	$\mathcal{I}$
Angle δ, θ, ξ	α, β	(D)	G, g	$\mathfrak{b}$	$\mathcal{B}$
v	v	(ᴅ)	γ	κ	κ
ω	ω	(R)	R, r	μ	μ
a	a	(ʀ)	ρ	L	L, l
F	F, f	W	W	Magnetomotive Force	$\mathcal{F}$
W	W	Activity	Power, P	Reluctance of Circuit	$\mathcal{R}$
Activity	Power, P			Reluctivity	$\nu = 1/\mu$
p	p				
N	K				

Dimensions.—In the Dimensions given in this volume from time to time, it will be observed that, for example, a Torque has Dimensions ML^2/T^2, while those of Energy are the same. There are other instances of the same kind. So long as we use the Dimensions only for checking our equations numerically, or for translating from one system of units to another, this identity of expression between physical quantities which, like the two in question, are truly dissimilar, is of no importance; but if we wish the Dimensional Equations to convey to us an idea as to the physical reality lying behind them, we must find some means of introducing into them a representation of the Directions involved. Now the symbol $\sqrt{-1}$ signifies a Rotation through 90°, for the operation which it represents would, if effected twice, convert a directed quantity $\mathbf{x}$ into $-\mathbf{x}$, that is, would turn its direction round through 180°; and it has been proposed to distinguish the dimensions of a Torque from those of Energy or Work, by introducing the factor $\sqrt{-1}$ into it. The expressions would then be $ML/T^2 \cdot L\sqrt{-1}$ and ML^2/T^2 respectively; and the former of these would show that the second L was at right angles to the first, while in the other case both the L's are in the same direction. Mr. Williams, in the paper referred to on p. 746, has shown that this idea is capable of great extension by means of keeping the three rectangular axes of direction, X, Y, and Z, entirely separate, so that the corresponding L's do not cancel on division or multiplication unless they be in the same direction. The Dimensional Equations thus acquire a deeper significance and an enhanced utility.

4π.—In the formulæ of Chap. xvi., the factor 4π, or some multiple or sub-multiple or power thereof, appears with painful frequency. Mr. Oliver Heaviside has pointed out that this is due to the total flux of force, or rather of induction, **I** or **B**, round a quantity Q or $\mathfrak{m}$, being taken as equal to $4\pi Q$ or $4\pi\mathfrak{m}$, as the case may be. This is itself a necessary consequence of so choosing our units that the force, F dynes, between two equal quantities Q (or $\mathfrak{m}$) is Q^2/Kd^2 (or $\mathfrak{m}^2/\mu d^2$). Mr. Heaviside proposes that we should, while

retaining the dyne as our unit of force, so alter our units of electrical and magnetic quantity as to make the force $F = Q^2/4\pi Kd^2$ or $\mathfrak{m}^2/4\pi\mu d^2$. In other words, he proposes to make the units such that not $4\pi Q$ or $4\pi\mathfrak{m}$, but Q or $\mathfrak{m}$ lines radiate from each quantity Q or $\mathfrak{m}$, as measured in the new units. Then the new numerical value Q_r or $\mathfrak{m}_r$, which stands in the place of the old Q or $\mathfrak{m}$, is equal to $Q\sqrt{4\pi}$ or to $\mathfrak{m}\sqrt{4\pi}$, as the case may be. Whence the new units, which Mr. Heaviside calls rational units, of quantity are smaller than the present air-units in the ratio of 1 to $\sqrt{4\pi}$. If this change were effected, most of the units used in electricity and magnetism would have to be changed at the same time. If we make the suffix $_r$ signify that the number indicated by the letter is now to be a number expressing the same physical quantity in terms of the new units, we find that $Q_r/Q = \sigma_r/\sigma = I_r/I = E/E_r = \phi/\phi_r = \sqrt{4\pi}$; $\mathfrak{m}_r/\mathfrak{m} = \mathfrak{h}/\mathfrak{h}_r = \mathfrak{b}/\mathfrak{b}_r = \mathfrak{s}_r/\mathfrak{s} = \Omega/\Omega_r = \sqrt{4\pi}$; and further, $R/R_r = C_r/C = L/L_r = M/M_r = 4\pi$; while K and μ remain the same. The fundamental electrostatic equations would then become $\phi_r = \sigma_r$, $\mathfrak{i}_r = K\phi_r$, $f = \sigma_r^2/2K$, $\phi_r = E_r/d$; or, dropping the suffixes and confining ourselves to air, $\phi = \sigma = \mathfrak{i}$; $f = \sigma^2/2$; $\phi = E/d$; whence $V_{\prime} - V_{\prime\prime} = E = d \cdot \sqrt{2f} = d \cdot \phi = d \cdot \sigma = d \cdot \mathfrak{i}$; $\phi = \sigma = \mathfrak{i} = \sqrt{2f} = 2f/\sigma = E/d = Q/A$; $f = E^2/2d^2 = \phi^2/2 = \sigma^2/2 = \mathfrak{i}^2/2 = \phi\sigma/2 = \mathfrak{i}\sigma/2 = \phi\mathfrak{i}/2 =$ Energy of Field per cub. cm.; and $C = KA/d$, or, for a unit cube condenser, $C = K$, while the Dielectric Elasticity $= 1/K$. In the magnetic and electromagnetic parts of the subject there would be corresponding simplifications. The factor 4π would, however, make its appearance in other formulæ where it does not at present occur, but only where the conditions of the problem are truly spherical and not merely superficial or linear; for example, instead of $\mathfrak{b} = \mathfrak{m}/r^2$ per sq. cm. at a distance r from a central pole $\mathfrak{m}$, we would have $\mathfrak{b}_r = \mathfrak{m}_r/4\pi r^2 = \mathfrak{B}_r/\text{area}$, which is a better representation of the fact. This change of the fundamental units of electrical and magnetic measurement would involve the study of an additional system of units. But it is not clear that, admitting the reasoning, the proposal goes far enough. The relation between the Unit of Mass and the Unit of Force is equally based upon neglect of the central nature of gravitational forces, and of the Field of Force, with its Lines of Force and Equipotential Surfaces, round an attracting mass. If the same reasoning were here applied, either the unit of mass or the unit of force, or both, would have to be changed. If the gramme were retained as our unit of mass, and if the Force of Gravitation between two equal masses, m grammes each, at distance d, were written $G_R = m^2/4\pi d^2$, the new unit of force would be equal to $4\pi\gamma$ dynes. Then, between two equal electrical quantities Q, as measured in the present C.G.S. units, the force would be $F_R (= F/4\pi\gamma) = Q_R^2/4\pi Kd^2$; and $Q_R^2 = F_R \cdot 4\pi Kd^2 = F/4\pi\gamma \times 4\pi Kd^2 = FKd^2/\gamma = Q^2/\gamma$; so that the effect would be to set up still another system of electric and magnetic measure, in which the units of electric and magnetic quantity would be equal to $\sqrt{\gamma} \times$ the present C.G.S. units; or else, if γ were left in the formula for G, with its present value unaltered ($G_R = \gamma \cdot m^2/4\pi d^2$), we would have our unit of force equal to 4π dynes, and our unit of mass equal to {one gramme $\div \sqrt{\gamma}$}, while we would at the same time restore the present C.G.S. units of electric and magnetic quantity, etc. If we took the "astronomical" unit of mass, $1/\gamma$ gramme, as our standard, our unit of force would be $4\pi/\gamma$ dynes, and the electrical and magnetic units of quantity would be equal to the present C.G.S. units $\div \sqrt{\gamma}$. If we adhered to the dyne as our unit of force, although we based it on the equation $F = ma$, we would have to make our standard unit of mass equal to {one

gramme $\div \sqrt{4\pi\gamma}\}$ and our unit of acceleration equal to $\sqrt{4\pi\gamma}$ cm.-per-sec. per second; and the electrical and magnetic units of quantity would then bear to the present C.G.S. units the ratio of $1 : \sqrt{4\pi}$, as in Mr. Heaviside's proposed system.

It seems at any rate clear that if the principle were thoroughly applied throughout the whole of Physics, then, though the electrical formulæ were simplified, the 4π would be transferred from the more remote parts of the subject to its very threshold; and it is doubtful whether this would not cause more inconvenience than it would remedy.

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